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Journal of Water Resources Planning and Management, 2004; 130(1):63-72

http://doi.org/10.1061/(ASCE)0733-9496(2004)130:1(63)

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Journal of Water Resources Management and Planning

Genetic Algorithms for Reliability-Based Optimization of Water Distribution Systems

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Abstract A new approach for reliability-based optimization of water distribution networks is presented. The approach links a genetic algorithm (GA) as the optimization tool with the first-order reliability method (FORM) for estimating network capacity reliability. Network capacity reliability in this case study refers to the probability of meeting minimum allowable pressure constraints across the network under uncertain nodal demands and uncertain pipe roughness conditions. The critical node capacity reliability approximation for network capacity reliability is closely examined and new methods for estimating the critical nodal and overall network capacity reliability using FORM are presented. FORM approximates Monte Carlo simulation reliabilities accurately and efficiently. In addition, FORM can be used to automatically determine the critical node location and corresponding capacity reliability. Network capacity reliability approximations using FORM are improved by considering two failure modes. This research demonstrates the novel combination of a GA with FORM as an effective approach for reliability-based optimization of water distribution networks. Correlations between random variables are shown to significantly increase optimal network costs.

Keywords - Algorithms; Water distribution; Optimization; Hydraulic networks; Network reliability.

Introduction

Water distribution networks (WDNs) are essential and costly infrastructure in every modern community. As WDNs continue to age and cities continue to grow, the design of new WDNs and the rehabilitation or upgrade of existing WDNs will continue to be an important problem. The general WDN design problem involves minimizing whole of life network costs (e.g., pipe and construction) subject to meeting minimum allowable pressure and/or maximum allowable velocity constraints under design demand levels. Traditionally, WDN design, upgrade, or rehabilitation has been based on engineering judgment. More recently, a significant amount of research has focused on the optimal design or upgrade of WDNs. Some of the first studies utilized linear programming (LP) (Alperovits and Shamir 1977; Quindry et al. 1981) while later studies applied nonlinear programming (NLP) (Su et al. 1987; Lansey and Mays 1989; Xu and Goulter 1999), or chance constrained approaches (Lansey et al. 1989) to the pipe network optimization problem. Much of the recent literature has utilized genetic algorithms (GAs) for the determination of low cost WDN designs and they have been shown to have several advantages over more traditional optimization methods (Simpson et al. 1994; Savic and Walters 1997).

The focus of a number of the above studies is the least-cost design of reliable WDNs. Goulter (1995) states that reliability is generally concerned with the ability of WDNs to provide an adequate level of service to consumers, under both normal and abnormal conditions. For example, failure to meet an adequate level of service would occur due to nodal flow demands being supplied at inadequate pressures or flow rates. In general, WDN reliability-based optimization is focused on either the mechanical failure of components (e.g., Goulter and Coals 1986; Su et al. 1987) such as pipe or pump failure, or the hydraulic failure of the system due to degraded pipe capacities and/or uncertain nodal demand flows (Lansey and Mays 1989; Xu and Goulter 1999). Goulter (1995) and Xu and Goulter (1999) provide reviews of reliability analysis methods.

Reliability-based optimization of WDNs requires the combination of an optimization algorithm with a method for estimating WDN reliability. In this paper, a novel approach to reliability-based optimization of WDNs is proposed and tested on a 14-pipe case study. This approach combines a GA as the optimization tool with improved methods for estimating different WDN reliability measures based on the first-order reliability method (FORM). It should be noted that the approach presented in this paper is restricted to ''capacity reliability'' estimation as in Xu and Goulter (1999), which refers to the probability that the minimum allowable nodal pressures are met under the assumption that the required nodal demand flows are satisfied, and is a function of the uncertain nodal demands and the uncertain degree to which pipe hydraulic capacities will be reduced over the design period.

Proposed Approach

Optimization

Although GAs have been found to be a robust technique for the optimization of deterministic WDNs (Simpson et al. 1994; Savic and Walters 1997), their application to reliability-based optimization of WDNs is scarce and they appear to have only been used in studies where a surrogate of reliability is considered in the optimization framework (Halhal et al. 1997; Shin and Park 1999). This is despite the fact that GAs have a number of advantages over the mathematical programming techniques traditionally used for WDN optimization (e.g., NLP), including (1) decision variables are represented as a discrete set of possible values, (2) the ability to find near globally optimum solutions, and (3) the generation of a range of good solutions in addition to one leading solution. Consequently, GAs are used as the optimization technique as part of the proposed approach.

GAs are robust heuristic iterative search methods that are based on Darwinian evolution and survival of the fittest (Holland 1975; Goldberg 1989). The GA techniques and mechanisms selected as part of the proposed approach are binary coding of the decision variables, tournament selection, uniform crossover, creep mutation, elitism, and the MicroGA technique. These combined GA procedures produce a relatively new and efficient GA called the small-elitist-creeping-uniform-restart GA or ''*securGA*'' that is built on the *MicroGA* technique (see Krishnakumar 1989) and is first introduced by Yang et al. (1998). Computational efficiency is important in the context of reliability-based optimization, as GAs generally require many more function evaluations compared with traditional optimization methods.

Briefly, the *securGA* works by using a smaller population size (relative to that which would be used in a traditional GA) that evolves like a traditional GA using uniform crossover, elitism, and creep mutation. When convergence is reached, however, a new random population is generated and combined with the elite individual from the previous generation and the evolution process repeats itself using a new pool of genetic information. This cycle continues until a maximum generation limit is reached.

Reliability Estimation

The performance of any engineered system can be expressed in terms of its load and resistance. If $\mathbf{X}=(X_1, X_2,\ldots, X_n)^T$ is the vector of random variables that influences a system's load (L) and/or resistance (R), the performance function, $G(\mathbf{X})$, is commonly written as

$$G(\mathbf{X}) = R - L \tag{1}$$

The failure (limit state) surface G=0 separates all combinations of $\mathbf{X}$ that lie in the failure domain (F) from those in the survival domain (S). Consequently, the probability of failure, p_f, is given as (Sitar et al. 1987)

$$p_f = Pr\{\mathbf{X} \in F\} = Pr\{G(\mathbf{X}) < 0\} = \int_{G(X)<0} f_X(\mathbf{X})\,\mathrm{dx} \tag{2}$$

where $f\,X(\mathbf{x})$ is the joint probability density function (PDF) of $\mathbf{X}$.

In most realistic applications, the integral in Eq. (2) is difficult to compute. Approximate solutions can be obtained by using a variety of techniques including Monte Carlo simulation (MCS) and the first-order reliability method (FORM) (Madsen et al. 1986). The most accurate reliability estimation method is MCS with a large number of realizations. Since all other reliability estimation techniques are generally developed to be more computationally efficient than MCS, their accuracy should always be assessed in comparison with MCS benchmark solutions.

MCS approximates the integral in Eq. (2) by repeatedly generating random realizations of the variables in **X** and then evaluating the performance function in Eq. (1) for each realization. The reliability measure as given by MCS is then the ratio of the number of realizations where $G(\mathbf{X})>0$ to the total number of MCS realizations evaluated. The MCS reliability estimate approaches the actual reliability as the number of MCS realizations used increases.

The objective of FORM is to compute the reliability index β, which is then used to obtain the reliability α, using

$$\alpha = 1 - p_f = 1 - \Phi(-\beta) = \Phi(\beta) \tag{3}$$

where F( )5standard normal cumulative distribution function (CDF). In the *n*-dimensional space of the *n* random variables, β can be interpreted as the minimum distance between the point defined by the values of the *n* variable means (mean point) and the failure surface. The point on the failure surface closest to the mean point is generally referred to as the design point, which may be thought of as the most likely failure point. The reliability obtained using FORM is only an approximation, unless the performance function is linear. The degree of non-linearity in the performance function, and hence the accuracy of FORM, is problem dependent (see Madsen et al. (1986) and Xu and Goulter (1999) for a more complete description of FORM).

Much of the research in WDN reliability studies has utilized Monte Carlo simulation (MCS) for WDN reliability estimation (Bao and Mays 1990; Gargano and Pianese 2000). However, as MCS is relatively computationally inefficient, Xu and Goulter (1998, 1999) pioneered the use of the first-order reliability method (FORM) for WDN capacity reliability estimation. This research builds on the work by Xu and Goulter (1999) on the use of FORM for approximating MCS predictions of WDN capacity reliability within an optimization framework by introducing new formulations for using FORM to estimate critical node and overall network capacity reliability.

Both MCS and FORM can be used to estimate nodal, critical node, and network capacity reliability. Nodal and critical node capacity reliabilities are defined with respect to a specified node
and the node in the network that has the worst nodal capacity reliability, respectively. In contrast, network capacity reliability can be defined as the probability that the minimum allowable pressures are met at *all* nodes. The way MCS and FORM can be used to estimate these different measures of reliability is discussed below.

Nodal Capacity Reliability

The performance function used to evaluate the capacity reliability of node *i*, by MCS or FORM is

$$G_i(\mathbf{X}) = H_i(\mathbf{X}) - H_i^{\min} \tag{4}$$

where $H_i(\mathbf{X})$=head predicted at node *i* as a function of the vector of both random nodal demands and pipe hydraulic capacities, **X**, and $H_i^{\min}$=minimum allowable specified head required at node *i*. In order to assess the reliability at all the nodes in the network using FORM, the performance function in Eq. (4) must be specified for each node and a separate FORM computational procedure is then required. In contrast, MCS can be used to estimate the reliability at all nodes in the network in the same computational procedure since the performance functions at all nodes can be evaluated for each MCS realization. Therefore, the relative computational advantage of FORM over MCS diminishes quickly if the reliability at many or all nodes in the network is of interest.

Critical Node Capacity Reliability

The most basic way that FORM or MCS can be used to determine the critical node capacity reliability is to evaluate Eq. (4) for each node in the network and then select the smallest nodal reliability. When MCS is used, this approach is no less efficient than a single nodal capacity reliability estimate. However, as recognized by Xu and Goulter (1999), the increased computational burden imposed by FORM for estimating multiple nodal capacity reliabilities makes this basic approach for critical node determination undesirable when FORM is the reliability estimation technique. Consequently, Xu and Goulter (1999) employed Eq. (4) to find one measure of nodal capacity reliability at the most critical node in the network. Xu and Goulter (1999) identified the most critical node in the network by analyzing the intermediate results of FORM and therefore observed the increase in computational time to be minimal. A detailed description of this approach is given by Xu and Goulter (1998).

Depending on the reliability analysis program utilized to implement FORM, the intermediate results of FORM may not be available to the program user. Therefore, an alternative approach that does not require intermediate FORM results is proposed and tested here for identification of the critical node in the network. The following performance function can be defined for use with FORM to identify the critical node in the network:

$$G_c(\mathbf{X}) = \min\left(H_i(\mathbf{X}) - H_i^{\min}\right), \quad 1, 2, \dots, I \tag{5}$$

where $G_c(\mathbf{X})$=performance function at the node that is most critical with respect to meeting its corresponding minimum pressure requirement and I=number of nodes considered. In this performance function, the location of the critical node is not fixed and can change locations with each FORM evaluation of the performance function. Thus, FORM is left to converge to the critical node location at the design point. This performance function is designed so that the critical node location and reliability at the critical node are determined without accessing the intermediate results of the FORM computational procedure.

Network Capacity Reliability

It is often convenient and sometimes more meaningful to estimate a single reliability measure that characterizes overall network performance. Previous approaches have proposed heuristic measures of network reliability such as the arithmetic mean or weighted average of all nodal reliabilities (e.g., Bao and Mays 1990). Another common heuristic measure is to use the critical node reliability as an approximation of network reliability (e.g., Bao and Mays 1990; Xu and Goulter 1999). While these heuristics can provide reasonable estimates of network reliability, it is important to realize that MCS can be used to directly estimate network reliability by treating WDNs as a series system in which failure to provide adequate heads at *any* one or more nodes with a minimum pressure requirement constitutes a network failure.

A direct estimate of the above definition of network capacity reliability can be found using MCS to evaluate the performance function in Eq. (5). Even though both FORM and MCS can be used to evaluate the performance function defined in Eq. (5), the reliability measures estimated by each method are not the same. When Eq. (5) is used as the FORM performance function, FORM searches for and converges to the critical node at the design point (i.e., the single most likely event to cause failure) and therefore estimates the capacity reliability only for that event at that node. If the same performance function is defined for MCS reliability estimation, failure is defined with respect to meeting the minimum pressure requirement at the most critical node in the system for *each* MCS realization. Thus, the MCS technique measures the reliability with respect to all failure events simultaneously, instead of just measuring the reliability with respect to the most likely failure event.

When FORM is used for reliability estimation in WDNs, a heuristic measure of network reliability must be employed. At present, the only proposed approach for approximating network reliability using FORM is to assume it is approximately equal to the critical node reliability (Xu and Goulter 1999). Relying on a heuristic measure of network reliability such as the critical node reliability is a significant disadvantage of FORM with respect to MCS for network reliability estimation unless the heuristic measure accurately approximates the MCS measure of network reliability. A common assumption in the literature is that the critical node reliability does in fact closely approximate the true network reliability. Although this may be the case and could even be evaluated in reliability studies dealing with a limited number of predefined network configurations, it is probably unreasonable to extend this assumption to WDN reliability-based design studies for two reasons. First, the critical node reliability approximation is non-conservative as it only considers failure at a single node in the network. Events leading to failure at other nodes in the network that are independent of failure at the critical node are not considered by this heuristic. Therefore, when the approximation is not accurate, the reliability of the network is overestimated. The other difficulty with this assumption is that in many or even all WDN design situations it is generally impossible to determine the appropriateness of the assumption for all or even a significant proportion of the vast number of possible network designs (often greater than a million). The combined effect that these two shortcomings have on the critical node approximation to network reliability is that when the approximation is employed in network optimization trials, there will most likely be a number of candidate network designs that are judged to have a significantly higher network reliability than their true network reliabilities. Consequently, if FORM is to be used to approximate network reliability in reliability-based optimization models, the FORM network reliability estimate should be improved.

The FORM measure of network capacity reliability can be enhanced if an additional failure mode is considered. For example, two failure modes could represent the capacity reliability at two nodes of interest in the network. In any system with two failure modes, the probability of system failure, p_{fs}, is

$$p_{fs} = p_{f1} - p_{f2} - p_{f12} = Pr\{G_1 < 0\} + Pr\{G_2 < 0\} - Pr\{G_1 < 0 \text{ and } G_2 < 0\} \tag{6}$$

where p_{f1} and p_{f2}=probabilities of failure due to failure Modes 1 and 2, respectively; p_{f12}=joint probability of failure for failure Modes 1 and 2 and G_1=G($\mathbf{X}_1$) and G_2=G($\mathbf{X}_2$) are the performance functions for failure Modes 1 and 2, respectively. The failure probabilities for the individual failure modes (p_{f1} and p_{f2}) can be obtained using Eq. (3) while the joint probability of failure, p_{f12}, is given by Madsen et al. (1986) as

$$p_{f12} = \Phi(-\beta_1, -\beta_2; \rho_{12}) = \Phi(-\beta_1)\Phi(-\beta_2) + \int_0^{\rho 12} \psi(-\beta_1, -\beta_2; \mathrm{y})\,\mathrm{dy} \tag{7}$$

where Φ(, ;ρ)=CDF for a bivariate normal vector with zero mean values and unit variances and correlation coefficient r and ψ(, ;ρ)5corresponding PDF. The integral in Eq. (7) is generally obtained numerically. The approximate correlation coefficient needed to evaluate this integral, ρ_{12}, is calculated using (Madsen et al. 1986)

$$\rho_{12} = \frac{\mathbf{V}_1^{*T}\mathbf{V}_2^*}{|\mathbf{V}_1^*||\mathbf{V}_2^*|} = \frac{1}{\beta_1\beta_2}\mathbf{V}_1^{*T}\mathbf{V}_2^* \tag{8}$$

where $\mathbf{V}_1^*$ and $\mathbf{V}_2^*$=design points in standard normal space for failure Modes 1 and 2, respectively.

Although Eq. (6) can be evaluated exactly when there are only two failure modes, only the bounds on the system probability of failure can be calculated when there are more than two modes of failure. In addition, consideration of each additional mode of failure adds one more FORM computational procedure. Therefore, it is proposed that a more accurate point estimate of network capacity reliability for a reasonable increase in computational cost can be evaluated using FORM for reliability estimation in the two most critical failure modes (i.e., at two nodes in the network) in conjunction with Eqs. (6), (7), and (8) to estimate the series system probability of failure. The basic idea underlying this approach is to identify the second most critical failure node, in addition to the most critical failure node in the network, to attempt to account for a significant fraction of the network failure events that may not lead to failure at the most critical node. The two most critical nodes in the network are determined by first estimating the most critical nodal capacity reliability using the performance function defined in Eq. (5) and then estimating the capacity reliability for the next most critical node in the network according to the following performance function:

$$G_{2c}(\mathbf{X}) = \min\left[H_i(\mathbf{X}) - H_i^{\min}\right], \quad 1, 2, \ldots, I \quad \text{and} \quad i \neq l \tag{9}$$

where the performance function in the second mode of failure is Eq. (9) and is defined as in Eq. (5) except that node *l*, the critical node determined in the first failure mode, is not considered as a possible location of failure in the second mode of failure.

Based on Eq. (6), the new system reliability measure, as, as estimated by FORM becomes

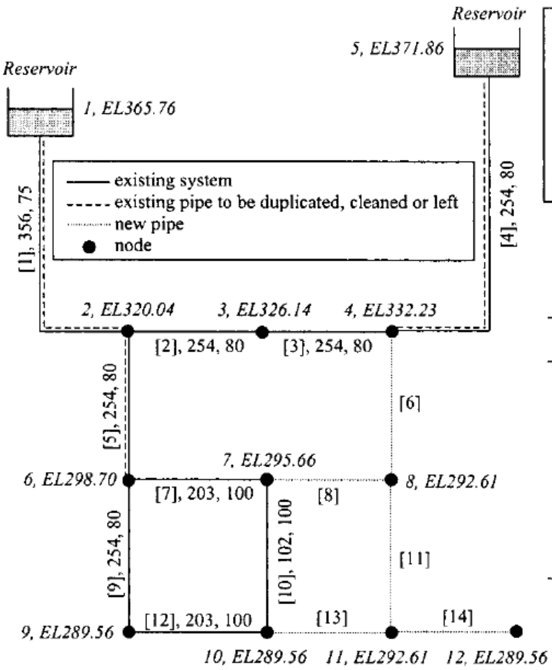
$$\alpha_s = 1 - p_{f1} - p_{f2} + p_{f12} = 1 - p_{fs} \tag{10}$$

where the individual probabilities of failure at the first and second most critical nodes in the network are *pf* 1 and *pf* 2 , respectively, and are calculated from Eq. (3), and the joint probability of failure between the two most critical nodes (*pf* 12) is calculated from Eq. (7).

Case Study

The novel combination of a GA with FORM for reliability-based optimization of WDNs is demonstrated for a case study that has been previously optimized under deterministic conditions (Simpson et al. 1994). The original design problem was to determine the required pipe sizes for an expansion to an existing WDN in order to minimize total pipe costs while still meeting the minimum nodal head requirements under three different flow demand patterns. The three demand patterns considered represent the peak-hour demand pattern and two fire-loading demand patterns that occur for a demand equal to the average peak-day demand pattern.

The layout of the case study WDN and the proposed expansion, other network characteristics, and the head requirements and mean nodal flows for each demand pattern, are summarized in Fig. 1. There are five new pipes to be sized (Pipes 6, 8, 11, 13, and 14) and three existing pipes (Pipes 1, 4, and 5) that can be left as is, cleaned, or duplicated with another pipe in parallel. New pipes and cleaned pipes are assumed to have a Hazen-Williams coefficient *C* value of 120. For the five new pipes, eight possible diameters from which to choose, in millimeters, are 152, 203, 254, 305, 356, 406, 457, and 508. The available pipe diameters considered in the duplication of the existing pipes are, in millimetres 152, 203, 254, 305, 356, and 406. Therefore, there are also eight possible decisions for Pipes 1, 4, and 5. The pipe costs per meter associated with each possible decision are listed in Table 1.



[1], 356, 75 [pipe number], diameter (mm), present day Hazen-Williams roughness coefficient (C)

2, EL320.04 node number, node elevation (m)

Note: 1. All pipe lengths are 1609m, except pipe [1] and pipe [4] which are 4828m and 6437m, respectively.
2. C is 120 for new and cleaned pipes.

	Demand Pattern 1		Demand Pattern 2		Demand Pattern 3	
Node	Mean Q (L/s)	H_{min} (m)	Mean Q (L/s)	H_{min} (m)	Mean Q (L/s)	H_{min} (m)
2	12.62	28.18	12.62	14.09	12.62	14.09
3	12.62	17.61	12.62	14.09	12.62	14.09
4	0	17.61	0	14.09	0	14.09
6	18.93	35.22	18.93	14.09	18.93	14.09
7	18.93	35.22	82.03	10.57	18.93	14.09
8	18.93	35.22	18.93	14.09	18.93	14.09
9	12.62	35.22	12.62	14.09	12.62	14.09
10	18.93	35.22	18.93	14.09	18.93	14.09
11	18.93	35.22	18.93	14.09	18.93	14.09
12	12.62	35.22	12.62	14.09	50.48	10.57

Figure 1. Layout of two-reservoir network upgrade problem under uncertainty

Table 1. Available Pipe Sizes and Associated Costs from Simpson et al. (1994)

Diameter (mm)	Cost of new pipe (\$/m)	Cost of cleaning existing pipe (\$/m)
152	49.5	47.6
203	63.3	51.5
254	94.8	55.1
305	132.9	58.1
356	170.9	60.7
406	194.9	63.0
457	231.3	66.3
508	262.5	69.2

Modifications to the Original Case Study

The original deterministic problem in Simpson et al. (1994) must be converted to a design problem under uncertain conditions. The objectives of the expansion of the WDN in Fig. 1 are to minimize total pipe costs and maximize network capacity reliability during the three critical demand patterns. In order to combine the network reliability values for each demand pattern, the three demand patterns are assumed to be of equal importance with respect to meeting the required nodal pressures. Therefore, the minimum network capacity reliability of the three demand patterns is used to represent the value of the reliability objective. In other words, the network reliability of each demand pattern is evaluated independently and then the minimum reliability is taken as the value of the reliability objective. This is just one way to generate a composite reliability measure characterizing multiple design events. It should be noted that the network capacity reliability measures outlined earlier are meant to estimate network reliability during a single design condition—not to combine reliabilities across multiple design conditions.

The sources of uncertainty considered are the uncertain values of roughness for each pipe and the uncertain nodal demand flows for each demand pattern. Estimation of WDN reliability during the critical design demand levels is complicated by the combination of the pipe hydraulic capacities and the nodal demand flows as uncertain variables because the time scales at which these sources of uncertainty vary are different. For example, pipe hydraulic capacities degrade slowly over time due to corrosion and deposition instead of varying randomly over time, as could be the case with the critical demand patterns. Therefore, the reliability in this study is calculated with respect to the uncertain conditions at the end of the design period under consideration and will thus, generally, be the worst-case network reliability during any point in time during the design period. It is assumed that a random amount of degradation in the pipe hydraulic capacities (i.e., random reductions in the *C* values from present day values) will occur over the design period and that the variability in the annual nodal demand patterns will be constant throughout the entire design period.

The reduction in *C* values of each pipe is assumed to be represented by a normal random variable with an average degradation of 10% over the design period and a coefficient of variation of the reduction in $C(\text{COV}_C)$ of 40%. For example, a pipe that has a present day *C* value of 120 is reduced on average by 0.1*120=12, with a standard deviation of 0.4*12=4.8, to C=120-N(12,4.8) at the end of the design period. The random amount of degradation in each pipe is assumed to be independent and is bounded such that the minimum and maximum *C* values at the end of the design period are 60 and the present day *C* value, respectively. It should be noted that the reliability analysis program used in this work for the implementation of MCS and FORM automatically adjusts the PDF for each bounded random variable so that the total probability is equal to one. The nodal demand flows in all three critical demand patterns are assumed to be normally distributed, with means as listed in Fig. 1 and a COV of 40% for all nonfire demand nodes. The fire demand nodes considered are Node 7, in Demand Pattern 2, and Node 12, in Demand Pattern 3, and both are assumed to have a reduced COV of 5%. All nodal demands are bounded to be greater than 0 and are assumed uncorrelated unless stated otherwise.

Model Formulation and Implementation

The optimization formulation presented in this section is defined in general terms such that the reliability objective can be based on any measure of reliability, estimated by either MCS or FORM, and the cost objective can be based on any network cost characterization. The reliability-constrained cost minimization model involves minimizing the total pipe costs of the WDN while meeting a specified minimum reliability constraint for the demand pattern with the lowest reliability and is given as follows:

$$\text{Maximize:} \quad \left\{k(\mathbf{Y}) + A\left[100\left(h^t\left(\mathbf{Y}, \mathbf{H}_{\min}^t, \mathbf{X}_{\text{par}}^t\right) - \alpha^*\right)\right]^B\right\}^{-1} \tag{11}$$

where $k(\mathbf{Y})$=total cost of the network as a function of **Y**, the vector of the selected decision variables in the system; $h^t\left(\mathbf{Y}, \mathbf{H}_{\min}^t, \mathbf{X}_{\text{par}}^t\right)$=estimated value of the reliability measure of interest for demand pattern *t*, which has the lowest reliability of the three demand patterns considered, and is a function of the vector of decision variables **Y**, the vector of minimum specified nodal head requirements for demand pattern $t\mathbf{H}_{\min}^t$, and the vector of probability distribution parameters for demand pattern $t\mathbf{X}_{\text{par}}^t$ that describe the random variables in the WDN model; α*=minimum desired reliability level for the WDN under the least reliable demand pattern and *A* and *B* are the penalty coefficient and exponent, respectively, in the GA penalty function. A penalty is only imposed, per reliability unit (i.e., per 1%) in which the reliability constraint is violated, if the estimated reliability is less than the specified minimum reliability level. Values of *A* and *B* need to be selected such that the penalty term in Eq. (11) drives the objective function value to very small values for unacceptable designs. The objective is inverted to become a maximization objective because the GA utilized in this work, as with most GAs, is coded as a maximization algorithm. Reliability-cost trade-off curves can be constructed by solving this model for a range of reliability constraints.

The general reliability-constrained cost minimization model is applied to the WDN case study. The mathematical formulation considers the objectives of minimizing total pipe costs and maximizing the minimum reliability of the three critical demand patterns. For each trial WDN design, the total pipe costs and the reliability under all three critical demand patterns must be estimated. The model is implemented in FORTRAN by linking a hydraulic network solver, a reliability analysis program, and a GA program together. All three of these programs, as well as all supplemental subroutines, such as that needed for the calculation of the WDN pipe costs, are also written in *FORTRAN*. The Wadiso hydraulic network simulation program (Gessler and Walski 1985) is used to simulate the hydraulic system for each set of random variables and pipe network configurations generated during the optimization trials. Wadiso assumes that the nodal demand flows are met and solves for the resultant nodal heads across the network. The original version of Wadiso is modified so that it can be repeatedly executed without the use of an input file and its accuracy is verified against a standard hydraulic simulation package (*EPANET2*). A slightly modified version of the general reliability analysis program *RELAN* (Foschi et al. 1993) is used to implement the FORM and MCS reliability estimation techniques. *RELAN* has been previously used and described in many studies such as Maier et al. (2001) and is generally robust for high reliability estimation given that it was developed to analyze very low probability structural failures. FORM and MCS as implemented in *RELAN* allow random variables to be drawn from a number of probability distributions and allow the user to specify correlations between any combination of the random variables. The *RELAN* implementation of FORM uses the Rackwitz-Fiessler method (Madsen et al. 1986) to find the minimum β.

The GA source code used in this study (*FORTRAN GA version 1.7.1*) after minor modifications is written by Dr. David Carroll and is available at http://cuaerospace.com/carroll/ga.html (also see Yang et al. 1998). An efficient set of GA parameters, as determined by Tolson (2000) for use in a different case study, is used. The *securGA* parameter values used here are a population size of 5, a maximum generation limit of 1,000, a uniform crossover probability of 0.5, a creep mutation probability of 0.1, and a single offspring per pair of parents. The binary coded values of the decision variables that are used by Simpson et al. (1994) are also adopted in this study. The deterministic network optimization problem solved by Simpson et al. (1994) and Simpson and Goldberg (1994) is solved again with the above GA parameter set with ten random seeds to ensure that the *securGA* with the above parameter settings is comparable in efficiency to the GAs used in these previous studies. Results show approximately the same or better performance in comparison with the previous GAs used to solve this problem.

Analyses Conducted

The first set of analyses conducted involves testing the various FORM and MCS reliability definitions on six example network designs and then further analyses demonstrating the application of the reliability-constrained cost minimization model follow. The six example designs are selected so as to cover network designs that span a range of reliability values. The pipe sizes and total cost of the six designs selected are summarized in Table 2. For each network design, the three demand patterns are analyzed. Consequently, the first sets of analyses are carried out on 18 different design conditions.

1. *Node Capacity Reliability Estimation*. To test whether FORM provides a good approximation to MCS reliabilities for the case study considered, FORM and MCS estimates of nodal reliability are obtained for each node across all 18 design conditions in accordance with Eq. (4). Initial testing is undertaken to determine the number of MCS realizations required to generate accurate MCS benchmark reliabilities. MCS nodal and network reliability estimates using 100,000 realizations converge to within approximately 0.001 of the MCS reliabilities found using 500,000 realizations. Therefore, all benchmark MCS reliabilities are generated using 100,000 realizations;
2. *Critical Node Capacity Reliability Identification*. To test whether FORM can be used to automatically determine the critical node, FORM is used to estimate the critical node reliabilities in accordance with Eq. (5) for all 18 design conditions. The results obtained are compared with the known FORM reliability predictions at the critical node obtained as part of the complete enumeration of the nodal capacity reliabilities in (1) above;

3. *Network Capacity Reliability Estimation*. To quantify the difference between critical node and network capacity reliability estimates for the case study under consideration, network capacity reliability is calculated using MCS in accordance with Eq. (5) for the 18 design cases and the results obtained are compared with the critical node capacity reliabilities obtained using MCS in (1) above.
4. *Network Capacity Reliability Approximation Using FORM*. To quantify the improvement associated with considering two failure modes when using FORM to approximate network capacity reliability, rather than using a single failure mode, estimates of network capacity reliability are obtained for the 18 design cases in accordance with Eq. (10) using FORM, and the results obtained are compared with the FORM results obtained using a single failure mode in (2) above and the true network capacity reliabilities obtained using MCS in (3) above;
5. *Reliability Constrained Cost Minimization*. The reliability-constrained cost minimization model is solved using the *securGA* to obtain minimum cost solutions for reliability constraints of 0.7, 0.8, 0.9, 0.95, 0.99, and 0.999. The FORM approximation of network capacity reliability [obtained using two failure modes—see (4) above] is utilized as the reliability constraint. The penalty exponent in Eq. (11) is set at 1.0 for all model evaluations to remain consistent with Simpson et al. (1994). However, the penalty coefficient is determined by trial and error for different reliability constraint levels. The penalty coefficients found to be reasonable are 0.1 million dollars for reliability constraints under 0.9, 0.5 million for constraints between 0.9 and 0.99, and 1.0 million for reliability constraints to 0.999. Since the operation of the GA is probabilistic in nature, at least two optimization trials, using different random GA seeds, are used to generate all model solutions; and
6. *Correlated Nodal Demands*. To investigate the impact nodal demand correlations have on the reliability-constrained cost minimization model solutions, the analyses in (5) above are repeated with a correlation coefficient of 0.5 (rather than 0.0) between the nodal demands. Correlations between pipe roughness coefficients are not considered.

Table 2. Water Distribution Network Designs Selected for Evaluation of Proposed Water Distribution

Design	Pipe diameter sizes (mm)								Total cost (10^6 \$/yr)
	Pipe [1]	Pipe [4]	Pipe [5]	Pipe [6]	Pipe [8]	Pipe [11]	Pipe [13]	Pipe [14]	
A	*dup*356	*dup*356	*dup*254	406	356	254	254	305	3.1860
B	*dup*254	*dup*356	*dup*305	254	508	305	203	356	2.9378
C	leave	*dup*406	clean	356	305	305	254	305	2.4035
D	*dup*406	*dup*305	*dup*152	254	305	254	203	254	2.6585
E	*dup*152	*dup*356	clean	305	203	254	152	254	2.1738
F	*dup*356	*dup*305	*dup*152	305	254	203	203	254	2.4922

Note: dup254 means the pipe is duplicated with a pipe 254 mm in diameter in parallel.

Results and Discussion

Nodal Capacity Reliability Estimation

The comparison of 180 MCS and FORM nodal capacity reliabilities for (1) above shows that, at worst, the absolute difference in the FORM and MCS estimates of nodal reliability is 0.026 at a reliability level of approximately 0.5. The magnitude of the absolute differences decreases as the reliability level increases. For example, above a reliability level of 0.95, the absolute differences between FORM and MCS are most often less than 0.001 and a maximum of 0.006. For nodal reliabilities between 0.8 and 0.95, the absolute differences in FORM and MCS reliabilities range between 0.000 and 0.017 with an average absolute difference of 0.007. Therefore, the case study is deemed appropriate for evaluating the new FORM reliability measures.

In 17 of the 18 design cases considered for (2) above, the use of Eq. (5) leads to the correct identification of the critical node and the correct reliability estimate for that node, indicating that FORM can be used to automatically determine the critical node. In the one case where the correct critical node is not identified, the third most critical node is identified as critical and the difference in reliabilities between the critical and third most critical node is less than 0.002. The average and worst observed increase in computational cost associated with using Eq. (5) instead of specifically identifying the critical node as in Eq. (4) is observed to be 11.4 and 31.0%, respectively. This small average increase in computational cost is deemed a reasonable trade-off for the simplicity, consistency, and accuracy with which the proposed performance function definition allows FORM to determine the critical node and its corresponding capacity reliability without accessing intermediate FORM results.

Network Capacity Reliability Estimation

The comparison of the MCS critical node and critical network capacity reliabilities for (3) above shows that for 11 of the 18 design cases investigated, the approximation to network capacity reliability given by the critical node capacity reliability is quite reasonable, as the critical node capacity reliability approximations fall within 0.001 of the actual network capacity reliability. Even though the results for the remaining seven cases remain close approximations, the differences in reliabilities in excess of 0.001 show that these two quantities can be dissimilar. The largest discrepancy in the approximation occurs for a case where the actual network capacity reliability of 0.943 is overestimated by 0.015 with a critical node capacity reliability of 0.958. Although this error may seem small, such a difference could have a significant impact on network cost. Therefore, FORM approximations of network reliability are necessary that go beyond the critical node capacity reliability.

Table 3 presents the FORM approximations to network capacity reliability for (4) above obtained using two failure modes (Column 3), as well as the MCS estimates of network capacity reliability (Column 2), and the FORM critical node reliability estimates (Column 4). Many cases show little or no absolute improvement by using the FORM estimate of network capacity reliability as can be seen by comparing Columns (3) and (4) in Table 3. However, if the relative reduction in the FORM approximation errors is considered (last column in Table 3 which compares Columns 3 and 4), then it becomes clear that for several designs the novel FORM approximation of network capacity reliability proposed here results in more accurate estimates of the MCS network capacity reliability. In fact, the average reduction in error for the eight cases where the reduction is greater than 0% is 35.2%.

Design B(2) in Table 3 shows the largest observed error reduction and can be analyzed further to show that the improvement is significant with respect to the approximate 95% confidence limits on the MCS reliability (a binomial proportion). Since the approximate confidence limits are calculated as (0.942,0.944) and do not contain the FORM critical node capacity reliability (0.958), the approximation using FORM with two failure modes (0.946) that is now closer to the upper bound of the confidence limits demonstrates that the improvement is significant. More evidence that the improvement in network reliability estimation by FORM is noteworthy is that in 8 of 13 design cases where improvement is possible (e.g., Column 4≠Column 2 in Table 3), at least some error reduction is observed. The increase in computational time required for FORM evaluation of two failure modes over the critical node approximation to network reliability is approximately 100% since this new measure of network reliability requires two FORM computational procedures instead of one.

Table 3. Comparison of Alternative Measures of Network Capacity

Design (demand pattern)	Monte Carlo simulation network capacity reliability	First-order reliability method network capacity reliability	First-order reliability method critical node capacity reliability	Percent reduction in First-order reliability method approximation error[a] (%)
A(1)	1.000	1.000	1.000	-
A(2)	0.981	0.982	0.982	0.0
A(3)	1.000	1.000	1.000	-
B(1)	0.995	0.996	0.998	66.7
B(2)	0.943	0.946	0.958	80.0
B(3)	0.995	0.996	0.996	0.0
C(1)	0.999	0.999	0.999	-
C(2)	0.924	0.924	0.924	-
C(3)	0.996	0.996	0.996	-
D(1)	0.954	0.957	0.957	0.0
D(2)	0.804	0.822	0.826	18.2
D(3)	0.818	0.835	0.835	0.0
E(1)	0.982	0.985	0.987	40.0
E(2)	0.689	0.709	0.719	33.3
E(3)	0.948	0.954	0.957	33.3
F(1)	0.911	0.919	0.919	0.0
F(2)	0.605	0.621	0.622	5.9
F(3)	0.524	0.549	0.55	3.8

Reliability-Constrained Cost Minimization

Reliability-cost trade-off curves obtained for (5) above by solving the reliability-constrained cost minimization model with the FORM network capacity reliability as estimated from Eq. (10) are shown in Fig. 2. The best solution of the two or more optimization trials used to solve the model at each reliability constraint is plotted. Fig. 2 shows that the model solutions found do not all occur adjacent to their respective reliability constraints. This suggests that the total number of noninferior solutions in this problem may be fairly limited. For each model solution in Fig. 2, the FORM network capacity reliabilities were checked using 100,000 MCS realizations. The magnitude of the absolute differences between FORM and MCS network reliability estimates are a maximum of 0.017 at approximately the 0.80 reliability level and remain below 0.004 for all solutions with reliability levels greater than or equal to 0.90. Therefore, FORM approximates MCS network reliability quite well for the trade-off solutions in the range where most realistic designs would be considered (e.g., above 0.9 reliability).

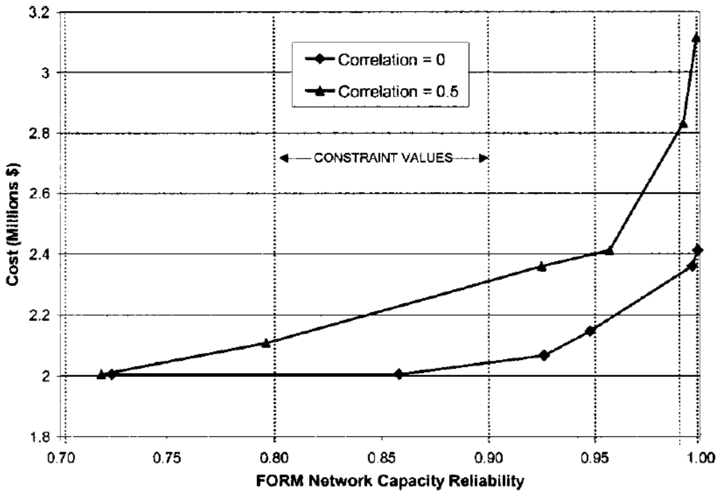


Figure 2. Reliability-cost trade-off curves generated by reliability-constrained cost minimization model with correlation coefficients between random nodal demands of 0 and 0.5

The typical execution time for one optimization model trial (i.e., the generation of one trade-off curve point) with 5,000 GA evaluations is approximately 75 min on a Pentium III, 666 MHz processor although some trials required in excess of 120 min for execution. The variance in the processor time is due to varying convergence rates of FORM for the trial network configurations. In comparison, if 2,000 MCS realizations were used to estimate network capacity reliability in the model instead of FORM, the execution time is increased approximately six times over the typical FORM execution time.

Comparison of the trade-off curves with and without demand correlations for (6) above in Fig. 2 shows that if correlation between nodal demands is considered, higher cost network designs are generally required to achieve a given desired level of network capacity reliability. Furthermore, the difference in cost between the trade-off curves increases as more reliable designs are required. Consequently, it is important that correlations between the random variables are taken into account.

Limitations and Future Work

There are some limitations to the ideas presented in this case study that are noteworthy. First of all, the accuracy of FORM relative to MCS is generally application specific and initial testing is required to determine if FORM is appropriate for other reliability-based optimization studies. Other limitations relate to the application of the ideas presented here to larger, more complex WDNs. Future work in this area is needed to extend the approaches outlined here to be applicable to large networks with hundreds and perhaps thousands of pipes. These limitations are discussed in more detail below to help guide future work.

Although convergence to the critical node is observed in this case study, future applications using Eq. (5) should initially be tested to ensure that FORM does converge to the correct critical node and critical node reliability. In practice, large networks are likely to require a modified approach to that proposed here. In particular, the new FORM methods proposed here could likely be applied to subsections of large networks such that FORM was constrained to find the critical node in a subsection of a network and then further constrained to find the second most critical node in another subsection of the network. This approach would function to find nonadjacent nodes as the two most critical nodes and would thus improve the accuracy of the FORM network reliability measure.

As network size increases, the number of failure events caused by independent failures at multiple nodes also increases. Therefore, the errors in any approximation of network reliability that does not capture some of the possible failure events will also generally increase. Studies that are focused on managing for network reliability should consider this since the result is that the critical node reliability approximation of network reliability, and even the approximation using the FORM network reliability measure with two failure modes, may not be sufficiently accurate. Consequently, when FORM is to be utilized, more than two failure modes may need to be considered. However, there is a trade-off between the accuracy of the network reliability approximation and required computational time using FORM.

Conclusions

This paper has introduced a number of new and useful techniques for consideration in future reliability-based optimization studies of WDNs. Some techniques proposed are specific to studies employing FORM for WDN reliability estimation while others are more general and should be applicable in future WDN reliability-based optimization studies regardless of the type of reliability measure considered. FORM has been applied to accurately estimate WDN nodal capacity reliability and a new FORM approach that is both accurate and efficient is proposed which automatically identifies the most critical node in the network. For this case study, it is demonstrated that the MCS critical node capacity reliability approximation can significantly underestimate the true MCS network capacity reliability. Considering that the critical node capacity reliability approximation to network capacity reliability estimation may not be reasonable in all WDN reliability-based optimization studies, a new and more accurate FORM approximation to network capacity reliability is developed that considers failure events at the two most critical nodes in the network. This work also demonstrates the novel combination of a GA with FORM as a reasonably efficient approach for reliability-based optimization of WDNs. Last, correlations between nodal demands are shown to significantly increase WDN costs designed to meet a specific reliability target.

Acknowledgments

Financial support for this project was provided by the University of Adelaide Small Grants Scheme. This support is gratefully acknowledged. The writers would also like to acknowledge the anonymous reviewers for their insightful comments that improved the manuscript.

Notation

The following symbols are used in this paper:

A = penalty multiplier;
B = penalty exponent;
C = Hazen-Williams coefficient;
F = failure domain;
$f_X(\mathbf{x})$ = is joint probability density function (PDF) of $\mathbf{X}$;
$G(\mathbf{X})$ = performance function, where failure (limit state) surface G=0 separates all combinations of $\mathbf{X}$ that lie in failure domain from those in survival domain;
$G_c(\mathbf{X})$, $G_{2c}(\mathbf{X})$ = performance function at node that is most critical with respect to meeting its corresponding minimum allowable pressure requirement, 2 refers to second most critical node;
$H_i(\mathbf{X})$ = head predicted at node i;
$H_i^{\min}$ = minimum allowable specified head required at node i;
$h^t\left(\mathbf{Y}, \mathbf{H}_{\min}^t, \mathbf{X}_{\mathrm{par}}^t\right)$= estimated value of reliability measure for demand pattern with lowest reliability of three demand patterns considered as function of vector of decision variables $\mathbf{Y}$, vector of minimum specified nodal head requirements $\mathbf{H}_{\min}^t$ and vector of probability distribution parameters $\mathbf{X}_{\mathrm{par}}^t$;
I = number of nodes in network with minimum pressure requirements;
i = index for specific node in network;
$k(\mathbf{Y})$ = total cost of network
L = system's load;
l = most critical node in network;
p_f ,p_{f1} = probability of failure, 1 refers to Mode 1;
p_{fs} = probability of system failure;
p_{f12} = joint probability of failure for failure Modes 1 and 2;
R = system's resistance;
S = survival domain;
$\mathbf{V}_1^*$ = design point in standard normal space for failure Mode 1;
$\mathbf{X}$ = vector of random variables that influences system's load and resistance;
$\mathbf{Y}$ = vector of decision variable values (pipe sizes);
α = reliability;
α_s = system reliability measure;
α^* = is minimum desired reliability level for WDN under any critical demand pattern;
β = reliability index;

Φ ( ) = standard normal cumulative density function;

Φ (, ;ρ) = CDF for bivariate normal vector with zero mean values, unit variances, and correlation coefficient ρ; and

ψ(, ;ρ) = PDF for bivariate normal vector with zero mean values, unit variances, and correlation coefficient ρ.

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