



THE UNIVERSITY  

---

of ADELAIDE

# Improved Determination of Hadron Matrix Elements using the Variational Method

Jack Dragos

*Supervisors:*

Assoc. Prof. Ross Young

Dr. James Zanotti

Faculty of Sciences  
School of Physical Sciences  
Department of Physics

September, 2016

# Abstract

Utilising lattice QCD to calculate nucleon matrix elements has had a huge impact on the knowledge of the structure of nucleons. From the comparison to experimental data, to the new insights into the structure of nucleons, the practices of lattice QCD has cemented itself as a fundamental field for particle physics. Some key contributions to the understanding of nucleon structure lattice QCD can provide are parameters needed for the *beyond standard model* (BSM) extensions, understanding the size of the nucleons via the charge radii and the decomposition of the spin and angular momentum of the quarks and gluons within the nucleon.

But the extraction of hadron matrix elements in lattice QCD using the standard two- and three-point correlator functions demands careful attention to systematic uncertainties. Although other systematics including discretisation, renormalisation and chiral extrapolation effects need to be analysed, one of the most recent and emerging sources of systematic error is contamination from excited-states.

This thesis applies the variational method to calculate the axial vector current  $g_A$ , the scalar current  $g_S$ , the tensor current  $g_T$  and the quark momentum fraction  $\langle x \rangle$  of the nucleon and we compare the results to the more commonly used summation and two-exponential fit methods. Proceeding with the same comparison of methods, we extend the calculation to non-zero momentum transfer to access the vector form factors for both the proton and neutron, as well as the iso-vector combination of the axial and induced pseudoscalar form factors for the proton. The results demonstrate how excited-states affect the extraction of nucleon matrix elements and in the process discovering that the variational approach offers a more efficient and robust method for the determination of nucleon matrix elements.

Through this demonstration of how excited-states impact lattice QCD calculation and how we can use methods to suppress these excited-states, we can hope to achieve higher and higher precision determinations of nucleon matrix elements from lattice QCD which will aid in our understanding of the structure of nucleons.



I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. The author acknowledges that copyright of published works contained within this thesis resides with the copyright holder(s) of those works. I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

Jack Dragos



# Acknowledgements

From the beginning, I thank Waseem Kamleh and Derek Leinweber for starting me off on my road to research through my pre-PhD years. Little did I know what I was getting into when you set me up on this research path, yet over the years you make it quite clear what the motivations and goals in this research topic was. I am grateful for your patience while I staggered over my self numerous times trying to get my head around lattice QCD concepts. None of the things I have achieved in my PhD would have been possible without the two of you.

In this period, I send my deepest gratitude to Ben Owen. Probably one of the best teachers I have ever had, along with his endless patience me, made my honours year and the years to follow the most inspiring and interesting years of my academic life. Never turning me away and always attempting to explain things I had troubles understanding in different and new ways, my understanding of lattice QCD would not be what it is today without his support. I wish him all the best in his life and will always be remembered.

A special thanks to Kaustubh Naik for the friendship we had during our under- and post-graduate years. You made sure I was well fed with lunch at mid day sharp, and I value all the random talks we had about physics, gaming and other topics.

Next in the time line comes James Zanotti. After sitting in on my honours meetings and helping me through this period, it started to come clear that I would eventually be working with James for my PhD. I thank you for this path you have shown to me, from the beginnings of creating the lattice data, to showing me the world of lattice QCD research at the lattice conference and to the patience you had with my bad writing skills for my publications.... Always making time to try to assist in my understanding of lattice QCD, James always made sure I fully understand what I was doing and explaining things in an understandable and relatable way. It always amazed me how much enthusiasm, interest and support he had with the work I was producing, always giving me the confidence to continue.

After working with Ross Young for some time, I started to understand how supportive he is to me and his colleagues as well as how passionate his about particle physics. My fear of showing Ross some writing or some results was a very real thing, as he will pick through every little detail and make sure everything is

exact and correct. I was very honoured to have Ross as my supervisor, someone who was always making sure I was working up to a high standard of research and always made me keep track of the key goals of my work.

To my fellow colleagues, Samuel Thomas, Adrian Kiratidis, Daniel Trewartha, Finn Stokes, Alexander Chambers, Ryan Bignell and Dylan Harries (and others I have talked with over the years). You made my PhD years interesting and insightful (even if it wasn't anything to do with physics) and helped me understand physics in a whole new way. Listening to you guys talking about random stuff made my days somehow. Whether it be listening to seminars at the conference, or just talking about physics and other random stuff at different events, you guys made my time here interesting and enjoyable. Along side this, the research you share and produce is always amassing and inspiring to me, pushing me to always better myself to keep up with your ever increasing standards.

A large thanks to the CSSM and Physical Sciences department. They both have made my PhD years new and exciting, whether it be the CSSM conferences they hosted and sent me to, or the Physical Sciences department showing me the teaching side of being in academia.

To all the people I meet at the international lattice conference in Japan. The conversations and presentations I was a part of gave my work a sense of purpose in the lattice QCD community. Special thanks to Andrea Shindler who I look forward to working with in the future.

To my loving and caring family, I thank whole-heartily. My mother Sophie Wait for supporting and taking care of me throughout my journey, to my brothers James and Jon Dragos for always listening and making me laugh over the years and to all my extended family for the fun gatherings and entertaining times.

Lastly to my fiancée Sarah Brown, for always being there, always listening, always supporting me, always making me laugh and for all the love you show to me and others.

Dedicated to my grandma (yiayia Marika),  
take care of Marie for us up there.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Quantum Chromodynamics (QCD)</b>	<b>3</b>
2.1	Quarks and Interactions	3
2.2	Lagrangian	4
2.3	Neutron $\beta$ -Decay	6
2.4	Form Factors	7
2.5	Deep Inelastic Scattering	10
<b>3</b>	<b>Quantum Chromodynamics on the Lattice</b>	<b>15</b>
3.1	Gluon Action	16
3.1.1	Gauge Field Mean-Field Improvement	18
3.2	Fermion Action	19
3.2.1	Improving the Action	20
3.2.2	Further Improving the Action	21
3.3	Gauge Field and Propagator Generating	22
3.4	Correlation Functions	24
3.4.1	Two-Point Functions	24
3.4.2	Three-Point Functions	28
<b>4</b>	<b>Renormalisation</b>	<b>37</b>
4.1	Vector Form Factor Renormalisation	37
4.2	General Non-Perturbative Renormalisation	38
4.2.1	Zero Transfer Momentum	38
4.2.2	Lattice Greens Function Calculation	40
4.2.3	Moving to the $\overline{MS}$ Scheme	40
4.3	Calculation	41
4.3.1	Lattice Parameters	42
4.3.2	Results	43



<b>5</b>	<b>Advanced Correlation Functions</b>	<b>47</b>
5.1	Smearing the Quark Fields . . . . .	47
5.1.1	Fixed Sink Propagator Smearing . . . . .	48
5.2	Variational Method for Correlation Functions . . . . .	49
5.2.1	Bases for the Variational Analysis . . . . .	51
5.2.2	Pencil of Function Basis . . . . .	52
5.2.3	Variational Method Optimised Fixed Sink Propagator . . . . .	53
<b>6</b>	<b>Analysis of Correlation Functions</b>	<b>55</b>
6.1	Two-Point Function Analysis . . . . .	56
6.2	Three-Point and Ratio Function Analysis . . . . .	59
6.3	Summing the Ratio Function . . . . .	63
6.4	Form Factor Extractions . . . . .	64
<b>7</b>	<b>Zero Momentum Transfer Results</b>	<b>66</b>
7.1	Lattice Details . . . . .	66
7.2	Mass Extraction from Two-Point Correlation Function . . . . .	68
7.3	Zero Momentum Proton Iso-Vector Matrix Elements . . . . .	72
7.4	Analysing the Variational Method Data . . . . .	73
7.4.1	Plateau Smearing/Variational comparison . . . . .	73
7.4.2	One-Exponential Comparison . . . . .	77
7.4.3	Source-Sink Separation Dependence . . . . .	80
7.4.4	Two-Exponential Comparison . . . . .	83
7.4.5	Pencil of Function Analysis . . . . .	86
7.5	Source-Sink Analysis . . . . .	89
7.5.1	Summing the Ratio Function . . . . .	89
7.5.2	Comparing Summation Method to the Correlators . . . . .	94
7.5.3	One-Exponential Comparison . . . . .	96
7.5.4	Two-Exponential Comparison . . . . .	99
7.6	Summary . . . . .	103
7.6.1	Summation Results . . . . .	107
7.6.2	One-Exponential Fit Results . . . . .	108
7.6.3	Two-Exponential Fit Results . . . . .	108
7.6.4	Variational Results . . . . .	109
7.6.5	Pencil of Function Results . . . . .	109
7.7	Cost Benefit Analysis . . . . .	110
<b>8</b>	<b>Form Factor Results</b>	<b>112</b>
8.1	Vector Form Factors . . . . .	112
8.2	Axial $G_A$ and $G_P$ . . . . .	122
8.3	Summary . . . . .	124

<b>9</b>	<b>Summary</b>	<b>126</b>
<b>A</b>	<b>Lattice QCD</b>	<b>129</b>
A.1	Sakurai Representation . . . . .	129
A.2	Bootstrap and Jackknife Ensembles of Correlation Functions . . . . .	129
A.3	Pencil of Function De-correlation . . . . .	131
<b>B</b>	<b>Perturbative Renormalisation Parameters</b>	<b>133</b>
<b>C</b>	<b>Tabulated Results</b>	<b>136</b>
C.1	Zero Momentum . . . . .	136
C.2	Form Factors . . . . .	138