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$g_{\pi\Lambda\Sigma}$ and $g_{K\Sigma\Xi}$ from QCD sum rules

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The coupling constants $g_{\pi\Lambda\Sigma}$ and $g_{K\Sigma\Xi}$ are calculated in the QCD sum rule approach using the three-point function method and taking into account the SU(3) symmetry breaking effects. The pattern of SU(3) breaking appears to be different from that based on SU(3) relations. [S0556-2813(98)04304-0]

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Understanding the explicit SU(3) symmetry breaking effects in physical quantities, such as mass splitting, coupling constants, and decay constants, has been a subject of research in models of QCD for many years. Among those models, the method of QCD sum rules [1–3] has proved to be a very effective tool to extract information about hadron properties. In the QCD sum rule approach the SU(3) breaking effects are included systematically in perturbative quark mass corrections (i.e., $m_u = m_d \neq m_s$) and the different quark condensates ($\langle\bar{u}u\rangle = \langle\bar{d}d\rangle \neq \langle\bar{s}s\rangle$). From fitting analyses of meson and baryon mass splittings it was found that the best fit was obtained with $m_s \sim 150$ MeV and $\gamma = \langle\bar{s}s\rangle/\langle\bar{u}u\rangle - 1 \sim -0.2$. However, it was not always possible to calculate all physical quantities in QCD sum rules, especially those related to Goldstone bosons because of small momentum transfer and possible direct instanton effects. However, by appropriately choosing the correlation function and improving the continuum part, we can estimate effects of explicit chiral symmetry breaking even for quantities related to the Goldstone bosons. For example, in Ref. [4] we calculated $g_{KN\Lambda}$, $g_{KN\Sigma}$[1] and compared to $g_{\pi NN}$, and in Ref. [6] we obtained the decay constants f_π, f_K, and their ratio using the correlation function of the axial vector currents, for which no contamination from direct instantons is expected.

In this work, we proceed along these line by presenting a QCD sum rule calculation for the coupling constants $g_{\pi\Lambda\Sigma}$ and $g_{K\Sigma\Xi}$ using the three-point correlation function. Comparing these coupling constants to each other can provide further insight into SU(3) symmetry breaking effects on physical quantities as in the case of $g_{KN\Lambda}$ and $g_{KN\Sigma}$.

We present sum rules for the coupling constants, taking into account the two SU(3) symmetry breaking parameters, m_s and γ; we discuss uncertainties in our calculations and the sign convention of the pole residues for $\frac{1}{2}^+$ octet baryons; and we summarize our results.

We will closely follow the procedures given in Refs. [7,2,4]. Consider the three point function constructed of the two baryon interpolating fields η_B, $\eta_{B'}$ and the pseudoscalar meson current j_5:

$$A(p,p',q) = \int dx\, dy\, \langle 0|T(\eta_{B'}(x) j_5(y) \times \overline{\eta}_B(0)|0\rangle\, e^{i(p'\cdot x - q\cdot y)}. \tag{1}$$

In order to obtain $g_{\pi\Lambda\Sigma}$ we will use the following interpolating fields for the Λ and the Σ as in Refs. [2,4]:

$$\eta_\Lambda = \sqrt{\frac{2}{3}}\epsilon_{abc}[(u_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu d_c - (d_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu u_c],$$

$$\eta_{\Sigma^\circ} = \sqrt{2}\epsilon_{abc}[(u_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu d_c + (d_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu u_c], \tag{2}$$

where u and d are the up and down quark fields, and a,b,c are color indices. T denotes the transpose in Dirac space, and C is the charge conjugation matrix. For the π^0 we choose the current

$$j_{\pi^0} = \bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d. \tag{3}$$

The sum rule after Borel transformation in $p^2 = p'^2$ is

$$\lambda_\Lambda\lambda_\Sigma \frac{M_B}{M_\Sigma^2 - M_\Lambda^2}(e^{-M_\Lambda^2/M^2} - e^{-M_\Sigma^2/M^2}) g_{\pi\Lambda\Sigma}\frac{f_\pi m_\pi^2}{\sqrt{2}m_q} = -\frac{2}{\sqrt{3}}\left(\frac{7}{12\pi^2}M^4 + \frac{m_s^2}{4\pi^2}M^2 - m_s\langle\bar{s}s\rangle\right)\langle\bar{q}q\rangle. \tag{4}$$

Note that in this first exploratory work the pole-continuum transition terms [8,9] have been neglected as was done in Ref. [4]. For λ_Λ and λ_Σ, we use the values obtained from the following baryon sum rules for the Λ and the Σ [2]:

$$M^6 + \frac{2}{3}am_s(1-3\gamma)M^2 + bM^2 + \frac{4}{9}a^2(3+4\gamma) = 2(2\pi)^4\lambda_\Lambda^2 e^{-M_\Lambda^2/M^2}, \tag{5}$$

$$M^6 - 2am_s(1+\gamma)M^2 + bM^2 + \frac{4}{3}a^2 = 2(2\pi)^4\lambda_\Sigma^2 e^{-M_\Sigma^2/M^2}, \tag{6}$$

again paralleling the procedure of Ref. [4]. Here $a \equiv -(2\pi)^2\langle\bar{q}q\rangle$, $b \equiv \pi^2\langle(\alpha_s/\pi)G^2\rangle$, and $\gamma \equiv \langle\bar{s}s\rangle/\langle\bar{q}q\rangle - 1 \simeq -0.2$. We take the strange quark mass m_s = 150 MeV, and the pion decay constant f_π = 133 MeV. The sum rule in Eq. (4) does not display a plateau as a function of the Borel mass. However, to gain some idea of the SU(3) symmetry

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[1]A recent status on these couplings is given in Ref. [5].

breaking effects we proceed by considering the value at Borel mass $M \simeq M_B = \frac{1}{2}(M_\Lambda + M_\Sigma)$, where M_Λ and M_Σ are the masses of the Λ and the Σ particle, respectively. This approach parallels to that of Ref. [4] for $g_{KN\Lambda}$ and $g_{KN\Sigma}$. At this Borel mass, in the right-hand side (RHS) of Eq. (4), the contribution from the s-quark mass correction is only 2% of the first term. Hence the leading order SU(3) breaking effects appear to be small.

Using the PCAC relation $m_\pi^2 f_\pi^2 = -4 m_q \langle \bar{q}q \rangle$, we obtain

$$g_{\pi\Lambda\Sigma} = 7.53 \tag{7}$$

for $\langle \bar{q}q \rangle = -(0.230\ \mathrm{GeV})^3$ and $\langle (\alpha_s/\pi) G^2 \rangle = (0.340\ \mathrm{GeV})^4$. We now check the dependence of our result on the SU(3) symmetry breaking parameters. If we take $\langle \bar{s}s \rangle = 0.6\ \langle \bar{q}q \rangle$, then the variation is within 0.3%. In addition, for $m_s = 180$ MeV the change of coupling constant is less than 0.8%. Thus, we find the coupling constant $g_{\pi\Lambda\Sigma}$ is very weakly dependent on the SU(3) symmetry breaking parameters. One should be cautious, however, that larger SU(3) symmetry breaking may be contained in higher order terms not considered here.

On the other hand, our result is more sensitive to different values of the quark condensate. We obtain $g_{\pi\Lambda\Sigma} = 7.35$ and 7.13 for $\langle \bar{q}q \rangle = -(0.240\ \mathrm{GeV})^3$ and $-(0.250\ \mathrm{GeV})^3$, respectively.

The interpolating fields of Σ^+ and Ξ° are defined by [2]

$$\eta_{\Sigma^+} = \epsilon_{abc}(u_a^T C \gamma_\mu u_b)\gamma_5 \gamma^\mu s_c,$$
$$\eta_{\Xi^\circ} = -\epsilon_{abc}(s_a^T C \gamma_\mu s_b)\gamma_5 \gamma^\mu u_c, \tag{8}$$

and we use

$$j_{K^-} = \bar{s} i \gamma_5 u. \tag{9}$$

Then the final expression is

$$\lambda_\Sigma \lambda_\Xi \frac{M_B}{M_\Xi^2 - M_\Sigma^2}(e^{-M_\Sigma^2/M^2} - e^{-M_\Xi^2/M^2})\sqrt{2} g_{K\Sigma\Xi} \frac{f_K m_K^2}{2 m_q}$$
$$= + \left(\frac{9}{10\pi^2} M^4 + \frac{7 m_s^2}{5\pi^2} M^2 - \frac{6}{5} m_s \langle \bar{s}s \rangle \right) \langle \bar{q}q \rangle. \tag{10}$$

In this case the contribution of the s-quark mass corrections is also small; about 3% of the first term.

For λ_Ξ, we use the following sum rule for Ξ [2]:

$$M^6 + bM^2 + \frac{4}{3} a^2 (1+\gamma)^2 = 2(2\pi)^4 \lambda_\Xi^2 e^{-M_\Xi^2/M^2}. \tag{11}$$

Then, the value of $g_{K\Sigma\Xi}$ is

$$g_{K\Sigma\Xi} = -7.02 \tag{12}$$

for $\langle \bar{q}q \rangle = -(0.230\mathrm{GeV})^3$ and $f_k = 160$ MeV. The variation of the coupling constant is within 0.5% if we take $\langle \bar{s}s \rangle = 0.6$ $\langle \bar{q}q \rangle$. On the other hand, we obtain $g_{K\Sigma\Xi} = -7.87$ and -8.75 for $\langle \bar{q}q \rangle = -(0.240\ \mathrm{GeV})^3$ and $-(0.250\ \mathrm{GeV})^3$, respectively. In addition the coupling constant is rather dependent on the s-quark mass. For example, if we take $m_s = 180$ MeV, then $g_{K\Sigma\Xi} = -8.55$ for $\langle \bar{q}q \rangle = -(0.230\mathrm{GeV})^3$. In the case of $g_{K\Sigma\Xi}$ we insert the values of the s-quark mass on the left-hand side of Eq. (10) and the quark condensate in the RHS directly instead of using the PCAC relation for the kaon. Therefore the variation of the coupling constant is much larger than that for the case of $g_{\pi\Lambda\Sigma}$.

We have calculated $g_{\pi\Lambda\Sigma}$ and $g_{K\Sigma\Xi}$ by following the same procedures in Ref. [4]. Here we discuss contributions not included in the previous calculations. First, the next-leading operator is dimension 5 $\langle g_s \bar{q} \sigma \cdot G q \rangle$, and it may contribute to the OPE side with considerable weight as in nucleon mass sum rules [10]. In addition, operators of dimension 7 may also be important in the OPE side as a further power correction. Second, the pole-continuum transition terms are neglected as we said previously.[2] Last, the contribution of pure continuum is not included.

While the inclusion of higher order power corrections would significantly complicate the exploratory analysis presented here, one can easily include the pure continuum contribution by considering the following factor in the OPE side:

$$E_i = 1 - \sum_{k=0}^{i} \frac{s_0^k}{k!(M^2)^k} e^{-s_0/M^2}, \tag{13}$$

where s_0 is a continuum threshold. For example, including the effect of the pure continuum Eq. (4) becomes

$$\lambda_\Lambda \lambda_\Sigma \frac{M_B}{M_\Sigma^2 - M_\Lambda^2}(e^{-M_\Lambda^2/M^2} - e^{-M_\Sigma^2/M^2}) g_{\pi\Lambda\Sigma} \frac{f_\pi m_\pi^2}{\sqrt{2} m_q}$$
$$= -\frac{2}{\sqrt{3}} \left(\frac{7}{12\pi^2} E_1 M^4 + \frac{m_s^2}{4\pi^2} E_0 M^2 - m_s \langle \bar{s}s \rangle \right) \langle \bar{q}q \rangle, \tag{14}$$

and Eqs. (5),(6) can be written as

$$E_2 M^6 + \frac{2}{3} a m_s (1 - 3\gamma) E_0 M^2 + b E_0 M^2 + \frac{4}{9} a^2 (3 + 4\gamma)$$
$$= 2(2\pi)^4 \lambda_\Lambda^2 e^{-M_\Lambda^2/M^2}, \tag{15}$$

$$E_2 M^6 - 2 a m_s (1+\gamma) E_0 M^2 + b E_0 M^2 + \frac{4}{3} a^2$$
$$= 2(2\pi)^4 \lambda_\Sigma^2 e^{-M_\Sigma^2/M^2}, \tag{16}$$

where we assume the same continuum threshold in Eqs. (14), (15), and (16). In Table I we present our previous results of $g_{KN\Lambda}$, $g_{KN\Sigma}$ and the present results of $g_{\pi\Lambda\Sigma}$, $g_{K\Sigma\Xi}$ without (and with) a continuum model. The previous analysis of Ref. [4] has been repeated with the continuum model correction. We take the continuum threshold to be $s_0 = 2.560$, 2.756, and 2.856 GeV^2 for the case of $g_{KN\Lambda}$, $g_{KN\Sigma}$ (and $g_{\pi\Lambda\Sigma}$), and $g_{K\Sigma\Xi}$, respectively, considering the next $\Lambda(1600)$, $\Sigma(1660)$, and $\Xi(1690)$ particle each other, although in the case of

[2] In the case of $g_{\pi NN}$ its contribution is at most 5% [11,12].

TABLE I. Coupling constants.

Coupling constants	$g_{KN\Lambda}$	$g_{KN\Sigma}$	$g_{\pi\Lambda\Sigma}$	$g_{K\Sigma\Xi}$
SU(3)	$-16.01 - -10.67$	3.01–4.51	7.94–11.90	$-16.12 - -10.74$
QSR (w/o cont.) [a]	-6.96	1.05	7.53	-7.02
QSR (with cont.) [a]	-8.34	1.26	10.79	-10.22
Exp. fit	-13.68 [b]	3.86 [b]	11.75 [c]	N/A

[a]We take $\langle \bar{q}q \rangle = -(0.230\ \text{GeV})^3$.
[b]Ref. [14].
[c]Ref. [15].

$\Xi(1690)$ its quantum number is not clarified in experiments [13]. In our calculation the continuum contribution is always less than 50% of the phenomenological side at the relevant Borel mass. A comparison to fitting analyses of experimental data [14,15] is also provided.

In the table the first row is a prediction from SU(3) relations between meson-baryon coupling constants. The SU(3) symmetry, using de Swart's convention [16], predicts

$$g_{KN\Lambda} = -\frac{1}{\sqrt{3}}(3-2\alpha_D)g_{\pi NN},$$

$$g_{KN\Sigma} = +(2\alpha_D - 1)g_{\pi NN},$$

$$g_{\pi\Lambda\Sigma} = \frac{2}{\sqrt{3}}\alpha_D g_{\pi NN},$$

$$g_{K\Sigma\Xi} = -g_{\pi NN}, \tag{17}$$

where α_D is the fraction of the D type coupling, $\alpha_D = D/(D+F)$. We take α_D from a recent analysis of hyperon semileptonic decay data by Ratcliffe, $\alpha_D = 0.64$ [17] while 7/12 in the SU(3) symmetric limit [7], and $g_{\pi NN}$ from an analysis of the np data by Ericson *et al.* [18], $g_{\pi NN} = 13.43$. We denote the error bar allowing for SU(3) symmetry breaking at the 20% level.

One can see that the results with a continuum model are larger than those without the continuum contribution. However, the corrections range from 20 to 45 % and suggest that further analysis is required before any firm conculsions may be drawn. Full quantitative analysis along the lines of Leinweber's work [10] would require all of the above mentioned corrections and is beyond the scope of this first exploratory calculation.

Let us comment on the sign convention of λ_B. Usually we construct the interpolating fields for the octet baryons by starting from the nucleon current and then making SU(3) rotations. Then the phase will be the same for all baryon states assuming exact SU(3) symmetry. But, in the real world the λ_Bs are not SU(3) symmetric and the phase can be changed according to the level of SU(3) symmetry breaking. However, our previous calculation of $g_{KN\Lambda}$ and $g_{KN\Sigma}$, and the present calculation of $g_{\pi\Lambda\Sigma}$ and $g_{K\Sigma\Xi}$ show that contribution of the s-quark mass corrections is very small compared to the leading term, and thus the relative signs of λ_B are the same for all octet baryons.

One can easily check this as below. In Ref. [4] the coupling constants in our diagram correspond to $-g_{KN\Lambda}$ and $-g_{KN\Sigma}$, respectively, according to de Swart's sign convention [16]. Then, our results can be rewritten as follows:

$$g_{KN\Lambda} \simeq \frac{-}{\lambda_N \lambda_\Lambda},$$

$$g_{KN\Sigma} \simeq \frac{+}{\lambda_N \lambda_\Sigma}, \tag{18}$$

where + and − on the RHS mean that the signs of numerators are + and −, respectively. Similarly our present results for $g_{\pi\Lambda\Sigma}$ and $g_{K\Sigma\Xi}$ give

$$g_{\pi\Lambda\Sigma} \simeq \frac{+}{\lambda_\Lambda \lambda_\Sigma},$$

$$g_{K\Sigma\Xi} \simeq \frac{-}{\lambda_\Sigma \lambda_\Xi}. \tag{19}$$

Assuming $g_{\pi NN} > 0$, one can see that the relative signs of λ_Bs are the same as can be seen by comparing Eqs. (18) and (19) to Eq. (17). This result follows from the fact that the SU(3) symmetry is slightly broken in our sum rules. In fact, there is another sign convention for meson-baryon coupling constants [15]. As emphasized in Ref. [19], however, both conventions lead to the same result for the only physically meaningful sign, $g_{KN\Lambda}$ and $g_{KN\Sigma} \cdot \mu(\Sigma^\circ \Lambda)$, where $\mu(\Sigma^\circ \Lambda)$ is the $\Sigma^\circ - \Lambda$ transition moment.

In summary, using the three-point correlation function method $g_{\pi\Lambda\Sigma}$ and $g_{K\Sigma\Xi}$ are obtained in the QCD sum rule approach. In both cases the contribution of SU(3) breaking effects in the leading order OPE side is less than 5%. The pattern of SU(3) breaking appears to be different from that based on SU(3) relations. Omission of continuum model contributions, as done in previous calculations, appears to be too crude. The couplings increase when the continuum model corrections are included, in some cases by nearly 50%. It would be interesting to further refine the QCD sum rule approach to allow a more depth study.

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