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Parameterised Complexity Analysis of Evolutionary Algorithms for Combinatorial Optimization Problems

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A thesis submitted in fulfilment for the
degree of Doctor of Philosophy

in the
Optimisation and Logistics
School of Computer Science

May 2017

Declaration of Authorship

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آمد سحری نداز میخانه ما
برخیز که پرکنیم چمانه زمر

*"Dreaming when Dawn's Left Hand was in the Sky
I heard a voice within the Tavern cry,
'Awake, my Little ones, and fill the Cup
Before Life's Liquor in its Cup be dry.' "*

Omar Khayyam
Translated into English in 1859 by Edward FitzGerald

UNIVERSITY OF ADELAIDE

Abstract

Engineering, Computer and Mathematical Sciences
School of Computer Science

Doctor of Philosophy

Parameterised Complexity Analysis of Evolutionary Algorithms for Combinatorial Optimization Problems

by Mojgan Pourhassan

Evolutionary algorithms are general problem solvers that have been successfully used in solving combinatorial optimization problems. However, due to the great amount of randomness in these algorithms, theoretical understanding of them is quite challenging. In this thesis we analyse the parameterized complexity of evolutionary algorithms on combinatorial optimization problems. Studying the parameterized complexity of these algorithms can help us understand how different parameters of problems influence the runtime behaviour of the algorithm and consequently lead us in finding better performing algorithms. We focus on two NP-hard combinatorial optimization problems; the generalized travelling salesman problem (GTSP) and the vertex cover problem (VCP). For solving the GTSP, two hierarchical approaches with different neighbourhood structures have been proposed in the literature. In this thesis, local search algorithms and simple evolutionary algorithms based on these approaches are investigated from a theoretical perspective and complementary abilities of the two approaches are pointed out by presenting instances where they mutually outperform each other. After investigating the runtime behaviour of the mentioned randomised algorithms on GTSP, we turn our attention to the VCP. Evolutionary multi-objective optimization for the classical vertex cover problem has been previously analysed in the context of parameterized complexity analysis. We extend the analysis to the weighted version of the problem. We also examine a dynamic version of the classical problem and analyse evolutionary algorithms with respect to their ability to maintain a 2-approximation. Inspired by the concept of duality, an edge-based evolutionary algorithm for solving the VCP has been introduced in the literature. Here we show that this edge-based EA is able to maintain a 2-approximation solution in the dynamic setting. Moreover, using the dual form of the problem, we extend the edge-based approach to the weighted vertex cover problem.

Acknowledgements

I would like to express my sincere appreciation to my principle supervisor, Prof. Frank Neumann, for his motivation, patience, support, and priceless guidance during my PhD study. Without his insights and advices, this journey would have been extremely difficult for me, if not impossible.

My grateful thanks also go to my co-supervisor, Dr. Markus Wagner, for his encouragement, useful comments, and his positive attitude towards solving problems that I faced.

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To my family