



**Spectral Finite Element Modelling and Damage  
Identification of Beam-like Structures Using  
Linear and Nonlinear Guided Waves**

**Shuai He**

Thesis submitted in fulfilment of the requirements for the degree  
of Doctor of Philosophy

The University of Adelaide  
Faculty of Engineering, Computer and Mathematical Sciences  
School of Civil, Environmental and Mining Engineering

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This thesis is dedicated to my beloved parents.



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of Beam-like Structures Using Linear and Nonlinear Guided  
Waves**

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# Abstract

This thesis contains a series of journal papers focused on the development of the model-based approach for damage identification using guided waves. The proposed approach requires no baseline data. It can identify multiple damages such as characterising the number, location and the size of cracks in isotropic beams and delaminations in composite beams efficiently and accurately with quantifying the associated uncertainties using linear guided waves. It also investigate the plausibility of using the nonlinear guided wave for damage identification. Based on the modelling ability, this approach is able to extend to different kinds of structures with various types of damages.

In utilising the linear guided wave for damage detection, the efficient spectral finite element (SFE) method is used to simulate the guided wave propagation in beams for both isotropic and composite materials. An SFE crack element is developed to simulate crack-wave interaction and the guided wave mode-conversion effect resulted from an asymmetric open crack in the isotropic beam. The delamination is simulated by duplicated the nodes of SFE elements in the delaminated regions. The proposed SFE model is verified using three-dimensional (3D) finite element (FE) method and good agreements are found in the results.

Stochastic methods are applied for the proposed model-based approach in the identification of multiple damages. The Bayesian model class selection algorithm is employed to determine the number of damages. The Bayesian model updating method implemented with efficient transitional Markov Chain Monte Carlo (TMCMC) sampler is proposed to identify the location and size of the crack. The Bayesian updating with structural reliability method (BUS) using the efficient and robust algorithm, Subset simulation, is proposed to identify the location, delaminated layer and length of the delaminations. The uncertainties of the identification are provided. For validation, the proposed methods are experimentally executed using Laser vibrometre and good agreements are obtained in the results.

The proposed SFE model is extended to simulate the nonlinear guided waves resulted from both classical and contact nonlinearity. Numerical case studies and parametric study highlight the potential of the SFE model in simulating nonlinear guided waves. This suggests that the model-based approach employed the nonlinear feature of guided waves to identify damages in further research.



# Statement of Originality

I, **Shuai He**, hereby declare that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution in my name and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and, where applicable, any partner institution responsible for the joint award of this degree.

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# Acknowledgments

I would like to sincerely thank my respectable parents for their financial support to my tuition. Without their unconditional love, I cannot go this far and this work would hardly be completed. I also grateful thank my beloved grandparents for their generous love and understanding. Nothing can I do in return but only keep climbing every mountain in my life.

I gratefully acknowledge my principle supervisor Dr Ching-Tai Ng, Senior Lecturer and my co-supervisors Abdul Hamid Sheikh, Associate Professor and Dr Togay Ozbakkaloglu, Senior Lecturer. Without their supervision and great contributions, this presented thesis would not be possible. I am particularly grateful to Dr Ching-Tai Ng for not only his unreserved guidance but also the encouragement and emotional supports during my candidature.

I would also like to thank those who accompany me sharing their happy life and hardworking time with me in Adelaide. It makes my PhD journey colourful and memorable.

Finally, I would like to thank the University of Adelaide for offering me the precious opportunity to seek the light in the deep ocean of knowledge.



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