ADELAIDE UNIVERSITY AUSTRALIA

SUB CRUCE LUMEN

Yang–Mills connections on $U(n)$–bundles over compact Riemann surfaces

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[1]Masters by research thesis supervised by Dr. Nicholas Buchdahl

Abstract

This thesis was submitted as a part of a Masters by research degree in the School of Mathematical and Computer Sciences at the University of Adelaide during March 2002. Its aim is to provide a detailed dissertation on the solutions to the *Yang–Mills equations* over compact Riemann surfaces analysed in terms of algebro–topological and differential–geometric structures on vector bundles over such manifolds, in the spirit of the paper *The Yang–Mills equations over compact Riemann surfaces* [2] by *Michael Atiyah* and *Raoul Bott.*

The introduction gives a physical motivation for the subject of *Yang–Mills connections* which is aimed at familiarising the reader, at an informal level, with preliminary concepts in differential geometry needed in exploring this topic. Subsequent chapters will make specific the preliminary material and will also serve as the introduction of the main analytical and algebraic methods implemented in the study of Yang–Mills connections on Riemann surfaces.

The project is structured as follows. Following a brief overview of connections, the Yang–Mills functional and the associated equations are given. A subsequent section on *equivariant Morse theory* sets the framework for the thesis, while the following sections on relations to *stable bundles* and certain *moduli spaces* of semi–stable bundles serve as descriptive methods of solutions of the Yang–Mills equations on compact Riemann surfaces.

By restricting our attention to bundles with structure group $U(n)$ we may apply *Morse theory* to the Yang–Mills functional and *stratify* the space of connections. With this we will deduce information about the Yang–Mills minima by computing the equivariant cohomology of the moduli spaces $N(n,k)$ of stable bundles of rank n and degree k in the coprime case $(n,k)=1$.

The level of complexity of this thesis is that understandable by honours graduate students of differential geometry and algebraic topology who have gone on to specialise in these fields.

1 Statement of submission.

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

The principal aid given to me in the production of this thesis was by *Dr. Nicholas Buchdahl* acting as primary supervisor, and *Dr. Michael Murray* as outside consultant, both members of the Pure Mathematics department in the University of Adelaide.

Mr. Peter Ernst Lawrence BSc(Ma & Comp Sc)(Hons)

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