Chiral extrapolation of lattice data for heavy baryons

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The masses of heavy baryons containing a b quark have been calculated numerically in lattice QCD with pion masses which are much larger than the physical value. In the present work we extrapolate these lattice data to the physical mass of the pion by applying an effective chiral Lagrangian for heavy baryons, which is invariant under chiral symmetry when the light quark masses go to zero and heavy quark symmetry when the heavy quark masses go to infinity. A phenomenological functional form with three parameters, which has the correct behavior in the chiral limit and appropriate behavior when the pion mass is large, is proposed to extrapolate the lattice data. It is found that the extrapolation deviates noticeably from the naive linear extrapolation when the pion mass is smaller than about 500 MeV. The mass differences between Σ_b and Σ_b^* and between $\Sigma_b^{(*)}$ and Λ_b are also presented. Uncertainties arising from both lattice data and our model parameters are discussed in detail. We also give a comparison of the results in our model with those obtained in the naive linear extrapolations.

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I. INTRODUCTION

The spectrum of some hadrons has been calculated numerically in lattice QCD over the past few years. These hadrons include light mesons and baryons [1], heavy mesons [2,3], and heavy baryons [2,4]. Using nonrelativistic QCD (NRQCD) on the lattice [5] for heavy quarks and the tadpole-improved clover action for light quarks, the authors of Refs. [2,3] studied extensively the spectra of heavy mesons and heavy baryons (including doubly heavy baryons). These lattice data were obtained in the region where the mass of the pion is much larger than the physical mass of the pion. Hence one needs to extrapolate these data to the physical pion mass in order to obtain the heavy hadron masses in the real world. Naively, this is done by linear extrapolations which are inconsistent with the model independent, nonanalytic behavior of hadron properties in the chiral limit. In order to overcome this problem, pion-hadron loops are included in the study of light hadron properties [6–9]. This yields the correct leading and next-to-leading nonanalytic terms in the light quark masses and leads to rapid variation at small pion masses. In general, lattice data extrapolated to the physical pion mass this way yield quite different results from linear extrapolations. In the light of these results, we considered previously the chiral extrapolation of the lattice data for heavy D and B mesons and discussed the important hyperfine splittings [10]. Here we generalize our approach to the case of heavy baryons and extrapolate the lattice data for heavy b baryons obtained in Ref. [2].

We are guided by the two opposite limits for the quark masses. One is the zero quark mass limit while the other is the infinite quark mass limit. When the masses of the light quarks, u, d, and s, go to zero the QCD Lagrangian has a chiral $SU(3)_L\times SU(3)_R$ symmetry, which is spontaneously broken into $SU(3)_V$ plus eight Goldstone bosons. When the masses of the heavy quarks c and b go to infinity, we have an effective theory, heavy quark effective theory (HQET), which is invariant under heavy quark flavor and heavy quark spin transformations, $SU(2)_f\times SU(2)_s$. Thus the interactions of heavy baryons with the light pseudoscalar mesons should be described by an effective chiral Lagrangian for heavy baryons which is invariant under both $SU(3)_L\times SU(3)_R$ and $SU(2)_f\times SU(2)_s$ transformations. This Lagrangian will be applied to the extrapolation of the lattice QCD data to the physical pion mass, with the chiral aspects being especially important in the small pion mass region.

The remainder of this paper is organized as follows. In Sec. II we give a brief review of the chiral Lagrangian for heavy baryons including the propagators of heavy baryons. In Sec. III we apply this Lagrangian to calculate pion loop contributions to the self-energy of heavy baryons. Then we propose a phenomenological functional form with three parameters for extrapolating the lattice data to the physical region. In Sec. IV we use this form to fit the lattice data and give numerical results. Finally, Sec. V contains a summary and discussion.

II. CHIRAL PERTURBATION THEORY FOR HEAVY BARYONS

When the light quark mass, m_q, approaches zero, the QCD Lagrangian possesses an $SU(3)_L\times SU(3)_R$ chiral symmetry. The light pseudo Goldstone bosons associated with spontaneous breaking of chiral symmetry can be incorporated into a 3×3 matrix

$$\Sigma=\exp\left(\frac{2iM}{f_\pi}\right), \tag{1}$$

where f_π is the pion decay constant, $f_\pi=132$ MeV, and M is a matrix which includes the eight Goldstone bosons

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$$M=\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \tag{2}$$

A heavy baryon is composed of a heavy quark Q ($Q=b$, or c) and two light quarks $q_a q_b$ [$a(b)$ equals 1, 2, 3 for u, d, s quarks, respectively]. When the heavy quark mass, m_Q, is much larger than the QCD scale, Λ_{QCD}, the light degrees of freedom in a heavy baryon become blind to the flavor and spin quantum numbers of the heavy quark because of the $SU(2)_f\times SU(2)_s$ symmetries. Therefore, the light degrees of freedom have good quantum numbers which can be used to classify heavy baryons. The angular momentum and parity J^P of the two light quarks may be 0^+ or 1^+, which correspond to $SU(3)_{L+R}$ antitriplet and sextet, respectively. The lowest-lying heavy baryons in the $\overline{3}$ representation have spin 1/2, and are denoted by fields which destroy these baryons, T_a ($T_3=\Lambda_Q$, $T_{1,2}=\Xi'_Q$). The lowest-lying heavy baryons in the 6 representation have spin 1/2 or 3/2, and are denoted by field operators S^{ab} and S^{ab}_μ, respectively, where $S^{(*)11,12,22}_{(\mu)}=\Sigma^{(*)}_Q$, $S^{(*)13,23}_{(\mu)}=\Xi^{(*)}_Q$, and $S^{(*)33}_{(\mu)}=\Omega^{(*)}_Q$.

It is convenient to combine S^{ab} and S^{*ab}_μ into the field S^{ab}_μ [11]:

$$S^{ab}_\mu=\frac{1}{\sqrt{3}}(\gamma_\mu+v_\mu)\gamma_5 S^{ab}+S^{*ab}_\mu. \tag{3}$$

Introducing a vector field V^μ_{ab},

$$V^\mu_{ab}=\frac{1}{2}(\xi^+\partial^\mu\xi+\xi\partial^\mu\xi^+)_{ab}, \tag{4}$$

where $\xi=\sqrt{\Sigma}$, and an axial-vector field A^μ_{ab},

$$A^\mu_{ab}=\frac{i}{2}(\xi^+\partial^\mu\xi-\xi\partial^\mu\xi^+)_{ab}, \tag{5}$$

one can define the covariant derivative

$$(D^\mu T)_a=\partial^\mu T_a-T_b(V^\mu)^b_a, \tag{6}$$

and

$$(D^\mu S_\nu)^{ab}=\partial^\mu S^{ab}_\nu+(V^\mu)^a_c S^{cb}_\nu+(V^\mu)^b_c S^{ac}_\nu. \tag{7}$$

In the limit where the light quarks have zero mass and the heavy quarks have infinite mass, the Lagrangian for the strong interactions of heavy baryons with pseudoscalar Goldstone bosons should be invariant under both chiral symmetry and heavy quark symmetry. It should also be invariant under Lorentz and parity transformations as required in general. The most general form for the Lagrangian satisfying these requirements is [12]

$$\begin{aligned}\mathcal{L}=&\,i\overline{T^a}v_\nu(D^\nu T)_a-i\overline{S^\mu_{ab}}v_\nu(D^\nu S_\mu)^{ab}+\Delta M\overline{S^\mu_{ab}}S^{ab}_\mu\\&+ig_1\epsilon_{\mu\nu\sigma\lambda}\overline{S^\mu_{ac}}v^\nu(A^\sigma)^a_b S^{\lambda bc}+g_2[\epsilon_{abc}\overline{T^a}(A^\mu)^b_d S^{cd}_\mu\\&+\epsilon^{abc}\overline{S^\mu_{cd}}(A_\mu)^d_b T_a],\end{aligned} \tag{8}$$

where g_1 and g_2 are coupling constants describing the interactions between heavy baryons and Goldstone bosons and ΔM is the mass difference between sextet and antitriplet heavy baryons in the heavy quark limit. As a consequence of heavy quark symmetry, g_1 and g_2 are universal for different heavy baryons. Since they contain information about the interactions at the quark and gluon level, they cannot be fixed from chiral perturbation theory, but should be determined by experiments.

In the limit $m_Q\to\infty$, the propagator for Λ_Q is

$$\frac{i}{v\cdot p}\frac{1+\not{v}}{2},$$

where p is the residual momentum of the heavy baryon. There is no mass difference between Σ_Q and Σ^*_Q when $m_Q\to\infty$. In HQET, the leading term which is responsible for a mass difference between Σ_Q and Σ^*_Q is the color-magnetic-moment operator, $(1/m_Q)\overline{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v$ (where h_v is the heavy quark field operator in HQET and $G^{\mu\nu}$ is the gluon field strength tensor). This term is singlet under $SU(3)_L\times SU(3)_R$ and leads to the following correction term to $\mathcal{L}$ in Eq. (8):

$$i\frac{\alpha}{m_Q}\overline{S^\mu_{ab}}\sigma_{\mu\nu}S^{\nu ab}, \tag{9}$$

where α is a constant which also contains interaction information at the quark and gluon level, and which is the same for Σ_Q and Σ^*_Q at the tree level because of heavy quark symmetry. When QCD loop corrections are included, α depends on m_Q logarithmically.

FIG. 1. Pion loop corrections to the propagator of $SU(3)$ sextet heavy baryons with spin-1/2, where $S^{(*)}_Q$ represent spin-1/2(3/2) $SU(3)$ sextet heavy baryons with heavy quark Q and T_Q represents $SU(3)$ antitriplet heavy baryons.

The term (9) enhances the mass of Σ_Q^* by α/m_Q and lowers that of Σ_Q by $2\alpha/m_Q$. Therefore, the propagators for Σ_Q and Σ_Q^* become

$$\frac{i}{v\cdot p-\Delta M+\dfrac{2\alpha}{m_Q}}\,\frac{1+\not{v}}{2},$$

and

$$\frac{1+\not{v}}{2}\,\frac{-i\left(g_{\mu\nu}-\dfrac{1}{3}\gamma_\mu\gamma_\nu-\dfrac{2}{3}v_\mu v_\nu\right)}{v\cdot p-\Delta M-\dfrac{\alpha}{m_Q}}\,\frac{1+\not{v}}{2},$$

respectively.

Substituting Eq. (3) into Eq. (8) we have the following explicit form for the interactions of heavy baryons with Goldstone bosons:

$$\begin{aligned}
&-\frac{i}{2f_\pi^2}\overline{T^a}[M,v\cdot\partial M]_a^b T_b+\frac{i}{2f_\pi^2}\{\overline{S}_{ab}[M,v\cdot\partial M]_c^a S^{cb}+\overline{S}_{ab}[M,v\cdot\partial M]_c^b S^{ac}-\overline{S}^{*\mu}_{ab}[M,v\cdot\partial M]_c^a S^{*cb}_\mu-\overline{S}^{*\mu}_{ab}[M,v\cdot\partial M]_c^b S^{*ac}_\mu\}\\
&\quad-\frac{i}{f_\pi}g_1\epsilon_{\mu\nu\sigma\lambda}v^\nu(\partial^\sigma M)_b^a\left[-\frac{1}{3}\overline{S}_{ac}\gamma^\mu\gamma^\lambda S^{bc}+\overline{S}^{*\mu}_{ac}S^{*\lambda bc}+\frac{1}{\sqrt{3}}\overline{S}^{*\mu}_{ac}\gamma^\lambda\gamma_5 S^{bc}-\frac{1}{\sqrt{3}}\overline{S}_{ac}\gamma_5\gamma^\mu S^{*\lambda bc}\right]\\
&\quad-\frac{g_2}{f_\pi}\left[\frac{1}{\sqrt{3}}\epsilon_{abc}\overline{T^a}(\partial^\mu M)_d^b(\gamma_\mu+v_\mu)\gamma_5 S^{cd}+\epsilon_{abc}\overline{T^a}(\partial^\mu M)_d^b S^{*cd}-\frac{1}{\sqrt{3}}\epsilon_{abc}\overline{S^{cd}}\gamma_5(\gamma^\mu+v^\mu)(\partial_\mu M)_d^b T^a\right.\\
&\quad\left.+\epsilon_{abc}\overline{S}^{*\mu cd}(\partial_\mu M)_d^b T^a\right],
\end{aligned}\tag{10}$$

where $O(M^3)$ terms are ignored.

Chiral symmetry can be broken explicitly by nonzero light quark masses. This leads to the following leading order terms in the explicit chiral symmetry breaking masses:

$$\begin{aligned}
&\lambda_1\overline{S}^\mu_{ab}(\xi m_q\xi+\xi^+m_q\xi^+)_c^b S^{ac}_\mu+\lambda_2\overline{S}^\mu_{ab}S^{ab}_\mu\mathrm{Tr}(m_q\Sigma^++\Sigma m_q)\\
&\quad+\lambda_3\mathrm{Tr}\,T_a(\xi m_q\xi+\xi^+m_q\xi^+)_b^a\overline{T}^b+\lambda_4\mathrm{Tr}(\overline{T}^a T_a)\\
&\quad\times\mathrm{Tr}(m_q\Sigma^++\Sigma m_q),
\end{aligned}\tag{11}$$

where $\lambda_i(i=1,2,3,4)$ are parameters which are also independent of the heavy quark mass in the limit $m_Q\to\infty$.

III. FORMALISM FOR THE EXTRAPOLATION OF LATTICE DATA FOR HEAVY BARYON MASSES

From the chiral Lagrangian for the interactions of heavy baryons with light Goldstone bosons, Eq. (10), we can calculate pion loop contributions to the heavy baryon propagators near the chiral limit—i.e., when the pion mass is not far from the chiral limit. This leads to a dependence of the heavy baryon masses on the pion mass. We will concentrate on Σ_Q and Σ_Q^*, but other heavy baryons can be treated in the same way.

From Eq. (10) we find four diagrams for pion loop corrections to the propagator of either Σ_Q or Σ_Q^*, and three diagrams for Λ_Q. These diagrams are shown in Fig. 1 for Σ_Q, in Fig. 2 for Σ_Q^*, and in Fig. 3 for Λ_Q. It can be easily seen that Fig. 1(a), Fig. 2(a) and Fig. 3(a) do not contribute [this is because the integrand is of the form $k^\mu f(k^2)$ where k is the momentum of the pion in the loop, and $f(k^2)$ is a function of k^2] and we will not consider them further.

Figure 1(b) arises from the $\Sigma_Q\pi\Sigma_Q$ vertex. In momentum space it can be expressed as

$$\frac{i}{v\cdot p-\Delta M+\dfrac{2\alpha}{m_Q}}(-i\Sigma_1)\frac{i}{v\cdot p-\Delta M+\dfrac{2\alpha}{m_Q}}\,\frac{1+\not{v}}{2},\tag{12}$$

where p is the residual momentum of the heavy baryon Σ_Q. From Eq. (10), Fig. 1(b) takes the following form:

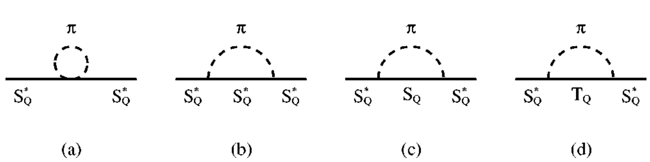

FIG. 2. Pion loop corrections to the propagator of $SU(3)$ sextet heavy baryons with spin-3/2. Same notation as in Fig. 1.

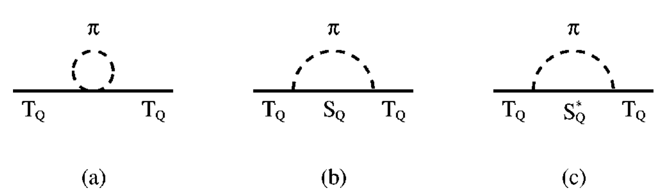

FIG. 3. Pion loop corrections to the propagator of $SU(3)$ antitriplet heavy baryons. Same notation as in Fig. 1.

$$-\frac{g_1^2}{18f_\pi^2}\epsilon_{\mu\nu\sigma\lambda}\epsilon_{\mu'\nu'\sigma'\lambda'}v^\nu v^{\nu'} \times\left(\frac{1+\not{v}}{2}\gamma^\mu\gamma^\lambda\frac{1+\not{v}}{2}\gamma^{\mu'}\gamma^{\lambda'}\frac{1+\not{v}}{2}\right) \times\left(\frac{i}{v\cdot p-\Delta M+\frac{2\alpha}{m_Q}}\right)^2 \times\int\frac{d^4k}{(2\pi)^4}\frac{k^\sigma k^{\sigma'}}{\left[v\cdot(p-k)-\Delta M+\frac{2\alpha}{m_Q}\right](k^2-m_\pi^2)}, \tag{13}$$

where again k is the momentum of the pion in the loop, and m_π is the pion mass.

As discussed in Ref. [10], the integral

$$X^{\mu\nu}\equiv\int\frac{d^4k}{(2\pi)^4}\frac{k^\mu k^\nu}{[v\cdot k-\delta](k^2-m_\pi^2)}, \tag{14}$$

where δ is some constant, can be written as

$$X^{\mu\nu}=X_1(\delta)g^{\mu\nu}+X_2(\delta)v^\mu v^\nu, \tag{15}$$

where X_1 and X_2 are Lorentz scalars, which are functions of δ. Obviously, only the X_1 term contributes in Eq. (13). In the evaluation of X_1, the integration over k_0 was made first by choosing the appropriate contour. Then a cutoff Λ, which characterizes the finite size of the source of the pion, was introduced in the three dimensional integration since pion loop contributions are suppressed when the Compton wavelength of the pion is smaller than the source of the pion. Since the leading nonanalytic contribution of these loops is associated with the infrared behavior of the integral, it does not depend on the details of the cutoff. In this way, $X_1(\delta)$ has the following expression [10]:

$$X_1(\delta)=\frac{i}{72\pi^2}\left\{12(m_\pi^2-\delta^2)^{3/2}\left[\arctan\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\delta}{\sqrt{m_\pi^2-\delta^2}}-\arctan\frac{m_\pi-\delta}{\sqrt{m_\pi^2-\delta^2}}\right]+3\delta(2\delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi} +3\delta\Lambda\sqrt{\Lambda^2+m_\pi^2}+6(\delta^2-m_\pi^2)\Lambda+2\Lambda^3\right\}, \tag{16}$$

when $m_\pi^2\geqslant\delta^2$;

$$X_1(\delta)=\frac{i}{72\pi^2}\left\{6(\delta^2-m_\pi^2)^{3/2}\ln\left(\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\delta-\sqrt{\delta^2-m_\pi^2}}{\Lambda+\sqrt{\Lambda^2+m_\pi^2}-\delta+\sqrt{\delta^2-m_\pi^2}}\left|\frac{m_\pi-\delta+\sqrt{\delta^2-m_\pi^2}}{m_\pi-\delta-\sqrt{\delta^2-m_\pi^2}}\right|\right)+3\delta(2\delta^2-3m_\pi^2)\ln\frac{\Lambda+\sqrt{\Lambda^2+m_\pi^2}}{m_\pi} +3\delta\Lambda\sqrt{\Lambda^2+m_\pi^2}+6(\delta^2-m_\pi^2)\Lambda+2\Lambda^3\right\}, \tag{17}$$

when $m_\pi^2\leqslant\delta^2$. In the case where $\delta=0$,

$$X_1=\frac{i}{36\pi^2}\left(3m_\pi^3\arctan\frac{\Lambda}{m_\pi}-3m_\pi^2\Lambda+\Lambda^3\right). \tag{18}$$

From Eqs. (12) and (13) we have

$$\Sigma_1=i\frac{2g_1^2}{3f_\pi^2}X_1(\Delta_1), \tag{19}$$

where $\Delta_1=v\cdot p-\Delta M+2\alpha/m_Q$.

Figures 1(c) and (d) have the same expression as in Eq. (12), except for Σ_1 being replaced by Σ_2 and Σ_3, respectively. In the same way, we have

$$\Sigma_2=i\frac{g_1^2}{3f_\pi^2}X_1(\Delta_2), \tag{20}$$

where $\Delta_2=v\cdot p-\Delta M-\alpha/m_Q$.

Figure 1(d) arises from the $\Sigma_Q\pi\Lambda_Q$ vertex. In this paper we only consider $\Sigma_b^{(*)}$ since lattice data are available for them. For $\Sigma_b^\pm$, $\pi^\pm$ appears in the pion loop, then we have

$$\Sigma_3^{\Sigma_b^\pm}=i\frac{g_2^2}{f_\pi^2}X_1(\Delta_3), \tag{21}$$

where $\Delta_3=v\cdot p$. For Σ_b^0, π^0 appears in the pion loop, and we have

$$\Sigma_3^{\Sigma_b^0}=i\frac{g_2^2}{2f_\pi^2}X_1(\Delta_3). \tag{22}$$

Defining Σ as the sum of Σ_1, Σ_2, and Σ_3, the propagator of Σ_b becomes

$$\frac{i}{v\cdot p-\Delta M+\dfrac{2\alpha}{m_Q}-\Sigma}\,\frac{1+\not v}{2}, \tag{23}$$

where

$$\Sigma^{\Sigma_b^{\pm}}=i\frac{2g_1^2}{3f_\pi^2}X_1(\Delta_1)+i\frac{g_1^2}{3f_\pi^2}X_1(\Delta_2)+i\frac{g_2^2}{f_\pi^2}X_1(\Delta_3), \tag{24}$$

and

$$\Sigma^{\Sigma_b^{0}}=i\frac{2g_1^2}{3f_\pi^2}X_1(\Delta_1)+i\frac{g_1^2}{3f_\pi^2}X_1(\Delta_2)+i\frac{g_2^2}{2f_\pi^2}X_1(\Delta_3). \tag{25}$$

Pion loop contributions to the propagator of Σ_b^* can be calculated in the same way. After Figs. 2(b), (c), (d) are included the propagator of Σ_b^* becomes

$$\frac{1+\not v}{2}\,\frac{-i\left(g_{\mu\nu}-\dfrac{1}{3}\gamma_\mu\gamma_\nu-\dfrac{2}{3}v_\mu v_\nu\right)}{v\cdot p-\Delta M-\dfrac{\alpha}{m_Q}-\Pi}\,\frac{1+\not v}{2}, \tag{26}$$

where

$$\Pi^{\Sigma_b^{*\pm}}=i\frac{5g_1^2}{6f_\pi^2}X_1(\Delta_2)-i\frac{g_1^2}{6f_\pi^2}X_1(\Delta_1)+i\frac{g_2^2}{f_\pi^2}X_1(\Delta_3), \tag{27}$$

and

$$\Pi^{\Sigma_b^{*0}}=i\frac{5g_1^2}{6f_\pi^2}X_1(\Delta_2)-i\frac{g_1^2}{6f_\pi^2}X_1(\Delta_1)+i\frac{g_2^2}{2f_\pi^2}X_1(\Delta_3). \tag{28}$$

Similarly, if we include Figs. 3(b) and (c) then the propagator of Λ_b becomes

$$\frac{i}{v\cdot p-K}\,\frac{1+\not v}{2}, \tag{29}$$

where

$$K=i\frac{3g_2^2}{f_\pi^2}X_1(\Delta_1)+i\frac{6g_2^2}{f_\pi^2}X_1(\Delta_2). \tag{30}$$

Consequently, the pion loop contribution to the mass of Σ_b, σ_{Σ_b}, has the following expression:

$$\sigma_{\Sigma_b^{\pm}}=i\frac{2g_1^2}{3f_\pi^2}X_1(0)+i\frac{g_1^2}{3f_\pi^2}X_1\left(-\frac{3\alpha}{m_b}\right)+i\frac{g_2^2}{f_\pi^2}X_1\left(\Delta M-\frac{2\alpha}{m_b}\right) \tag{31}$$

and

$$\sigma_{\Sigma_b^{0}}=i\frac{2g_1^2}{3f_\pi^2}X_1(0)+i\frac{g_1^2}{3f_\pi^2}X_1\left(-\frac{3\alpha}{m_b}\right)+i\frac{g_2^2}{2f_\pi^2}\times X_1\left(\Delta M-\frac{2\alpha}{m_b}\right). \tag{32}$$

For Σ_b^*, we have

$$\sigma_{\Sigma_b^{*\pm}}=i\frac{5g_1^2}{6f_\pi^2}X_1(0)-i\frac{g_1^2}{6f_\pi^2}X_1\left(\frac{3\alpha}{m_b}\right)+i\frac{g_2^2}{f_\pi^2}X_1\left(\Delta M+\frac{\alpha}{m_b}\right) \tag{33}$$

and

$$\sigma_{\Sigma_b^{*0}}=i\frac{5g_1^2}{6f_\pi^2}X_1(0)-i\frac{g_1^2}{6f_\pi^2}X_1\left(\frac{3\alpha}{m_b}\right)+i\frac{g_2^2}{2f_\pi^2}X_1\left(\Delta M+\frac{\alpha}{m_b}\right). \tag{34}$$

For Λ_b,

$$\sigma_{\Lambda_b}=i\frac{3g_2^2}{f_\pi^2}X_1\left(-\Delta M+\frac{2\alpha}{m_b}\right)+i\frac{6g_2^2}{f_\pi^2}X_1\left(-\Delta M-\frac{\alpha}{m_b}\right). \tag{35}$$

In Eqs. (31)–(35), X_1 is given by Eqs. (16)–(18).

In order to extrapolate the lattice data from large m_π to the physical value of the pion mass, we follow the arguments proposed in Ref. [10] where we dealt with heavy mesons. These arguments can be generalized to the case of heavy baryons straightforwardly. Equations (31)–(35) are valid when m_π is not far away from the chiral limit—i.e., when $m_\pi\leqslant\Lambda$. As pointed out in Refs. [6–10], pion loop contributions vanish in the limit $m_\pi\to\infty$, and the heavy baryon mass becomes proportional to m_π^2 when m_π becomes large (at least up to ~ 1 GeV^2). This behavior is consistent with lattice simulations. Following Refs. [6–10], we propose the following phenomenological, functional form for the extrapolation of lattice data for heavy baryons:

$$m_B=a_B+b_Bm_\pi^2+\sigma_B, \tag{36}$$

for $B=\Sigma_b$, Σ_b^* or Λ_b.

The advantage of fitting the lattice data in this way is that we can guarantee that our formalism has both the correct chiral limit behavior and the appropriate behavior when m_π is large, with only three parameters (a, b, and Λ) to be determined in the fit.

Chiral symmetry is explicitly broken by the terms in Eq. (11). Substituting Eqs. (1), (3) into Eq. (11) we have the following explicit expression:

$$2\lambda_1\sum_{a,b=1}^{3}[m_{q^a}(-\bar{S}_{ab}S^{ab}+\bar{S}^{*\mu}_{ab}S^{*ab}_{\mu})]+2\lambda_2\sum_{a=1}^{3}m_{q^a}\times\sum_{a,b=1}^{3}(-\bar{S}_{ab}S^{ab}+\bar{S}^{*\mu}_{ab}S^{*ab}_{\mu})+2\lambda_3\times\sum_{a=1}^{3}m_{q^a}\bar{T}_aT^a+2\lambda_4\sum_{a=1}^{3}m_{q^a}\sum_{a=1}^{3}\bar{T}_aT^a, \tag{37}$$

where we have made a Taylor expansion for ξ and omitted $O(1/f_\pi^2)$ terms. It can be seen that Eq. (37) does not contribute to the mass difference between Σ_Q and Σ_Q^* to order m_q. Corrections to this statement are of order $m_qO(1/f_\pi^2)$, with extra suppression from m_q with respect to the pion loop effects. They will therefore be ignored. Equation (37) may contribute to the mass different between $\Sigma_Q^{(*)}$ and Λ_Q. Such effects will be considered to be effectively included in the parameter ΔM in Eq. (8).

IV. EXTRAPOLATION OF LATTICE DATA FOR HEAVY BARYON MASSES

The masses of Σ_b, Σ_b^*, and Λ_b were calculated with the aid of NRQCD in quenched approximation in Ref. [2]. Since the mass of the heavy quark is much larger than $\Lambda_{\rm QCD}$, it becomes an irrelevant scale for the dynamics inside a heavy hadron and is removed from NRQCD. This makes it possible to simulate heavy baryons when the lattice spacing is larger than the Compton wavelength of the heavy quark. The lattice spacing used is $1/a=1.92$ GeV. For light quarks the tadpole-improved clover action was used which has discretization errors of order $\alpha_s a$. The value of β which is related to the bare gauge coupling is 6.0 and the lattice size is $16^3\times48$. In the simulations, three different values for the hopping parameter κ, 0.1369, 0.1375, and 0.13808, were used. The light quark mass is related to κ through the definition $m_q=(1/2a)(1/\kappa-1/\kappa_c)$, with $\kappa_c=0.13917$. These three hopping parameters correspond to three values of m_π^2: 0.6598 GeV^2, 0.4833 GeV^2, and 0.3141 GeV^2, respectively.

The heavy baryon masses were calculated for five different values of aM^0 (M^0 is the bare heavy quark mass): 1.6, 2.0, 2.7, 4.0, 7.0, and 10.0, where the data for the last two values are less reliable because of large discretization errors [2]. The best estimate for aM_b^0, 2.31, was obtained by matching the lattice data to the mass of the B meson. Consequently, in our fit we first extrapolate the lattice data for $aM^0=1.6$, 2.0, 2.7, 4.0 to $aM_b^0=2.31$. This can be done by linear extrapolation with respect to $1/M^0$ with the form $c+d/M^0$, where c and d are constants. This is because $aE_{\rm sim}$, which is the simulation mass in NRQCD and which is related to the heavy baryon mass, depends on $1/M^0$ linearly [note that in the case of b baryons, $O((1/M^0)^2)$ can be safely ignored]. Then from the data in Table XV of Ref. [2], we obtain the values of $aE_{\rm sim}$ for the three hopping parameters at $aM_b^0=2.31$, which are shown in Table I. In the following, we will extrapolate these values to the physical pion mass with the formulas in Eq. (36).

TABLE I. Extrapolated values of $aE_{\rm sim}$ at $aM_b^0=2.31$.

κ	$aE_{\rm sim}(\Lambda_b)$	$aE_{\rm sim}(\Sigma_b)$	$aE_{\rm sim}(\Sigma_b^*)$
0.13690	0.816(33)	0.877(28)	0.889(27)
0.13750	0.779(43)	0.845(32)	0.856(33)
0.13808	0.733(63)	0.818(40)	0.827(37)

In our fit we have to determine three parameters in our formalism [a_{Σ_b}, b_{Σ_b}, and Λ in Eq. (36), for example]. These parameters are related to ΔM, α, g_1, and g_2, which represent interactions at the quark and gluon level and cannot be determined from the chiral Lagrangian for heavy baryons. In our fit, we treat them as effective parameters and assume that their possible slight m_Q dependence, which results from QCD corrections and $1/m_Q$ corrections, has been taken into account effectively in this way.

ΔM is the mass difference between sextet and antitriplet heavy baryons. Since we do not have experimental data for the masses of $\Sigma_b^{(*)}$, we use the data for $\Sigma_c^{(*)}$ to determine ΔM [13]. The spin-averaged mass of $\Sigma_c^{(*)}$ is $\frac{1}{6}(2m_{\Sigma_c}+4m_{\Sigma_c^*})$, which is bigger than m_{Λ_c} by 0.213 GeV. In our fit, we let ΔM vary between 0.17 GeV and 0.23 GeV, which are given by $m_{\Sigma_c}-m_{\Lambda_c}$ and $m_{\Sigma_c^*}-m_{\Lambda_c}$, respectively. The mass difference $m_{\Sigma_c^*}-m_{\Sigma_c}$, which is equal to $3\alpha/m_c$ to order $1/m_c$, leads to $\alpha=0.032$ GeV^2 if we choose $m_c=0.15$ GeV. To see the dependence of our fit on α, we let it vary from 0.025 GeV^2 to 0.035 GeV^2.

The coupling constant g_2 can be determined from the decay width for $\Sigma_c^*\to\Lambda_c\pi$, which has the following explicit form:

$$\Gamma_{\Sigma_c^*\to\Lambda_c\pi}=\frac{g_2^2}{12\pi f_\pi^2}\times\left[\frac{(m_{\Sigma_c^*}^2-m_{\Lambda_c}^2)^2-2m_\pi^2(m_{\Sigma_c^*}^2+m_{\Lambda_c}^2)+m_\pi^4}{4m_{\Sigma_c^*}^2}\right]^{3/2}\times\frac{(m_{\Sigma_c^*}+m_{\Lambda_c})^2-m_\pi^2}{m_{\Sigma_c^*}^2}. \tag{38}$$

From $\Gamma_{\Sigma_c^{*++}\to\Lambda_c\pi^+}=18\pm5$ GeV, we have $g_2^2=0.559\pm0.155$, while from $\Gamma_{\Sigma_c^{*0}\to\Lambda_c\pi^-}=13\pm5$ GeV, we have $g_2^2=0.404\pm0.155$. Hence, in our fit we choose the range $0.249\leqslant g_2^2\leqslant0.714$.

Since Σ_c^* cannot decay to $\Sigma_c\pi$, we cannot fix g_1 from decays. However, g_1 can be related to the matrix of the axial-vector current between sextet heavy baryon states where a $u \rightarrow d$ transition is involved. By assuming $g_A^{ud} = 0.75$, which corresponds to $g_A^{\mathrm{nucleon}} = 1.25$ in neutron β decays and using spin-flavor wave functions for heavy baryons, the authors in Ref. [14] found that $g_1 = 0.38$. Based on this, we let g_1^2 vary from 0.1 to 0.2 in our fit.

As discussed in Sec. III, the parameter Λ characterizes the size of the source of the pion. In principle, the value of Λ can be determined by fitting the lattice data. However, since Λ is mainly related to the data at small pion masses and the current lattice data are only available at large pion masses, the error in the determination of Λ is very large. The size difference between Σ_b and Σ_b^* is caused by effects of order $1/m_b$ which are small. The size difference between Σ_b and Λ_b is caused by the difference between 0^+ and 1^+ light degrees of freedom, which is also the main reason for a size difference between N and Δ. It has been pointed out that the values of Λ for N and Δ are very close to each other [6]. Hence we expect that the difference between the values of Λ for Σ_b and Λ_b should also be small. Since the integrand in X_1 becomes small near the cutoff Λ, a small variation in Λ will only lead to an even smaller change in X_1. Based on these arguments, we will ignore the differences among the values of Λ for Σ_b, Σ_b^*, and Λ_b. To see the dependence of our analysis on Λ, we let Λ vary between 0.4 GeV and 0.6 GeV.

Whereas the formulas in Eq. (36) correspond to the infinite volume limit, in practice the lattice simulations are performed on a finite volume. As a result, the lowest nonzero pion momentum sampled in the lattice calculation is $2\pi/aL$ [L is the number of lattice sites in x (y, z) direction], which is almost 0.8 GeV for the data considered here. This effectively means that the entire low momentum region is not sampled by the lattice simulation and this in turn means that the chiral behavior is modified. This problem has already been addressed within the context of the nucleon and Δ masses in Refs. [6] and [15]. The idea is to replace the continuum self-energy integral in Eq. (36) by a sum over the discrete pion momenta allowed on the lattice. In the case of the N and Δ it was found that, because of the suppression of the self-energy terms for pion mass larger than 0.5 GeV (observed for *all* hadron properties), the effect of this correction *where the lattice data exists* was relatively small. Indeed, as a crude first approximation one could simply ignore it in that region. On the other hand, a better estimate would be obtained by replacing the infinite volume self-energy when fitting the data by a discrete sum:

$$\int d^3k \approx \frac{1}{V}\left(\frac{2\pi}{a}\right)^3 \sum_{k_x,k_y,k_z}, \tag{39}$$

where V is the spatial volume of the lattice and the discrete momenta k_x, k_y, k_z are given by

$$\frac{2\pi n}{aL}. \tag{40}$$

Here again L is the number of lattice sites in x (y, z) direction and the integer n satisfies the constraint

$$-\frac{L}{2} < n \leqslant \frac{L}{2}. \tag{41}$$

With $1/a = 1.92$ GeV and $L = 16$, the smallest momentum allowed on the lattice, $2\pi/aL$, equals 0.75 GeV which is bigger than the sharp cutoff Λ employed in our formalism. Consequently the discrete sum in Eq. (39) becomes zero if we take the finite lattice volume effects into account. One could, of course, choose a more complicated method of ultraviolet regularization such as a dipole. In that case the finite volume self-energy would not vanish exactly, but it would still be very strongly suppressed in the region where the lattice data exists (at large pion mass). As a result we believe that the results obtained with the θ function should be a good representation of the physics involved. In fact, Young, Leinweber, and Thomas gave a detailed discussion of the dependence of chiral extrapolation of the nucleon mass on different ultraviolet regularization schemes in Ref. [16]. They carefully analyzed four different functional forms for the finite-ranged, ultraviolet regulator including the sharp-cutoff, monopole, dipole, and Gaussian, finding that these four regulators produce model-independent chiral extrapolations agreeing at the level of 1% over a wide range of quark mass (up to $m_\pi^2 = 0.8$ GeV^2).

Using the three masses for Σ_b, Σ_b^*, and Λ_b in Table I, we fix the other two parameters besides Λ (a_{Σ_b} and b_{Σ_b} for Σ_b, for example) in Eq. (36) through a least squares fit. The values for the parameters obtained in this way are then used in Eq. (36) to obtain the results in the infinite volume limit, which should be compared with experiment at the physical pion mass.

Since the self-energy integral is zero when the finite lattice size effects are taken into account, the values for a_B and b_B are just those in the naive linear extrapolation. Further-

TABLE II. Fitted parameters, extrapolated masses of $\Sigma_b^{\pm}$, $\Sigma_b^{*\pm}$, and Λ_b and mass differences at m_π^{phys}. Numbers in brackets are errors caused by the errors in the lattice data.

	$\Sigma_b^{\pm}$	$\Sigma_b^{*\pm}$	Λ_b
a(GeV)	1.465(0.143)	1.479(0.187)	1.263(0.208)
b(GeV^{-1})	0.330(0.265)	0.346(0.326)	0.460(0.366)
m(GeV)	1.4506(0.1384)	1.4638(0.1803)	1.2239(0.2008)
$m_{\Sigma_b^{*\pm}} - m_{\Sigma_b^{\pm}}$(GeV)		0.0132(0.2272)	
$m_{\Sigma_b^{\pm}}^{\mathrm{ave}} - m_{\Lambda}$(GeV)		0.2355(0.2385)	
$(m_{\Sigma_b^{*\pm}} - m_{\Sigma_b^{\pm}})^*$(GeV)		0.0182(0.0025)	

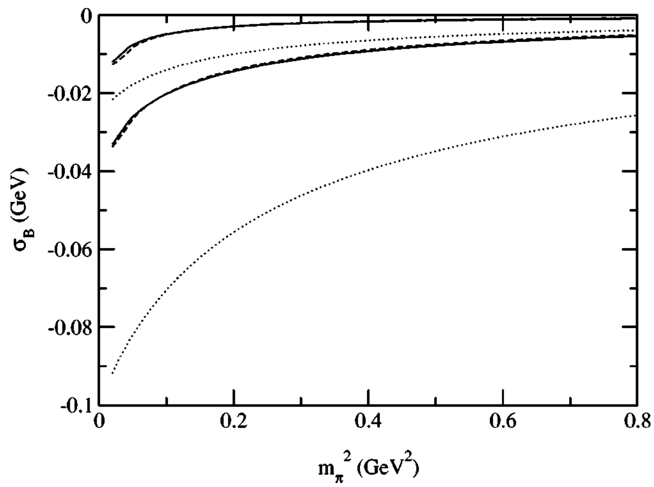

FIG. 4. Pion loop contributions to the masses of $\Sigma_b^\pm$, $\Sigma_b^{*\pm}$, and Λ_b as a function of the pion mass. The upper (lower) solid line is for $\Sigma_b^\pm$ with $\Lambda=0.4$ GeV (0.6 GeV). The upper (lower) dashed line is for $\Sigma_b^{*\pm}$ with $\Lambda=0.4$ GeV (0.6 GeV). The upper (lower) dotted line is for Λ_b with $\Lambda=0.4$ GeV (0.6 GeV).

more, these two parameters are the same for $\Sigma_b^{(*)\pm}$ and $\Sigma_b^{(*)0}$. The values for a_B and b_B are shown in Table II, where we choose $\Lambda=0.5$ GeV, $\alpha=0.032$ GeV^2, $\Delta M=0.213$ GeV, $g_1^2=0.15$, and $g_2^2=0.48$. The extrapolated masses for $\Sigma_b^\pm$, $\Sigma_b^{*\pm}$, and Λ_b at the physical pion mass are also shown in this table. The spin-averaged mass $m_{\Sigma_b}^{\text{ave}}$ is defined as $\frac{1}{6}(2m_{\Sigma_b}+4m_{\Sigma_b^*})$.

With the parameters in Table II we obtain the masses of $\Sigma_b^\pm$, $\Sigma_b^{*\pm}$, and Λ_b as a function of the pion mass. They are the result of linear extrapolations modified by the pion loop contributions. These pion loop contributions are shown in Fig. 4 for $\Lambda=0.4$ GeV and 0.6 GeV, respectively. In Fig. 5,

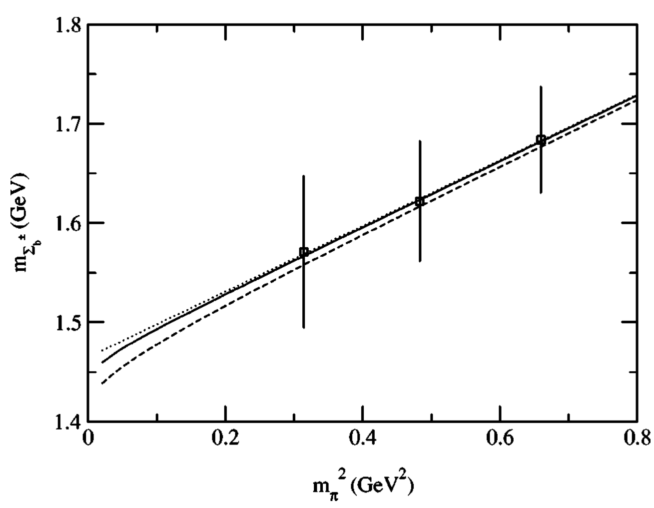

FIG. 5. Phenomenological fits to the lattice data for the masses of $\Sigma_b^\pm$ as a function of the pion mass. The solid (dashed) line corresponds to $\Lambda=0.4(0.6)$ GeV with finite lattice volume effects being taken into account and the dotted line represents the fit using a linear extrapolation.

TABLE III. Extrapolated values of ΔE, the mass difference between Σ_b^* and Σ_b, at $aM_b^0=2.31$ using Eq. (42).

κ	0.13690	0.13750	0.13808
$a\Delta E$	0.0093(5)	0.0095(7)	0.0096(7)
ΔE	0.0179(10)	0.0182(13)	0.0184(14)

we take $\Sigma_b^\pm$ as an example to show the dependence of the heavy baryon mass on the pion mass.

It can be seen from Table II that the extrapolated mass difference between $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ has a very large error. This is caused by taking the difference between two masses calculated in lattice QCD which have a significant error. A better way to obtain the mass difference between Σ_b and Σ_b^* is to extrapolate the lattice data for this mass difference itself, which were obtained from ratio fits—since these data have much smaller errors. The mass difference between Σ_b and Σ_b^*, ΔE, was also given in Ref. [2] for five different values of aM^0. We use the data at $aM^0=1.6$, 2.0, 2.7, and 4.0 to obtain the value of ΔE at $aM^0=2.31$ with the formula

$$a\Delta E=\frac{e}{aM^0}, \tag{42}$$

where e is a constant. Equation (42) is motivated by the idea that the mass splitting between Σ_b and Σ_b^* is caused primarily by effects of order $1/m_Q$. With the least squares fitting method we obtain results for ΔE at $aM^0=2.31$, for different values of κ. These are shown in Table III.

In order to extrapolate the values in Table III to the physical mass of the pion, we use the following formula:

$$m_{\Sigma_b^*}-m_{\Sigma_b}=\bar{a}+\bar{b}m_\pi^2+\sigma_{\Sigma_b^*}-\sigma_{\Sigma_b}. \tag{43}$$

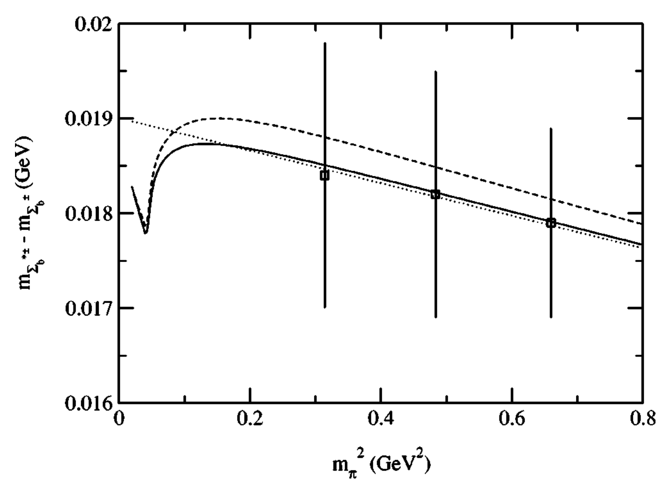

FIG. 6. Phenomenological fits to the lattice data for the mass difference between $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ as a function of the pion mass (same notation as in Fig. 5).

TABLE IV. Uncertainties for the extrapolated quantities for $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ which are caused by the uncertainties associated with parameters in the fitting function.

Quantities	α	ΔM	g_1^2	g_2^2	Λ
$m_{\Sigma_b^{\pm}}$	0.0%	0.3%	0.1%	1.2%	1.4%
$m_{\Sigma_b^{*\pm}}$	0.0%	0.3%	0.0%	1.3%	1.4%
m_{Λ_b}	0.0%	0.4%	0.0%	3.7%	5.6%
$m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}}$	3.9%	0.3%	4.2%	11.4%	0.1%
$m_{\Sigma_b^{\pm}}^{\mathrm{ave}}-m_{\Lambda_b}$	0.0%	4.1%	0.6%	11.2%	18.4%
$(m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}})^*$	2.8%	0.3%	3.0%	8.4%	0.1%

With $\Lambda=0.5$ GeV, $\alpha=0.032$ GeV^2, $\Delta M=0.213$ GeV, $g_1^2=0.15$, and $g_2^2=0.48$, we obtain $\overline{a}=0.0190(26)$, $\overline{b}=-0.00172(470)$, and the extrapolated mass difference between $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ $m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}}=0.0182(25)$, which is listed in Table II as $(m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}})^*$. In Fig. 6, we show $m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}}$ obtained in this way as a function of the pion mass. From Table II we can see that the result for $m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}}$ which is obtained from Eq. (36) is consistent with the extrapolation based directly on the lattice data for the mass difference between Σ_b and Σ_b^*, because of its large error.

In addition to the uncertainties which are caused by the errors in the lattice data, the fitted results can also vary a little in the range of the parameters α, ΔM, g_1^2, g_2^2, and Λ. In Table IV we list these uncertainties.

In the naive linear extrapolations pion loop corrections are ignored. Hence the results do not depend on the parameters α, ΔM, g_1^2, g_2^2, and Λ. In Table V we list the results of linear extrapolations for comparison. We note that there is no difference between the results for $\Sigma_b^{(*)\pm}$ and $\Sigma_b^{(*)0}$ in the linear extrapolations.

Comparing the uncertainties listed in Table II and Table IV we can see clearly that the main uncertainties in our fit are caused by the errors in the lattice data. In fact, the errors of lattice data for heavy baryons are much larger than those for heavy mesons [3]. Indeed, the uncertainties in the extrapolated heavy baryon masses are about one order larger than those in the case of heavy mesons. However, because of the small errors in the lattice data for the mass splitting between $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$, in this case the extrapolated mass difference at the physical pion mass also has a smaller error, about 27%.

From Figs. 4–7 we see that when the pion mass is smaller than about 500 MeV the extrapolations begin to deviate from linear behavior. This is because the pion loop corrections begin to affect the extrapolations around this point. As the pion mass becomes smaller and smaller, pion loop corrections become more and more important. For the masses of $\Sigma_b^{\pm}$, $\Sigma_b^{*\pm}$, Λ_b, and the mass difference between $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ the extrapolated values are smaller than those obtained by linear extrapolation. For the difference between the spin-averaged mass of $\Sigma_b^{(*)\pm}$ and the mass of Λ_b, the extrapolated value is larger than that obtained by linear extrapolation. We have checked that this behavior is independent of the uncertainties in the parameters in our model.

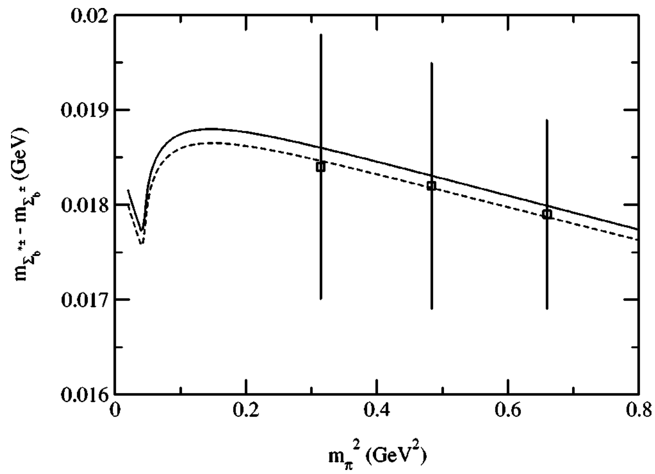

FIG. 7. Comparison between $m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}}$ obtained from Eq. (43) with (solid line) and without (dashed line) finite lattice volume effects being taken into account for $\Lambda=0.5$ GeV.

Comparing the results in the naive linear extrapolations with those with pion loop corrections being included we find that the splitting between $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ is only about 4% smaller if pion loop effects are taken into account, while the hyperfine splitting in the case of B mesons is about 20% when pion loop effects are taken into account [10]. Hence when we extrapolate hyperfine splittings, the linear extrapolation is a better approximation in the case of heavy baryons than in the case of heavy mesons.

For Σ_b^0 and Σ_b^{*0} we should use Eqs. (32), (34) in the extrapolation of lattice data. Repeating the same procedure as that for $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ we find that, apart from some minor changes in numerical results, the quantitative results remain essentially the same. In Tables VI and VII we list our numerical results for Σ_b^0 and Σ_b^{*0}. Comparing the results in Table VI with those in Tables II and V we can see that the naive linear extrapolations work even better for the extrapo-

TABLE V. Extrapolated masses of Σ_b, Σ_b^*, and Λ_b and mass differences at m_π^{phys} for linear extrapolations. Numbers in brackets are the errors caused by the errors in the lattice data.

	Σ_b	Σ_b^*	Λ_b
m(GeV)	1.4714(0.1384)	1.4854(0.1803)	1.2724(0.2008)
$m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}}$(GeV)		0.0140(0.2272)	
$m_{\Sigma_b^{\pm}}^{\mathrm{ave}}-m_{\Lambda}$(GeV)		0.2084(0.2385)	
$(m_{\Sigma_b^{*\pm}}-m_{\Sigma_b^{\pm}})^*$(GeV)		0.0190(0.0025)	

TABLE VI. Extrapolated masses of Σ_b^0 and Σ_b^{*0} and mass differences evaluated at $m_\pi^{\rm phys}$. Numbers in brackets are errors caused by the errors in the lattice data.

	Σ_b^0	Σ_b^{*0}
m(GeV)	1.4598(0.1384)	1.4738(0.1803)
$m_{\Sigma_b^{*0}}-m_{\Sigma_b^0}$(GeV)	0.0140(0.2272)	
$m_{\Sigma_b^0}^{\rm ave}-m_\Lambda$(GeV)	0.2452(0.2385)	
$(m_{\Sigma_b^{*0}}-m_{\Sigma_b^0})^*$(GeV)	0.0190(0.0025)	

TABLE VII. Uncertainties for the extrapolated quantities for Σ_b^0 and Σ_b^{*0} which are caused by the uncertainties in the parameters of the fitting function.

Quantities	α	ΔM	g_1^2	g_2^2	Λ
$m_{\Sigma_b^0}$	0.0%	0.2%	0.1%	0.6%	0.8%
$m_{\Sigma_b^{*0}}$	0.0%	0.2%	0.0%	0.6%	0.8%
m_{Λ_b}	0.0%	0.4%	0.0%	3.7%	5.6%
$m_{\Sigma_b^{*0}}-m_{\Sigma_b^0}$	2.1%	0.0%	4.2%	5.6%	4.2%
$m_{\Sigma_b^0}^{\rm ave}-m_{\Lambda_b}$	0.0%	3.0%	0.5%	14.1%	20.7%
$(m_{\Sigma_b^{*0}}-m_{\Sigma_b^0})^*$	1.6%	0.0%	3.1%	4.1%	3.1%

lation of hyperfine splittings in the case of Σ_b^0 and Σ_b^{*0} than in the case of $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$.

In order to see the effects of finite lattice volume, we also made a fit to the lattice data with Eqs. (36), (43) directly, without replacing the continuum self-energy integration by the discrete sum over the pion momenta allowed on the lattice. This leads to a few percent changes in the quantities we considered. For example, with $\Lambda=0.5$ GeV, $\alpha=0.032$ GeV^2, $\Delta M=0.213$ GeV, $g_1^2=0.15$, and $g_2^2=0.48$, we find that the finite volume effect is about 1% for the extrapolated mass difference between $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ obtained from Eq. (43). (This is completely consistent with the general finding that pion loop contributions to all hadron properties are highly suppressed for m_π exceeding 0.4–0.5 GeV [17].) The results are shown in Fig. 7. For other quantities, the finite volume effect is about 0.5% for the masses of $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$, 2% for the mass of Λ_b, 3% for the mass difference between $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ obtained from Eq. (36), and 8% for the difference between the averaged mass of $\Sigma_b^{*\pm}$ and the mass of Λ_b.

V. SUMMARY AND DISCUSSION

The masses of heavy baryons Σ_b, Σ_b^*, Λ_b, and the mass difference between Σ_b and Σ_b^* have been calculated numerically in lattice QCD with unphysical pion masses which are larger than about 560 MeV. In order to extrapolate these data to the physical mass of the pion in a consistent way, we included pion loop effects on the heavy baryon masses by applying the effective chiral Lagrangian for heavy baryons when the pion mass is smaller than the inverse radii of heavy baryons. This chiral Lagrangian is invariant under both chiral symmetry (when the light quark masses go to zero) and heavy quark symmetry (when the heavy quark masses go to infinity). In order to study mass difference between Σ_b and Σ_b^*, we took the color-magnetic-moment operator at order $1/m_Q$ in HQET into account since this operator is the leading one to cause splitting between Σ_b and Σ_b^*. When m_π becomes large, lattice data show that heavy baryon masses depend on m_π^2 linearly in the range of interest. Based on these considerations, we proposed a phenomenological functional form to extrapolate the lattice data.

The advantage of our formalism is that it has the correct chiral limit behavior as well as the appropriate behavior when m_π is large and that there are only three parameters to be determined in the fit to lattice data. Since lattice simulations are performed on the finite lattice grid, we fit the lattice data by replacing the continuum integration in the self-energy with the sum over the allowed discrete momenta of the pion in the loop. We find that the finite lattice volume effects are at most a few percent for the quantities we studied. It is found that when the pion mass is smaller than about 500 MeV the extrapolations begin to deviate from the naive linear extrapolations. For the hyperfine splitting between $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$, the differences between the extrapolations with and without pion loop effects being included is smaller than those in the case of B mesons. Hence for hyperfine splittings, the linear extrapolation is a better approximation in the case of heavy baryons. We carefully analyzed uncertainties in our extrapolations which are caused by both lattice data errors and uncertainties in several parameters in our model and found that the main uncertainties are caused by the errors of the current lattice data. The uncertainties associated with the parameters in our model are mostly a few percent and do not exceed 20%. By directly extrapolating the lattice data for $m_{\Sigma_b^*}-m_{\Sigma_b}$, which has much smaller errors, we found that the extrapolated mass difference between $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ at the physical mass of the pion is 18.2 MeV, with an uncertainty of 27% caused by lattice data errors. For Σ_b^0 and Σ_b^{*0} this difference is 19.0 MeV with 26% uncertainty from lattice data errors. For the mass difference between $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ obtained from Eq. (36), and the difference between the spin-averaged mass of $\Sigma_b^{(*)\pm}$ and the mass of Λ_b, the extrapolated values have very large errors. These need to be improved when the lattice data become more accurate. Furthermore, we should bear in mind that our extrapolations are based on the lattice data in the quenched approximation. From our experience in the cases of light and heavy mesons [18,10], the quenched approximation may affect the mass splitting between Σ_b and Σ_b^*. In addition, the lattice results for $m_{\Sigma_b^*}-m_{\Sigma_b}$ may be sensitive to both the coefficient of the $\sigma\cdot\mathbf{B}$ term in NRQCD [3] and the clover coefficient in the clover action for light quarks. This may also influence the lattice data and consequently affect our extrapolations.

ACKNOWLEDGMENT

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