Direct CP violation in charmed hadron decays via ρ-ω mixing

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(Received 12 July 1999; published 10 May 2000)

We study the possibility of obtaining large direct CP violation in the charmed hadron decays $D^+ \to \rho^+\rho^0(\omega) \to \rho^+\pi^+\pi^-$, $D^+ \to \pi^+\rho^0(\omega) \to \pi^+\pi^+\pi^-$, $D^0 \to \phi\rho^0(\omega) \to \phi\pi^+\pi^-$, $D^0 \to \eta\rho^0(\omega) \to \eta\pi^+\pi^-$, $D^0 \to \eta'\rho^0(\omega) \to \eta'\pi^+\pi^-$, $D^0 \to \pi^0\rho^0(\omega) \to \pi^0\pi^+\pi^-$, and $\Lambda_c \to p\rho^0(\omega) \to p\pi^+\pi^-$ via ρ-ω mixing. The analysis is carried out in the factorization approach. The CP violation parameter depends on the effective parameter N_c which is relevant to the hadronization dynamics of each decay channel and should be determined by experiment. It is found that for fixed N_c the CP violation parameter reaches its maximum value when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. For most of the parameter space explored the CP violating asymmetry is of order 10^{-4}. However, over a small range, $1.98 \leqslant N_c \leqslant 1.99$ and $1.95 \leqslant N_c \leqslant 2.02$, the asymmetries for $D^0 \to \pi^0\rho^0(\omega) \to \pi^0\pi^+\pi^-$ and $\Lambda_c \to p\rho^0(\omega) \to p\pi^+\pi^-$ (respectively) can exceed 1%, at the cost of a small branching ratio. We also estimate the decay branching ratios for $D^0 \to \pi^0\rho^0$ and $\Lambda_c \to p\rho^0$ for these values of N_c, which should be tested by future experimental data.

PACS number(s): 11.30.Er, 12.39.−x, 13.20.Fc, 14.20.Lq

I. INTRODUCTION

Although CP violation has been known in the neutral kaon system for more than three decades its dynamical origin still remains an open problem. In addition to the kaon system, the study of CP violation in heavy quark systems has been a subject of intense interest and is important in understanding whether the standard model provides a correct description of this phenomenon through the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Actually there have been many theoretical studies in the area of CP violation in b-flavored and charm systems and some experimental projects have been proposed [1].

Recent studies of direct CP violation in the B meson system [2] have suggested that large CP-violating asymmetries should be observed in forthcoming experiments. However, in the charm sector, CP violation is usually predicted to be small. A rough estimate of CP violation in charmed systems gives an asymmetry parameter which is typically smaller than 10^{-3} due to the suppression of the CKM matrix elements [3]. By introducing large final-state-interaction phases provided by nearby resonances, Buccella *et al.* predicted larger CP violation, namely, a few times 10^{-3} [4]. On the other hand, experimental measurements in some decay channels are consistent with zero asymmetry [5].

Direct CP violation occurs through the interference of two amplitudes with different weak and strong phases. The weak phase difference is determined by the CKM matrix elements and the strong phase is usually very uncertain. In Refs. [6,7], the authors studied direct CP violation in hadronic B decays through the interference of tree and penguin diagrams, where ρ-ω mixing was used to obtain a large strong phase (as required for large CP violation). This mechanism was also applied to the hadronic decays of the heavy baryon, Λ_b, where even larger CP violation may be possible [8]. In the present paper we will investigate direct CP violation in the hadronic decays of charmed hadrons, involving the same mechanism, with the aim of finding channels which may exhibit large CP asymmetry.

Since we are considering direct CP violation, we have to consider hadronic matrix elements for both tree and penguin diagrams which are controlled by the effects of nonperturbative QCD and hence are uncertain. In our discussions we will use the factorization approximation so that one of the currents in the nonleptonic decay Hamiltonian is factorized out and generates a meson. Thus the decay amplitude of the two body nonleptonic decay becomes the product of two matrix elements, one related to the decay constant of the factorized meson and the other to the weak transition matrix element between two hadrons. There have been some discussions of the plausibility of factorization [9,10], and this approach may be a good approximation in energetic decays. In some recent work corrections to the factorization approximation have also been considered by introducing some phenomenological nonfactorizable parameters which depend on the specific decay channels and should be determined by experimental data [11–14].

The effective Hamiltonian for the $\Delta S = 1$, weak, nonleptonic decays has been discussed in detail in Refs. [15,16], where the Wilson coefficients for the tree and penguin operators were obtained to the next-to-leading order QCD and QED corrections by calculating the 10×10, two-loop,

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anomalous dimension matrix. The dependence of the Wilson coefficients on renormalization scheme, gauge and infra-red cutoff was also discussed. The formalism can be extended to the charmed hadron nonleptonic decays in a straightforward way.

The remainder of this paper is organized as follows. In Sec. II we calculate the six Wilson coefficients of tree and QCD penguin operators to the next-to-leading order QCD corrections by applying the results of Refs. [15,16]. Then in Sec. III we give the formalism for the CP-violating asymmetry in charmed hadron nonleptonic decays and show numerical results. Finally, Sec. IV is reserved for a summary and some discussion.

II. THE EFFECTIVE HAMILTONIAN FOR NONLEPTONIC CHARMED HADRON DECAYS

In order to calculate direct CP violation in nonleptonic, charmed hadron decays we use the following effective weak Hamiltonian, which is Cabibbo first-forbidden, based on the operator product expansion:

$$\mathcal{H}_{\Delta C=1}=\frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s}V_{uq}V_{cq}^{*}(c_1O_1^q+c_2O_2^q)-V_{ub}V_{cb}^{*}\sum_{i=3}^{6}c_iO_i\right]+\text{H.c.} \tag{1}$$

Here $c_i(i=1,\ldots,6)$ are the Wilson coefficients and the operators O_i have the following expressions:

$$\begin{aligned}
O_1^q&=\bar{u}_\alpha\gamma_\mu(1-\gamma_5)q_\beta\bar{q}_\beta\gamma^\mu(1-\gamma_5)c_\alpha,\\
O_2^q&=\bar{u}\gamma_\mu(1-\gamma_5)q\bar{q}\gamma^\mu(1-\gamma_5)c,\\
O_3&=\bar{u}\gamma_\mu(1-\gamma_5)c\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_4&=\bar{u}_\alpha\gamma_\mu(1-\gamma_5)c_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,\\
O_5&=\bar{u}\gamma_\mu(1-\gamma_5)c\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_6&=\bar{u}_\alpha\gamma_\mu(1-\gamma_5)c_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,
\end{aligned} \tag{2}$$

where α and β are color indices, and $q'=u,\ d,\ s$. In Eq. (2) O_1 and O_2 are the tree operators, while O_3-O_6 are QCD penguin operators. In the Hamiltonian we have omitted the operators associated with electroweak penguin diagrams.

The Wilson coefficients $c_i(i=1,\ldots,6)$, are calculable in perturbation theory by using the renormalization group. The solution has the following form:

$$\mathbf{C}(\mu)=U(\mu,m_W)\mathbf{C}(m_W), \tag{3}$$

where $U(\mu,m_W)$ describes the QCD evolution which sums the logarithms $[\alpha_s\ln(m_W^2/\mu^2)]^n$ (leading-log approximation) and $\alpha_s[\alpha_s\ln(m_W^2/\mu^2)]^n$ (next-to-leading order). In Refs. [15,16] it was shown that $U(m_1,m_2)$ can be written as

$$U(m_1,m_2)=\left(1+\frac{\alpha_s(m_1)}{4\pi}J\right)U^0(m_1,m_2)\left(1-\frac{\alpha_s(m_2)}{4\pi}J\right), \tag{4}$$

where $U^0(m_1,m_2)$ is the evolution matrix in the leading-log approximation and the matrix J summarizes the next-to-leading order corrections to this evolution.

The evolution matrices $U^0(m_1,m_2)$ and J can be obtained by calculating the appropriate one- and two-loop diagrams, respectively. The initial conditions $\mathbf{C}(m_W)$ are determined by matching the full theory and the effective theory at the scale m_W. At the scale m_c the Wilson coefficients are given by

$$\mathbf{C}(m_c)=U_4(m_c,m_b)M(m_b)U_5(m_b,m_W)\mathbf{C}(m_W), \tag{5}$$

where $U_f(m_1,m_2)$ is the evolution matrix from m_2 to m_1 with f active flavors and $M(m_b)$ is the quark-threshold matching matrix at m_b. Since the strong interaction is independent of quark flavors, the matrices $U_4(m_c,m_b)$, $U_5(m_b,m_W)$, and $M(m_b)$ are the same as those in b decays. Hence, using the expressions for $U^0(m_1,m_2)$, J and $M(m_b)$ given in Refs. [15,16], we can obtain $\mathbf{C}(m_c)$.

In general, the Wilson coefficients depend on the renormalization scheme. The scheme-independent Wilson coefficients $\overline{\mathbf{C}}(\mu)$ are introduced by the following equation:

$$\overline{\mathbf{C}}(\mu)=\left(1+\frac{\alpha_s}{4\pi}R^T\right)\mathbf{C}(\mu), \tag{6}$$

where R is the renormalization matrix associated with the four-quark operators $O_i(i=1,\ldots,6)$ in Eq. (2), at the scale m_W. The scheme-independent Wilson coefficients have been used in the literature [4,17,18]. However, since R depends on the infrared regulator [15], $\overline{\mathbf{C}}(\mu)$ also carries such a dependence. In the present paper we have chosen to use the scheme-independent Wilson coefficients.

From Eqs. (5),(6) and the expressions for the matrices $U_f(m_1,m_2)$, $M(m_b)$, and R in Refs. [15,16] we obtain the following scheme-independent Wilson coefficients for c decays at the scale $m_c=1.35$ GeV:

$$\begin{aligned}
&\bar{c}_1=-0.6941, \quad \bar{c}_2=1.3777, \quad \bar{c}_3=0.0652,\\
&\bar{c}_4=-0.0627, \quad \bar{c}_5=0.0206, \quad \bar{c}_6=-0.1355.
\end{aligned} \tag{7}$$

In obtaining Eq. (7) we have taken $\alpha_s(m_Z)=0.118$ which leads to $\Lambda_{\rm QCD}^{(5)}=0.226$ GeV and $\Lambda_{\rm QCD}^{(4)}=0.329$ GeV. To be consistent, the matrix elements of the operators O_i should also be renormalized to the one-loop order since we are working to the next-to-leading order for the Wilson coefficients. This results in effective Wilson coefficients, c_i', which satisfy the constraint

$$c_i(m_c)\langle O_i(m_c)\rangle=c_i'\langle O_i\rangle^{\rm tree}, \tag{8}$$

where $\langle O_i(m_c)\rangle$ are the matrix elements, renormalized to the one-loop order. The relations between c_i' and c_i read [17,18]

$$\begin{aligned} &c_1'=\overline{c_1}, \quad c_2'=\overline{c_2}, \quad c_3'=\overline{c_3}-P_s/3, \\ &c_4'=\overline{c_4}+P_s, \quad c_5'=\overline{c_5}-P_s/3, \quad c_6'=\overline{c_6}+P_s, \end{aligned} \tag{9}$$

where

$$P_s=[\alpha_s(m_c)/8\pi][10/9+G(m,m_c,q^2)]\overline{c_2},$$

with

$$G(m,m_c,q^2)=4\int_0^1 \mathrm{d}x x(1-x)\ln\frac{m^2-x(1-x)q^2}{m_c^2}.$$

Here q^2 is the momentum transfer of the gluon in the penguin diagram and m is the mass of the quark in the loop of the penguin diagram.[1] $G(m,m_c,q^2)$ has the following explicit expression [19]:

$$\mathrm{Re}\,G=\frac{2}{3}\left[\ln\frac{m^2}{m_c^2}-\frac{5}{3}-4\frac{m^2}{q^2}+\left(1+2\frac{m^2}{q^2}\right)\right.$$

$$\left.\times\sqrt{1-4\frac{m^2}{q^2}}\ln\frac{1+\sqrt{1-4\frac{m^2}{q^2}}}{1-\sqrt{1-4\frac{m^2}{q^2}}}\right],$$

$$\mathrm{Im}\,G=-\frac{2}{3}\pi\left(1+2\frac{m^2}{q^2}\right)\sqrt{1-4\frac{m^2}{q^2}}. \tag{10}$$

Based on simple arguments at the quark level, the value of q^2 is chosen in the range $0.3<q^2/m_c^2<0.5$ [6,7]. From Eqs. (7), (9), and (10) we can obtain numerical values of c_i'. When $q^2/m_c^2=0.3$,

$$\begin{aligned} &c_1'=-0.6941, \quad c_2'=1.3777, \\ &c_3'=0.07226+0.01472i, \quad c_4'=-0.08388-0.04417i, \\ &c_5'=0.02766+0.01472i, \quad c_6'=-0.1567-0.04417i, \end{aligned} \tag{11}$$

and when $q^2/m_c^2=0.5$,

$$\begin{aligned} &c_1'=-0.6941, \quad c_2'=1.3777, \\ &c_3'=0.06926+0.01483i, \quad c_4'=-0.07488-0.04448i, \\ &c_5'=0.02466+0.01483i, \quad c_6'=-0.1477-0.04448i. \end{aligned} \tag{12}$$

In calculating the matrix elements of the Hamiltonian (1), we can then simply use the effective Wilson coefficients in Eqs. (11)(12) to multiply the tree-level matrix elements of the operators $O_i(i=1,\ldots,6)$.

III. CP VIOLATION IN CHARMED HADRON DECAYS

A. Formalism for CP violation in charmed hadron decays

The formalism for CP violation in B and Λ_b hadronic decays [6–8] can be generalized to the case of charmed hadrons in a straightforward manner. Let H_c denote a charmed hadron which could be $D^{\pm}$, D^0, or Λ_c. The amplitude A for the decay $H_c\to f\pi^+\pi^-$ (f is a decay product) is

$$A=\langle\pi^+\pi^- f|\mathcal{H}^{\mathrm{T}}|H_c\rangle+\langle\pi^+\pi^- f|\mathcal{H}^{\mathrm{P}}|H_c\rangle, \tag{13}$$

where $\mathcal{H}^{\mathrm{T}}$ and $\mathcal{H}^{\mathrm{P}}$ are the Hamiltonians for the tree and penguin operators, respectively.

The relative magnitude and phases of these two diagrams are defined as follows:

$$\begin{aligned} &A=\langle\pi^+\pi^- f|\mathcal{H}^{\mathrm{T}}|H_c\rangle[1+re^{i\delta}e^{i\phi}], \\ &\overline{A}=\langle\pi^+\pi^-\overline{f}|\mathcal{H}^{\mathrm{T}}|\overline{H_c}\rangle[1+re^{i\delta}e^{-i\phi}], \end{aligned} \tag{14}$$

where δ and ϕ are strong and weak phases, respectively. ϕ arises from the CP-violating phase in the CKM matrix, and it is $\arg[V_{ub}V_{cb}^*/(V_{uq}V_{cq}^*)]$ for the $c\to q$ transition ($q=d$ or s). The parameter r is defined as

$$r\equiv\left|\frac{\langle\pi^+\pi^- f|\mathcal{H}^{\mathrm{P}}|H_c\rangle}{\langle\pi^+\pi^- f|\mathcal{H}^{\mathrm{T}}|H_c\rangle}\right|. \tag{15}$$

The CP-violating asymmetry, a, can be written as

$$a\equiv\frac{|A|^2-|\overline{A}|^2}{|A|^2+|\overline{A}|^2}=\frac{-2r\sin\delta\sin\phi}{1+2r\cos\delta\cos\phi+r^2}. \tag{16}$$

It can be seen from Eq. (16) that both weak and strong phases are needed to produce CP violation. Since in r there is strong suppression from the ratio of the CKM matrix elements, $[V_{ub}V_{cb}^*/(V_{uq}V_{cq}^*)]$, which is of the order 10^{-3} [3] [for both $q=d$ and $q=s$ this suppression is 0.62×10^{-3}, see Eqs. (32) and (43) in Sec. III B], usually the CP violation in charmed hadron decays is predicted to be small.

The weak phase ϕ for a specific physical process is fixed. In order to obtain possible large CP violation, we need some mechanism to produce either large $\sin\delta$ or large r. ρ-ω mixing has the dual advantages that the strong phase difference is large (passing through 90° at the ω resonance) and well known. In this scenario one has [7,8]

$$\langle\pi^+\pi^- f|\mathcal{H}^{\mathrm{T}}|H_c\rangle=\frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}t_\omega+\frac{g_\rho}{s_\rho}t_\rho, \tag{17}$$

[1] m could be m_d or m_s. However, the numerical values of c_i' change by at most 4% when we change m from m_d to m_s. Therefore, we ignore this difference in our calculations, setting $m=m_s$.

$$\langle \pi^+\pi^- f|\mathcal{H}^{\rm P}|H_c\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega}p_\omega + \frac{g_\rho}{s_\rho}p_\rho, \tag{18}$$

where t_V ($V=\rho$ or ω) is the tree and p_V is the penguin amplitude for producing a vector meson V by $H_c\to fV$; g_ρ is the coupling for $\rho^0\to\pi^+\pi^-$; $\tilde{\Pi}_{\rho\omega}$ is the effective ρ-ω mixing amplitude and s_V^{-1} is from the propagator of V, $s_V=s-m_V^2+im_V\Gamma_V$, with $\sqrt{s}$ being the invariant mass of the $\pi^+\pi^-$ pair. The numerical values for the ρ-ω mixing parameter are [7,20,21] $\mathrm{Re}\,\tilde{\Pi}_{\rho\omega}(m_\omega^2)=-3500\pm300$ MeV^2, $\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}(m_\omega^2)=-300\pm300$ MeV^2. The direct coupling $\omega\to\pi^+\pi^-$ is effectively absorbed into $\tilde{\Pi}_{\rho\omega}$ [21].

Defining

$$\frac{p_\omega}{t_\rho}\equiv r' e^{i(\delta_q+\phi)}, \quad \frac{t_\omega}{t_\rho}\equiv\alpha e^{i\delta_\alpha}, \quad \frac{p_\rho}{p_\omega}\equiv\beta e^{i\delta_\beta}, \tag{19}$$

where δ_α, δ_β, and δ_q are strong phases, one has the following expression for r and δ,

$$re^{i\delta}=r' e^{i\delta_q}\frac{\tilde{\Pi}_{\rho\omega}+\beta e^{i\delta_\beta}s_\omega}{s_\omega+\tilde{\Pi}_{\rho\omega}\alpha e^{i\delta_\alpha}}. \tag{20}$$

It will be shown that in the factorization approach, for all the decay processes $H_c\to f\pi^+\pi^-$ we are considering, $\alpha e^{i\delta_\alpha}$ is real (see Sec. III B for details). Therefore, we let

$$\alpha e^{i\delta_\alpha}=g, \tag{21}$$

where g is a real parameter. Letting

$$\beta e^{i\delta_\beta}=b+ci, \quad r' e^{i\delta_q}=d+ei, \tag{22}$$

and using Eq. (20), we obtain the following result when $\sqrt{s}\sim m_\omega$:

$$re^{i\delta}=\frac{C+Di}{(s-m_\omega^2+g\,\mathrm{Re}\,\tilde{\Pi}_{\rho\omega})^2+(g\,\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}+m_\omega\Gamma_\omega)^2}, \tag{23}$$

where

$$\begin{aligned}C=&(s-m_\omega^2+g\,\mathrm{Re}\,\tilde{\Pi}_{\rho\omega})\{d[\mathrm{Re}\,\tilde{\Pi}_{\rho\omega}+b(s-m_\omega^2)-cm_\omega\Gamma_\omega]\\&-e[\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}+bm_\omega\Gamma_\omega+c(s-m_\omega^2)]\}\\&+(g\,\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}+m_\omega\Gamma_\omega)\{e[\mathrm{Re}\,\tilde{\Pi}_{\rho\omega}+b(s-m_\omega^2)-cm_\omega\Gamma_\omega]\\&+d[\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}+bm_\omega\Gamma_\omega+c(s-m_\omega^2)]\},\\D=&(s-m_\omega^2+g\,\mathrm{Re}\,\tilde{\Pi}_{\rho\omega})\{e[\mathrm{Re}\,\tilde{\Pi}_{\rho\omega}+b(s-m_\omega^2)-cm_\omega\Gamma_\omega]\\&+d[\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}+bm_\omega\Gamma_\omega+c(s-m_\omega^2)]\}\\&-(g\,\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}+m_\omega\Gamma_\omega)\{d[\mathrm{Re}\,\tilde{\Pi}_{\rho\omega}+b(s-m_\omega^2)\\&-cm_\omega\Gamma_\omega]-e[\mathrm{Im}\,\tilde{\Pi}_{\rho\omega}+bm_\omega\Gamma_\omega+c(s-m_\omega^2)]\}.\end{aligned} \tag{24}$$

The weak phase comes from $[V_{ub}V_{cb}^*/(V_{uq}V_{cq}^*)]$. If the operators O_1^d, O_2^d contribute to the decay processes we have

$$\begin{aligned}\sin\phi|_d&=\frac{\eta}{\sqrt{[\rho+A^2\lambda^4(\rho^2+\eta^2)]^2+\eta^2}},\\\cos\phi|_d&=-\frac{\rho+A^2\lambda^4(\rho^2+\eta^2)}{\sqrt{[\rho+A^2\lambda^4(\rho^2+\eta^2)]^2+\eta^2}},\end{aligned} \tag{25}$$

while if O_1^s and O_2^s contribute, we have

$$\begin{aligned}\sin\phi|_s&=-\frac{\eta}{\sqrt{\rho^2+\eta^2}},\\\cos\phi|_s&=\frac{\rho}{\sqrt{\rho^2+\eta^2}},\end{aligned} \tag{26}$$

where we have used the Wolfenstein parametrization [22] for the CKM matrix elements. In order to obtain $r\sin\delta$, $r\cos\delta$, and r we need to calculate $\alpha e^{i\delta_\alpha}$, $\beta e^{i\delta_\beta}$, and $r' e^{i\delta_q}$. This will be done in the next subsection.

B. CP violation in $H_c\to f\pi^+\pi^-$

In the following we will calculate the CP-violating asymmetries in $H_c\to f\pi^+\pi^-$. In the factorization approximation $\rho^0(\omega)$ is generated by one current which has the proper quantum numbers in the Hamiltonian in Eq. (1). In the following we will consider the decay processes $D^+\to\rho^+\rho^0(\omega)\to\rho^+\pi^+\pi^-$, $D^+\to\pi^+\rho^0(\omega)\to\pi^+\pi^+\pi^-$, $D^0\to\phi\rho^0(\omega)\to\phi\pi^+\pi^-$, $D^0\to\eta\rho^0(\omega)\to\eta\pi^+\pi^-$, $D^0\to\eta'\rho^0(\omega)\to\eta'\pi^+\pi^-$, $D^0\to\pi^0\rho^0(\omega)\to\pi^0\pi^+\pi^-$, and $\Lambda_c\to p\rho^0(\omega)\to p\pi^+\pi^-$, individually.

(1) $D^+\to\rho^+V(V=\rho^0$ or $\omega)$. First we consider $D^+\to\rho^+\rho^0(\omega)$. After factorization, the contribution to $t_\rho^{\rho^+}$ (the superscript denotes the decay product f in $H_c\to f\pi^+\pi^-$) from the tree level operator O_1^d is

$$\langle\rho^+\rho^0|O_1^d|D^+\rangle=\langle\rho^0|(\bar{d}d)|0\rangle\langle\rho^+|(\bar{u}c)|D^+\rangle\equiv T_1, \tag{27}$$

where $(\bar{d}d)$ and $(\bar{u}c)$ denote the V-A currents. If we ignore isospin violating effects, then the matrix element of O_2^d is the same as that of O_1^d. After adding the contributions from Fierz transformation of O_1^d and O_2^d we have

$$t_\rho^{\rho^+}=(c_1'+c_2')(1+1/N_c)T_1, \tag{28}$$

where we have omitted the CKM matrix elements in the expression of $t_\rho^{\rho^+}$. Since in Eq. (28) we have neglected the color-octet contribution, which is nonfactorizable and difficult to calculate, N_c should be treated as an effective parameter which depends on the hadronization dynamics of different decay channels. In the same way we find that $t_\omega^{\rho^+}=-t_\rho^{\rho^+}$, so that, from Eq. (19), we have

$$(\alpha e^{i\delta_\alpha})^{\rho^+}=-1. \tag{29}$$

The penguin operator contributions, $p_\rho^{\rho^+}$ and $p_\omega^{\rho^+}$, can be evaluated in the same way with the aid of the Fierz identities. From Eq. (19) we have

$$(\beta e^{i\delta_\beta})^{\rho^+}=0 \tag{30}$$

and

$$(r' e^{i\delta_q})^{\rho^+}=2\frac{(c_3'+c_4')\left(1+\frac{1}{N_c}\right)+c_5'+\frac{1}{N_c}c_6'}{(c_1'+c_2')\left(1+\frac{1}{N_c}\right)}\left|\frac{V_{ub}V_{cb}^*}{V_{ud}V_{cd}^*}\right|, \tag{31}$$

where

$$\left|\frac{V_{ub}V_{cb}^*}{V_{ud}V_{cd}^*}\right|=\frac{A^2\lambda^4}{1-\lambda^2/2}\sqrt{\frac{\rho^2+\eta^2}{(1+A^2\lambda^4\rho)^2+A^4\lambda^8\eta^2}}. \tag{32}$$

(2) $\Lambda_c\to pV$. Next we consider $\Lambda_c\to p\rho^0(\omega)$. Defining

$$\langle\rho^0 p|O_1^d|\Lambda_c\rangle=\langle\rho^0|(\bar{d}d)|0\rangle\langle p|(\bar{u}c)|\Lambda_c\rangle\equiv T_2, \tag{33}$$

we have

$$t_\rho^p=\left(c_1'+\frac{1}{N_c}c_2'\right)T_2. \tag{34}$$

After evaluating t_ω^p and the penguin diagram contributions we obtain the following results:

$$(\alpha e^{i\delta_\alpha})^p=-1, \tag{35}$$

$$(\beta e^{i\delta_\beta})^p=\frac{c_4'+\frac{1}{N_c}c_3'}{\left(2+\frac{1}{N_c}\right)c_3'+\left(1+\frac{2}{N_c}\right)c_4'+2\left(c_5'+\frac{1}{N_c}c_6'\right)}, \tag{36}$$

$$(r' e^{i\delta_q})^p=\frac{\left(2+\frac{1}{N_c}\right)c_3'+\left(1+\frac{2}{N_c}\right)c_4'+2\left(c_5'+\frac{1}{N_c}c_6'\right)}{c_1'+\frac{1}{N_c}c_2'}\times\left|\frac{V_{ub}V_{cb}^*}{V_{ud}V_{cd}^*}\right|. \tag{37}$$

(3) $D^0\to\phi V$. For the decay channel $D^0\to\phi\rho^0(\omega)\to\phi\pi^+\pi^-$ the operators O_1^s and O_2^s contribute to the decay matrix elements. If we define

$$\langle\rho^0\phi|O_1^s|D^0\rangle=\langle\phi|(\bar{s}s)|0\rangle\langle\rho^0|(\bar{u}c)|D^0\rangle\equiv T_3, \tag{38}$$

we have

$$t_\rho^\phi=\left(c_1'+\frac{1}{N_c}c_2'\right)T_3, \tag{39}$$

and

$$(\alpha e^{i\delta_\alpha})^\phi=1, \tag{40}$$

$$(\beta e^{i\delta_\beta})^\phi=1, \tag{41}$$

$$(r' e^{i\delta_q})^\phi=-\frac{c_3'+\frac{1}{N_c}c_4'+c_5'+\frac{1}{N_c}c_6'}{c_1'+\frac{1}{N_c}c_2'}\left|\frac{V_{ub}V_{cb}^*}{V_{us}V_{cs}^*}\right|, \tag{42}$$

where

$$\left|\frac{V_{ub}V_{cb}^*}{V_{us}V_{cs}^*}\right|=\frac{A^2\lambda^4\sqrt{\rho^2+\eta^2}}{1-\lambda^2/2}. \tag{43}$$

(4) $D^0\to\eta(\eta')V$. For the decay channels $D^0\to\eta\rho^0(\omega)\to\eta\pi^+\pi^-$ and $D^0\to\eta'\rho^0(\omega)\to\eta'\pi^+\pi^-$, things become a little complicated. It is known that η and η' have both $\bar{u}u+\bar{d}d$ and $\bar{s}s$ components. The decay constants, $f^u_{\eta(\eta')}$ and $f^s_{\eta(\eta')}$, defined as

$$\begin{aligned}\langle 0|\bar{u}\gamma_\mu\gamma_5 u|\eta(\eta')\rangle&=if^u_{\eta(\eta')}p_\mu,\\ \langle 0|\bar{s}\gamma_\mu\gamma_5 s|\eta(\eta')\rangle&=if^s_{\eta(\eta')}p_\mu,\end{aligned} \tag{44}$$

are different. After straightforward derivations we have

$$(\alpha e^{i\delta_\alpha})^{\eta(\eta')}=1, \tag{45}$$

$$(\beta e^{i\delta_\beta})^{\eta(\eta')}=1, \tag{46}$$

$$(r' e^{i\delta_q})^{\eta(\eta')}=-\frac{2f^u_{\eta(\eta')}+f^s_{\eta(\eta')}}{f^u_{\eta(\eta')}-f^s_{\eta(\eta')}}\times\frac{c_3'+\frac{1}{N_c}c_4'-c_5'-\frac{1}{N_c}c_6'}{c_1'+\frac{1}{N_c}c_2'}\left|\frac{V_{ub}V_{cb}^*}{V_{us}V_{cs}^*}\right|. \tag{47}$$

In the derivations of Eqs. (45)–(47) we have made the approximation that $V_{ub}V_{cb}^*/V_{ud}V_{cd}^*=-V_{ub}V_{cb}^*/V_{us}V_{cs}^*$. It is noted that the minus signs associated with c_5' and c_6' in Eq. (47) arise because $\eta(\eta')$ are pseudoscalar mesons. Since the imaginary part of $c_3'(c_4')$ is the same as that of $c_5'(c_6')$, δ_q is zero. This leads to the strong phase, δ, being zero, in combination with Eqs. (45), (46).

The decay constants $f^u_{\eta(\eta')}$ and $f^s_{\eta(\eta')}$ were calculated phenomenologically in Ref. [23], based on the assumption that the decay constants in the quark flavor basis follow the pattern of particle state mixing. It was found that

$$f_\eta^u = 78\ \text{MeV}, \quad f_\eta^s = -112\ \text{MeV}, \quad f_{\eta'}^u = 63\ \text{MeV}, \quad f_{\eta'}^s = 137\ \text{MeV}. \tag{48}$$

(5) $D \to \pi V$. For the decay process $D^+ \to \pi^+ \rho^0(\omega) \to \pi^+ \pi^+ \pi^-$, two kinds of matrix element products are involved after factorization, i.e.,

$$\langle \rho^0(\omega) | (\overline{d}d) | 0 \rangle \langle \pi^+ | (\overline{u}c) | D^+ \rangle$$

and

$$\langle \pi^+ | (\overline{u}d) | 0 \rangle \langle \rho^0(\omega) | (\overline{d}c) | D^+ \rangle.$$

These two quantities cannot be related to each other by symmetry. Therefore, we have to evaluate them in some phenomenological quark models and hence more uncertainties are involved. Similarly for $D^0 \to \pi^0 \rho^0(\omega) \to \pi^0 \pi^+ \pi^-$ we have to evaluate $\langle \rho^0(\omega) | (\overline{d}d) | 0 \rangle \langle \pi^0 | (\overline{u}c) | D^0 \rangle$ and $\langle \pi^0 | (\overline{d}d) | 0 \rangle \langle \rho^0(\omega) | (\overline{u}c) | D^0 \rangle$ separately.

The matrix elements for $D \to X$ and $D \to X^*$ (X and X^* denote pseudoscalar and vector mesons, respectively) can be decomposed as [24]

$$\langle X | J_\mu | D \rangle = \left(p_D + p_X - \frac{m_D^2 - m_X^2}{k^2} k \right)_\mu F_1(k^2) + \frac{m_D^2 - m_X^2}{k^2} k_\mu F_0(k^2), \tag{49}$$

$$\begin{aligned}\langle X^* | J_\mu | D \rangle = {} & \frac{2}{m_D + m_{X^*}} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p_D^\rho p_{X^*}^\sigma V(k^2) \\ & + i \left[\epsilon_\mu^* (m_D + m_{X^*}) A_1(k^2) - \frac{\epsilon \cdot k}{m_D + m_{X^*}} \right. \\ & \left. \times (p_D + p_{X^*})_\mu A_2(k^2) - \frac{\epsilon \cdot k}{k^2} 2 m_{X^*} k_\mu A_3(k^2) \right] \\ & + i \frac{\epsilon \cdot k}{k^2} 2 m_{X^*} k_\mu A_0(k^2),\end{aligned} \tag{50}$$

where J_μ is the weak current, $k = p_D - p_{X(X^*)}$ and ϵ_μ is the polarization vector of X^*. The form factors satisfy the relations $F_1(0) = F_0(0)$, $A_3(0) = A_0(0)$ and $A_3(k^2) = [(m_D + m_{X^*})/2m_{X^*}] A_1(k^2) - [(m_D - m_{X^*})/2m_{X^*}] A_2(k^2)$.

Using the decomposition in Eqs. (49), (50), we have for $D^+ \to \pi^+ \rho^0(\omega)$,

$$t_\rho^{\pi^+} = -\sqrt{2} m_D |\vec{p}_\rho| \left[\left(c_1' + \frac{1}{N_c} c_2' \right) f_\rho F_1(m_\rho^2) + \left(c_2' + \frac{1}{N_c} c_1' \right) f_\pi A_0(m_\pi^2) \right], \tag{51}$$

where f_ρ and f_π are the decay constants of the ρ and π, respectively, and $\vec{p}_\rho$ is the three momentum of the ρ.

It can be shown that $t_\omega^{\pi^+} = -t_\rho^{\pi^+}$. After calculating the penguin operator contributions, we have

$$(\alpha e^{i\delta_\alpha})^{\pi^+} = -1, \tag{52}$$

$$(\beta e^{i\delta_\beta})^{\pi^+} = \frac{[f_\rho F_1(m_\rho^2) - f_\pi A_0(m_\pi^2)] \left(c_4' + \frac{1}{N_c} c_3' \right) - \frac{2 m_\pi^2 f_\pi A_0(m_\pi^2)}{(m_c + m_d)(m_u + m_d)} \left(c_6' + \frac{1}{N_c} c_5' \right)}{x}, \tag{53}$$

$$(r' e^{i\delta_q})^{\pi^+} = \frac{x}{\left[f_\rho F_1(m_\rho^2) + \frac{1}{N_c} f_\pi A_0(m_\pi^2) \right] c_1' + \left[\frac{1}{N_c} f_\rho F_1(m_\rho^2) + f_\pi A_0(m_\pi^2) \right] c_2'} \left| \frac{V_{ub} V_{cb}^*}{V_{ud} V_{cd}^*} \right|, \tag{54}$$

where x is defined as

$$\begin{aligned} x = {} & \left[2 f_\rho F_1(m_\rho^2) + \frac{f_\rho F_1(m_\rho^2) + f_\pi A_0(m_\pi^2)}{N_c} \right] c_3' + \left[\frac{2 f_\rho F_1(m_\rho^2)}{N_c} + f_\rho F_1(m_\rho^2) + f_\pi A_0(m_\pi^2) \right] c_4' \\ & + 2 \left[f_\rho F_1(m_\rho^2) - \frac{m_\pi^2 f_\pi A_0(m_\pi^2)}{N_c (m_c + m_d)(m_u + m_d)} \right] c_5' + 2 \left[\frac{f_\rho F_1(m_\rho^2)}{N_c} - \frac{m_\pi^2 f_\pi A_0(m_\pi^2)}{(m_c + m_d)(m_u + m_d)} \right] c_6'. \end{aligned} \tag{55}$$

We can consider the process $D^0 \to \pi^0 \rho^0(\omega) \to \pi^0 \pi^+ \pi^-$ in the same way. We find

$$(\alpha e^{i\delta_\alpha})^{\pi^0} = -\frac{f_\rho F_1(m_\rho^2) - f_\pi A_0(m_\pi^2)}{f_\rho F_1(m_\rho^2) + f_\pi A_0(m_\pi^2)}, \tag{56}$$

$$(\beta e^{i\delta_\beta})^{\pi^0} = \frac{[f_\rho F_1(m_\rho^2)+f_\pi A_0(m_\pi^2)]\left(c_4'+\frac{1}{N_c}c_3'\right) - \frac{2m_\pi^2 f_\pi A_0(m_\pi^2)}{(m_c+m_d)(m_u+m_d)}\left(c_6'+\frac{1}{N_c}c_5'\right)}{x}, \tag{57}$$

$$(r' e^{i\delta_q})^{\pi^0} = \frac{x}{[f_\rho F_1(m_\rho^2)+f_\pi A_0(m_\pi^2)]\left(c_1'+\frac{1}{N_c}c_2'\right)}\left|\frac{V_{ub}V_{cb}^*}{V_{ud}V_{cd}^*}\right|. \tag{58}$$

In Eqs. (52)–(58) the form factors $F_1(m_\rho^2)$ and $A_0(m_\pi^2)$ depend on the inner structure of the hadrons. Under the nearest pole dominance assumption, the k^2 dependence of these form factors are

$$F_1(k^2) = \frac{h_1}{1-k^2/m_1^2}, \quad A_0(k^2) = \frac{h_{A_0}}{1-k^2/m_{A_0}^2}, \tag{59}$$

where $m_1 = 2.01$ GeV and $m_{A_0} = 1.87$ GeV [24] and h_1 and h_{A_0} are given by the overlap integrals of the hadronic wave functions of D and $X(X^*)$ [24,25].

Having obtained the expressions for $\alpha e^{i\delta_\alpha}$, $\beta e^{i\delta_\beta}$, and $r' e^{i\delta_q}$, for different decay processes, we may substitute them into Eq. (23) to obtain ($r \sin\delta$) and ($r \cos\delta$) for each channel. Then, in combination with with Eqs. (25) and (26), the CP-violating asymmetries a can be obtained from Eq. (16).

C. Numerical results

In the numerical calculations, we have several parameters: q^2, N_c, and the CKM matrix elements in the Wolfenstein parametrization. As mentioned in Sec. II, the value of q^2 is conventionally chosen to be in the range $0.3 < q^2/m_c^2 < 0.5$. For the CKM matrix elements, which should be determined from experiment, we use $\lambda = 0.221$, $\eta = 0.34$ and $\rho = -0.12$ as in Ref. [8].

The value of the effective N_c should also be determined by experiment. Since the hadronization information is included in N_c, the value of N_c may be different for different decay channels. Furthermore, since the color-octet contribution associated with each operator in the Hamiltonian (1) can vary, the effective N_c in the Fierz transformation for each operator may be different. In general, nonfactorizable effects can be absorbed into the effective parameters a_i^{eff} after the Fierz transformation

$$a_{2i}^{\mathrm{eff}} = c_{2i}' + \frac{1}{(N_c)_{2i}}c_{2i-1}', \quad a_{2i-1}^{\mathrm{eff}} = c_{2i-1}' + \frac{1}{(N_c)_{2i-1}}c_{2i}' \quad (i=1,2,3), \tag{60}$$

where

$$\frac{1}{(N_c)_i} \equiv \frac{1}{3} + \xi_i \quad (i=1,\dots,6), \tag{61}$$

with ξ_i being the nonfactorizable effects, which may be different for each operator. However, since we do not have enough information about the operator dependence of ξ_i, we assume ξ_i is universal for each operator [13] and hence for each operator we use the same effective N_c $[=(N_c)_i]$.

In the numerical calculations, it is found that for a fixed N_c there is a maximum point, $a_{\max}$, for the CP violating parameter a, when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. We have calculated $a_{\max}$ in the range $N_c > 0$ for different decay channels. In the calculations we use the following two sets of form factors [24].

$$\text{Set 1:} h_1 = 0.69, \quad h_{A_0} = 0.67, \quad h_V = 1.23,$$

$$h_{A_1} = 0.78, \quad h_{A_2} = 0.92;$$

$$\text{Set 2:} h_1 = 0.78, \quad h_{A_0} = 0.77, \quad h_V = 1.55,$$

$$h_{A_1} = 0.98, \quad h_{A_2} = 1.27.$$

The above two sets of parameters correspond to taking the average transverse momentum of the constituents in the meson to be 400 or 500 MeV, respectively [24]. The k^2 dependence of $A_1(k^2)$, $A_2(k^2)$, and $V(k^2)$ are the same as in Eq. (59).

The numerical results show that for $D^+ \to \rho^+\rho^0(\omega) \to \rho^+\pi^+\pi^-$, in the whole range $N_c > 0$, we have $a_{\max} \leqslant 3 \times 10^{-4}$, which is small. For $D^0 \to \eta\rho^0(\omega) \to \eta\pi^+\pi^-$ and $D^0 \to \eta'\rho^0(\omega) \to \eta'\pi^+\pi^-$, from Eqs. (45)–(47) it can be seen that the strong phase δ is zero. Therefore, we do not have CP violation in these decays in our approach. However, for other processes there is a small range of N_c in which we may have large $a_{\max}$ ($\geqslant 1\%$).

For $D^0 \to \phi\rho^0(\omega) \to \phi\pi^+\pi^-$, the range of N_c for $a_{\max} \geqslant 1\%$ is $1.98 \leqslant N_c \leqslant 1.99$, while for $\Lambda_c \to p\rho^0(\omega) \to p\pi^+\pi^-$ the range is $1.95 \leqslant N_c \leqslant 2.02$. For $D^+ \to \pi^+\rho^0(\omega) \to \pi^+\pi^+\pi^-$ we find that for the first set of form factors when $N_c \geqslant 56$, $a_{\max} \geqslant 1\%$, while for the second set of form factors when $N_c \geqslant 136$, $a_{\max} \geqslant 1\%$, in the range $0.3 \leqslant q^2/m_c^2 \leqslant 0.5$. For $D^0 \to \pi^0\rho^0(\omega) \to \pi^0\pi^+\pi^-$ we find that when $1.98 \leqslant N_c \leqslant 1.99$ we have $a_{\max} \geqslant 1\%$ in the range $0.3 \leqslant q^2/m_c^2 \leqslant 0.5$ for both sets of form factors.

The above ranges for N_c were obtained by the requirement that we have large CP violation in this range. However, whether N_c can be in this range should be determined by the experimental data for the branching ratio of each decay channel. Usually the decay rate for $D \to f\rho^0$ is determined primarily by the tree operators, O_1 and O_2, which are related to t_ρ^f. In fact, the reason why we can find large CP

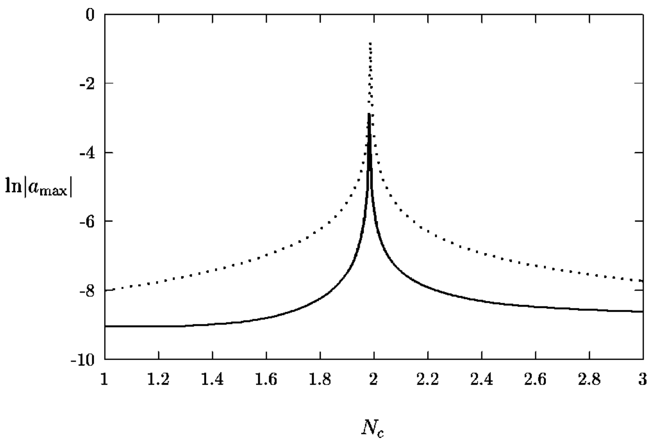

FIG. 1. The variation of $\ln|a_{\max}|$ over the range $1\leqslant N_c\leqslant 3$. The solid line is for $D^0\to\pi^0\rho^0(\omega)\to\pi^0\pi^+\pi^-$, with the first set of form factors and $q^2/m_c^2=0.3$, while the dotted line is for $\Lambda_c\to p\rho^0(\omega)\to p\pi^+\pi^-$, with $q^2/m_c^2=0.3$.

violation in some range of N_c, is that in this range t_ρ^f becomes small enough so that r', and hence r, becomes large [see Eqs. (19), (20)]. However, if t_ρ^f is too small the decay rate it yields for $D\to f\rho^0$ may be smaller than the experimental data. In such a case, the range of N_c in which we could have large CP violation will be excluded by the data.

The decay widths for nonleptonic decays of the D meson can be calculated straightforwardly in the quark model of Refs. [24,25]. Since we are considering the range for N_c in which t_ρ^f is small, we have to take into account the penguin contributions p_ρ^f as well when we calculate the decay widths. In the calculations of the decay width for $D^0\to\phi\rho^0$ we use $f_\phi=237$ MeV. We find that for the first set of form factors the branching ratio is smaller than 2.3×10^{-8} and for the second set the branching ratio is smaller than 3.6×10^{-8} in the range $1.98\leqslant N_c\leqslant 1.99$. The dependence of the branching ratio on q^2/m_c^2 is negligible. These branching ratios are much smaller than the experimental data $(6\pm3)\times10^{-4}$ [26] which corresponds to $1.31\leqslant N_c\leqslant 1.53$ $(1.41\leqslant N_c\leqslant 1.60)$ for the first (second) set of form factors. Similarly, when $N_c\geqslant 56$ the branching ratio for $D^+\to\pi^+\rho^0$ is smaller than 1.0×10^{-5}, while the experimental data is $(1.05\pm0.31)\times10^{-3}$ [26] corresponding to $2.1\leqslant N_c\leqslant 2.9$ $(2.5\leqslant N_c\leqslant 3.4)$ for the first (second) set of form factors. Therefore, we cannot have large CP violation in $D^0\to\phi\rho^0(\omega)\to\phi\pi^+\pi^-$ and $D^+\to\pi^+\rho^0(\omega)\to\pi^+\pi^+\pi^-$.

TABLE I. Values of $\mathrm{Br}(D^+\to\rho^+\rho^0)$ with the first (second) set of form factors and $a_{\max}$ for $D^+\to\rho^+\rho^0(\omega)\to\rho^+\pi^+\pi^-$, with $q^2/m_c^2=0.3(0.5)$.

N_c	$\mathrm{Br}(D^+\to\rho^+\rho^0)$	$a_{\max}$
0.5	$6.3(9.6)\times10^{-2}$	$2.0(1.8)\times10^{-4}$
1.0	$2.5(3.8)\times10^{-2}$	$1.4(1.3)\times10^{-4}$
1.5	$1.6(2.5)\times10^{-2}$	$1.1(0.96)\times10^{-4}$
2.0	$1.3(1.9)\times10^{-2}$	$8.7(7.7)\times10^{-5}$
3.0	$0.95(1.5)\times10^{-2}$	$6.1(5.3)\times10^{-5}$

TABLE II. Values of $\mathrm{Br}(D^0\to\phi\rho^0)$ and $a_{\max}$ for $D^0\to\phi\rho^0(\omega)\to\phi\pi^+\pi^-$, with $q^2/m_c^2=0.3(0.5)$.

	First set of form factors	
N_c	$\mathrm{Br}(D^0\to\phi\rho^0)$	$a_{\max}$
1.31	9.0×10^{-4}	$1.2(1.3)\times10^{-4}$
1.36	7.2×10^{-4}	$1.3(1.3)\times10^{-4}$
1.41	5.7×10^{-4}	$1.4(1.4)\times10^{-4}$
1.46	4.4×10^{-4}	$1.5(1.5)\times10^{-4}$
1.53	3.0×10^{-4}	$1.6(1.6)\times10^{-4}$
	Second set of form factors	
N_c	$\mathrm{Br}(D^0\to\phi\rho^0)$	$a_{\max}$
1.41	8.9×10^{-4}	$1.4(1.4)\times10^{-4}$
1.46	6.9×10^{-4}	$1.5(1.5)\times10^{-4}$
1.51	5.3×10^{-4}	$1.6(1.6)\times10^{-4}$
1.56	4.0×10^{-4}	$1.7(1.7)\times10^{-4}$
1.60	3.0×10^{-4}	$1.8(1.8)\times10^{-4}$

However, for the decay processes $D^0\to\pi^0\rho^0$ and $\Lambda_c\to p\rho^0$ there are no experimental data at present [26]. Therefore, there is still a possibility that N_c could be in the range required for large CP violation for $D^0\to\pi^0\rho^0(\omega)\to\pi^0\pi^+\pi^-$ or $\Lambda_c\to p\rho^0(\omega)\to p\pi^+\pi^-$. In Fig. 1 we plot $\ln|a_{\max}|$ over the range $1\leqslant N_c\leqslant 3$ for these two processes, with $q^2/m_c^2=0.3$ and the first set of form factors for $D^0\to\pi^0\rho^0(\omega)\to\pi^0\pi^+\pi^-$. In fact, the difference between the results for $q^2/m_c^2=0.3$ and 0.5, and for the first and second set of form factors for $D^0\to\pi^0\rho^0(\omega)\to\pi^0\pi^+\pi^-$, are small. This can be seen more clearly from Tables IV and V.

The decay width for $D^0\to\pi^0\rho^0$ is calculated in the same way and we find that for the first set of of form factors the branching ratio is $1.4(1.7)\times10^{-8}$, while for the second set the branching ratio is $1.8(2.1)\times10^{-8}$ for $N_c=1.98(1.99)$. This prediction is almost independent of q^2/m_c^2.

The branching ratio for $\Lambda_c\to p\rho^0$ can be calculated with the same method as that in Ref. [8], where we worked in the heavy quark limit $m_c\to\infty$ and used the diquark model hadronic wave functions for both the heavy baryon, Λ_c, and the proton, p. As in the neutron case, in the diquark model the Clebsch-Gordan coefficient of the $u[ud]$ component ($[ud]$

TABLE III. Values of $\mathrm{Br}(D^+\to\pi^+\rho^0)$ and $a_{\max}$ for $D^+\to\pi^+\rho^0(\omega)\to\pi^+\pi^+\pi^-$, with $q^2/m_c^2=0.3(0.5)$.

	First set of form factors	
N_c	$\mathrm{Br}(D^+\to\pi^+\rho^0)$	$a_{\max}$
2.1	1.4×10^{-3}	$-3.0(-3.0)\times10^{-4}$
2.5	9.8×10^{-4}	$-3.9(-4.0)\times10^{-4}$
2.9	7.3×10^{-4}	$-4.9(-4.9)\times10^{-4}$
	Second set of form factors	
N_c	$\mathrm{Br}(D^+\to\pi^+\rho^0)$	$a_{\max}$
2.5	1.3×10^{-3}	$-4.0(-4.0)\times10^{-4}$
3.0	9.2×10^{-4}	$-5.2(-5.1)\times10^{-4}$
3.4	7.3×10^{-4}	$-6.1(-6.0)\times10^{-4}$

TABLE IV. Values of $\mathrm{Br}(D^0 \to \pi^0 \rho^0)$ with the first (second) set of form factors and $a_{\max}$ for $D^0 \to \pi^0 \rho^0(\omega) \to \pi^0 \pi^+ \pi^-$, with $q^2/m_c^2 = 0.3(0.5)$.

N_c	$\mathrm{Br}(D^0 \to \pi^0 \rho^0)$	$a_{\max}$ (set 1)	$a_{\max}$ (set 2)
0.5	$2.1(2.8)\times 10^{-2}$	$1.0(0.94)\times 10^{-4}$	$1.1(0.93)\times 10^{-4}$
1.0	$2.3(3.0)\times 10^{-3}$	$9.5(8.2)\times 10^{-5}$	$9.3(8.1)\times 10^{-5}$
1.5	$2.5(3.3)\times 10^{-4}$	$9.7(8.7)\times 10^{-5}$	$9.3(8.4)\times 10^{-5}$
1.9	$4.8(6.2)\times 10^{-5}$	$-7.1(-8.3)\times 10^{-4}$	$-7.4(-8.6)\times 10^{-4}$
1.98	$1.4(1.8)\times 10^{-8}$	$-1.4(-1.6)\times 10^{-2}$	$-1.5(-1.7)\times 10^{-2}$
1.99	$1.7(2.1)\times 10^{-8}$	$1.4(1.6)\times 10^{-2}$	$1.4(1.7)\times 10^{-2}$
2.1	$7.3(9.4)\times 10^{-6}$	$7.2(8.1)\times 10^{-4}$	$7.5(8.3)\times 10^{-4}$
2.5	$1.0(1.3)\times 10^{-4}$	$2.5(2.8)\times 10^{-4}$	$2.7(2.9)\times 10^{-4}$
3.0	$2.8(3.6)\times 10^{-4}$	$2.0(2.1)\times 10^{-4}$	$2.1(2.1)\times 10^{-4}$
10.0	$1.6(2.0)\times 10^{-3}$	$1.5(1.4)\times 10^{-4}$	$1.5(1.4)\times 10^{-4}$

is the scalar diquark) is also $1/\sqrt{2}$ for the proton [27]. We find that for $N_c = 1.95(2.02)$ the branching ratio for $\Lambda_c \to p\rho^0$ is $7.9(7.5)\times 10^{-9}$ for $b = 1.77\ \mathrm{GeV}^{-1}$, corresponding to $\langle k_\perp^2 \rangle^{1/2} = 400$ MeV, and $6.9(7.0)\times 10^{-9}$ for $b = 1.18\ \mathrm{GeV}^{-1}$, corresponding to $\langle k_\perp^2 \rangle^{1/2} = 600$ MeV, where $\langle k_\perp^2 \rangle$ is the average transverse momentum of the c quark in the Λ_c. Again the branching ratio is very insensitive to q^2/m_c^2.

In Tables I–V we list numerical results for $a_{\max}$ and $\mathrm{Br}(H_c \to f\rho^0)$ for various processes, with different values of N_c and q^2/m_c^2. It should be noted that $\mathrm{Br}(H_c \to f\rho^0)$ is almost same for $q^2/m_c^2 = 0.3$ and $q^2/m_c^2 = 0.5$. Table I shows explicitly that, for $D^+ \to \rho^+ \rho^0(\omega) \to \rho^+ \pi^+ \pi^-$, $a_{\max}$ is at most $\sim 10^{-4}$ no matter what N_c is. From Tables II and III we can see that in the region of N_c allowed by the experimental data, $a_{\max}$ is of the order 10^{-4} for $D^0 \to \phi \rho^0(\omega) \to \phi \pi^+ \pi^-$ and $D^+ \to \pi^+ \rho^0(\omega) \to \pi^+ \pi^+ \pi^-$. It can also be seen explicitly from Tables IV and V that there is a range for N_c in which $a_{\max}$ may be bigger than 1% for $D^0 \to \pi^0 \rho^0(\omega) \to \pi^0 \pi^+ \pi^-$ and $\Lambda_c \to p \rho^0(\omega) \to p \pi^+ \pi^-$.

In Fig. 2 we plot the numerical values of the CP-violating asymmetries, a, for $D^0 \to \pi^0 \rho^0(\omega) \to \pi^0 \pi^+ \pi^-$ with $N_c = 1.99$ and $q^2/m_c^2 = 0.3$, 0.5 (for $N_c = 1.98$ we have similar results) as a function of the invariant mass of the $\pi^+ \pi^-$ pair.

TABLE V. Values of $\mathrm{Br}(\Lambda_c \to p\rho^0)$ with $\langle k_\perp^2 \rangle^{1/2}$ = 400 MeV (600 MeV) and $a_{\max}$ for $\Lambda_c \to p \rho^0(\omega) \to p \pi^+ \pi^-$, with $q^2/m_c^2 = 0.3(0.5)$.

N_c	$\mathrm{Br}(\Lambda_c \to p\rho^0)$	$a_{\max}$
0.5	$2.2(1.9)\times 10^{-4}$	$2.0(1.8)\times 10^{-4}$
1.0	$2.4(2.1)\times 10^{-5}$	$3.3(3.0)\times 10^{-4}$
1.5	$2.6(2.3)\times 10^{-6}$	$7.3(6.7)\times 10^{-4}$
1.9	$4.9(4.3)\times 10^{-8}$	$3.9(4.1)\times 10^{-3}$
1.95	$7.9(6.9)\times 10^{-9}$	$1.0(1.1)\times 10^{-2}$
2.02	$7.5(7.0)\times 10^{-9}$	$-1.1(-1.0)\times 10^{-2}$
2.1	$7.5(6.5)\times 10^{-8}$	$-3.4(-3.1)\times 10^{-3}$
2.5	$1.1(0.92)\times 10^{-6}$	$-8.0(-7.4)\times 10^{-4}$
3.0	$2.8(2.5)\times 10^{-6}$	$-4.3(-4.0)\times 10^{-4}$
10.0	$1.6(1.4)\times 10^{-5}$	$-1.1(-1.1)\times 10^{-4}$

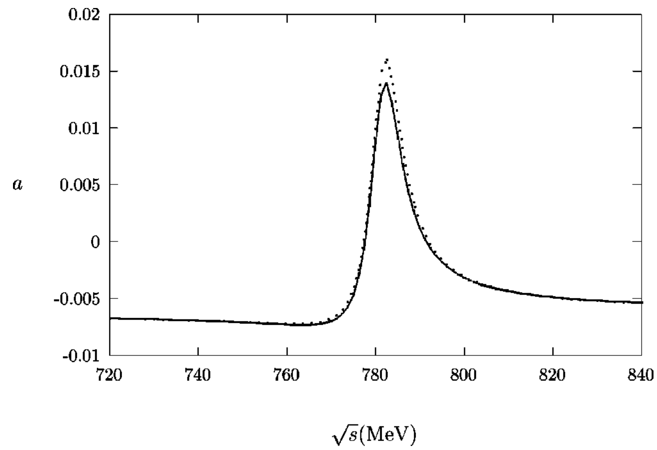

FIG. 2. The CP-violating asymmetry for $D^0 \to \pi^0 \rho^0(\omega) \to \pi^0 \pi^+ \pi^-$ (with $N_c = 1.99$) as a function of the invariant mass of the $\pi^+ \pi^-$ pair. The solid (dotted) line is for $q^2/m_c^2 = 0.3$ (0.5).

It should be noted that in Fig. 2 we used the first set of form factors. The results for the second set change very little. In Fig. 3 we plot the results for $\Lambda_c \to p \rho^0(\omega) \to p \pi^+ \pi^-$, with $N_c = 2.02$ and $q^2/m_c^2 = 0.3$, 0.5 (for $N_c = 1.95$ we have similar results). In both of these plots we find that we can have $a_{\max} \geqslant 1\%$.

IV. SUMMARY AND DISCUSSIONS

The aim of the present work was to look for possibilities of large CP violation in charmed meson or baryon nonleptonic decays, $H_c \to f \pi^+ \pi^-$. Since CP violation in the charm sector is usually estimated to be very small (less than 10^{-3}), it would be fascinating to find cases where the CP violation is large ($>1\%$). Following our previous work on CP violation in the b-quark system [7,8], we have studied direct CP violation in $D^+ \to \rho^+ \rho^0(\omega) \to \rho^+ \pi^+ \pi^-$, $D^+ \to \pi^+ \rho^0(\omega) \to \pi^+ \pi^+ \pi^-$, $D^0 \to \phi \rho^0(\omega) \to \phi \pi^+ \pi^-$, $D^0 \to \eta \rho^0(\omega) \to \eta \pi^+ \pi^-$, $D^0 \to \eta' \rho^0(\omega) \to \eta' \pi^+ \pi^-$, $D^0 \to \pi^0 \rho^0(\omega) \to \pi^0 \pi^+ \pi^-$, and $\Lambda_c \to p \rho^0(\omega) \to p \pi^+ \pi^-$ via ρ-ω mixing.

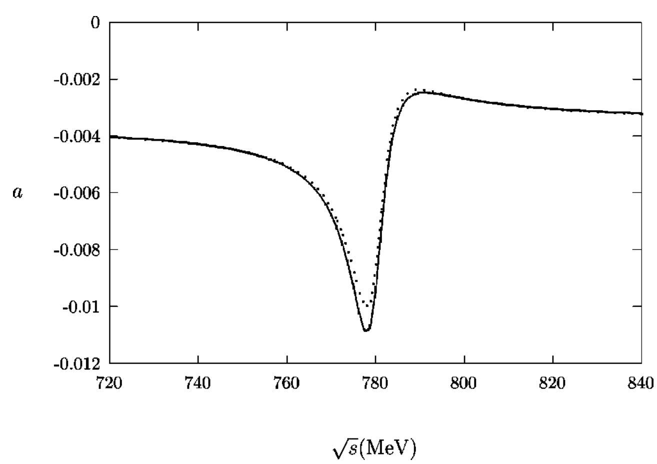

FIG. 3. The CP-violating asymmetry for $\Lambda_c \to p \rho^0(\omega) \to p \pi^+ \pi^-$, with $N_c = 2.02$. The solid (dotted) line is for $q^2/m_c^2 = 0.3$ (0.5).

The advantage of ρ-ω mixing is that the strong phase difference is large (passing through 90° at the ω resonance), for some fixed N_c. As a result, the CP-violating asymmetry a has a maximum $a_{\max}$ when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance. It was found that $a_{\max}$ depends strongly on the effective parameter N_c. For $D^+\to\rho^+\rho^0(\omega)\to\rho^+\pi^+\pi^-$, $D^0\to\eta\rho^0(\omega)\to\eta\pi^+\pi^-$, and $D^0\to\eta'\rho^0(\omega)\to\eta'\pi^+\pi^-$, $a_{\max}$ is small over the whole range, $N_c\geqslant 0$. However, for other processes we found that in order to have $a_{\max}\geqslant 1\%$, N_c should be in a particular range in which the amplitude t_ρ^f becomes small enough so that we can have large CP violation. This is because when t_ρ^f is small r can become large, leading to large CP violation. However, whether or not N_c can be in such a range is determined by the decay branching ratios for $H_c\to f\rho^0$.

The experimental data exclude the possibility of large CP violation in $D^+\to\pi^+\rho^0(\omega)\to\pi^+\pi^+\pi^-$ and $D^0\to\phi\rho^0(\omega)\to\phi\pi^+\pi^-$. However, since we do not have data for $D^0\to\pi^0\rho^0$ and $\Lambda_c\to p\rho^0$ at present, it is still possible that we could have $a_{\max}\geqslant 1\%$ for $D^0\to\pi^0\rho^0(\omega)\to\pi^0\pi^+\pi^-$ and $\Lambda_c\to p\rho^0(\omega)\to p\pi^+\pi^-$ via ρ-ω mixing in some small range of N_c. We estimated that in order to have large CP violation, the branching ratios for $D^0\to\pi^0\rho^0$ and $\Lambda_c\to p\rho^0$ should be around $10^{-9}-10^{-8}$ ($N_c=1.98$ or 1.99 for $D^0\to\pi^0\rho^0$ and $N_c=1.95$ or 2.02 for $\Lambda_c\to p\rho^0$). It will be very interesting to look for such large CP-violating asymmetries in the experiments in order to get a deeper understanding of the mechanism for CP violation. On the other hand, as explained in Sec. III C, the larger asymmetries are associated with smaller branching ratios and this will make the measurements more difficult. Furthermore, the study of CP violation in Λ_c decays may provide insight into the baryon asymmetry phenomena required for baryogenesis.

Our analysis can be extended straightforwardly to say, $\Xi_c'\to\Lambda\rho^0(\omega)\to\Lambda\pi^+\pi^-$, and also $D_s\to K^*\rho^0(\omega)\to K^*\pi^+\pi^-$, if we assume SU(3) flavor symmetry. In the calculations of CP violating asymmetry parameters we need the Wilson coefficients for the tree and penguin operators at the decay scale m_c. We calculated the six Wilson coefficients to the next-to-leading order by applying the formalism developed in Refs. [15,16] and the relevant anomalous dimension matrix elements. Since we only considered strong penguin operators, and since the strong interaction is independent of flavor, the relevant formulas in Refs. [15,16] can be applied to c decays directly. We worked with the renormalization-scheme-independent Wilson coefficients. Furthermore, to be consistent, we introduced the effective Wilson coefficients by taking into account the operator renormalization to the one-loop order.

There are some uncertainties in our calculations. While discussing direct CP violation, we have to evaluate hadronic matrix elements where nonperturbative QCD effects are involved. We have worked in the factorization approximation, which has not been justified completely up to now. It has been pointed out that this approximation may be quite reliable in energetic weak decays [9,10]. There has also been some discussion on nonfactorizable contributions. In Ref. [12] the authors introduced two phenomenological parameters, ϵ_1 and ϵ_8, which are scale dependent to parametrize nonfactorizable effects. The scale dependence of ϵ_1 and ϵ_8 cancels that of the Wilson coefficients c_1 and c_2 and it leads to $a_1^{\rm eff}$ and $a_2^{\rm eff}$. In Refs. [11,13], renormalization-scheme-independent coefficients are used and with the definition in Eq. (60) an effective N_c is introduced to describe nonfactorizable effects. On the other hand, Buras and Silvestrini [14] demonstrated that in the approach of Ref. [12], it is possible to find a renormalization scheme in which the nonfactorizable parameters ϵ_1 and ϵ_8 vanish at any chosen decay scale. In principle, such a scheme can be determined by experimental data. However, the present data is not accurate enough.

We can see from these investigations that more work is needed before we can judge the factorization approach. Since c decays are less energetic than b decays, we expect even more nonfactorizable effects. In the present work, as in Refs. [11,13], we introduced an effective value of N_c in Eq. (60) and assumed that it is the same for each $a_i^{\rm eff}$ ($i=1,\ldots,6$). The value of N_c should be determined by experimental data and it will, in general, depend on the decay channel, since hadronization dynamics can be different for each channel. Furthermore, its value depends on the Wilson coefficients to be used. We avoid the scheme dependence in Wilson coefficients by using the scheme independent ones. However, such coefficients do depend on infrared regulators and gauge. In principle, this dependence should be canceled by the matrix elements of the operators. Furthermore, while discussing the processes $D^+\to\pi^+\rho^0(\omega)\to\pi^+\pi^+\pi^-$ and $D^0\to\pi^0\rho^0(\omega)\to\pi^0\pi^+\pi^-$ we have to evaluate the matrix elements in some phenomenological quark model. All these factors may lead to some uncertainty in our numerical results. However, as pointed out earlier, since the large CP violation we predict is mainly caused by small t_ρ^f in some range of the phenomenological parameter associated with the breakdown of factorization, we expect that our predictions should still provide useful guidance for future investigations.

ACKNOWLEDGMENTS

One of us (Guo) would like to thank A.J. Buras for stimulating communications. This work was supported in part by the Australian Research Council and the National Science Foundation of China.

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