$1/m_Q$ corrections to the Bethe-Salpeter equation for Λ_Q in the diquark picture

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Corrections of order $1/m_Q$ ($Q=b$ or c) to the Bethe-Salpeter (BS) equation for Λ_Q are analyzed on the assumption that the heavy baryon Λ_Q is composed of a heavy quark and a scalar, light diquark. It is found that in addition to the one BS scalar function in the limit $m_Q\to\infty$, two more scalar functions are needed at the order $1/m_Q$. These can be related to the BS scalar function in the leading order in our model. The six form factors for the weak transition $\Lambda_b\to\Lambda_c$ are expressed in terms of these wave functions and the results are consistent with HQET to order $1/m_Q$. Assuming the kernel for the BS equation in the limit $m_Q\to\infty$ to consist of a scalar confinement term and a one-gluon-exchange term we obtain numerical solutions for the BS wave functions, and hence for the $\Lambda_b\to\Lambda_c$ form factors to order $1/m_Q$. Predictions are given for the differential and total decay widths for $\Lambda_b\to\Lambda_c l\bar{\nu}$, and also for the nonleptonic decay widths for $\Lambda_b\to\Lambda_c$ plus a pseudoscalar or vector meson, with QCD corrections being also included.

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I. INTRODUCTION

Heavy flavor physics provides an important area within which to study many important physical phenomena in particle physics, such as the structure and interactions inside heavy hadrons, the heavy hadron decay mechanism, and the plausibility of present nonperturbative QCD models. Heavy baryons have been studied much less than heavy mesons, both experimentally and theoretically. However, more experimental data for heavy baryons is being accumulated [1–6] and we expect that the experimental situation for them will continue to improve in the near future. On the theoretical side, heavy quark effective theory (HQET) [7] provides a systematic way to study physical processes involving heavy hadrons. With the aid of HQET heavy hadron physics is simplified when $m_Q \gg \Lambda_{QCD}$. In order to get the complete physics, HQET is usually combined with some nonperturbative QCD models which deal with dynamics inside heavy hadrons.

As a formally exact equation to describe the hadronic bound state, the Bethe-Salpeter (BS) equation is an effective method to deal with nonperturbative QCD effects. In fact, in combination with HQET, the BS equation has already been applied to the heavy meson system [8–10]. The Isgur-Wise function was calculated [8,10] and $1/m_Q$ corrections were also considered [8]. In previous work [11–13], we established the BS equations in the heavy quark limit ($m_Q\to\infty$) for the heavy baryons Λ_Q and $\omega_Q^{(*)}$ (where $\omega=\Xi$, Σ or Ω and $Q=b$ or c). These were assumed to be composed of a heavy quark, Q, and a light scalar and axial-vector diquark, respectively. We found that in the limit $m_Q\to\infty$, the BS equations for these heavy baryons are greatly simplified. For example, only one BS scalar function is needed for Λ_Q in this limit. By assuming that the BS equation's kernel consists of a scalar confinement term and a one-gluon-exchange term we gave numerical solutions for the BS wave functions in the covariant instantaneous approximation, and consequently applied these solutions to calculate the Isgur-Wise functions for the weak transitions $\Lambda_b\to\Lambda_c$ and $\Omega_b^{(*)}\to\Omega_c^{(*)}$.

In reality, the heavy quark mass is not infinite. Therefore, in order to give more exact phenomenological predictions we have to include $1/m_Q$ corrections, especially $1/m_c$ corrections. It is the purpose of the present paper to analyze the $1/m_Q$ corrections to the BS equation for Λ_Q and to give some phenomenological predictions for its weak decays. As in the previous work [11–14], we will still assume that Λ_Q is composed of a heavy quark and a light, scalar diquark. In this picture, the three body system is simplified to a two body system.

In the framework of HQET, the eigenstate of HQET Lagrangian $|\Lambda_Q\rangle_{\rm HQET}$ has 0^+ light degrees of freedom. This leads to only one Isgur-Wise function $\xi(\omega)$(ω is the velocity transfer) for $\Lambda_b\to\Lambda_c$ in the leading order of the $1/m_Q$ expan-

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sion [15–20]. When $1/m_Q$ corrections are included, another form factor in HQET and an unknown flavor-independent parameter which is defined as the mass difference $m_{\Lambda_Q}-m_Q$ in the heavy quark limit are involved [19]. This provides some relations among the six form factors for $\Lambda_b\to\Lambda_c$ to order $1/m_Q$. Consequently, if one form factor is determined, the other five form factors can be obtained.

Here we extend our previous work to solve the BS equation for Λ_Q to order $1/m_Q$, in combination with the results of HQET. It can be shown that two BS scalar functions are needed at the order $1/m_Q$, in addition to the one scalar function in the limit $m_Q\to\infty$. The relationship among these three scalar functions can be found. Therefore, our numerical results for the BS wave function in the order $m_Q\to\infty$ can be applied directly to obtain the $1/m_Q$ corrections to the form factors for the weak transition $\Lambda_b\to\Lambda_c$. It can be shown that the relations among all the six form factors for $\Lambda_b\to\Lambda_c$ in the BS approach are consistent with those from HQET to order $1/m_Q$. We also give phenomenological predictions for the differential and total decay widths for $\Lambda_b\to\Lambda_c l\bar{\nu}$, and for the nonleptonic decay widths for $\Lambda_b\to\Lambda_c$ plus a pseudoscalar or vector meson. Since the QCD corrections are comparable with the $1/m_Q$ corrections, we also include QCD corrections in our predictions. Furthermore, we discuss the dependence of our results on the various input parameters in our model, and present the comparison of our results with those of other models.

The remainder of this paper is organized as follows. In Sec. II we discuss the BS equation for the heavy quark and light scalar diquark system to order $1/m_Q$ and introduce the two BS scalar functions appearing at this order. We also discuss the constraint on the form of the kernel. In Sec. III we express the six form factors for $\Lambda_b\to\Lambda_c$ in terms of the BS wave function. The consistency of our model with HQET is discussed. We also present numerical solutions for these form factors. In Sec. VI we apply the solutions for the $\Lambda_b\to\Lambda_c$ form factors, with QCD corrections being included, to the semileptonic decay $\Lambda_b\to\Lambda_c l\bar{\nu}$, and the nonleptonic decays $\Lambda_b\to\Lambda_c$ plus a pseudoscalar or vector meson. Finally, Sec. VI contains a summary and discussion.

II. THE BS EQUATION FOR Λ_Q TO $1/m_Q$

Based on the picture that Λ_Q is a bound state of a heavy quark and a light, scalar diquark, its BS wave function is defined as [11]

$$\chi(x_1,x_2,P)=\langle 0|T\psi_Q(x_1)\varphi(x_2)|\Lambda_Q(P)\rangle, \tag{1}$$

where $\psi_Q(x_1)$ and $\varphi(x_2)$ are the field operators for the heavy quark Q and the light, scalar diquark, respectively, $P=m_{\Lambda_Q}v$ is the total momentum of Λ_Q and v is its velocity. Let m_Q and m_D be the masses of the heavy quark and the light diquark in Λ_Q, p be the relative momentum of the two constituents, and define $\lambda_1=m_Q/(m_Q+m_D),\lambda_2=m_D/(m_Q+m_D)$. The BS wave function in momentum space is defined as

$$\chi(x_1,x_2,P)=e^{iPX}\int\frac{d^4p}{(2\pi)^4}e^{ipx}\chi_P(p), \tag{2}$$

where $X=\lambda_1x_1+\lambda_2x_2$ is the coordinate of the center of mass and $x=x_1-x_2$. The momentum of the heavy quark is $p_1=\lambda_1P+p$ and that of the diquark is $p_2=-\lambda_2P+p$. $\chi_P(p)$ satisfies the following BS equation [21]:

$$\chi_P(p)=S_F(\lambda_1P+p)\int\frac{d^4q}{(2\pi)^4}K(P,p,q)\chi_P(q)\times S_D(-\lambda_2P+p), \tag{3}$$

where $K(P,p,q)$ is the kernel, which is defined as the sum of all the two particle irreducible diagrams with respect to the heavy quark and the light diquark. For convenience, in the following we use the variables

$$p_l\equiv v\cdot p-\lambda_2m_{\Lambda_Q},\quad p_t\equiv p-(v\cdot p)v. \tag{4}$$

It should be noted that p_l and p_t are of the order Λ_{QCD}. The mass of Λ_Q can be written in the following form with respect to the $1/m_Q$ expansion (from HQET):

$$m_{\Lambda_Q}=m_Q+m_D+E_0+\frac{1}{m_Q}E_1+O(1/m_Q^2), \tag{5}$$

where E_0 and E_1/m_Q are binding energies at the leading and first order in the $1/m_Q$ expansion, respectively. m_D, E_0 and E_1 are independent of m_Q.

Since we are considering $1/m_Q$ corrections to the BS equation, we expand the heavy quark propagator $S_F(\lambda_1P+p)$ to order $1/m_Q$. We find

$$S_F=S_{0F}+\frac{1}{m_Q}S_{1F}, \tag{6}$$

where S_{0F} is the propagator in the limit $m_Q\to\infty$ [11]

$$S_{0F}=i\frac{1+\not{v}}{2(p_l+E_0+m_D+i\epsilon)}, \tag{7}$$

and

$$S_{1F}=i\left[\frac{(-E_1+p_t^2/2)(1+\not{v})}{2(p_l+E_0+m_D+i\epsilon)^2}+\frac{\not{p}_t}{2(p_l+E_0+m_D+i\epsilon)}-\frac{1-\not{v}}{4}\right]. \tag{8}$$

It can be shown that the light diquark propagator to $1/m_Q$ still keeps its form in the limit $m_Q\to\infty$,

$$S_D=\frac{i}{p_l^2-W_p^2+i\epsilon}, \tag{9}$$

where $W_p\equiv\sqrt{p_t^2+m_D^2}$ (we have defined $p_t^2\equiv-p_t\cdot p_t$).

Similarly to Eq. (6), we write $\chi_P(p)$ and $K(P,p,q)$ in the following form (to order $1/m_Q$):

$$\chi_P(p)=\chi_{0P}(p)+\frac{1}{m_Q}\chi_{1P}(p),$$
$$K(P,p,q)=K_0(P,p,q)+\frac{1}{m_Q}K_1(P,p,q), \tag{10}$$

where $\chi_{1P}(p)$ and $K_1(P,p,q)$ arise from $1/m_Q$ corrections. As in our previous work, we assume the kernel contains a scalar confinement term and a one-gluon-exchange term. Hence we have

$$-iK_0=I\otimes IV_1+v_\mu\otimes(p_2+p_2')^\mu V_2,$$
$$-iK_1=I\otimes IV_3+\gamma_\mu\otimes(p_2+p_2')^\mu V_4, \tag{11}$$

where v_μ in K_0 appears because of the heavy quark symmetry.

Substituting Eqs. (6) and (10) into the BS equation (3) we have the integral equations for $\chi_{0P}(p)$ and $\chi_{1P}(p)$

$$\chi_{0P}(p)=S_{0F}(\lambda_1P+p)\int\frac{d^4q}{(2\pi)^4}K_0(P,p,q)\times\chi_{0P}(q)S_D(-\lambda_2P+p), \tag{12}$$

and

$$\chi_{1P}(p)=S_{0F}(\lambda_1P+p)\int\frac{d^4q}{(2\pi)^4}K_1(P,p,q)\times\chi_{0P}(q)S_D(-\lambda_2P+p)+S_{1F}(\lambda_1P+p)\times\int\frac{d^4q}{(2\pi)^4}K_0(P,p,q)\chi_{0P}(q)S_D(-\lambda_2P+p)+S_{0F}(\lambda_1P+p)\int\frac{d^4q}{(2\pi)^4}K_0(P,p,q)\times\chi_{1P}(q)S_D(-\lambda_2P+p). \tag{13}$$

Equation (12) is what we obtained in the limit $m_Q\to\infty$, which together with Eq. (7) gives

$$\not{v}\chi_{0P}(p)=\chi_{0P}(p), \tag{14}$$

since $\not{v}\not{v}=v^2=1$ and so $\not{v}S_{0F}=S_{0F}$. Therefore, $S_{0F}(\lambda_1P+p)\gamma_\mu\chi_{0P}(q)=S_{0F}(\lambda_1P+p)v_\mu\chi_{0P}(q)$ in the first term of Eq. (13). So to order $1/m_Q$, the Dirac matrix γ_μ from the one-gluon-exchange term in $K_1(P,p,q)$ can still be replaced by v_μ.

We divide $\chi_{1P}(p)$ into two parts by defining

$$\chi_{1P}(p)=\chi^+_{1P}(p)+\chi^-_{1P}(p),\quad \not{v}\chi^\pm_{1P}(p)=\pm\chi^\pm_{1P}(p), \tag{15}$$

i.e., $\chi^+_{1P}(p)\equiv\frac{1}{2}[\chi_{1P}(p)+\not{v}\chi_{1P}(p)]$ and $\chi^-_{1P}(p)\equiv\frac{1}{2}[\chi_{1P}(p)-\not{v}\chi_{1P}(p)]$. Multiplying Eq. (13) with either $(1+\not{v})/2$ or $(1-\not{v})/2$ and using Eqs. (7),(8),(9),(11) and (14), we obtain the following integral equations for $\chi^+_{1P}(p)$ and $\chi^-_{1P}(p)$:

$$\chi^+_{1P}(p)=-\frac{1}{(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}\times\int\frac{d^4q}{(2\pi)^4}K_0(P,p,q)\chi^+_{1P}(q)-\frac{1}{(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}\times\int\frac{d^4q}{(2\pi)^4}\left[K_1(P,p,q)+\frac{p_t^2/2-E_1}{p_l+E_0+m_D+i\epsilon}\times K_0(P,p,q)\right]\chi_{0P}(q), \tag{16}$$

$$\chi^-_{1P}(p)=-\frac{\not{p}_t}{2(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}\times\int\frac{d^4q}{(2\pi)^4}K_0(P,p,q)\chi_{0P}(q). \tag{17}$$

After writing down all the possible terms for $\chi_{0P}(p)$ and $\chi^\pm_{1P}(p)$, and considering the constraints on them, Eqs. (14) and (15), we obtain that

$$\chi_{0P}(p)=\phi_{0P}(p)u_{\Lambda_Q}(v,s),$$
$$\chi^+_{1P}(p)=\phi_{1P}(p)u_{\Lambda_Q}(v,s), \tag{18}$$
$$\chi^-_{1P}(p)=\phi_{2P}(p)\not{p}_tu_{\Lambda_Q}(v,s),$$

where $\phi_{0P}(p)$, $\phi_{1P}(p)$ and $\phi_{2P}(p)$ are Lorentz scalar functions.

Substituting Eq. (18) into Eqs. (12),(16),(17) and using Eqs. (7),(8),(9),(11) and (14) we have

$$\phi_{0P}(p)=-\frac{1}{(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}\times\int\frac{d^4q}{(2\pi)^4}K_0(P,p,q)\phi_{0P}(q), \tag{19}$$

$$\phi_{1P}(p)=-\frac{1}{(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}\times\int\frac{d^4q}{(2\pi)^4}K_0(P,p,q)\phi_{1P}(q)-\frac{1}{(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}$$

$$\times \int \frac{d^4q}{(2\pi)^4}\left[K_1(P,p,q)+\frac{p_t^2/2-E_1}{p_l+E_0+m_D+i\epsilon}\right.$$
$$\left.\times K_0(P,p,q)\right]\phi_{0P}(q), \tag{20}$$

and

$$\phi_{2P}(p)=-\frac{1}{2(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}$$
$$\times \int \frac{d^4q}{(2\pi)^4}K_0(P,p,q)\phi_{0P}(q). \tag{21}$$

Equations (19) and (21) lead to

$$\phi_{2P}(p)=\frac{1}{2}\phi_{0P}(p). \tag{22}$$

$\phi_{0P}(p)$ is the BS scalar function in the leading order of the $1/m_Q$ expansion, which was calculated in [11]. From Eq. (22) $\phi_{2P}(p)$ can be given in terms of $\phi_{0P}(p)$. The numerical solutions for $\phi_{0P}(p)$ and $\phi_{1P}(p)$ can be obtained by discretizing the integration region into n pieces (with n sufficiently large). In this way, the integral equations become matrix equations and the BS scalar functions $\phi_{0P}(p)$ and $\phi_{1P}(p)$ become n dimensional vectors. Thus $\phi_{0P}(p)$ is the solution of the eigenvalue equation $(A-I)\phi_0=0$, where A is an $n\times n$ matrix corresponding to the right hand side of Eq. (19). In order to have a unique solution for the ground state, the rank of $(A-I)$ should be $n-1$. From Eq. (20), $\phi_{1P}(p)$ is the solution of $(A-I)\phi_1=B$, where B is an n dimensional vector corresponding to the second integral term on the right hand side of Eq. (20). In order to have solutions for $\phi_{1P}(p)$, the rank of the augmented matrix $(A-I,B)$ should be equal to that of $(A-I)$, i.e., B can be expressed as linear combination of the $n-1$ linearly independent columns in $(A-I)$. This is difficult to guarantee if $B\neq 0$, since the way to divide $(A-I)$ into n columns is arbitrary. Therefore, we demand the following condition in order to have solutions for $\phi_{1P}(p)$

$$\int \frac{d^4q}{(2\pi)^4}\left[K_1(P,p,q)+\frac{p_t^2/2-E_1}{p_l+E_0+m_D+i\epsilon}\right.$$
$$\left.\times K_0(P,p,q)\right]\phi_{0P}(q)=0. \tag{23}$$

Equation (23) is a constraint we impose on the $O(1)$ and $O(1/m_Q)$ parts of the BS equation kernel, $K_1(P,p,q)$ and $K_0(P,p,q)$, under which we have solutions for $\phi_{1P}(p)$. In fact, $K_0(P,p,q)$ and $K_1(P,p,q)$ are the sum of two particle irreducible diagrams and both of them are determined by the complicated nonperturbative interactions between the heavy quark and the light diquark. As we cannot solve these kernels from the first principles of QCD we have to make a phenomenological model. The form assumed for $K_0(P,p,q)$ was given in [11] in the covariant instantaneous approximation. Equation (23) constraints the form of $K_1(P,p,q)$. The simplest $K_1(P,p,q)$ which satisfies Eq. (23) is $[(a-p_t^2/2)/(p_l+E_0+m_D+i\epsilon)]K_0(P,p,q)$, where a is a parameter in $K_1(P,p,q)$ which should be equal to E_1. However, as will be seen later, once Eq. (23) is satisfied, the physical results do not depend on the explicit form of $K_1(P,p,q)$.

We would like to stress that Eq. (23) is just one possibility we have found which guarantees that we have solutions for $\phi_{1P}(p)$. There may be other possible forms for $K_1(P,p,q)$ which can also lead to solutions for $\phi_{1P}(p)$, and whether or not Eq. (23) is a reasonable hypothesis should be tested by experiments.

With Eq. (23), $\phi_{1P}(p)$ satisfies the same eigenvalue equation as $\phi_{0P}(p)$. Therefore, we have

$$\phi_{1P}(p)=\sigma\phi_{0P}(p), \tag{24}$$

where σ is a constant of proportionality, with mass dimension, which can be determined by Luke's theorem [22] at the zero-recoil point in HQET. We will discuss it in the next section.

Since both $\phi_{1P}(p)$ and $\phi_{2P}(p)$ can be related to $\phi_{0P}(p)$, we can calculate the $1/m_Q$ corrections without explicitly solving the integral equations for $\phi_{1P}(p)$ and $\phi_{2P}(p)$. In the previous work [11] $\phi_{0P}(p)$ was solved by assuming that V_1 and V_2 in Eq. (11) arise from linear confinement and one-gluon-exchange terms, respectively. In the covariant instantaneous approximation, $\tilde{V}_i\equiv V_i|_{p_l=q_l}$, $i=1,2$, we find

$$\begin{aligned}\tilde{V}_1&=\frac{8\pi\kappa}{[(p_t-q_t)^2+\mu^2]^2}-(2\pi)^3\delta^3(p_t-q_t)\\&\quad\times\int\frac{d^3k}{(2\pi)^3}\frac{8\pi\kappa}{(k^2+\mu^2)^2},\\ \tilde{V}_2&=-\frac{16\pi}{3}\frac{\alpha_s^{(\mathrm{eff})2}Q_0^2}{[(p_t-q_t)^2+\mu^2][(p_t-q_t)^2+Q_0^2]},\end{aligned} \tag{25}$$

where κ and $\alpha_s^{(\mathrm{eff})}$ are coupling parameters related to scalar confinement and the one-gluon-exchange diagram, respectively. They can be related to each other when we solve the eigenvalue equation for $\phi_{0P}(p)$. The parameter μ is introduced to avoid the infrared divergence in numerical calculations, and the limit $\mu\to 0$ is taken in the end. It should be noted that in $\tilde{V}_2$ we introduced an effective form factor, $F(Q^2)=\alpha_s^{(\mathrm{eff})}Q_0^2/(Q^2+Q_0^2)$, to describe the internal structure of the light diquark [23].

Defining $\tilde{\phi}_{0P}(p_t)=\int(dp_l/2\pi)\phi_{0P}(p)$ the BS equation for $\tilde{\phi}_{0P}(p_t)$ is [11]

$$\tilde{\phi}_{0P}(p_t)=-\frac{1}{2(E_0-W_p+m_D)W_p}\int\frac{d^3q_t}{(2\pi)^3}$$
$$\times(\tilde{V}_1-2W_p\tilde{V}_2)\tilde{\phi}_{0P}(q_t), \tag{26}$$

in the covariant instantaneous approximation. The numerical results for $\tilde{\phi}_{0P}(k_t)$ can be obtained from Eq. (26), with the overall normalization constant being fixed by the normalization of the Isgur-Wise function at the zero-recoil point [11]. Furthermore, $\phi_{0P}(p)$ is expressed in terms of $\tilde{\phi}_{0P}(q_t)$:

$$\phi_{0P}(p)=\frac{i}{(p_l+E_0+m_D+i\epsilon)(p_l^2-W_p^2+i\epsilon)}\times\int\frac{d^3q_t}{(2\pi)^3}(\tilde{V}_1+2p_l\tilde{V}_2)\tilde{\phi}_{0P}(q_t). \tag{27}$$

III. $\Lambda_b\to\Lambda_c$ FORM FACTORS TO $1/m_Q$

In this section we will express the six form factors for the $\Lambda_b\to\Lambda_c$ weak transition in terms of the BS wave function and show the consistency between our model and HQET.

On the grounds of Lorentz invariance, the matrix element for $\Lambda_b\to\Lambda_c$ can be expressed as

$$\langle\Lambda_c(v')|J_\mu|\Lambda_b(v)\rangle=\bar{u}_{\Lambda_c}(v')[F_1(\omega)\gamma_\mu+F_2(\omega)v_\mu+F_3(\omega)v'_\mu-(G_1(\omega)\gamma_\mu+G_2(\omega)v_\mu+G_3(\omega)v'_\mu)\gamma_5]u_{\Lambda_b}(v), \tag{28}$$

where J_μ is the $V-A$ weak current, v and v' are the velocities of Λ_b and Λ_c, respectively, and $\omega=v'\cdot v$.

The form factors F_i and $G_i(i=1,2,3)$ are related to each other by the following equations, to order $1/m_Q$, when HQET is applied [19]:

$$F_1=G_1\left[1+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\frac{\bar{\Lambda}}{1+\omega}\right],$$

$$F_2=G_2=-G_1\frac{1}{m_c}\frac{\bar{\Lambda}}{1+\omega}, \tag{29}$$

$$F_3=-G_3=-G_1\frac{1}{m_b}\frac{\bar{\Lambda}}{1+\omega},$$

where $\bar{\Lambda}$ is an unknown parameter which is defined as the mass difference $m_{\Lambda_Q}-m_Q$ in the limit $m_Q\to\infty$.

On the other hand, the transition matrix element of $\Lambda_b\to\Lambda_c$ is related to the BS wave functions of Λ_b and Λ_c by the following equation:

$$\langle\Lambda_c(v')|J_\mu|\Lambda_b(v)\rangle=\int\frac{d^4p}{(2\pi)^4}\bar{\chi}_{P'}(p')\gamma_\mu(1-\gamma_5)\chi_P(p)S_D^{-1}(p_2), \tag{30}$$

where $P(P')$ is the momentum of Λ_b (Λ_c) and p' is the relative momentum defined in the BS wave function of $\Lambda_c(v')$, $\bar{\chi}_{P'}(p')$, which can also be expressed in terms of the three BS scalar functions $\phi_{0P}(p)$, $\phi_{1P}(p)$ and $\phi_{2P}(p)$ in Eq. (18)

$$\bar{\chi}_P(p)=\bar{u}_{\Lambda_Q}(v,s)\left\{\phi_{0P}(p)+\frac{1}{m_Q}[\phi_{1P}(p)+\phi_{2P}(p)\not{p}_t]\right\}. \tag{31}$$

Substituting Eqs. (18) and (31) into Eq. (30) and using the relations in Eq. (29) we find the following results by comparing the γ_μ, $\gamma_\mu\gamma_5$, $v_\mu(1-\gamma_5)$ and $v'_\mu(1+\gamma_5)$ terms, respectively:

$$G_1\left[1+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\frac{\bar{\Lambda}}{1+\omega}\right]=-i\int\frac{d^4k}{(2\pi)^4}\left\{\phi_{0P'}(k')\phi_{0P}(k)(k_l^2-W_k^2)+\frac{1}{m_c}[\phi_{1P'}(k')-(k'_l+m_D)\phi_{2P'}(k')]\times\phi_{0P}(k)(k_l^2-W_k^2)+\frac{1}{m_c}(-f_1+f_2+2m_DF)+\frac{1}{m_b}(f_1-f_2)+\frac{1}{m_b}\phi_{0P'}(k')\times[\phi_{1P}(k)-(k_l+m_D)\phi_{2P}(k)](k_l^2-W_k^2)\right\}+O(1/m_Q^2), \tag{32}$$

$$G_1=-i\int\frac{d^4k}{(2\pi)^4}\left\{\phi_{0P'}(k')\phi_{0P}(k)(k_l^2-W_k^2)+\frac{1}{m_c}(f_1+f_2)+\frac{1}{m_c}[\phi_{1P'}(k')-(k'_l+m_D)\phi_{2P'}(k')]\phi_{0P}(k)(k_l^2-W_k^2)+\frac{1}{m_b}(f_1+f_2)+\frac{1}{m_b}\phi_{0P'}(k')[\phi_{1P}(k)-(k_l+m_D)\phi_{2P}(k)](k_l^2-W_k^2)\right\}+O(1/m_Q^2), \tag{33}$$

$$\frac{1}{m_c}\left[iG_1\frac{\bar{\Lambda}}{1+\omega}+2(f_1-m_DF)\right]=O(1/m_Q^2), \tag{34}$$

$$\frac{1}{m_b}\left[iG_1\frac{\overline{\Lambda}}{1+\omega}+2f_2\right]=O(1/m_Q^2), \tag{35}$$

where we have defined f_1, f_2 and F by the following equations, on the grounds of Lorentz invariance:

$$\int\frac{d^4k}{(2\pi)^4}\phi_{2P'}(k')\phi_{0P}(k)(k_l^2-W_k^2)=F, \tag{36}$$

$$\int\frac{d^4k}{(2\pi)^4}\phi_{2P'}(k')\phi_{0P}(k)k^\mu(k_l^2-W_k^2)=f_1v^\mu+f_2v'^\mu. \tag{37}$$

Equation (37) leads to

$$f_1+f_2=\frac{1}{1+\omega}\int\frac{d^4k}{(2\pi)^4}\phi_{2P'}(k')\phi_{0P}(k)(k_l^2-W_k^2)(v\cdot k+v'\cdot k). \tag{38}$$

Equations (32) and (33) give the expression for G_1 to order $1/m_Q$. From Eqs. (34) and (35) we can see that Eq. (32) is the same as Eq. (33). Therefore, we can calculate G_1 to $1/m_Q$ from either of these two equations. This indicates that our model is consistent with HQET to order $1/m_Q$.

Substituting Eq. (38) into Eq. (33) and using Eq. (22) we have

$$\begin{aligned}G_1=&-i\int\frac{d^4k}{(2\pi)^4}\Bigg\{\phi_{0P'}(k')\phi_{0P}(k)(k_l^2-W_k^2)+\frac{1}{m_c}\left[\phi_{1P'}(k')-\frac{1}{2}(k_l'+m_D)\phi_{0P'}(k')\right]\phi_{0P}(k)(k_l^2-W_k^2)+\frac{1}{m_b}\phi_{0P'}(k')\\&\times\left[\phi_{1P}(k)-\frac{1}{2}(k_l+m_D)\phi_{0P}(k)\right](k_l^2-W_k^2)+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\frac{1}{2(1+\omega)}\phi_{0P'}(k')\phi_{0P}(k)(k_l^2-W_k^2)(v\cdot k+v'\cdot k)\Bigg\}.\end{aligned} \tag{39}$$

The first term in Eq. (39) gives the Isgur-Wise function which was calculated in our earlier work [11]. In order to obtain the $1/m_Q$ corrections, we have to fix $\phi_{1P}(k)$. Fortunately, this can be done by applying Luke's theorem [22]. The conservation of vector current in the case of equal masses for the initial and final heavy quarks leads to

$$G_1(\omega=1)=1+O(1/m_Q^2). \tag{40}$$

Now we consider Eq. (39) at the zero-recoil point, $\omega=1$, at which $P'=P$ and $k'=k$. Since the first term in Eq. (39) is the Isgur-Wise function, this term is 1 when $\omega=1$. From Eq. (40) the $1/m_c$ and $1/m_b$ terms in Eq. (39) should be zero at the zero-recoil point. Substituting Eq. (24) into Eq. (39) and noticing that $v\cdot k+v'\cdot k=2(k_l+m_D)+O(1/m_Q)$ when $\omega=1$ [see Eq. (4)], we can show that

$$\sigma=0. \tag{41}$$

Therefore, $\phi_{1P}(k)$ does not contribute to G_1.

Now we calculate G_1 through Eq. (39). Since in the weak transition the diquark acts as a spectator, its momentum in the initial and final baryons should be the same, $p_2=p_2'$. Then we can show that to order $1/m_Q$

$$k_l'v'+k_t'=k_lv+k_t. \tag{42}$$

From Eq. (42) we can obtain relations between k_l', k_t' and k_l, k_t straightforwardly:

$$\begin{aligned}k_l'&=k_l\omega-k_t\sqrt{\omega^2-1}\cos\theta,\\k_t'^2&=k_t^2+k_t^2(\omega^2-1)\cos^2\theta+k_l^2(\omega^2-1)\\&\quad-2k_lk_t\omega\sqrt{\omega^2-1}\cos\theta,\end{aligned} \tag{43}$$

where θ is defined as the angle between k_t and v_t'.

Substituting the relation between $\phi_{0P}(p)$ and $\tilde{\phi}_{0P}(p_t)$ [Eq. (27)] into Eq. (39), using the BS equation (26), and integrating the k_l component by selecting the proper contour we have

$$G_1(\omega)=\xi(\omega)+\frac{1}{m_c}A_c(\omega)+\frac{1}{m_b}A_b(\omega), \tag{44}$$

where

$$\xi(\omega)=-\int\frac{d^3k_t}{(2\pi)^3}F(\omega,k_t), \tag{45}$$

$$\begin{aligned}A_c(\omega)=&-\int\frac{d^3k_t}{(2\pi)^3}\frac{(\omega^2-1)W_k+\omega k_t\sqrt{\omega^2-1}\cos\theta}{2(\omega+1)}\\&\times F(\omega,k_t),\end{aligned} \tag{46}$$

$$A_b(\omega)=\int\frac{d^3k_t}{(2\pi)^3}\frac{k_t\sqrt{\omega^2-1}\cos\theta}{2(\omega+1)}F(\omega,k_t), \tag{47}$$

and $F(\omega,k_t)$ is defined as

$$F(\omega,k_t)=\frac{\tilde{\phi}_{0P}(k_t)}{E_0+m_D-\omega W_k-k_t\sqrt{\omega^2-1}\cos\theta}\times\int\frac{d^3r_t}{(2\pi)^3}\tilde{\phi}_{0P'}(r_t)[\tilde{V}_1(k_t'-r_t)-2(\omega W_k+k_t\sqrt{\omega^2-1}\cos\theta)\tilde{V}_2(k_t'-r_t)]|_{k_l=-W_k}. \quad (48)$$

The three dimensional integrations in Eqs. (45)–(47) can be reduced to one dimensional integrations by using the following identities:

$$\int\frac{d^3q_t}{(2\pi)^3}\frac{\rho(q_t^2)}{[(p_t-q_t)^2+\mu^2]^2}=\int\frac{q_t^2dq_t}{4\pi^2}\frac{2\rho(q_t^2)}{(p_t^2+q_t^2+\mu^2)^2-4p_t^2q_t^2}, \quad (49)$$

and

$$\int\frac{d^3q_t}{(2\pi)^3}\frac{\rho(q_t^2)}{(p_t-q_t)^2+\delta^2}=\int\frac{q_t^2dq_t}{4\pi^2}\frac{\rho(q_t^2)}{2|p_t||q_t|}\ln\frac{(|p_t|+|q_t|)^2+\delta^2}{(|p_t|-|q_t|)^2+\delta^2}, \quad (50)$$

where $\rho(q_t^2)$ is some arbitrary function of q_t^2.

In our model we have several parameters, $\alpha_s^{(\mathrm{eff})}$, κ, Q_0^2, m_D, E_0 and E_1. The parameter Q_0^2 can be chosen as 3.2 GeV^2 from the data for the electromagnetic form factor of the proton [23]. As discussed in Ref. [11], we let κ vary in the region between 0.02 GeV^3 and 0.1 GeV^3. In HQET, the binding energies should satisfy the constraint Eq. (5). Note that m_D+E_0 and E_1 are independent of the flavor of the heavy quark. From the BS equation solutions in the meson case, it has been found that the values $m_b=5.02$ GeV and $m_c=1.58$ GeV give predictions which are in good agreement with experiments [8]. Since in the b-baryon case the $O(1/m_b^2)$ corrections are very small, we use the following equation to discuss the relations among m_D, E_0 and E_1,

$$m_D+E_0+\frac{1}{m_b}E_1=0.62\ \ \mathrm{GeV}, \quad (51)$$

where we have used $m_{\Lambda_b}=5.64$ GeV. The parameter m_D cannot be determined, although there are suggestions from the analysis of valence structure functions that it should be around 0.7 GeV for non-strange scalar diquarks [24]. Hence we let it vary within some reasonable range, 0.65–0.75 GeV. In the expansion with respect to the heavy quark mass, we roughly expect $[(1/m_b)E_1]/E_0\sim\Lambda_{\mathrm{QCD}}/m_b$. Therefore, E_1 should be of the order $\Lambda_{\mathrm{QCD}}E_0$. In our numerical calculations, we let $\beta(=E_1/E_0)$ change between 0.2 and 1.0. Then

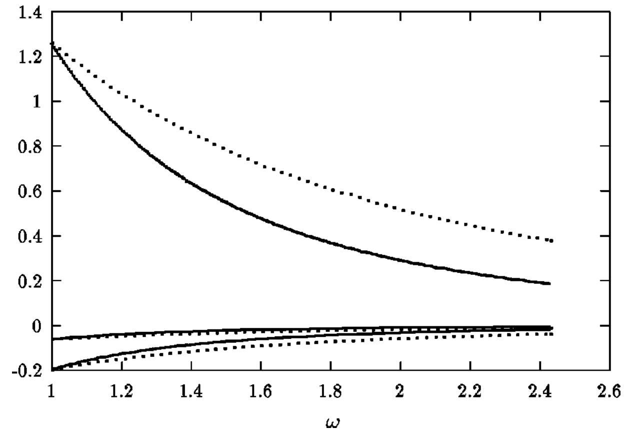

FIG. 1. The numerical results for $F_i(i=1,2,3)$ for $\kappa=0.02$ GeV^3 (solid lines) and $\kappa=0.10$ GeV^3 (dotted lines), with $m_D=0.7$ GeV. From top to bottom we have F_1, F_3, and F_2, respectively.

for some values of m_D and β we can determine E_0. Using Eqs. (44)–(50) and Eq. (29) we obtain numerical results for the weak decay form factors F_i, $G_i(i=1,2,3)$ to order $1/m_Q$. It turns out that the numerical results are very insensitive to the value of β, so we ignore this dependence. We also find that the dependence of F_i, G_i $(i=1,2,3)$ on the diquark mass m_D is not strong. In Fig. 1 we plot the numerical results for $F_i(i=1,2,3)$ for $\kappa=0.02$ GeV^3 and $\kappa=0.10$ GeV^3, respectively, with $m_D=0.7$ GeV.

IV. APPLICATIONS TO $\Lambda_b\to\Lambda_c l\bar{\nu}$ AND $\Lambda_b\to\Lambda_c P(V)$

With the numerical results for F_i, $G_i(i=1,2,3)$ to $1/m_Q$ obtained in Sec. III, we can predict the $\Lambda_b\to\Lambda_c$ semileptonic and nonleptonic weak decay widths to order $1/m_Q$. Since the QCD corrections to these form factors are comparable with the $1/m_Q$ effects, we will include both of them to give phenomenological predictions.

Neubert [25] has shown that the QCD corrections to the weak decay form factors can be written in the following form (up to corrections of the order $\alpha_s\bar{\Lambda}/m_Q$):

$$\Delta F_1=\xi\frac{\alpha_s(\bar{m})}{\pi}v_1, \quad \Delta G_1=\xi\frac{\alpha_s(\bar{m})}{\pi}a_1,$$
$$\Delta F_i=-\xi\frac{\alpha_s(\bar{m})}{\pi}v_i, \quad \Delta G_i=-\xi\frac{\alpha_s(\bar{m})}{\pi}a_i, \quad (i=2,3), \quad (52)$$

where $v_i=v_i(\omega)$ and $a_i=a_i(\omega)(i=1,2,3)$ are the QCD corrections calculated from the next-to-leading order renormalization group improved perturbation theory. The scale $\bar{m}$ is chosen such that higher-order terms $(\alpha_s\ln(m_b/m_c))^n(n>1)$ do not contribute. Consequently, it is not necessary to apply a renormalization group summation as far as only numerical evaluations are concerned. It is shown that $\bar{m}$ can be chosen as $2m_bm_c/(m_b+m_c)\simeq 2.3$ GeV. The detailed formulas for v_i and a_i can be found in [25], which also includes a discussion on the infrared cutoff employed in the calculation of the vertex corrections. As in [25], we choose this cutoff to

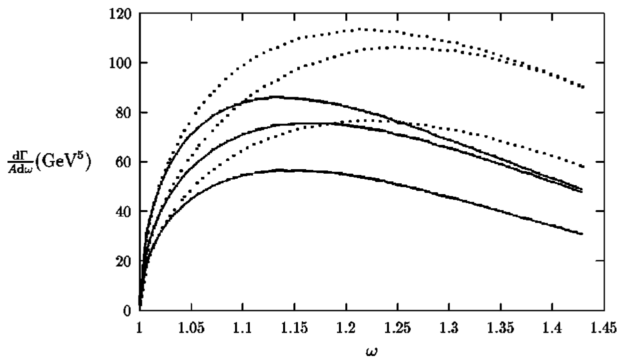

FIG. 2. The numerical results for $A^{-1}(\mathrm{d}\Gamma/\mathrm{d}\omega)$ for $\kappa=0.02$ GeV^3 (solid lines) and $\kappa=0.10$ GeV^3 (dotted lines), with $m_D=0.7$ GeV. From top to bottom we have the predictions without $1/m_Q$ and QCD corrections, with $1/m_Q$ corrections, and with both $1/m_Q$ and QCD corrections, respectively.

be 200 MeV which is a fictitious gluon mass. Furthermore, we use $\Lambda_{\mathrm{QCD}}=200$ MeV in our numerical calculations.

A. Semileptonic decays $\Lambda_b\to\Lambda_c l\bar{\nu}$

Making use of the general kinematical formulas by Köner and Krämer [26], we find for the differential decay width of $\Lambda_b\to\Lambda_c l\bar{\nu}$ [14]

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}\omega}=&\frac{2}{3}m_{\Lambda_c}^4 m_{\Lambda_b}AF_1^2\sqrt{\omega^2-1}\Bigg\{3\omega(\eta+\eta^{-1})-2-4\omega^2\\&-\bar{\Lambda}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)[3(\eta+\eta^{-1})-4-2\omega]+\frac{\alpha_s}{\pi}v_1(\omega-1)\\&\times[3(\eta+\eta^{-1})+2-4\omega]+\frac{\alpha_s}{\pi}a_1(\omega+1)[3(\eta+\eta^{-1})\\&-2-4\omega]-\frac{\alpha_s}{\pi}(\omega^2-1)[v_2(1+\eta)+v_3(1+\eta^{-1})\\&+a_2(1-\eta)+a_3(\eta^{-1}-1)]\Bigg\},\end{aligned}\tag{53}$$

where $\eta=m_{\Lambda_c}/m_{\Lambda_b}$ and $A=[G_F^2/(2\pi)^3]|V_{cb}|^2B(\Lambda_c\to ab)$, with $|V_{cb}|$ being the Kobayashi-Maskawa matrix element. $B(\Lambda_c\to ab)$ is the branching ratio for the decay $\Lambda_c\to a(\frac{1}{2}^+)+b(0^-)$ through which Λ_c is detected, since the structure for such decays is already well known. It should be noted that in Eq. (53) $O(\alpha_s\bar{\Lambda}/m_Q)$ corrections have been ignored and the lepton mass is set to zero. The plot for $A^{-1}(\mathrm{d}\Gamma/\mathrm{d}\omega)$ is shown in Fig. 2 for $m_D=700$ MeV, where we also show explicitly the effects of both $1/m_Q$ and QCD corrections. For other values of m_D the results change only a little.

After integrating ω in Eq. (53) we have the total decay width for $\Lambda_b\to\Lambda_c l\bar{\nu}$. The numerical results are shown in Table I for $m_D=650$ MeV, 700 MeV, 750 MeV and for $\kappa=0.02$ GeV^3 (0.10 GeV^3). Γ_0, Γ_{1/m_Q} and $\Gamma_{1/m_Q+\mathrm{QCD}}$ are the decay widths without $1/m_Q$ and QCD corrections, with $1/m_Q$ corrections, and with both $1/m_Q$ and QCD corrections, respectively. We have used $V_{cb}=0.045$ in the numerical calculations.

TABLE I. Predictions for the decay rates for $\Lambda_b\to\Lambda_c l\bar{\nu}$, in units 10^{10} s$^{-1}B(\Lambda_c\to ab)$.

m_D(GeV)	Γ_0	Γ_{1/m_Q}	$\Gamma_{1/m_Q+\mathrm{QCD}}$
0.65	4.77 (7.20)	4.26 (6.62)	3.10 (4.76)
0.70	5.12 (7.12)	4.60 (6.56)	3.34 (4.72)
0.75	5.40 (7.02)	4.89 (6.50)	3.54 (4.67)

We can see from Fig. 2 and Table I that both $1/m_Q$ and QCD corrections reduce the decay width for $\Lambda_b\to\Lambda_c l\bar{\nu}$, and the QCD effects are even bigger. From Table I we can also see that the dependence of our predictions on m_D is not strong.[1]

B. Nonleptonic decays $\Lambda_b\to\Lambda_c P(V)$

In this subsection we will apply the numerical solutions for the form factors F_i, $G_i(i=1,2,3)$ to the nonleptonic decays $\Lambda_b\to\Lambda_c P(V)$($P$ and V stand for pseudoscalar and vector mesons respectively). The Hamiltonian describing such decays reads

$$H_{\mathrm{eff}}=\frac{G_F}{\sqrt{2}}V_{cb}V_{UD}^*(a_1O_1+a_2O_2),\tag{54}$$

with $O_1=(\bar{D}U)(\bar{c}b)$ and $O_2=(\bar{c}U)(\bar{D}b)$, where U and D are the fields for light quarks involved in the decay, and $(\bar{q}_1q_2)=\bar{q}_1\gamma_\mu(1-\gamma_5)q_2$ is understood. The parameters a_1 and a_2 are treated as free parameters since they involve hadronization effects. Since Λ_b decays are energetic, the factorization assumption is applied so that one of the currents in the Hamiltonian (54) is factorized out and generates a meson [27,28]. Thus the decay amplitude of the two body nonleptonic decay becomes the product of two matrix elements, one is related to the decay constant of the factorized meson (P or V) and the other is the weak transition matrix element between Λ_b and Λ_c,

$$\begin{aligned}M^{\mathrm{fac}}(\Lambda_b\to\Lambda_cP(V))=&\frac{G_F}{\sqrt{2}}V_{cb}V_{UD}^*a_1\langle P(V)|A_\mu(V_\mu)|0\rangle\\&\times\langle\Lambda_c(P')|J^\mu|\Lambda_b(P)\rangle,\end{aligned}\tag{55}$$

where $\langle 0|A_\mu(V_\mu)|P(V)\rangle$ are related to the decay constants of the pseudoscalar meson or vector meson by

$$\langle 0|A_\mu|P\rangle=if_Pq_\mu,\tag{56}$$

[1] We note that the results without either $1/m_Q$ and QCD corrections in Table I are bigger than those presented in Ref. [11] by about 18%. This is because we employed a cutoff in the numerical integrations in Ref. [11], while the integrations are carried out to infinity in the present work.

$$\langle 0|V_\mu|V\rangle = f_V m_V \epsilon_\mu,$$

where q_μ is the momentum of the meson emitted from the W-boson and ϵ_μ is the polarization vector of the emitted vector meson. It is noted that in the two-body nonleptonic weak decays $\Lambda_b \to \Lambda_c P(V)$ there is no contribution from the a_2 term since such a term corresponds to the transition of Λ_b to a light baryon instead of Λ_c.

On the other hand, the general form for the amplitudes of $\Lambda_b \to \Lambda_c P(V)$ are

$$\begin{aligned} M(\Lambda_b \to \Lambda_c P) &= i\bar{u}_{\Lambda_c}(P')(A + B\gamma_5)u_{\Lambda_b}(P), \\ M(\Lambda_b \to \Lambda_c V) &= \bar{u}_{\Lambda_c}(P')\epsilon^{*\mu}(A_1\gamma_\mu\gamma_5 + A_2 P'_\mu\gamma_5 \\ &\quad + B_1\gamma_\mu + B_2 P'_\mu)u_{\Lambda_b}(P). \end{aligned} \tag{57}$$

Alternatively, the matrix element for $\Lambda_b \to \Lambda_c$ can be expressed as the following on the ground of Lorentz invariance:

$$\begin{aligned} \langle \Lambda_c(P')|J_\mu|\Lambda_b(P)\rangle = \bar{u}_{\Lambda_c}(P')[&f_1(q^2)\gamma_\mu + i f_2(q^2)\sigma_{\mu\nu}q^\nu \\ &+ f_3(q^2)q_\mu - (g_1(q^2)\gamma_\mu \\ &+ i g_2(q^2)\sigma_{\mu\nu}q^\nu \\ &+ g_3(q^2)q_\mu)\gamma_5]u_{\Lambda_b}(P), \end{aligned} \tag{58}$$

where f_i, $g_i (i=1,2,3)$ are the Lorentz scalars. The relations between f_i, g_i and F_i, G_i are

$$\begin{aligned} f_1 &= F_1 + \frac{1}{2}(m_{\Lambda_b} + m_{\Lambda_c})\left(\frac{F_2}{m_{\Lambda_b}} + \frac{F_3}{m_{\Lambda_c}}\right), \\ f_2 &= \frac{1}{2}\left(\frac{F_2}{m_{\Lambda_b}} + \frac{F_3}{m_{\Lambda_c}}\right), \\ f_3 &= \frac{1}{2}\left(\frac{F_2}{m_{\Lambda_b}} - \frac{F_3}{m_{\Lambda_c}}\right), \\ g_1 &= G_1 - \frac{1}{2}(m_{\Lambda_b} - m_{\Lambda_c})\left(\frac{G_2}{m_{\Lambda_b}} + \frac{G_3}{m_{\Lambda_c}}\right), \\ g_2 &= \frac{1}{2}\left(\frac{G_2}{m_{\Lambda_b}} + \frac{G_3}{m_{\Lambda_c}}\right), \\ g_3 &= \frac{1}{2}\left(\frac{G_2}{m_{\Lambda_b}} - \frac{G_3}{m_{\Lambda_c}}\right). \end{aligned} \tag{59}$$

The decay widths and the up-down asymmetries for $\Lambda_b \to \Lambda_c P(V)$ are available in Refs. [29,30]:

$$\begin{aligned} \Gamma(\Lambda_b \to \Lambda_c P) &= \frac{|\vec{P}'|}{8\pi}\left[\frac{(m_{\Lambda_b} + m_{\Lambda_c})^2 - m_P^2}{m_{\Lambda_b}^2}|A|^2 + \frac{(m_{\Lambda_b} - m_{\Lambda_c})^2 - m_P^2}{m_{\Lambda_b}^2}|B|^2\right], \\ \alpha(\Lambda_b \to \Lambda_c P) &= -\frac{2|\vec{P}'|\mathrm{Re}(A^*B)}{(E_{\Lambda_c} + m_{\Lambda_c})|A|^2 + (E_{\Lambda_c} - m_{\Lambda_c})|B|^2}, \end{aligned} \tag{60}$$

where A and B are related to the form factors by

$$\begin{aligned} A &= \frac{G_F}{\sqrt{2}} V_{cb} V_{UD}^* a_1 f_P [(m_{\Lambda_b} - m_{\Lambda_c}) f_1(m_P^2) + m_P^2 f_3(m_P^2)], \\ B &= \frac{G_F}{\sqrt{2}} V_{cb} V_{UD}^* a_1 f_P [(m_{\Lambda_b} + m_{\Lambda_c}) g_1(m_P^2) - m_P^2 g_3(m_P^2)], \end{aligned} \tag{61}$$

and

$$\begin{aligned} \Gamma(\Lambda_b \to \Lambda_c V) &= \frac{|\vec{P}'|}{8\pi}\frac{E_{\Lambda_c} + m_{\Lambda_c}}{m_{\Lambda_b}}\left[2(|S|^2 + |P_2|^2) + \frac{E_V^2}{m_V^2}(|S+D|^2 + |P_1|^2)\right], \\ \alpha(\Lambda_b \to \Lambda_c V) &= \frac{4m_V^2 \mathrm{Re}(S^* P_2) + 2E_V^2 \mathrm{Re}(S+D)^* P_1}{2m_V^2(|S|^2 + |P_2|^2) + E_V^2(|S+D|^2 + |P_1|^2)}, \end{aligned} \tag{62}$$

TABLE II. Predictions for the decay rates [in units $10^{10}\ \mathrm{s}^{-1} a_1^2$, which is defined in Eq. (54)], and the asymmetry parameters for $\Lambda_b \to \Lambda_c P(V)$.

Process	Γ_0	Γ_{1/m_Q}	$\Gamma_{1/m_Q+\mathrm{QCD}}$	$\alpha_{1/m_Q+\mathrm{QCD}}$
$\Lambda_b^0 \to \Lambda_c^+ \pi^-$	0.30 (0.56)	0.36 (0.67)	0.29 (0.55)	−1.00
$\Lambda_b^0 \to \Lambda_c^+ \rho^-$	0.44 (0.78)	0.51 (0.94)	0.42 (0.77)	−0.89
$\Lambda_b^0 \to \Lambda_c^+ D_s^-$	1.03 (1.57)	1.16 (1.81)	1.02 (1.59)	−0.98
$\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}$	0.78 (1.17)	0.89 (1.35)	0.76 (1.15)	−0.38
$\Lambda_b^0 \to \Lambda_c^+ K^-$	0.022 (0.039)	0.026 (0.048)	0.021 (0.039)	−1.00
$\Lambda_b^0 \to \Lambda_c^+ K^{*-}$	0.023 (0.041)	0.027 (0.049)	0.022 (0.040)	−0.85
$\Lambda_b^0 \to \Lambda_c^+ D^-$	0.037 (0.057)	0.042 (0.066)	0.036 (0.057)	−0.98
$\Lambda_b^0 \to \Lambda_c^+ D^{*-}$	0.027 (0.041)	0.031 (0.048)	0.026 (0.040)	−0.42

where

$$
\begin{aligned}
S &= -A_1,\\
D &= -\frac{|\vec{P}'|^2}{E_V(E_{\Lambda_c}+m_{\Lambda_c})}(A_1 - m_{\Lambda_b}A_2),\\
P_1 &= -\frac{|\vec{P}'|}{E_V}\left(\frac{m_{\Lambda_b}+m_{\Lambda_c}}{E_{\Lambda_c}+m_{\Lambda_c}}B_1 + m_{\Lambda_b}B_2\right),\\
P_2 &= \frac{|\vec{P}'|}{E_{\Lambda_c}+m_{\Lambda_c}}B_1,
\end{aligned}
\tag{63}
$$

with

$$
\begin{aligned}
A_1 &= -\frac{G_F}{\sqrt{2}}V_{cb}V_{UD}^* a_1 f_V m_V[g_1(m_V^2)+g_2(m_V^2)(m_{\Lambda_b}-m_{\Lambda_c})],\\
A_2 &= -2\frac{G_F}{\sqrt{2}}V_{cb}V_{UD}^* a_1 f_V m_V g_2(m_V^2),\\
B_1 &= \frac{G_F}{\sqrt{2}}V_{cb}V_{UD}^* a_1 f_V m_V[f_1(m_V^2)-f_2(m_V^2)(m_{\Lambda_b}+m_{\Lambda_c})],\\
B_2 &= 2\frac{G_F}{\sqrt{2}}V_{cb}V_{UD}^* a_1 f_V m_V f_2(m_V^2).
\end{aligned}
\tag{64}
$$

Then from Eqs. (59)–(64), we obtain the numerical results for the decay widths and asymmetry parameters. In Table II we list the results for $m_D = 0.70$ GeV. For other values of m_D, the results change only a little. The numbers without (with) brackets correspond to $\kappa = 0.02\ \mathrm{GeV}^3$ ($\kappa = 0.10\ \mathrm{GeV}^3$). Again, the subscripts "0", "$1/m_Q$", and "$1/m_Q + \mathrm{QCD}$" stand for the results without $1/m_Q$ and QCD corrections, with $1/m_Q$ corrections, and with both $1/m_Q$ and QCD corrections, respectively. In the calculations we have taken the following decay constants:

$$
f_\pi = 132\ \mathrm{MeV},\quad f_K = 156\ \mathrm{MeV},\quad f_D = 200\ \mathrm{MeV},
$$
$$
f_{D_s} = 241\ \mathrm{MeV},\quad f_\rho = 216\ \mathrm{MeV},\quad f_{K^*} = f_\rho,
$$
$$
f_D = f_{D^*},\quad f_{D_s} = f_{D_s^*}.
$$

Since the changes for the up-down asymmetries caused by $1/m_Q$ and QCD corrections are very small, in Table II we only listed $\alpha_{1/m_Q+\mathrm{QCD}}$. Furthermore, since to order $O(\alpha_s\bar{\Lambda}/m_Q)$ all the six form factors F_i, $G_i (i=1,2,3)$ can be expressed by one form factor, say F_1, which is canceled in α, the up-down asymmetries are model independent. Therefore, α does not depend on κ. It can be seen from Table II that the predictions for the decay widths show a strong dependence on the parameters κ in our model. In the future the experimental data will be used to fix this parameter and test our model.

In our previous work [13,14], the $\Lambda_b \to \Lambda_c$ semileptonic and nonleptonic decay widths were calculated using a hadronic wave function model in the infinite momentum frame by combining the Drell-Yan type overlap integrals and the results from HQET to order $1/m_Q$. Comparing the results in our present BS model with those in Refs. [13,14], we find that there is overlap between these two model predictions. The results with $\kappa = 0.02\ \mathrm{GeV}^3$ in the present model are close to those in Refs. [13,14] if the average transverse momentum of the heavy quark is chosen as 400 MeV.

The Cabibbo-allowed nonleptonic decay widths have also been calculated in the nonrelativistic quark model approach [29], where the form factors are calculated at the zero-recoil point and then extrapolated to other ω values under the assumption of a dipole behavior. It seems that the predictions in this model are close to those in our present work if we choose $\kappa = 0.02\ \mathrm{GeV}^3$.

V. SUMMARY AND DISCUSSION

In the present work, we assume that a heavy baryon Λ_Q is composed of a heavy quark, Q, and a scalar light diquark. Based on this picture, we analyze the $1/m_Q$ corrections to the BS equation for Λ_Q which was established in the limit $m_Q \to \infty$ in previous work [11]. We find that in addition to the one BS scalar function when $m_Q \to \infty$, two more scalar functions, $\phi_{1P}(p)$ and $\phi_{2P}(p)$, are needed at order $1/m_Q$. $\phi_{2P}(p)$ is related to $\phi_{0P}(p)$ directly [Eq. (22)]. Furthermore, with the aid of the reasonable constraint on the BS kernel at order $1/m_Q$, Eq. (23), and Luke's theorem, $\phi_{1P}(p)$ can also be related to the BS scalar function in the leading order.

Hence we do not need to solve explicitly for $\phi_{1P}(p)$ and $\phi_{2P}(p)$ anymore. The BS wave function in the leading order of $1/m_Q$ expansion was obtained numerically by assuming the kernel for the BS equation in the limit $m_Q \to \infty$ to consist of a scalar confinement term and a one-gluon-exchange term. On the other hand, all the six form factors for $\Lambda_b \to \Lambda_c$ are related to each other to order $1/m_Q$, as indicated from HQET. We determine these form factors by expressing them in terms of the BS wave functions. We also show explicitly that the results from our model are consistent with HQET to order $1/m_Q$. We also discuss the dependence of our numerical results on the various parameters in our model. It is found that F_i, $G_i(i=1,2,3)$ are insensitive to the binding energy, at order $1/m_Q$, and their dependence on the diquark mass, m_D, is mild. However, the numerical solutions are very sensitive to the parameter κ.

Furthermore, we apply our solutions for the weak decay form factors to calculate the differential and total decay widths for the semileptonic decays $\Lambda_b \to \Lambda_c l \bar{\nu}$, and the nonleptonic decay widths for $\Lambda_b \to \Lambda_c P(V)$. The QCD corrections are also included, and found to be comparable with the $1/m_Q$ corrections. Again the numerical results for the decay widths mostly depend on κ. We also compare our results with other models, including the hadronic wave function model and the norelativistic quark model, where $1/m_Q$ corrections are also included. Generally predictions from these models are consistent with each other if we take into account the range of model parameters. Data from the future experiments will help to fix the parameters and allow one to test these models.

Besides the uncertainties from the parameters in our model, higher order corrections such as $O(1/m_Q^2)$ and $O(\alpha_s \bar{\Lambda}/m_Q)$ will modify our results. However, we expect them to be small. Furthermore, we take a phenomenologically inspired form for the kernel of the BS equation and use the covariant instantaneous approximation while solving the BS equation. In addition, when we consider the BS equation at order $1/m_Q$, in order to have solutions for $\phi_{1P}(p)$, we assume Eq. (23) which gives the relation between the kernels $K_1(P,p,q)$ and $K_0(P,p,q)$. All these *Ansätze* should be tested by the forthcoming experiments.

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[1] OPAL Collaboration, R. Akers *et al.*, Z. Phys. C **69**, 195 (1996); Phys. Lett. B **353**, 402 (1995); OPAL Collaboration, K. Ackerstaff *et al.*, *ibid.* **426**, 161 (1998).
[2] UA1 Collaboration, C. Albarjar *et al.*, Phys. Lett. B **273**, 540 (1991).
[3] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **47**, 2639 (1993).
[4] S.E. Tzmarias, invited talk presented in the 27th International Conference on High Energy Physics, Glasgow, 1994; P. Abreu *et al.*, Phys. Lett. B **374**, 351 (1996).
[5] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **55**, 1142 (1997).
[6] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[7] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); H. Georgi, *ibid.* **264**, 447 (1991); see also M. Neubert, Phys. Rep. **245**, 259 (1994) for the review.
[8] H.-Y. Jin, C.-S. Huang, and Y.-B. Dai, Z. Phys. C **56**, 707 (1992); Y.-B. Dai, C.-S. Huang, and H.-Y. Jin, *ibid.* **60**, 527 (1993); **65**, 87 (1995).
[9] F. Hussain and G. Thompson, Phys. Lett. B **335**, 205 (1994).
[10] A. Abd El-Hady, K.S. Gupta, A.J. Sommerer, J. Spence, and J.P. Vary, Phys. Rev. D **51**, 5245 (1995).
[11] X.-H. Guo and T. Muta, Phys. Rev. D **54**, 4629 (1996); Mod. Phys. Lett. A **11**, 1523 (1996).
[12] X.-H. Guo, A.W. Thomas, and A.G. Williams, Phys. Rev. D **59**, 116007 (1999).
[13] X.-H. Guo, Mod. Phys. Lett. A **13**, 2265 (1998).
[14] X.-H. Guo and P. Kroll, Z. Phys. C **59**, 567 (1993).
[15] N. Isgur and M.B. Wise, Nucl. Phys. **B348**, 276 (1991).
[16] H. Georgi, Nucl. Phys. **B348**, 293 (1991).
[17] T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. **B355**, 38 (1991).
[18] F. Hussain, J.G. Körner, M. Krämmer, and G. Thompson, Z. Phys. C **51**, 321 (1991).
[19] H. Georgi, B. Grinstein, and M.B. Wise, Phys. Lett. B **252**, 456 (1990).
[20] A. Falk, Nucl. Phys. **B378**, 79 (1992).
[21] D. Lurie, *Particles and Fields* (Interscience Publishers, John Wiley & Sons, New York, London, Sydney, 1968); C. Itzykson and J.B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
[22] M.E. Luke, Phys. Lett. B **252**, 447 (1990).
[23] M. Anselmino, P. Kroll, and B. Pire, Z. Phys. C **36**, 89 (1987); P. Kroll, B. Quadder, and W. Schweiger, Nucl. Phys. **B316**, 373 (1988).
[24] F. Close and A.W. Thomas, Phys. Lett. B **212**, 227 (1988).
[25] M. Neubert, Nucl. Phys. **B371**, 149 (1992).
[26] J.G. Körner and M. Krämer, Phys. Lett. B **275**, 495 (1992).
[27] J.D. Bjorken, Nucl. Phys. B (Proc. Suppl.) **11**, 325 (1989).
[28] M.J. Dugan and B. Grinstein, Phys. Lett. B **255**, 583 (1991).
[29] H.Y. Cheng, Phys. Lett. B **289**, 455 (1992); Phys. Rev. D **56**, 2799 (1997); H.Y. Cheng and B. Tseng, *ibid.* **53**, 1457 (1996).
[30] S. Pakvasa, S.F. Tuan, and S.P. Rosen, Phys. Rev. D **42**, 3746 (1990).