Evaluation of Flow Forecasting Models for Adelaide Hills Catchments

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This Thesis embodies the results of supervised project work making up all of the work for the degree.

ERRATA

To qualify the comments made in Section [6.5.2] of the thesis, the following derivation is given for the solution of the [**A**] and [**B**] matrices for an AR(1) multisite model.

Chapter [3] sets out the development of the multisite generation equation, for both the annual case, and further, for the multiperiod, multisite case. The following derivation for the solution of the [**A**] and [**B**] matrices used in an AR(1) model is shown for the exact case to highlight the difference between the exact solution and the solution given in the thesis.

The general multisite model is given as –

$$[\mathbf{Z_t}] = [\mathbf{A}][\mathbf{Z_{t-1}}] + [\mathbf{B}][\epsilon_\mathbf{t}] \quad (1)$$

To derive the solution for the [**A**] matrix, both sides of Equation [1] are post-multiplied by $[\mathbf{Z_{t-1}}]^\mathbf{T}$ and expectations taken.

Thereby giving (using the notation given in Chapter 3) –

$$\mathbf{E}\{[\mathbf{Z_t}][\mathbf{Z_{t-1}}]^\mathbf{T}\} = [\mathbf{A}]\mathbf{E}\{[\mathbf{Z_{t-1}}][\mathbf{Z_{t-1}}]^\mathbf{T}\} + \mathbf{E}\{[\mathbf{B}][\epsilon_\mathbf{t}][\mathbf{Z_{t-1}}]^\mathbf{T}\} \quad (2)$$

$$\Rightarrow [\mathbf{A}] = \mathbf{E}\{[\mathbf{Z_t}][\mathbf{Z_{t-1}}]^\mathbf{T}\}\mathbf{E}\{[\mathbf{Z_{t-1}}][\mathbf{Z_{t-1}}]^\mathbf{T}\}^{-1} \quad (3)$$

$$[\mathbf{A}] = [\mathbf{M_1}][\mathbf{M_0^*}]^{-1}$$

$$where \quad [\mathbf{M_0^*}] = \mathbf{E}\{[\mathbf{Z_{t-1}}][\mathbf{Z_{t-1}}]^\mathbf{T}\}$$

Similarly, for the solution of the [**B**] matrix, both sides of Equation [1] are postmultiplied by $[\mathbf{Z_t}]^\mathbf{T}$ and expectations taken, giving –

$$\mathbf{E}\{[\mathbf{Z_t}][\mathbf{Z_t}]^\mathbf{T}\} = [\mathbf{A}]\mathbf{E}\{[\mathbf{Z_{t-1}}][\mathbf{Z_t}]^\mathbf{T}\} + \mathbf{E}\{[\mathbf{B}][\epsilon_\mathbf{t}][\mathbf{Z_t}]^\mathbf{T}\} \quad (4)$$

$$= [\mathbf{A}]\mathbf{E}\{[\mathbf{Z_{t-1}}][\mathbf{Z_t}]^\mathbf{T}\} + [\mathbf{B}]\mathbf{E}\{[\epsilon_\mathbf{t}][\mathbf{Z_{t-1}}]^\mathbf{T}[\mathbf{A}]^\mathbf{T} + [\epsilon_\mathbf{t}][\epsilon_\mathbf{t}]^\mathbf{T}[\mathbf{B}]^\mathbf{T}\} \quad (5)$$

By substituting Equation [3] for [A], and rearranging,

$$\Rightarrow [\mathbf{B}][\mathbf{B}]^{\mathbf{T}} = \mathbf{E}\{[\mathbf{Z_t}][\mathbf{Z_t}]^{\mathbf{T}}\} - \mathbf{E}\{[\mathbf{Z_t}][\mathbf{Z_{t-1}}]^{\mathbf{T}}\}\mathbf{E}\{[\mathbf{Z_{t-1}}][\mathbf{Z_{t-1}}]^{\mathbf{T}}\}^{-1}\mathbf{E}\{[\mathbf{Z_{t-1}}][\mathbf{Z_t}]^{\mathbf{T}}\}$$

$$[\mathbf{B}][\mathbf{B}]^{\mathbf{T}} = [\mathbf{M_0}] - [\mathbf{M_1}][\mathbf{M_0^*}]^{-1}[\mathbf{M_1}]^{\mathbf{T}}$$

The general matrix solution given by Equations [3.19] & [3.22], and further developed for the multiperiod model given by Equations [3.43] & [3.44] have assumed that the underlying process describing the distribution is stationary. This implies that the covariance matrix calculated using $[\mathbf{Z_t}][\mathbf{Z_t}]^{\mathbf{T}}$ is equal to the covariance matrix calculated using $[\mathbf{Z_{t-1}}][\mathbf{Z_{t-1}}]^{\mathbf{T}}$.

There is a subtle difference in using the latter form, and it has been noted by Kuzera [31] that for problems with a limited length of data, the above asymptotic solution may not hold true, and may lead to significantly different results.

By replacing $\quad [\mathbf{M_0^*}] = \mathbf{E}\{[\mathbf{Z_{t-1}}][\mathbf{Z_{t-1}}]^{\mathbf{T}}\}$

in lieu of $\quad [\mathbf{M_0}] = \mathbf{E}\{[\mathbf{Z_t}][\mathbf{Z_t}]^{\mathbf{T}}\}$

in Equations [3.19], [3.22],[3.43] & [3.44] the result will overcome matrix inconsistency for the solution of the [A] and [B] matrices.

The revised equations are thus –

For the general case,

$$[\mathbf{A}] = [\mathbf{M_1}][\mathbf{M_0^*}]^{-1} \tag{6}$$

$$[\mathbf{B}][\mathbf{B}]^{\mathbf{T}} = [\mathbf{M_0}] - [\mathbf{M_1}][\mathbf{M_0^*}]^{-1}[\mathbf{M_1}]^{\mathbf{T}} \tag{7}$$

and for the multiperiod case,

$$[\mathbf{A}_\tau] = [\mathbf{M}_{1,\tau}][\mathbf{M}^*_{0,\tau-1}]^{-1} \tag{8}$$

$$[\mathbf{B}_\tau][\mathbf{B}_\tau]^{\mathbf{T}} = [\mathbf{M}_{0,\tau}] - [\mathbf{M}_{1,\tau}][\mathbf{M}^*_{0,\tau-1}]^{-1}[\mathbf{M}_{1,\tau}]^{\mathbf{T}} \tag{9}$$

Kuczera [31] outlines a method to obtain consistent estimates of the [A] and [B] matrices when there is missing data in any of the records. This approach may have made better use of the streamflow records available in this study.

Furthermore, Crosby & Maddock [13] offer a solution technique to produce a consistent [A] and [B] matrix given a monotone sample (*i.e.* when continuous records have different sarting times).

REVISED TEXT

Page 3, Section 1.3, Paragraph 1: delete "at any point in time"

Page 40, Section 4.1, Paragraph 3: Replace "rain" with "precipitation".

Page 41, Replace "1700's (or 1800's)" with "1700s (or 1800s") respectively &,
Replace "world war one" (or two) with "World War One" (or Two).
(Also occurs on page 46)

Page 42, Figure [4.1]: Reference, South Australian Engineering &
Water Supply Department, publicity material (*Water Supply System)*.

Page 44, Figure [4.2]: Reference [12]
Crawley P.D. & Dandy G.C. (1989)
Optimal Operating Policies for Multiple Reservoir Systems
(University of Adelaide - Civil Engineering Department Report)

Page 58, Section 5.6.1.1, Paragraph 5:
Replace, "Although will not occur ...",
with "This will not occur ...".

Page 82, Figure [5.8], "Yields" measured in (Ml).

Page 106, Section 7.2.1, Paragraph 1: Remove "in toto".

Figures [5.3] to [5.5], The horizontal axis has the non dimensional units of
"Number of Standard Deviations from the Mean".

Tables [5.9] to [5.13], "Absolute Error" units are (Ml) for use in Tables.

Table [5.14], "Units of Yield" are in (Ml).

Chapter [2], The reference for the Air Passenger Data is -
Hyndman R.J. (1990) *PEST - User Manual*
(University of Melbourne)

Chapter [5], When referring to the "Warren" station, it has been incorrectly referred
to as the "Warren River" station. The Warren station gauges the
South Para River at the Warren Reservoir. (Occurs on pages 60 & 62).

Appendix [D], Units for all plots -
Horizontal axis - Number of Standard Deviations from the Mean
Vertical axis - Observed Yields, (Ml).
- Transformed Yields, (Non dimensional).

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