



AN ELEMENTARY CHARACTERIZATION OF  
THE SIMPLE GROUPS  $PSL(3,3)$   
AND  $M_{11}$  IN TERMS OF THE  
CENTRALIZER OF AN INVOLUTION

BY

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SUMMARY

In this thesis we investigate groups of even order containing an involution whose centralizer is isomorphic to  $GL(2,3)$ . The aim of the research was to give an elementary proof (that is, without the use of character theory) that the only such groups with the additional property of having no subgroup of index 2 are the simple groups  $PSL(3,3)$  and  $M_{11}$ .

Following the introduction, chapter one consists of a few preliminary general results together with some properties of the group  $GL(2,3)$ .

In chapter two we prove a few results about a group  $G$  satisfying the above two properties. In particular we show that there are four possibilities for the structure of the normalizer of a group of order 3 contained in the centralizer of an involution. Each of these cases is dealt with separately in the ensuing chapters.

STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University, and to the best of my knowledge and belief, contains no material previously published or written by another person, except when due reference is made in the text of this thesis.

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