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Convergence Properties of Surface Conductivity Characterization Method for Thin Conductive Strips

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***Abstract*—A new method to measure the surface conductivity σ_s of thin conductors has been recently proposed, that requires placement of a narrow strip inside a rectangular waveguide. Infinite series appear in the calculations required to extract σ_s from the measured scattering parameters. In this paper, we explore the convergence of the different series that are associated with two choices of basis functions (uniform & $\cosh$). We also show the impact of the series truncation on the final computed values of the complex surface conductivity of $10 - j10\,\mathrm{mS/Sq}$ at $10\,\mathrm{GHz}$.**

***Index Terms*—Surface conductivity, surface resistivity, measurement methods, convergent series, convergence error**

I. Introduction

Conventional methods used to measure bulk three-dimensional materials are generally not applicable for accurate characterization of two-dimensional materials. Therefore, the rise of the 2D materials demands development of procedures for the reliable and accurate assessment of their electrical properties. Surface conductivity (or alternatively surface resistivity) is of special interest since if characterized properly, it provides sufficient information to model the interaction of electromagnetic waves with the material.

Recently, laser beams have been used to reduce graphene dioxide to graphene on substrate [1], [2]. The method basically draws strips (lines with finite width) of graphene. Arbitrary-shaped surfaces can be patterned by repeated zig-zag movements of the beam that fill the desired area.

Many different methods were proposed for the measurement of the material properties [3]. Two-dimensional materials are often measured by transmission-reflection methods either inside a waveguide [4] or in free space [5] noting that both methods need a large patch of the material under test. Laser induced graphene often has a large length to width ratio, hence, its characterization by aforementioned methods is not recommended.

An alternative method of examination was proposed in [6] where only a narrow conductive strip was placed inside a rectangular waveguide. The method requires semi-analytical computation of infinite series to extract the surface conductivity of the unknown sample under test. In this paper, we examine the convergence of the infinite series and how the truncation can impact the final values of the extracted complex surface conductivity.

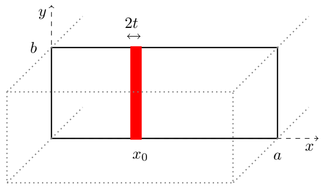


Fig. 1. Geometry and placement of the conductive strip within a rectangular waveguide with profile dimensions $a \times b$.

II. Measurement method

The strip is assumed to have a width of $2t$ and it is placed at the distance x_0 from the side wall while a is the length of the long side of the waveguide (see Fig. 1). By utilizing the Green's function inside the waveguide and enforcing the boundary conditions, one finds that the discontinuity due to the strip can be considered as a shunt element

$$Z_{cs} + jX_a = -\frac{S_{ii}+1}{2S_{ii}} \tag{1}$$

where S_{ii} is the measured or simulated complex reflection coefficient and Z_{cs} and X_a are [6]:

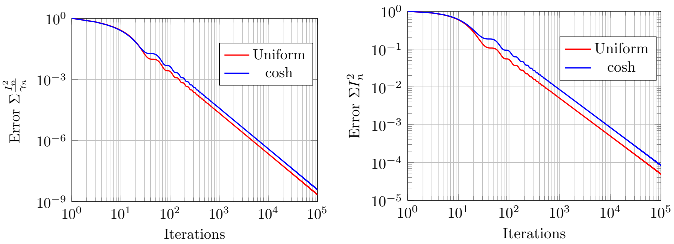
$$Z_{cs} = \frac{\gamma_1}{j4\omega\mu_0\sigma_s}\frac{\sum_{n=1}^{\infty}\left[\int J(x)\sin(\frac{n\pi x}{a})\mathrm{d}x\right]^2}{\left[\int J(x)\sin(\frac{\pi x}{a})\mathrm{d}x\right]^2}, \tag{2}$$

$$jX_a = \frac{\gamma_1}{2}\frac{\sum_{n=2}^{\infty}\frac{1}{\gamma_n}\left[\int J(x)\sin(\frac{n\pi x}{a})\mathrm{d}x\right]^2}{\left[\int J(x)\sin(\frac{\pi x}{a})\mathrm{d}x\right]^2}. \tag{3}$$

In the above, γ_1 and γ_n are the complex propagation constant of the first and $n^{\mathtt{th}}$ modes, respectively, and ω, μ_0 are the radian frequency and permeability of the free space while σ_s is the complex surface conductivity. Furthermore, the distribution of the surface current is $J(x)$ which should be approximated using basis functions and $I_n = \int J(x)\sin(n\pi x/a)dx$. Here, we consider two test functions, namely the uniform and $\cosh$ distributions. Performing the integration in (2) and (3), the coefficients of I_{n_U} and I_{n_C} for uniform and $\cosh$ distributions are [6]:

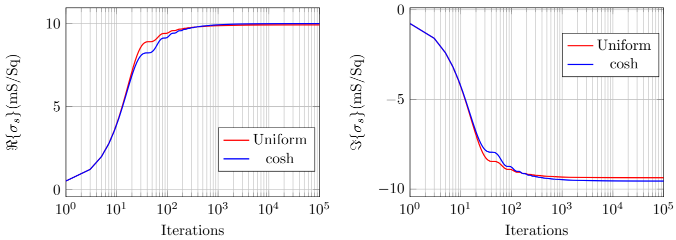
$$I_{n_U} = \frac{1}{n}\sin(\frac{n\pi x_0}{a})\sin(\frac{n\pi t}{a}) \tag{4}$$

$$I_{n_C} = \frac{a\cos\frac{n\pi t}{a}\sinh 1 + n\pi t\sin\frac{n\pi t}{a}\cosh 1}{2(\pi^2n^2t^2+a^2)(at\sin\frac{n\pi x_0}{a})^{-1}} \tag{5}$$



(a) S_X (b) S_Z

Fig. 2. Relative error in computation of the series (a) $\sum I_n^2/\gamma_n$ (b) $\sum I_n^2$



(a) $\Re\sigma_s$ (b) $\Im\sigma_s$

Fig. 3. Computed surface conductivity over the course of the iterative computation (a) real part (b) imaginary part.

III. Series Convergence

In this section, convergence of the following infinite series that appear in the relations of Z_{cs} and jX are explored

$$S_{Z_U} = \sum_{n=1}^{\infty} I_{n_U}^2, \quad S_{Z_C} = \sum_{n=1}^{\infty} I_{n_C}^2 \tag{6a}$$

$$S_{X_U} = \sum_{n=2}^{\infty} \frac{I_{n_U}^2}{\gamma_n}, \quad S_{X_C} = \sum_{n=2}^{\infty} \frac{I_{n_C}^2}{\gamma_n} \tag{6b}$$

where $\gamma_n = \sqrt{n^2\pi^2/a^2 - \omega^2\mu\epsilon}$.

Before attempting to compute the series numerically, tests from calculus [7] are used to ensure that the series are not diverging. In fact, it is possible to prove that the series in (6) are absolutely convergent. The sequence of elements in all series (6) approaches zero for large values of n which is essential for the convergence of any series. Since sin and cos are bounded functions, one can show for large n and a proper choice of constant C that $|I_n|^2 \leq Cn^{-2}$. The convegence of $\sum n^{-2}$ is obvious, therefore, one concludes that series in (6) are absolutely convergent.

We demonstrate convergence of the series by illustration of the error from the final result (obtained after 10^6 iterations) versus the iteration number. Here, we assume a standard WR90 waveguide, choose an operating frequency of 10 GHz, and set the width of the strip to $2t = 1$ mm. It is initially noted that S_X converges much quicker than S_Z due to the $1/\gamma_n$ terms which resembles a factor of $1/n$ for large values of n. This point is clearly illustrated by comparing Fig. 2a and Fig. 2b.

It is seen from Fig. 2 that series arising from the cosh test function are slightly slower to converge when compared with those from the uniform distributions. The numerator of I_{n_C} has two trigonometric functions of $n\pi t/a$ that makes the sequence even more oscillatory, leading to a slower convergence. Over the considered range, the relative error of both series drop similarly to a straight line on a logarithmic scale suggesting dependence $n^{-\alpha}$.

The truncation error in the computation of the infinite series is one of the sources of error in the extraction of σ_s from measured dataset. However, it can be controlled and reduced at the cost of extra iterations. For example, with 1000 iterations that only takes a total of 30 ms, one can compute σ_s with an accuracy of two significant digits which is bounded by the calculation error in S_Z.

IV. Impact on the Calculation of σ_s

Figure 3 demonstrates how the complex conductivity converges when more iterations are used in its computation. A similar structure with $\sigma_s = 10 - j10$mS/Sq was modelled with the FEM solver of CST MWS. The simulation estimated the reflection coefficient as $S_{11} = -0.1409 + 0.1476j$. As illustrated in Fig. 3, both real and imaginary parts of σ_s quickly approach their final expected values. It should be noted that the method does not an initial guess and the first iteration is computed by the first two terms in the S_Z and S_X.

V. Conclusion

In this paper, we examined the computational aspects of finding the complex surface conductivity of a strip of 2D-material in a rectangular waveguide. Absolute and unconditional convergence of the series that appear in the calculations have been analytically proved and numerically demonstrated. It was illustrated that to keep the computational error below 1 % at least 1000 iterations are needed. Furthermore, the impact of the convergence on the final computed conductivity has been illustrated in an example.

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