Item Group Recommendation: A Method Based on Game Theory

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ABSTRACT

In this paper, we focus on recommending an item set to multiple users. Group recommender systems are designed to deal with the issue of recommending items for a user group. However, in some scenarios where different items are packed together as a gift set, such as gift set promotion, album promotion, we need to focus on consumers' preferences to multiple items rather than to some specific item. To deal with this issue, we pioneer a Nash equilibrium based Item Group Recommendation approach (NIGR). Specifically, we evaluate each consumer's preference to an item group from two perspectives, attraction part from the customer herself and social affection from her friends. Then, we model the recommending process as a game to achieve Nash equilibrium. Finally, we demonstrate the effectiveness of our approach with extensive experiments.

Keywords

item group recommendation; group recommender system; Nash equilibrium; game theory

1. INTRODUCTION

Traditional recommender systems, usually referred as individual recommender systems, aim to provide information items (movies, products, web pages, etc.) for a user based on her profile of interests constructed from past behaviors [23]. However, in recent years, we are facing a increasing number of scenarios in which the recommended items are consumed by a group of users rather than individuals [13]. In the scenarios such as music selection in the public gym [18], TV selection in family [17], tourists arrangement in attractions [19], recommender systems should evaluate the preferences of all members in the group, and this type of recommender systems is called group recommender system[13].
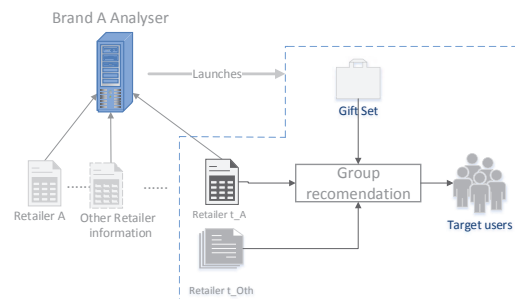
Figure 1: Framework of item group recommendation

Group recommender systems are designed to deal with multiple users and multiple items. Nowadays, group recommender systems are usually designed to recommend a specific item to a group of users. Research on these systems mainly falls into two categories based on their targets: generating recommendations for a group [18, 5, 13, 12] and group formulation [6, 8]. Many research works have provided solutions to those two kinds of group recommender systems [18, 5, 13, 12, 19]. Two common methods in group recommendation are: aggregating individual models into group models and aggregating individual predictions into group predictions [18, 17, 20, 14]. Roy et al. introduced a method to achieve user groups based on personal ranking lists, and the formed user groups have similar ranking sequences for some items [6]. This method does help to form a user group. However, the formed groups only concentrate on some pieces of ranking list rather than the whole item set, besides it neglects mutual influence among users such as social influence.

A running retailer example: Here is a running retailer example. Figure 1 shows the relationships between the brand products, retailers and the gift set produced by some brand. The trademark company launches a new gift set for promotion by analysing its sales information from all retailers including retailer A, some other retailers and our target retailer t. For the retailer t herself, she may know nothing about the gift set and has some sales information about the items of brand t_A and some other brands t_Oth in her store. Our work in this paper is to help retailer t find who are the suitable consumers to buy this new gift set.

Different from the former group recommender systems, we solve the problem of recommending an item set to multiple users. Traditional group recommender systems focus on the
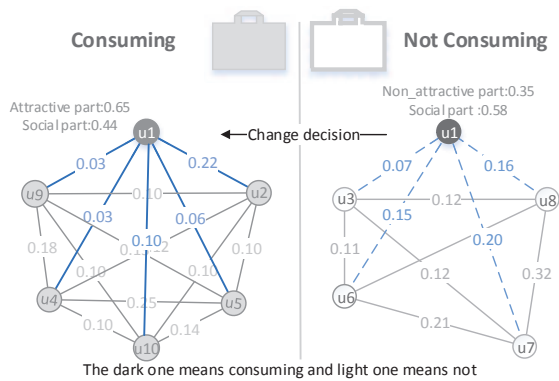
Figure 2: user decision process

overall group member preference to a specific item, failing to deal with issues of an item set.

In our research, we give the solution by dealing with the obstacles of modeling the user' preference to the overall item set and social influence when making recommendations. we model each user's preference to the item set by adapting some existing technologies in user group recommendation. Specifically, we measure each user's willingness W_u to the item set from two parts: the attraction degree of the item set to the user (namely, attractive part or non-attractive part) $A(u)$ and the social influence from user's friends $S(d_u, D)$. The main notations used are summarized in Table 1. The willingness of users to an item set can be simplified as the aggregation of these two parts

$$W_u = A(u) + S(d_u, D) \qquad (1)$$

The willingness shows her own interest in this gift set and the influence from her friends. If the willingness of "consuming" is higher than "not consuming", she will consume this gift set, and vise versa. In Figure 2, when user u_1 chooses not to consume, her willingness $W_u = 0.35 + 0.58 = 0.93$; if she chooses to consume, the willingness $W'_u = 0.65 + 0.44 = 1.09$. Under this situation, she has higher willingness when choose "consuming". Then she will make her decision to consume this item set.

Each user is influenced by the decisions of other users. They influence each other, as a result, the decision making process is like a game among multiple players and these users try to achieve an agreement. The final recommendation results can be determined when every user reaches her best decision, that is, the decision set made by users would not be optimal if any of users changes her decision. Such an optimal system state is called Nash Equilibrium in game theory. In this game, each candidate user acts as a player, each user has an action set (consuming or not consuming) and makes decision based on her willingness. The decision set corresponds to the strategy set in game theory, and the recommendation process is modeled as a problem of finding the Nash equilibrium in noncooperative game theory.

The contribution of this paper can be summarized as follows.

- We pioneer a game theoretic framework to resolve the problem of recommending an item group to multiple users.

- We introduce the influence of the conflict and collaborative relations among users on making decisions.

- We study the performance of our proposed methods with extensive experiments by comparing with state-of-the-art methods in real datasets.

The rest of the paper is structured as follows. We discuss related work in Section 2 where we summarize the developments in both individual recommender systems and group recommender systems. After that, we introduce our proposed method in Section 3, and our experiments to compare our method with the state-of-the-art methods on real datasets in Section 4. Finally, we present the conclusion of our work in section 5.

2. RELATED WORK

In this section, we give a brief introduction in both individual recommender systems and group recommender systems.

2.1 Individual Recommender System

Traditional recommender systems aim to provide information items (movies, products, web pages, etc.) to a user based on her past behaviors [23]. The work [10] classified six different classes of recommendation approaches: *content-based*, the system recommend items which are similar to the ones that user liked in the past [4, 21]; *collaborative filtering*, collaborative filtering systems generate recommendations based on the assumption that users with similar interests in the past will share common interests in the future [22, 15]; *demographic*, different recommendations should be generated for different demographic niches [16]; *knowledge-based*, specific domain knowledge is involved in modeling users' preferences [9]; *community-based*, this type of system recommendations follows the epigram "I will identify you if you tell me who are your friends" [2]; *hybrid recommender systems*, it is the combination of collaborative filtering and content-based. Recently, more information has been added to recommendation such as social information [5], tags [26], etc, and these methods have achieved numerous improvements of accuracy in recommendation [24].

2.2 Group Recommender System

Group recommender systems are designed to deal with the problem of satisfying a group of users with potentially conflicting interests. Depending on the targets, group recommender systems fall into two categories.

One is to make recommendations for a given group users. The most two common strategies are aggregating individual models into group models and aggregating individual predictions into group predictions. The former strategy generates recommendations for each group member and then combine the recommendation results for the group [20, 25]. The second strategy first aggregate the profiles of each member into one pre-do individual model, then make recommendations for this model [14]. In recent years, user-based group recommender systems attract more researches in modeling group preference with some new methods. The work [1] modeled the group preference from the relevant part and the disagreement part. The relevant part measures the aggregation over members' relevance of one candidate item, and the disagreement part scales the variance of the relevances for

the candidate item among group members. The work [7] advised to switch recommendation methods based on the density of the user's data. According to two predefined density thresholds, the system automatically toggles between the general and group recommendation aggregation strategies, and between the group aggregation strategies and CF strategies, respectively. The work [11] introduced noncooperative game theory to make recommendations for groups. For a group with m users, every user gets her favourite k items. These k items constitute her decision set. Among all the m^k cases, those reaching Nash Equilibrium are final recommendation lists.

Another category is about group formulation. With the development of internet, numerous people tend to attend activities in groups, which forms a new social network like EBSN (Event-based Social Networks) [3], and it has attracted a vast number of research. However, in group recommender system, it seems that it does not get enough attention. Most of the experiments just choose to form a group by randomly selecting k users or selecting the k most similar users. These methods only considered the partial sets of the entire item set or some partial information. The work [6] is a method which constructs a group based on personal ranking lists and the similarity of the ranking lists controls the probability users fall into one group. It does help to form a group. However, when given a set of items, the method only focuses on dealing with partial item set and neglects interactions between users.

3. METHODOLOGY

Let $I^0 = \{i_1, i_2, ..., i_n\}$ and $U = \{u_1, u_2, ..., u_m\}$ denote the set of all items and all users. Given a target item group, the aim of group recommendation is to identify the potential consumers. In this paper, we use I as the items which form the item group, and $|I|$ is defined as the number of items in a group. For example, if a group consists of items i_1, i_2 and i_5, then it can be expressed as $I = \{i_1, i_2, i_5\}$ and $|I| = 3$.

In our paper, we suppose all the users have given their ratings to each item. Based on this full rating matrix, we help retailer to find her potential consumers.

Table 1: Notations

Notation	Explanation
U,I	user set, item set
$s(u,v)$	the similarity between user u and v
d_u	the decision made by user $u, d_u \in \{0,1\}$
U^+, U^-	user set with $d_u = 1$ and $d_u = 0$
D	a decision vector constructed by users
$U_u^{d_u}$	the friend set of user u
r^0	the full mark in data set
$A^+(u), A^-(u)$	the attractive and non-attractive part
$S(d_u, D)$	the social effect of u with decision d_u
$B(u, d_u)$	the benefit of u with decision d_u

3.1 Game Theoretic Approach

When people make decisions, they are influenced by their fondness and the viewpoints from their friends. The positive friends inspire them to attend, and the negative friends discourage them to attend, which means that people with the same choice influence each other through their social relationship. It is called a game in the field of economics,

where each player makes decision influenced by others and influence others to make decision. In the problem discussed in our paper, each customer is a player, and each user has a specific strategy $d_u \in \{0,1\}$ representing her decision on whether to consume this item set, 1 standing for consuming and 0 for not. And the benefit function $B(u, d_u)$ carves her preference for this item set under her strategy d_u. The strategy set D represents the decision set of all the customers. In our formwork, we use U^+ to represent the player set with $d_u = 1$ and U^- for the player set with $d_u = 0$.

3.1.1 Modeling the user's preference

We model the user's benefit under different strategy $d_u \in \{0,1\}$ using Equation 2. In our paper, if two users have the same choice then they are considered as friends. If one user chooses to consume this item set, we will evaluate two parts, the attractive part from the item set itself $A^+(u)$ and her social part affected by her friends $S(d_u, D)$ under her strategy $d_u = 1$. When not consuming the item set, the willingness can also be divided into her non-attractive part $A^-(u)$ to the item set and the social part $S(d_u, D)$ under her strategy $d_u = 0$. In our paper, we model the social affection to user u by aggregating her social relation with each of her friend $v \in U_u^{d_u}$.

$$B(d_u) = \begin{cases} \omega_1 \cdot A^+(u) + \omega_2 \cdot \sum_{\substack{v \in U^+ \\ v \neq u}} \mathtt{soc}(u,v) & d_u = 1 \\ \omega_1 \cdot A^-(u) + \omega_2 \cdot \sum_{\substack{v \in U^- \\ v \neq u}} \mathtt{soc}(u,v) & d_u = 0 \end{cases}$$

(2)

We scale the social relation of two users $\mathtt{soc}(u,v)$ by their jaccard similarity. In real life, the more frequently they interact with each other, the closer their relationship is.

$$\mathtt{soc}(u,v) = \frac{I^0{}_u \bigcap I^0{}_v}{I^0{}_u \bigcup I^0{}_v}$$

(3)

We measure the attraction part using Equation 4. For the attractive part $A^+(u)$ for user u, we aggregate her ratings for each item in the item set. And for the non-attractive part, we introduce the parameter full mark r^0. Her dissatisfaction for this item set is the difference between full mark and actual rating. Then we aggregate the difference ratings for all the items in the group as the non-attractive factor.

$$A^+(u) = \sum_{i \in I} r(u,i), A^-(u) = \sum_{i \in I} (r^0 - r(u,i))$$

(4)

The parameters, which satisfy $\omega_1, \omega_2 \in (0,1)$ and $\omega_1 + \omega_2 = 1$, control the weight of different parts. For the overall benefit of this system as shown in Equation 5, it represents the sum of the attraction part and the inner social connections between different users. The factor 1/2 controls that $\mathtt{soc}(u,v)$ and $\mathtt{soc}(u,v)$ will only contribute once in the total benefit.

$$\begin{aligned} \Phi = {} & \omega_1 \cdot \sum_{u \in U^+} A^+(u) + \frac{1}{2}\omega_2 \cdot \sum_{\substack{u,v \in U^+ \\ v \neq u}} \mathtt{soc}(u,v) \\ & + \omega_1 \cdot \sum_{u \in U^-} A^-(u) + \frac{1}{2}\omega_2 \cdot \sum_{\substack{u,v \in U^- \\ v \neq u}} \mathtt{soc}(u,v) \end{aligned}$$

(5)

3.1.2 Nash Solution

In strategic games, each player chooses a strategy to maximize her benefit under others' decisions. If each player has chosen a strategy and no player can benefit more by changing her strategy while the other players remain unchanged, then the current set of strategy choices reaches a pure strategy Nash equilibrium. In our paper, each candidate is one player with strategy set $\{0, 1\}$. Each player u in U' makes decision based on her benefit $B(u, d_u)$ and holds the position that maximizes her benefit. Nash equilibrium will be reached if the following condition is true for all players:

$$B(u, d_u) > B(u, d'_u) \qquad d_u, d'_u \in \{0, 1\} \tag{6}$$

d_u is the current decision made by u, and d'_u is the opposite decision.

Algorithm 1 illustrates the process of the dynamic process of decision-making in a strategic game. First, system assigns a random strategy to each player initializing the decision set D. Then in line $3 \sim 5$, each player calculates her benefit $B(u, d_u)$ at current decision and the opposite decision, and chooses the strategy that maximizes her benefit. Finally, no player can benefit by changing strategies while the other players keep theirs unchanged, which means no player has an incentive to switch her strategy given that the strategies of the other players are fixed, the Nash equilibrium is reached.

Algorithm 1 Best-Response Dynamics

Input: Strategic game U',I,$d_u \in 0, 1$
Output: Nash equilibrium
1: Assign a random strategy to each player
2: **repeat**
3:    For each player u in U'
4:    compute best strategy of u wrt the other player strategies
5:    let u follow her best strategy
6: **until** Nash equilibrium//no player has incentive to change her strategy
7: **return** The strategy vector at Nash equilibrium

We first analyze the feasibility of Nash equilibrium framework in our recommendation. In order to gain the optimal total overall benefit, we will show that NIGR is an exact potential game. An exact potential game must have a Nash equilibrium.

THEOREM 1. *Finding consumers for a given item set in our paper constitutes an exact potential game.*

PROOF. In a strategic game, a game is an exact potential game if after a strategy change of an individual, the individual benefit difference equals the overall benefit of changing this strategy. For example, for any user u in candidate U', she switches her strategy from $d_u = 1$ to $d'_u = 0$. The user benefit $B(u, d_u)$ and overall benefit Φ_{u,d_u} with the strategy

of user u when $d_u = 1$:

$$B(u, d_u) = \omega_1 \cdot A^+(u) + \omega_2 \cdot \sum_{\substack{v \in U^+ \\ v \neq u}} \mathrm{soc}(u, v) \tag{7}$$

$$\Phi_{u,d_u} = \omega_1 \sum_{\substack{v \in U^+ \\ v \neq u}} A^+(v) + \frac{1}{2}\omega_2 \sum_{\substack{v,m \in U^+, \\ v \neq u, m \neq u}} \mathrm{soc}(m, v)$$
$$+ \omega_1 \sum_{v \in U^-} A^-(v) + \frac{1}{2}\omega_2 \sum_{\substack{v,m \in U^- \\ v \neq m}} \mathrm{soc}(m, v)$$
$$+ \omega_1 A^+(u) + 2 \cdot \frac{1}{2}\omega_2 \sum_{\substack{v \in U^+ \\ v \neq u}} \mathrm{soc}(u, v) \tag{8}$$

user benefit $B'(u, d'_u)$ and overall benefit Φ_{u,d'_u} at $d'_u = 0$

$$B'(u, d'_u) = \omega_1 A^-(u) + \omega_2 \sum_{\substack{v \in U^- \\ v \neq u}} \mathrm{soc}(u, v)$$

$$\Phi_{u,d'_u} = \omega_1 \sum_{\substack{v \in U^+ \\ v \neq u}} A^+(v) + \frac{1}{2}\omega_2 \sum_{\substack{v,m \in U^+, \\ v \neq u, m \neq u}} \mathrm{soc}(m, v)$$
$$+ \sum_{v \in U^-} \omega_1 A^-(v) + \frac{1}{2}\omega_2 \sum_{\substack{v,m \in U^- \\ v \neq m}} \mathrm{soc}(m, v)$$
$$+ \omega_1 A^-(u) + 2 \cdot \frac{1}{2}\omega_2 \cdot \sum_{\substack{v \in U^- \\ v \neq u}} \mathrm{soc}(u, v)$$

then

$$\Phi_{u,d'_u} - \Phi_{u,d_u} = \omega_1 \cdot A^-(u) + \omega_2 \sum_{\substack{v \in U^- \\ v \neq u}} \mathrm{soc}(u, v)$$
$$- \omega_1 A^+(u) + \omega_2 \cdot \sum_{\substack{v \in U^+ \\ v \neq u}} \mathrm{soc}(u, v)$$
$$= B'(u, d_{u'}) - B(u, d_u)$$

$\square$

3.2 Normalization Issues

The aim of normalization is to regularize the attraction part and the social part. For instance, value of attraction part ranges in $[0, 25]$, the social relationship between users only ranges in $[0, 1]$. Without normalization, compared to the attraction part, the social part may only take little influence on decision except that the weight of social part is much higher than the attraction part weight. We utilise the full mark $r^0|I|$ to normalize the willingness parts into $[0, 1]$.

$$A^+(u) = \frac{A^+(u)}{r^0|I|}, A^-(u) = \frac{A^-(u)}{r^0|I|} \tag{9}$$

4. EXPERIMENTS

In this section, we present some details in experimental studies evaluating our approach, including the development experiment and data source. Then we introduce the overall strategies and metrics in the experiment. After that we give a running example of our proposed methods in real datasets to help understand the process. Finally we give the experimental results and analyse the performance of our proposed method.

4.1 Experiment Setup

4.1.1 Experiment Development

All the algorithms used in this paper have been coded in Matlab and all the performance experiments were conducted on an Intel machine with dual-core 2.90GHz, 8G Memory running Windows 7. We use the MovieLens rating dataset which is widely used in both individual recommendation and group recommendation. We extracted our date from the 100K dataset, which consists of 943 users, 1682 movies and 100,000 movie ratings ranging on a 0-5 scale. As the real dataset is very sparse, in our experiment we construct our dataset by completing the rating matrix first. There are numerous ways to do rating prediction, such as collaborative filtering, matrix factorization and so on. In our paper we adopt the simple method nearest neighbor algorithm to fill the missing ratings.

4.1.2 Strategies and Metrics

As there are little evaluation metrics in item group recommender systems, here we choose two popular aggregation strategies (LM and AR) in traditional group recommender to explain the reasonable results generated by NIGR.

1. Least Misery (LM), which uses the least rating some user has given for the item group as the user's preference for this whole item set.

2. Average Relevance (AR), which model the preference of a user through her average ratings to the item set.

3. Nash equilibrium based Item Group Recommendation (NIGR), which computes the user's preference from two perspectives, each user's preference consists of attraction part and social part, and two parameters adjusts the weight of these two parts.

4.2 Case Study

After matrix completion, we get the ratings from these 10 customers set $U' = \{u_1, u_2, \ldots, u_{10}\}$ on the give item set $I = \{i_1, i_2, i_3, i_4, i_5\}$ as shown in Table 2. Then we calculate the attraction (including the attractive and the non-attractive part) and social part of these 10 customers using Equation 2 as shown in Table 3 and Table 4. Here we define the weight of attraction part $\omega_1 = 0.2$ and social weight $\omega_2 = 0.8$. After that, we will use Algorithm 1 introduced in section 3 to get Nash solution.

Table 2: user item rating matrix.

	i_1	i_2	i_3	i_4	i_5
u_1	4.53	1.00	3.81	2.00	5.00
u_2	4.43	5.00	3.71	3.00	4.00
u_3	5.00	3.67	4.00	5.00	4.00
u_4	3.00	5.00	4.00	3.60	3.66
u_5	5.00	1.00	3.00	3.65	3.69
u_6	4.55	1.00	5.00	5.00	5.00
u_7	4.52	5.00	3.81	2.00	2.00
u_8	4.54	4.00	3.82	3.00	4.00
u_9	4.38	5.00	3.71	3.50	4.00
u_{10}	4.55	3.64	3.86	4.00	5.00

In each round, each player computes her best strategy under the other player strategies in turn and the Nash equi-

Table 3: user's interest part.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
A^+	0.65	0.81	0.87	0.77	0.65	0.82	0.69	0.77	0.82	0.87
A^-	0.35	0.19	0.13	0.23	0.35	0.18	0.31	0.23	0.18	0.13

Table 4: user social matrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
u_1	1.00	0.22	0.07	0.03	0.06	0.15	0.20	0.16	0.03	0.10
u_2	0.22	1.00	0.11	0.12	0.10	0.12	0.18	0.14	0.10	0.10
u_3	0.07	0.11	1.00	0.20	0.24	0.11	0.12	0.12	0.11	0.13
u_4	0.03	0.12	0.20	1.00	0.25	0.06	0.05	0.06	0.18	0.10
u_5	0.06	0.10	0.24	0.25	1.00	0.12	0.09	0.10	0.13	0.14
u_6	0.15	0.12	0.11	0.06	0.12	1.00	0.21	0.28	0.03	0.19
u_7	0.20	0.18	0.12	0.05	0.09	0.21	1.00	0.32	0.05	0.25
u_8	0.16	0.14	0.12	0.06	0.10	0.28	0.32	1.00	0.03	0.27
u_9	0.03	0.10	0.11	0.18	0.13	0.03	0.05	0.03	1.00	0.04
u_{10}	0.10	0.10	0.13	0.10	0.14	0.19	0.25	0.27	0.04	1.00

librium is reached when each player gets her best strategy and keeps unchanged.

4.2.1 Modeling the user's preference

D is a strategy set representing current decision made by all the users in U'. In Table 5, $Innitialset = \{1,1,0,1,1,0, 0,0,1,1\}$ which represents $d_{u_3} = d_{u_6} = d_{u_7} = d_{u_8} = 0$ and the other 6 users choose not consuming at the initial stage. For user u_1, when $d_{u_1} = 1$ the attractive part can be obtained by Equation 2, $A^-(u) = 16.34$. Then we use Equation 9 to normalize the attractive part, $A^+(u_1) = 16.34/25 = 0.65$, and the whole attraction part is shown in Table 2. Besides, she has a friend set $\{u_2, s_4, s_5, s_9, u_{10}\}$ who have the same choice, so the social part $soc(u, d_u) = \sum_{u \in \{u_2, s_4, s_5, s_9, u_{10}\}} \mathtt{soc}(u_1, u) = 0.44$, therefore the benefit is $B(u_1, d_{u_1}) = 0.48$; If she changes her decision, $d'_{u_1} = 0$, the non-attractive part is $A^-(u) = 0.35$, and social part is $\mathtt{soc}(u_1, d'_{u_1}) = \sum_{u \in \{u_3, u_6, u_7, u_8\}} \mathtt{soc}(u_1, u) = 0.57$, thus the benefit is $B'(u_1, d'_{u_1}) = 0.53$.

4.2.2 Nash solution

Comparing to the older benefit $B(u_1, d_{u_1}) = 0.48$, the new benefit $B'(u_1, d'_{u_1}) = 0.53$. u_1 can gain more benefit so u_1 chooses to change the decision, and the new decision state is $D_{new} = \{\boldsymbol{0}, 1, 0, 1, 1, 0, 0, 1, 1, 1\}$. Then other users choose their best decisions in turn. After round R1, there still exist some users changing their decisions, another round is required. Finally in some round, every user holds her position and remains unchanged, which means decision state reaches a Nash equilibrium and then the process terminates. Decision set $D = \{0, 0, 1, 1, 1, 0, 0, 0, 1, 0\}$ is the final recommendation result. In other words, we will recommend user set $\{u_3, u_4, u_5, u_9\}$ to consume this packed item set $\{i_1, i_2, i_3, i_4, i_5\}$.

4.3 Results and Analysis

In this section, we compare the different recommendation lists under LM, AR and NIGR strategies, then we report the performance under different parameters.

4.3.1 Reasonability

Table 5: Nash Process when attraction weight $\omega_1 = 0.2$. In u(num1,num2), num1 represent the willingness when $d(u) = 1$ and num2 for $d(u) = 0$

	Steps	Attendance D
	InitialSet	$\{1,1,0,1,1,0,0,0,1,1\}$
R1	$u_1(0.48, 0.53)$	$\{\boldsymbol{0},1,0,1,1,0,0,0,1,1\}$
	$u_2(0.50, 0.65)$	$\{0,\underline{0},0,1,1,0,0,0,1,1\}$
	$u_3(0.72, .45)$	$\{0,0,\boldsymbol{\underline{1}},1,1,0,0,0,1,1\}$
	$u_4(0.73, 0.30)$	$\{0,0,1,\underline{1},1,0,0,0,1,1\}$
	$u_5(0.74, 0.45)$	$\{0,0,1,1,\underline{1},0,0,0,1,1\}$
	$u_6(0.58, 0.63)$	$\{0,0,1,1,1,\underline{0},0,0,1,1\}$
	$u_7(0.58, 0.79)$	$\{0,0,1,1,1,0,\underline{0},0,1,1\}$
	$u_8(0.61, 0.76)$	$\{0,0,1,1,1,0,0,\underline{0},1,1\}$
	$u_9(0.53, 0.22)$	$\{0,0,1,1,1,0,0,0,\underline{1},1\}$
	$u_{10}(0.50, 0.75)$	$\{0,0,1,1,1,0,0,0,1,\boldsymbol{\underline{0}}\}$
R2	$u_1(0.28, 0.73)$	$\{\underline{0},0,1,1,1,0,0,0,1,0\}$
	$u_2(0.50, 0.65)$	$\{0,\underline{0},1,1,1,0,0,0,1,0\}$
	$u_3(0.62, 0.55)$	$\{0,0,\underline{1},1,1,0,0,0,1,0\}$
	$u_4(0.66, 0.38)$	$\{0,0,1,\underline{1},1,0,0,0,1,0\}$
	$u_5(0.62, 0.56)$	$\{0,0,1,1,\underline{1},0,0,0,1,0\}$
	$u_6(0.43, 0.79)$	$\{0,0,1,1,1,\underline{0},0,0,1,0\}$
	$u_7(0.39, 0.99)$	$\{0,0,1,1,1,0,\underline{0},0,1,0\}$
	$u_8(0.40, 0.98)$	$\{0,0,1,1,1,0,0,\underline{0},1,0\}$
	$u_9(0.50, 0.25)$	$\{0,0,1,1,1,0,0,0,\underline{1},0\}$
	$u_{10}(0.50, 0.75)$	$\{0,0,1,1,1,0,0,0,1,\underline{0}\}$

Table 6: recommendation results

Result	AR		LM		GNR
u_1	3.27	9	1	8	0
u_2	3.27	5	3	4	0
u_3	4.33	2	3.67	2	1
u_4	3.85	7	3	5	1
u_5	5	10	1	9	1
u_6	4.11	4	1	10	0
u_7	3.47	8	2	7	0
u_8	3.87	6	3	6	0
u_9	4.12	3	3.5	3	1
u_{10}	4.36	1	3.80	1	0

Table 7: Recommendation vs. ω_1

ω_1	Recommendation_list
0	$\{0,0,0,0,0,0,0,0,0,0\}$
0.1	$\{0,0,0,0,0,0,0,0,0,0\}$
0.2	$\{0,0,1,1,1,0,0,0,1,0\}$
0.3	$\{0,0,1,1,1,0,0,0,1,0\}$
0.4	$\{1,1,1,1,1,1,1,1,1,1\}$
0.5	$\{1,1,1,1,1,1,1,1,1,1\}$
0.6	$\{1,1,1,1,1,1,1,1,1,1\}$
0.7	$\{1,1,1,1,1,1,1,1,1,1\}$
0.8	$\{1,1,1,1,1,1,1,1,1,1\}$
0.9	$\{1,1,1,1,1,1,1,1,1,1\}$
1	$\{1,1,1,1,1,1,1,1,1,1\}$

Table 6 shows the recommendation results under strategy AR, LM and NIGR when $\omega_1 = 0.2$. For each strategy, the first columns refers to the scores under this strategy and the second refers to the ranking position (in AR and LM) and decision for NIGR. NIGR recommendation result is $Rec\_NIGR = \{u_3, u_4, u_5, u_9\}$. If taking NIGR result as the user number base, which means we also select four users as the final recommendation list for each strategy, then AR and LM will also have 4 candidates as the result. We get the two lists $Rec\_AR = \{u_{10}, u_3, u_9, u_6\}$ and $Rec\_LM = \{u_{10}, u_3, u_9, u_2\}$. AR is usually selected as an important factor to measure the recommendation precision as it reflects the average attitudes of the group. However, it neglects the interaction among users. NIGR imports the social influence in decision. We get the distinct results by using NIGR and AR, $\{u_4, u_5\}$ in NIGR and $\{u_{10}, u_6\}$ in AR. $\{u_{10}, u_6\}$ have higher user willingness, but most of their companions choose the opposite decision so that they gain lower support than $\{u_4, u_5\}$. Eventually they are not the right most appropriate users to be recommended under the affection of the attraction and social influence.

4.3.2 NIGR under different attraction weight

Table 7 shows the recommendation results under different attraction weight ω_1. The initial consuming state $D = \{1,0,0,1,1,0,0,0,0,1\}$. If attraction weight matches $\omega_1 < 0.2$, the social part predominates the decision of each user. When $\omega_1 \in (0.2, 0.4)$, the attraction part and social influence jointly control decision process. When $\omega_1 > 0.4$ interest part takes control of process. In other words, ω_1 adjusts the importance of social affection during a group decision.

4.3.3 NIGR with customer size and item size

In NIGR, customer size controls the number of players attending a game. In a team decision, if the initial state is very close to an agreement, the discussion will be terminated quickly. Otherwise, it may be a long negotiating process. Algorithm 1 also has this limitation, its computational time largely depend on its initial state. In this part, we analyse the computational complexity of one iteration. For an iteration in Algorithm 1 with $|U|$ users and $|I|$ items, each user should calculate her attraction part to $|I|$ items and social influence from other $(|U|-1)$ individuals, as well as her non-attractive part to $|I|$ items and social influence from other $(|U|-1)$ individuals. During an iteration, the computational process is $O(|U||I|+|U|^2)$. In our experiment, the attractive part and non-attractive part can be calculated offline as well as the similarity of two users. Therefore, the computational process can be approximately decreased to $O(|U|^2)$.

5. CONCLUSIONS

In this paper, we focus on the issue of potential user mining for a given item set in group recommendation. To solve the cooperation and conflict problems in group decision, we proposed a method based on Nash equilibrium. However, our solution still has some limitations. In future work, we plan to investigate new techniques to tackle larger customer size and optimize algorithm process.

6. ACKNOWLEDGMENTS

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