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# Teaching Music with Mathematics: A Pilot Study

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**Abstract.** We detail a recently conducted teaching intervention involving the use of mathematics and associated software to teach rhythm and meter to Year 9 pupils. This intervention served as a feasibility and pilot study within a broader project related to the mutual teaching of mathematics and music. Causal conclusions cannot be made due to the lack of a control group, but questionnaires show that 81% of the pupils found interacting with software helped them to understand and visualize mathematical theories of rhythm and meter, and the same percentage think that mathematics and music are related. The two teachers who delivered the program enjoyed the experience and felt the software was beneficial.

**Keywords:** Music education · XronoBeat · STEAM education · Rhythm · Meter · Set theory · Maximal evenness · Modulo small · Cyclic graph.

## 1 Introduction

In educational settings, there are widely reported correlations between mathematical and musical abilities [24, 10, 8, 15, 2–4] (including music theory [1, 14, 12, 13, 21]). These correlations remain after controlling for other factors such as general intelligence and academic aptitude [23]. Converging evidence suggests that both musical and mathematical skills are informed by core *geometric skills* for perceiving, recognising, and mentally manipulating spatial patterns [22].

The authors of this paper, in collaboration with Richard Cohn, Tara Hamilton, Courtney Hilton, and many others too numerous to mention, are conducting an ongoing applied research project called "Teaching Mathematics with Music and Music with Mathematics". We are developing a suite of educational materials including lesson plans and software designed to improve educational outcomes in both mathematics and music. To achieve this aim, we are (a) teaching mathematical and musical skills simultaneously; (b) using novel software applications that visualize musical structures with geometrical (typically, isomorphic) forms that can be manipulated and sonified in musically appealing ways [16]; (c) using analogical comparisons between multimodal representations of the same underlying concept to help students draw out the commonalities of the musical and mathematical representations and generalize them [9, 20].

This paper focuses on a recent pilot intervention at Sydney Grammar School that used music to teach mathematics – specifically rhythm and meter. It forms a counterpoint with two other recent pilot studies. One study was conducted at Andrea Calilhanna's private music tuition practice, and focused on the use of ski-hill graphs [5] and the associated SkiHill App [16] to teach and explain musical meter. Qualitative results were detailed in [16] but, in summary, both students (aged 7 and 13) found the approach enhanced their understanding of meter as well as helping them to play previously challenging passages. The other intervention used musical rhythms to teach mathematics to Year 7 pupils in the second-lowest mathematics stream at Bankstown Girls High School (BGHS). This Western Sydney suburb has low socio-economic status (SES), and many of the pupils have English as a second language. For the three lessons, an adapted and simplified version of the music software application XronoMorph [19, 18, 17], called XronoBeat (developed by the first author, and pictured in [17]), was used to teach basic fraction skills – understanding what fractions are, and learning how to order them. The results were promising, with average test scores rising from 51% to 75% after just three lessons [11].

The study reported here provides a useful counterpoint to the BGHS study. Sydney Grammar is a private school for boys in central Sydney whose pupils are generally of high SES and, instead of teaching mathematics with music, we taught music with mathematics. In addition to XronoBeat, we used teaching materials specially developed by Richard Cohn, which were based on a rhythm-centred version of his "modulo small" teaching program [6, 7].

## 2 Methods

The two classroom music teachers were provided with Cohn's lesson plans to teach to their Year 9 music students. There were a total of 28 boys, aged 14–15, who were all music pupils in the classes ordinarily taught by the two teachers. The lesson plans were designed for six 45-minutes lessons and comprised written descriptions of the lessons and explanatory diagrams and cyclic graphs. XronoBeat was installed on the computers in each classroom and there was one computer per child. This study was approved by the Human Research Ethics Committee of Western Sydney University. Below is a summary of the lessons:

**Lesson 1** A 2-point cycle. A zero-based counting system for "beats". Basic set-theory relations: inclusion, rotation, and complementation. Basic musical terms: "time point" (= "beat"), "pulse", "meter".

**Lesson 2** A 3-point cycle. Application of basic terms learned last time. A 6-point cycle as a product of a 2-cycle and a 3-cycle. Exploration of hemiola and basic polymeter (3:2).

**Lesson 3** Introduction to *maximally even* (ME) sets; how they apply to embeddings of the 2- and 3-cycle into the 6-cycle. The notation $\mathrm{ME}(c, d)$. Explore $\mathrm{ME}(8, d)$, that is, maximally even sets for an 8-point cycle but restricting to values of $d$ that divide 8; that is, $\mathrm{ME}(8, 2)$ and $\mathrm{ME}(8, 4)$ (embedding of 2- and 4- point cycle within an 8-point cycle).

**Lesson 4** The distinction between perfect and imperfect ME sets. Exploration of imperfect ME sets in an 8-point cycle: ME$(8, 3)$ and ME$(8, 5)$ – the Cuban *tresillo* and *cinquillo*. Rotation, inclusion, and complementation relations.

**Lesson 5** The 12-point cycle. Perfect ME sets ($d = 2, 3, 4, 6$) and their interaction. A lesson on polyrhythms, building on Lesson 2. Imperfect ME sets in a 12-point cycle – ME$(12, 5)$ and ME$(12, 7)$. African bell patterns. Application of rotation, inclusion, and complementation relations, building on Lesson 4.

**Lesson 6** Application of imperfect ME$(12, 5)$ and ME$(12, 7)$ in pitch: major and pentatonic scales. A 16-point cycle. The focus here is on imperfect ME sets, and their application to popular music. ME$(16, 5)$ yields the prime-generated double tresillo $(3, 3, 3, 3, 4)$; ME$(16, 7)$ yields the hyperdiatonic rhythm $(2, 2, 3, 2, 2, 2, 3)$; rock-n-roll examples of these.

## 3 Results

Students and teachers were assessed through questionnaires and observations made by both authors. This study was intended primarily to test the feasibility of the intervention and to obtain indications of possible effects bearing in mind that, due to the lack of a control group, causal claims cannot be asserted. The students were asked to write answers for seven post-test questions:

1. Which parts of the lessons did you enjoy the most or find the most interesting?
2. Which parts of the lessons did you enjoy the least or find the least interesting?
3. In what ways did you find the software help or hinder your learning of musical or mathematical concepts?
4. Which parts of the lessons or software did you not understand?
5. Did you learn anything that you had previously struggled to understand? If so, can you explain how?
6. Do you think mathematics and music are related subjects? Has your opinion changed since these lessons?
7. Do you have any suggestions for improving the lessons?

A statistical analysis of the questionnaires was undertaken by a research assistant not involved with the project. In summary, 73% of the pupils stated a general preference for computer interaction, and 69% expressed that the maths theory relevant to rhythms was the least interesting part of the lessons. However, 81% of the pupils stated that the interaction with the software helped with understanding and/or visualising the maths theory, and 23% of students understood complex rhythms more thoroughly as a result of the lessons. The majority of students (81%) thought mathematics and music were related subjects, with 42% of pupils expressing that they better understood the relation or saw a relation where they did not before as a result of the lessons. Although 38% of pupils did not express any difficulties with the lessons, 35% of pupils struggled with the theory, with 27% of pupils recommending more detailed explanations as a way of improving future lessons.

The teachers also filled in post-test questionnaires. Teacher A stated that the class were "at the strong end academically and musically" and taught the materials at length, extending some of the lessons to two lessons per written lesson. Teacher B expressed that it was challenging teaching new terminology to the students during the first lessons. However, both teachers expressed the efficacy of having XronoBeat in the classroom for students to interact with and to reinforce the concepts being taught. Teacher A explained that the students liked learning with XronoBeat because it was game-like. Teacher A noted how efficacious learning about mathematics and music could be in helping students with strong mathematical ability who were weak performers in that their new knowledge about mathematics and music would give those students a "leg up". Of particular interest to Teacher B was "applying the theory to existing musical examples, the mathematical aspects of the course and seeing the students brainstorm in a mathematical manner and explaining their ideas during discussions".

The teachers approached the pedagogy of rhythm and meter through applying visualizations and sonifications of the mathematical principles of basic set theory. In the classes, the students learned to identify rhythms and meter (sets and their relations) from experiencing polyrhythms and polymeter in Latin American, Afro-Caribbean, West African, Western art, and Western rock music. Both teachers drew cyclic graphs on the board, and cycles were also composed by the students using XronoBeat. The students of Teacher A used XronoBeat to produce rhythms in different tempos so as to observe, and comment on, the effect tempo can have on the perception of meter. Both teachers taught their students about different degrees of evenness (perfect evenness, maximal evenness, neither) in rhythms and the effect this has on the listening experience. For example, Teacher A's students reported that some rhythms, or sequences of rhythms, were "smooth" while others were "jarring" according to the amount of evenness and inclusion. Notably, neither teacher told the students how to hear the rhythms or meter; rather, the focus of lessons was on what the students heard and for them to bring to the class their observations of their own listening experiences. By mapping rhythms and meter to cyclic graphs to discuss inclusion, rotation and complementation, we observed students learned more deeply about why a piece of music has a certain "feel" or "groove".

Teacher A acknowledged that the students in their class knew "very little – both formally and informally – about Western popular musics," and added, "undoubtedly, this methodology provided a glimpse into that world and how it might be better comprehended academically." Both teachers saw the potential for XronoBeat to be included among the classroom music activities where theoretical materials were taught. Through applying visualizations and sonifications of mathematical music theory to their understanding of rhythm and meter, the students in both classes learned that, unlike traditional understandings, meter is not notation-based; rather, it is experienced and initially located in the mind of the listener. Students displayed impressive knowledge of the mathematics they had learned from prior classes in this pilot project and its relationship to their experience of music. Notably, students were focused, engaged, and eager when learning about mathematics of rhythm and meter through the application of set theory with visualizations and sonifications of cyclic graphs.

## 4 Conclusion

The results demonstrate that this type of music-teaching intervention, which integrates music software with mathematically based teaching materials, is feasible – the program ran successfully to completion over six weeks and both the pupils and teachers enjoyed the experience. It seems that the software helped the students to better understand mathematical concepts behind rhythm and meter, and they found it useful for providing immediate visualizations and sonifications of theoretical concepts as well as for exploring rhythms in a more creative way. One student noted "I legitimately learned so much just from trying out different combinations and ideas, 'what would happen if ...' and I think many other students would benefit from this powerful kind of learning". It is also interesting to note that such a high proportion of the students felt mathematics and music were related, with 42% of students suggesting that this intervention helped them to strengthen or to make new such connections. As with many novel interventions, there will always be practical difficulties related to integrating into standard curricula; however, given the desire in many education authorities for greater emphasis on inter-disciplinary activities and skills, we suggest that combining mathematics and music is a model that is both feasible and fruitful.

## References


1. Bahna-James, T.: The relationship between mathematics and music: Secondary school student perspectives. The Journal of Negro Education **60**(3), 477–485 (1991)
2. Bahr, N., Christensen, C.A.: Inter-domain transfer between mathematical skill and musicianship. Journal of Structural Learning and Intelligence Systems **14**, 187–197 (2000)
3. Brochard, R., Dufour, A., Després, O.: Effect of musical expertise on visuospatial abilities: Evidence from reaction times and mental imagery. Brain and Cognition **54**, 103–109 (2004)
4. Cheek, J.M., Smith, L.R.: Music training and mathematics achievement. Adolescence **34**, 759–761 (1999)
5. Cohn, R.: Complex hemiolas, ski-hill graphs and metric spaces. Music Analysis **20**(3), 295–326 (2001)
6. Cohn, R.: Teaching atonal and beat-class theory, modulo small. MusMat: Brazilian Journal of Music and Mathematics **1**(1), 15–24 (2016)
7. Cohn, R.: Scaling up to atonality: The pedagogy of small cyclic universes. In: Montiel, M., Gómez, F. (eds.) Visualizing and sonifying mathematical music theory with software applications: Implications of computer-based models for practice and education, chap. 6, pp. 127–149. World Scientific (2019)
8. Cox, H.A., Stephens, L.J.: The effect of music participation on mathematical achievement and overall academic achievement of high school students. International Journal of Mathematical Education in Science and Technology **37**(7), 757–763 (2004)

9. Gentner, D., Loewenstein, J., Thompson, L.: Learning and transfer: A general role for analogical encoding. Journal of Educational Psychology **95**(2), 393–408 (2003)
10. Haimson, J., Swaina, D., Winner, E.: Do mathematicians have above average musical skill? Music Perception **29**(2), 203–213 (2011)
11. Hamilton, T.J., Doai, J., Milne, A.J., Saisanas, V., Calilhanna, A., Hilton, C., Goldwater, M., Cohn, R.: Teaching mathematics with music: A pilot study. In: Proceedings of IEEE International Conference on Teaching, Assessment, and Learning for Engineering (TALE 2018). University of Wollongong, NSW, Australia (2018)
12. Harrison, C.S.: Relationships between grades in the components of freshman music theory and selected background variables. Journal of Research in Music Education **38**(3), 175–186 (1990)
13. Harrison, C.S.: Relationships between grades in music theory for nonmusic majors and selected background variables. Journal of Research in Music Education **44**(4), 341–352 (1996)
14. Harrison, C.: Predicting music theory grades: The relative efficiency of academic ability, music experience, and musical aptitude. Journal of Research in Music Education **38**(2), 124–137 (1990)
15. Helmrich, B.H.: Window of opportunity? adolescence, music, and algebra. Journal of Adolescent Research **25**(4), 557–577 (2010)
16. Hilton, C., Calilhanna, A., Milne, A.J.: Visualizing and sonifying mathematical music theory with software applications: Implications of computer-based models for practice and education. In: Montiel, M., Gómez, F. (eds.) Theoretical and Practical Pedagogy of Mathematical Music Theory: Music for Mathematics and Mathematics for Musicians, From School to Postgraduate Levels, chap. 9, pp. 201–236. World Scientific (2019)
17. Milne, A.J.: XronoMorph: Investigating paths through rhythmic space. In: Holland, S., Mudd, T., Wilkie-McKenna, K., McPherson, A.P., Wanderley, M.M. (eds.) New Directions in Music and Human-Computer Interaction. Springer (2019)
18. Milne, A.J., Bulger, D., Herff, S.A.: Exploring the space of perfectly balanced rhythms and scales. Journal of Mathematics and Music **11**(2–3), 101–133 (2017). https://doi.org/10.1080/17459737.2017.1395915
19. Milne, A.J., Herff, S.A., Bulger, D., Sethares, W.A., Dean, R.T.: XronoMorph: Algorithmic generation of perfectly balanced and well-formed rhythms. In: Proceedings of the 2016 International Conference on New Interfaces for Musical Expression (NIME 2016). pp. 388–393. Griffith University, Brisbane, Australia (2016)
20. Richland, L.E., Zur, O., Holyoak, K.J.: Cognitive supports for analogies in the mathematics classroom. Science **316**, 1128–1129 (2007)
21. Rogers, N., Clendinning, J.P.: Music theory ability correlates with mathematical ability. In: Meeting of the Society for Music Perception and Cognition. Nashville, TN, US (2015)
22. Rogers, N., Clendinning, J.P., Ganley, S.H.C.: Specific mathematical and spatial abilities correlate with music theory abilities. In: Proceedings of the 14th International Conference on Music Perception and Cognition. pp. 537–543 (2016)
23. Rogers, N., Clendinning, J.P., Hart, S., Ganley, C.: Specific correlations between abilities in mathematics and music theory. In: Society for Music Theory: Fortieth Annual Meeting. Arlington, VA, US (November 2017)
24. Schmithorst, V.J., Holland, S.K.: The effect of musical training on the neural correlates of math processing: A functional magnetic resonance imaging study in humans. Neuroscience Letters **354**, 193–196 (2004)