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Parametric study for an ant algorithm applied to water distribution system optimization

Aaron C. Zecchin, Angus R. Simpson, Holger R. Maier and John B. Nixon

Abstract—Much research has been carried out on the optimization of Water Distribution Systems (WDSs). Within the last decade, the focus has shifted from the use of traditional optimization methods, such as linear and non-linear programming, to the use of evolutionary algorithms (EAs), namely genetic algorithms, simulated annealing and more recently, ant colony optimization (ACO), an optimization algorithm based on the foraging behavior of ants. EAs have been seen to perform better than more traditional optimization methods and amongst the EAs applied to WDS optimization, a recent study found ACO to outperform other EAs for two well-known case studies. One of the major problems that exists with the use of EAs, particularly ACO, is that their searching behavior, and hence performance, is governed by a set of user-selected parameters. Consequently, a large calibration phase is required for successful application to new problems. The aim of this paper is to provide a deeper understanding of ACO parameters and to develop parametric guidelines for the application of ACO to WDS optimization. For the adopted ACO formulation, seven parameters are used: two decision policy control parameters a and b , initial pheromone value t_0 , pheromone persistence factor r , number of ants m , pheromone addition factor Q , and the penalty factor PEN . Deterministic and semi-deterministic expressions for Q and PEN are developed. For the remaining non-deterministic parameters a parametric study is performed, from which guidelines for appropriate parameter settings are developed. Based on the use of these heuristics, the ACO algorithm used achieves the best performance presented in the literature, in terms of efficiency and solution quality, for one of the well-known case studies, the New York Tunnels Problem.

Index Terms—ant colony optimization, parameter guidelines, optimization, water distribution systems.

I. INTRODUCTION

Water distribution systems (WDSs) are systems that are designed to transport water from water sources to consumers. They typically involve a complex network of electromechanical components such as pipes, pumps, valves, and tanks. Optimization of WDSs usually focuses on the minimization of the material and installation costs of a WDS design, where the WDS components (e.g. the diameters of pipes) are treated as the decision variables and the constraints that determine the feasibility of a design are the design requirements (e.g. the provision of adequate pressure). Much research has gone into the optimization of WDSs. Within the last decade, the focus has shifted from the use of methods based on traditional optimization theory, such as linear programming [1]–[3], two-phase decomposition methods [4], [5], and non-linear programming [6] to the use of evolutionary algorithms (EAs), namely genetic algorithms (GAs) [7]–[13], simulated annealing [14], and more recently ant colony optimization

(ACO) [15]–[18]. It should be noted that ACO is often classified as a population-based metaheuristic, rather than an EA. However, for the sake of simplicity, ACO will be classed as an EA for the remainder of this paper.

EAs have been shown to perform better than the more traditional optimization methods [8]. The advantages of EAs for application to WDS optimization are that they: (i) are a discrete combinatorial optimization algorithm and as such deal only with realistic commercial pipe diameter sizes; (ii) perform a global search; and (iii) have a simpler application as they deal only with objective function values, and hence information such as first and second derivatives need not be computed. In a recent study [17], ACO was found to be extremely competitive and outperformed GAs for two well known case studies: the Two-Reservoir Problem; and the New York Tunnels Problem. As ACO is a relatively new discrete combinatorial optimization technique that has shown great potential for other optimization problems [19], this research focuses further on its application to WDS optimization.

One issue that exists with the use of EAs, particularly ACO, is that their searching behavior, and hence performance, is governed by a set of non-deterministic (i.e. user-selected) parameters. For ACO, other papers have offered guidelines as to appropriate parameter settings [20], but optimal or near-optimal settings of these parameters are heavily dependent on the properties of the multi-dimensional objective function surface associated with the optimization problem. Consequently, recommendations for parameter guidelines can only be considered within the context in which they were derived. This determination of appropriate or optimal settings for different and new problems can be an optimization problem in itself. As ACO has only received little application to WDS optimization, no guidelines exist for its application to this problem.

The objective of this paper is to develop parametric guidelines for the application of ACO algorithms to WDS optimization. The guidelines have been derived based on only one algorithm, but with other optimization problems (e.g. the traveling salesperson problem) other authors [22]–[25] have effectively utilized parameter guidelines that have been derived from simpler ACO algorithms [20] for the application of more advanced ACO algorithms.

This paper is structured as follows. In section II WDS optimization is explained and is formally defined as an optimization problem. In section III the basic concept of the foraging behavior of ants is explained and, based on this, the mathematical formulation of the adopted ACO algorithm is given. This is followed by a brief discussion of

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the ACO parameters. In section IV the transformations of the search space and conversion of ACO elements required for ACO to be applied to WDS optimization is given. The three case studies are outlined in section V. In section VI the derivations for the deterministic equation and the semi-deterministic equation for two of the parameters are given. The results from the parametric studies for the remaining parameters are presented in section VII. Parameter guidelines are derived from these results. Section VIII presents an assessment of the performance of ACO, utilizing the parameter guidelines derived in section VII, with the best performing algorithms in the literature for the given case studies. The summary and conclusions are given in section IX.

II. THE WATER DISTRIBUTION SYSTEM OPTIMIZATION PROBLEM

The optimization of a WDS is defined as the selection of the lowest cost combination of appropriate component sizes and component settings such that the criteria of demands and other design constraints are satisfied. A simple example of this is as follows. Consider two networks, the first comprising pipes with small diameters and the second comprising pipes with large diameters. The first network has a low cost but as the pipe diameters are small the headloss¹ through the network will be greater, which is likely to result in insufficient pressure at the demand points (nodes). The second system is likely to provide more than adequate pressure, as the pipe diameters are large, but is also more expensive. The optimal design is then the least cost combination of diameter sizes that are able to provide adequate pressure at each of the nodes.

In practice, the optimization of a WDS can take many forms, as WDSs are comprised of many different components and have many different performance criteria. For example, the decision variables within the optimization problem could involve the selection of diameter sizes for all the pipes, the sizing and locating of tanks, valve pressure settings and valve locations, and pump types and locations. In addition to these potential decision variables, the demands on the system could involve a range of demand cases including peak hour, fire loading, and an extended period simulation. The constraints on the system could be specified to include minimum and maximum allowable pressures at each demand point, a maximum velocity constraint for each of the pipes, and water quality requirements. In addition to all of these demands and constraints, for the system to be properly assessed as to whether it meets the

¹ Headloss through a pipe is the engineering term given to the reduction in pressure of a fluid as it flows along a pipe due to the energy losses from the friction of the fluid flow against the pipe wall and the resulting turbulence within the fluid body. It is typically expressed in units of meters (i.e. head = pressure / specific weight).

design criteria the inherent uncertainty that exists within the system (e.g. the realistic randomness of nodal demands and stochastic performance of the components) would also need to be quantified [26].

In the literature, the optimization of WDSs has traditionally dealt with a much more simplistic and idealized problem. The decision variables have primarily been related to the pipes within the system where, more specifically, the decision options have been the selection of: (i) a diameter for a new pipe; (ii) a diameter for a parallel pipe; and (iii) the cleaning of an existing pipe to reduce the hydraulic resistance. The constraints on the system have usually been the minimum allowable pressures at each of the nodes. This form of the optimization of WDSs is used within this paper.

A semi-formal expression of the optimization problem, entitled the Water Distribution System Problem (WDSP) is given in the following sections, which expands on previous formulations [11], [13], as multiple demand patterns and pipe rehabilitation options are included (similarly to [13]) such that the formulation encompasses problems such as the Two-Reservoir Problem [9]. The following sub-sections more clearly provide a definition of a “design” as used in the WDSP and the constraints for a feasible design. From these the optimization problem is formulated.

A. Definition of a design

A WDSP typically falls into one of two categories: (i) “new” design; or (ii) “rehabilitation” of an existing network. A new design problem consists of a network layout where all of the pipes have to be assigned a diameter (i.e. the locations of all pipes are a given). A rehabilitation problem contains an existing network that requires an increase in its hydraulic capacity (e.g. via replacement, cleaning, or parallelization of existing pipes) and/or provision of water to new consumers (e.g. via expansion with new pipes). This rehabilitation involves one or all of the three decision options outlined in the introduction of this section. Due to the generality of the problem formulation presented here, both types of problems are encompassed within the same framework and from hereon are referred to as a design.

For the WDSP, a design option involves the selection of rehabilitation and/or duplication options for all or some of the pipes within the network. From the framework that has been qualitatively established earlier in this section, a WDS design Ω is defined as a set of n decisions where n is the number of pipes to be sized and/or rehabilitated, that is

$$\Omega = \{decision_1, \dots, decision_n\} \quad (1)$$

where $decision_i$ is the selected option for pipe i , and is defined as

$$decision_i \in \{option_{i,1}, \dots, option_{i,NO_i}\} \quad \forall i = 1, \dots, n \quad (2)$$

where $\{option_{i,1}, \dots, option_{i,NO_i}\}$ is the set of all NO_i options available for pipe i . For each option there is an associated cost, $c_{i,j}$ of implementing that option, a diameter and roughness coefficient for the new (or parallel) pipe, and a roughness coefficient for the existing pipe (i.e. $dia_{i,j}^{new}, RC_{i,j}^{new}, RC_{i,j}^{exist}$). The inclusion of $RC_{i,j}^{exist}$ as a variable within the alternatives allows for rehabilitation options that change the roughness coefficient only (e.g. for problems like that in [9]). Clearly, for rehabilitation options $dia_{i,j}^{new} = 0$. Similarly, existing pipe roughnesses $RC_{i,j}^{exist}$ will remain constant for all non-rehabilitation options. By definition, for new design problems $dia_{i,j}^{exist} = 0 \quad \forall i = 1, \dots, n$ and $\forall j = 1, \dots, NO_i$ as there are no existing pipes (i.e. the existing pipe diameters are zero). Option j for pipe i is given by the set of elements

$$option_{i,j} = \{c_{i,j}, dia_{i,j}^{new}, RC_{i,j}^{exist}, RC_{i,j}^{new}\}. \quad (3)$$

B. Constraints for a feasible design

The constraints on a solution are categorized as design constraints and hydraulic constraints. A design constraint is an inequality constraint that defines the minimum acceptable performance of a design, whereas hydraulic constraints are equality constraints that describe the distribution of the flow of water through the WDS (based on the fundamental equations for fluid flow within a closed conduit and the governing equations for fluid flow within a looped network). The design constraint for the WDSP specifies the minimum allowable pressure at each node, and is given as

$$H_i^j(\Omega) \geq \overline{H_i^j} \quad \forall i = 1, \dots, N_{node} \quad \forall j = 1, \dots, N_{pattern} \quad (4)$$

where $H_i^j(\Omega)$ is the actual head at node i for demand pattern j and design Ω , $\overline{H_i^j}$ is the minimum allowable head at node i for demand pattern j , N_{node} is the total number of nodes and $N_{pattern}$ is the number of demand patterns.

The hydraulic equations for fluid flow within a WDS are the conservation of mass, the conservation of energy, and the pipe headloss equations (note: energy losses due to junctions, bends, and valves, etc. are generally small in comparison to frictional losses for WDSs and are consequently ignored). These equations can be formulated in many different ways but the authors have selected the following formulation for ease of understanding. As the fluid is assumed to be incompressible, the conservation of mass equations dictate that the flow rate into a node is equal to the flow rate out of a node. This can be expressed as

$$DM_i^j + \sum_{k \in \Theta_i^+(\Omega)} Q_k^j(\Omega) - \sum_{k \in \Theta_i^-(\Omega)} Q_k^j(\Omega) = 0 \quad \forall i = 1, \dots, N_{node} \quad \forall j = 1, \dots, N_{pattern} , \quad (5)$$

where DM_i^j is the demand for node i and demand pattern j (by definition, a positive demand is one that draws water from the node), $Q_k^j(\Omega)$ is the flow in pipe k for design Ω for demand pattern j , $\Theta_i^-(\Omega)$ is the set of all pipes that provide flow into node i for design Ω , and $\Theta_i^+(\Omega)$ is the set of pipes that provide flow out of node i for design Ω .

The conservation of energy equations ensure that the net frictional energy loss around a closed loop is equal to zero. This is expressed in terms of a summation of the headlosses around a closed loop

$$\sum_{k \in \Psi_i^+(\Omega)} h_k^j(\Omega) - \sum_{k \in \Psi_i^-(\Omega)} h_k^j(\Omega) = 0 \quad \forall i = 1, \dots, N_{loop} \quad \forall j = 1, \dots, N_{pattern} \quad (6)$$

where $\Psi_i^+(\Omega)$ is the set of pipes whose flow direction is in a positive rotation about loop i for design Ω (note: it is convention to take clockwise as being a positive rotation), $\Psi_i^-(\Omega)$ is the set of pipes whose flow direction is in a negative rotation about loop i for design Ω , $h_k^j(\Omega)$ is the headloss in pipe k for demand pattern j and design Ω (by definition headloss is positive along the direction of flow) and N_{loop} is the number of loops. A special case of (6) is when the loop is an unclosed path between sources of fixed head (i.e. reservoirs). In this situation the summation is

equal to the head difference between the two sources. The general headloss equation, the last of the hydraulic constraints, is written as

$$h_i^j(\Omega) = r_i^j(\Omega) [Q_i^j(\Omega)]^a \quad \forall i = 1, \dots, N_{pipe} \quad \forall j = 1, \dots, N_{pattern} \quad (7)$$

where $Q_i^j(\Omega)$ is the flow rate in pipe i for demand pattern j and design Ω , $r_i^j(\Omega)$ is a hydraulic resistance term dependent on the type of headloss equation used, a is the flow exponent which is also dependent on the type of headloss equation used, and N_{pipe} is the number of pipes including new pipes. The headloss equation used within this study is the standard Hazen-Williams equation, therefore, $r_i^j(\Omega)$ is expressed as

$$r_i^j(\Omega) = A \frac{L_i}{[C_i(\Omega)]^a [D_i(\Omega)]^b} \quad \forall i = 1, \dots, N_{pipe} \quad \forall j = 1, \dots, N_{pattern} \quad (8)$$

where L_i is the length of pipe i , $D_i(\Omega)$ is the diameter of pipe i for design Ω , $C_i(\Omega)$ is the Hazen-Williams coefficient for pipe i for design Ω (for the Hazen-Williams formulation the RC values are the Hazen-Williams coefficients), A is a constant that is dependent on the units used, and a and b are regression coefficients. The Hazen-Williams formulation has been assumed for the sake of continuity with other studies, but the Darcy-Weisbach equation could have just as easily been used. The adopted values of A , a , and b are found to vary throughout the literature [11], [6], [13], [17], bringing about a variety of optimum solutions for various problems. In an attempt to standardize, the values adopted in this paper are based on those used in the hydraulic solver software EPANET2 (as this is based on the original formulation by Hazen and Williams). These are: $A = 10.6744$ for SI units and 4.7270 for US customary units, $a = 1.852$, and $b = 4.871$.

For the design to be feasible it must satisfy both sets of constraints. In practice, only the design constraints need to be considered, as the flow regime determined by the hydraulic solver automatically satisfies the hydraulic constraints.

C. Formulation of the Water Distribution Problem

As the objective is the minimization of the material and installation costs, the WDSP can be expressed as

$$\min C(\Omega) = \sum_{i=1}^n L_i c(\text{decision}_i) \quad (9)$$

Subject to (4), (5), (6), and (7).

where $C(\Omega)$ is the cost of design Ω and $c(\text{decision}_i)$ is the unit length cost of decision_i .

III. ANT COLONY OPTIMIZATION

A. Basis of Algorithm

Ant Colony Optimization (ACO) is a discrete combinatorial optimization algorithm based on the foraging behavior of ants. Over a period of time ants are able to determine the shortest path from their home to a food source. This shortest-path-finding process of the colony can be viewed as a form of swarm intelligence [21]. This process is achieved by the colony's accumulation of information about the surrounding area, which is communicated to the individual ants in the form of trails of pheromone, a chemical substance laid by the ants themselves.

As explained in [20], isolated ants essentially wander randomly until they come across a previously laid pheromone path, which they will, by instinct, be more inclined to follow as opposed to continuing to wander randomly. As an ant traverses the path, it too lays pheromone, thus reinforcing the existing pheromone strength of the current path and hence attracting further ants to follow it. Gradually over time the shorter paths between destinations will increase in pheromone intensity due to lower traverse times, and so the colony gradually determines the optimum route between the destinations. This phenomenon is best illustrated by an example.

In Fig. 1 a system comprising a colony home (H), a food source (F), and an obstacle (A–B) is depicted. The obstacle is placed such that there are two paths of unequal lengths, and it is assumed that path HAF takes two time steps to traverse and path HBF takes a single time step to traverse. At time $t = 0$ (Fig. 1(a)), eight ants are placed into the system with the objective of getting food from F back to H. As their initial selection of the path they take is random, it is assumed that 4 ants select each of the two alternate paths.

At $t = 1$ (Fig. 1(b)), the ants that traversed HBF have acquired food and begin to journey back to H. As there is existing pheromone on FBH the ants have a higher probability of utilizing this path, consequently three ants select FBH and one ant selects FAH. The ants that are traversing HAF are only half way along this path.

At $t = 2$ (Fig. 1(c)), the three ants that traversed FBH are home again while the ant that embarked on FAH is only

half way along this path. The four ants that were traversing HAF have made it to F and embark on their journey back to H via either FAH or FBH. Path FBH has a greater amount of pheromone on it (note that the pheromone intensity is represented by the darkness of the path) as it has been traversed seven times, whereas FAH has only been traversed five times. Consequently, by probability, three ants select path FBH and one ant selects FAH.

At $t = 4$ (Fig. 1(d)), all ants have returned to H (note that the ant that embarked on FBH at $t = 2$ arrived at H at $t = 3$). From Fig. 1(d) it is seen that the shorter path HBF has a greater amount of pheromone on it as it has been traversed ten times in total, while the longer path HAF has been traversed only six times. Future ants entering the system will have a higher probability of selecting path HBF. In the pattern illustrated here the operation of swarm intelligence to determine the shortest path is seen.

In addition to the positive feedback strategy illustrated in the example, the pheromone trails also decay with time. This means that paths that are not regularly given additional pheromone will eventually decay to zero intensity. This decaying quality of pheromone also aids in the ability of the ant colony to find the shorter paths. The longer paths, which receive less pheromone, will decay more rapidly enabling shorter paths to have a higher probability of being selected.

B. Formulation of Ant Colony Optimization

In applying the ant colony analogy to develop ACO, [20] highlighted some differences between the analogy and ACO. These are that, as distinct from real ants, artificial ants: (i) have some memory; (ii) are not completely blind; and (iii) live in a discrete time environment. In addition to this, the formulation of ACO used in this paper includes the following differences: (iv) ants enter the network consecutively; (v) pheromone is laid on each ant's path when all ants have completed their tours; and (vi) the amount of pheromone placed on a path is non-constant but is dependent on the length of that path (as represented by the objective function value).

The ACO algorithm operates by iteratively generating a population of solutions where each solution is representative of the path that a single ant has traveled. An ant generates a solution by selecting an edge at each decision point based upon a decision policy [20]. Once each ant has generated a solution, an amount of pheromone proportional to the quality of the solution is deposited upon the path. In this way, good solution components are reinforced with greater amounts of pheromone.

At each decision point, an ant probabilistically selects an edge governed by the decision policy. This policy considers a trade-off between the pheromone intensity on a particular edge and the desirability of that edge with respect to its isolated impact on the objective function. The desirability of an edge has different definitions for different problems. For example, if the objective is to minimize cost, the desirability of an edge may be set equal to the inverse of the cost associated with that edge. Taking these two properties of an edge into account, ACO algorithms effectively utilize heuristic information that has been learned (i.e. represented as pheromone intensity) in addition to incorporating a bias towards edges that are of a greater desirability or local optimality. The most common form of the decision policy is [20]

$$p_{i,j}(t) = \frac{[\mathbf{t}_{i,j}(t)]^a [\mathbf{h}_{i,j}]^b}{\sum_{l \in \mathbf{q}_i} [\mathbf{t}_{i,l}(t)]^a [\mathbf{h}_{i,l}]^b} \quad (10)$$

where $p_{i,j}(t)$ is the probability that edge (i,j) is chosen at iteration t , $\mathbf{t}_{i,j}(t)$ is the concentration of pheromone associated with edge (i,j) at iteration t , $\mathbf{h}_{i,j}$ is the desirability of edge (i,j) , \mathbf{a} is the parameter controlling the relative importance of pheromone in the decision making process, \mathbf{b} is the parameter controlling the relative importance of the desirability, and \mathbf{q}_i is the set of edges l available at decision point i .

As stated above, the way the colony utilizes information about the optimization problem is via the addition of pheromone to the graph to reinforce good edges. The common form of the pheromone update equation is [20]

$$\mathbf{t}_{i,j}(t+1) = \mathbf{r}\mathbf{t}_{i,j}(t) + \Delta\mathbf{t}_{i,j}(t) \quad (11)$$

where \mathbf{r} is the coefficient representing pheromone persistence (note: $0 \leq \mathbf{r} \leq 1$) and $\Delta\mathbf{t}_{i,j}(t)$ is the pheromone addition on edge (i,j) . The pheromone persistence factor allows for the modeling of pheromone evaporation (mentioned in the preceding example), which is analogous to a gradual loss of memory. This is important as it allows ACO algorithms to forget poor information that was learned early on in the search, and to focus on using the better information that has presumably been gained later on in the search.

Many authors have proposed different formulations of $\Delta\mathbf{t}_{i,j}(t)$ [20], [22]–[25] as this function dictates how the ACO

algorithm utilizes the learned information. In this research, a simple formulation is used based on the work of [17], [24], and [25]. This formulation is given the title Iteration-Best Ant System ($AS_{i\text{-best}}$) as it updates only the path of the ant that finds the best solution within a given iteration. The advantage of this formulation is that only the best information is retained and reinforced. The authors in [17], when applying ACO to the WDSP, found that updating only the iteration-best ant's path worked better than updating all ant paths. Within $AS_{i\text{-best}}$, $\Delta t_{i,j}(t)$ is defined as

$$\Delta t_{i,j}(t) = \begin{cases} \frac{Q}{f(S_{best}(t))} & \text{if edge } (i, j) \in S_{best}(t) \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

where Q is the phomone reward factor (a constant), $f(\cdot)$ is the objective function, and $S_{best}(t)$ is the set of edges contained within the best solution (i.e. the solution with the minimum objective function value) found within iteration t . From (12) it is clear that the amount of phomone that is awarded to a path is inversely proportional to the objective function value. This is important as solutions that give better results are awarded greater amounts of phomone (analogous to the shorter path receiving more phomone from the preceding example) and hence are more likely to be reselected by future ants.

C. Summary of ACO parameters

In this section, a summary of the ACO parameters for which guidelines are derived in sections VI and VII is provided.

1) Decision policy control parameters, \mathbf{a} and \mathbf{b}

As stated previously, \mathbf{a} and \mathbf{b} are the coefficients that determine the relative importance of phomone intensity and desirability, respectively, in an ant's decision process, as given by (10). The phomone intensity is representative of the learned attractiveness of an option, whereas the desirability is representative of the inherent attractiveness of an option (i.e. lower cost options are more attractive). If $\mathbf{a} \gg \mathbf{b}$ then the algorithm will make decisions based mainly on the learned information, as represented by the phomone. If $\mathbf{b} \gg \mathbf{a}$ the algorithm will act as a greedy heuristic, selecting mainly the shortest or cheapest edges, disregarding the impact of these decisions on the final solution quality.

For the traveling salesperson problem, the recommended values are $\mathbf{a} = 1.0$ and $\mathbf{b} = 5.0$, as originally suggested in [20]. These values have been successfully used by other authors [23]–[25]. In the application of ACO to the WDSP, [16] and [17] used values of $\mathbf{a} = 5.0$ and 3.5 and $\mathbf{b} = 0.5$ and 3.5 , respectively.

2) Pheromone initialization value, \mathbf{t}_0

The pheromone initialization value is the pheromone intensity that all paths are initially set at (i.e. $\mathbf{t}_{i,j}(1) = \mathbf{t}_0$, for all i and j). This is an important parameter in the solution evolution of ACO as it affects the relative importance of pheromone additions $\Delta\mathbf{t}_{ij}(t)$ [24]. This effect is especially exaggerated in the early stages of the search, when good solutions have not yet been established. If \mathbf{t}_0 is set too small the relative difference in pheromone levels as a result of an addition of $\Delta\mathbf{t}_{ij}(t)$ will be given too much importance in the probability determination, and premature convergence can result. Conversely, if \mathbf{t}_0 is set too high, a difference of $\Delta\mathbf{t}_{ij}(t)$ will not be given sufficient importance.

Appropriate settings for \mathbf{t}_0 are largely un-discussed in the literature on ACO parameters. When applying the Ant Colony System (a version of ACO) to the symmetric and asymmetric traveling salesperson problem, the authors of [22] suggest the use of

$$\mathbf{t}_0 = (n \cdot L_{mn})^{-1}, \quad (13)$$

where n is the number of decision points and L_{mn} is the tour length produced by the nearest neighbor heuristic. In a non-problem-specific sense, (13) is analogous to a quantity equal to the pheromone addition based on the objective function value of some near-optimal solution divided by n (the number of decision points). This extrapolation to the general case is justified by recognizing that, for the Ant Colony System applied to the traveling salesperson problem, the pheromone additions are equal to the inverse of the tour lengths.

3) Pheromone persistence factor, \mathbf{r}

The pheromone persistence factor \mathbf{r} regulates the decay of the pheromone trails (as implemented in (11)), which results in the gradual decrease of the probability of selection of paths that are not regularly traversed. This decay is important as it allows for new, and better, information to guide the search. For $\mathbf{r} \approx 1$ only small amounts of pheromone are decayed between iterations and the convergence rate is slower, whereas for $\mathbf{r} \approx 0$ more pheromone

is decayed resulting in faster convergence.

Values suggested in the literature range from $r = 0.5$ [20], [23], to $r = 0.75$ [24], and $r = 0.8$ [25]. For application of ACO to the WDSP, [16] and [17] found that higher values of $r = 0.8$ and 0.95 resulted in improved performance.

4) *Number of ants, m*

The number of ants, m , indicates the number of trial solutions generated before new information (i.e. pheromone addition) is utilized to further guide the search. The parameter m is also effectively equal to the number of random starting positions that the algorithm has in objective function space. For the traveling salesperson problem it is recommended that $m = n$ [20], which has been used effectively by numerous authors [22]–[25]. For applications to the WDSP, m has tended to be set higher than the value given by this recommendation, where a value of $m = 100$ was assumed for problem sizes of $n = 14$ and 21 [17]. However, no relationship between the search space characteristics and m has been determined for the WDSP.

5) *Pheromone reward factor, Q*

The reward factor Q is used as a scaling factor for the pheromone additions (see (12)). The parameter Q quantifies the influence of new information relative to the pheromone initialization value t_0 [24]. The authors of [20] and [24] stated that the performance of the ACO algorithms used in their studies were relatively insensitive to Q , and furthermore that it is not a crucial parameter. Typically $Q = 100$ for the application of ACO to the traveling salesperson problem [20], [23], [24]. The authors of [17] also found that a similar performance was achieved for a broad range of values of Q when applying ACO to the WDSP.

IV. INTEGRATION OF ACO AND THE WDSP

As ACO is a discrete combinatorial optimization algorithm that can only deal with unconstrained search spaces [19], transformations of the WDSP from a constrained optimization problem to an unconstrained problem must be performed. Similarly, for application to different problems ACO elements such as desirability must be redefined.

A. *Transformation From Constrained To Unconstrained Problem*

ACO, like all EAs, performs the optimization process by navigating the n -dimensional solution space, where searching behavior is governed by the value of the objective function at each point. It is desired to guide the search

of the EA into the feasible region and to do this it is necessary to transfer information about the feasibility of the solution to the EA via the objective function value. As performed previously in the application of EAs to the WDSP, [9], [10], [16], [17], this can be achieved by the use of a penalty function [27]. A penalty function is a function that makes infeasible solutions unattractive (in terms of objective function value) relative to the optimum solution such that they are treated by the algorithm as being poor, non-optimal solutions. As the objective function in the WDSP is the minimization of cost, the cost of infeasible solutions is increased via the use of a penalty cost. Therefore, the optimization problem in its transformed unconstrained state takes the form

$$\min \quad NC(\Omega) = C(\Omega) + PC(\Omega), \quad (14)$$

where $NC(\Omega)$ is the total network cost for design Ω , $C(\Omega)$ is the material cost for design Ω (i.e. the objective for the constrained problem), and $PC(\Omega)$ is the penalty cost incurred by design Ω . There are many different forms that $PC(\Omega)$ can take that account for the number of constraints violated, the degree of violation, and extent of the search (see [27] for a state-of-the-art overview of penalty functions). Here $PC(\Omega)$ is defined similarly to [9] and [17], in that it is proportional to the maximum pressure violation. That is

$$PC(\Omega) = \begin{cases} 0 & H_i^j(\Omega) \geq \overline{H}_i^j \quad \forall i = 1, \dots, N_{node} \quad \forall j = 1, \dots, N_{pattern} \\ \max_{\substack{i=1, \dots, N_{node} \\ j=1, \dots, N_{pattern}}} \{ \overline{H}_i^j - H_i^j(\Omega) \} \cdot PEN & \text{otherwise} \end{cases}, \quad (15)$$

where PEN is the penalty factor (constant) with units of dollars per meter of pressure violation. The parameter PEN is a user-defined parameter, as appropriate values of PEN are different for each case study [17]. Typically PEN requires calibration [17], however a semi-deterministic expression for PEN is derived in section VI.

B. Conversion From the General ACO Problem Formulation to the WDSP

In addition to a transformation of the optimization problem, a redefinition of the desirability of an option is required. For the WDSP, the objective is to minimize the total network cost. Therefore, the desirability for each option is equal to the inverse of the unit cost of implementing that option. To generalize the formulation given in [17], the desirability of option j at pipe i is defined as

$$\mathbf{h}_{i,j} = \frac{1}{c_{i,j}}. \quad (16)$$

From (16), as lower cost options are more desirable, a higher bias in the probability of selection of lower cost options results. An issue arises with this definition of desirability when a pipe has a ‘null’ option (i.e. the option to take no action), as the cost is zero and consequently the desirability factor is infinity. To overcome this, a ‘virtual-zero-cost’ [17] for these options was used. The value of the virtual-zero-cost was selected such that it was approximately one third of the lowest cost option.

Table 1 gives a summary of the conversion of the problem elements from the general ACO problem formulation to the WDSP.

V. CASE STUDIES

A. *The New York Tunnels Problem*

The New York Tunnels Problem (NYTP), originally considered in [1], involves the rehabilitation of a WDS by selecting design options for the 21 tunnels in the system. The network has 20 nodes, including a reservoir, and a single demand case. In this paper, the metric version of the problem as defined in [10] was used. There are 16 design options for each tunnel (i.e. parallelization with one of 15 tunnel sizes or the null option) creating a search space size of 16^{21} (approximately 1.9342×10^{25}). As there is the null option, a virtual-zero-cost of \$110 per meter was used.

The NYTP has been considered as a continuous [1], [29], [3], [4], split pipe [30], and discrete [31], [32], [10]–[13], [17] WDS optimization problem. The current feasible known-optimum (i.e. current lowest cost solution) to the discrete version of this problem is \$38.638 million (i.e. \$38,638,000) found by [17].

B. *Hanoi Problem*

Originally considered in [4], the Hanoi Problem (HP) involves the selection of diameters for 34 new pipes within a WDS consisting of 32 nodes, including a reservoir. There are six diameter options for each pipe, creating a total search space size of 6^{34} (approximately 2.8651×10^{26}) designs. The HP has been considered as a continuous [4], [6],

split-pipe [4], [33], [5], [6], and discrete pipe [11], [14], [13] optimization problem. (The reader is referred to [13] for case study details.)

As $AS_{i\text{-best}}$ deals only with discrete optimization problems, only the discrete solutions to this network are considered for comparison in this paper. As a range of coefficients for the Hazen-Williams head-loss equation were used in different studies, all reported solutions have been analyzed using EPANET2, which was the benchmark hydraulic analysis tool used in this research. The resulting heads for the critical nodes are given in Table 2. It can be seen that the \$6.073 million solution (from [11]) fails the design pressure constraint at nodes 13 and 30, whilst the \$6.065 million solution (from [14]) fails at nodes 13, 16, 27, and 29 and thus these two solutions are deemed infeasible. The \$6.195 million solution (from [11]) and \$6.182 million solution (from [13]) were found to be feasible solutions, and thus the current known-optimum cost is \$6.182 million.

C. Doubled New York Tunnels Problem

The Doubled New York Tunnels Problem (NYTP2), which has not been considered previously in the literature, consists of two NYTP networks connected via the single reservoir at node 1 (see [10] for network data). Similarly to the NYTP, there are 16 design options, creating a search space size of 16^{42} (approximately 3.7414×10^{50}) designs. The minimum cost, although never found previously, is calculated as double the cost of the known-optimum solution for the NYTP (\$77.276 million). It can be determined this way because the two sub-networks are hydraulically independent of one another as they are connected only by a source node (reservoir).

VI. DERIVATIONS FOR DETERMINISTIC AND SEMI-DETERMINISTIC PARAMETERS

If possible, it is advantageous to develop deterministic expressions for parameters of algorithms like ACO as this decreases the calibration requirements. For a parameter to be a candidate for the development of a deterministic expression it must have certain properties. In the case of Q , it was realized that a relationship exists between Q and another parameter (t_0) such that the other parameter could be calibrated to any value of Q without any loss in the algorithm's performance. In the case of PEN , as this is the parameter that determines the penalty cost, a conservative approach was taken such that all infeasible solutions were guaranteed to assume a network cost higher than the optimum. This formulation assumes no *a priori* knowledge of the optimum except that it has a lower cost than the

maximum material cost $C(\Omega^{\max})$, where

$$C(\Omega^{\max}) = \sum_{i=1}^n L_i \max\{c_{i,1}, \dots, c_{i,NO_i}\}. \quad (17)$$

Clearly, deterministic expressions cannot be developed for all parameters, as not all parameters are dependent on other parameters (as with Q) or have a physical meaning that can be exploited (as with PEN).

A. Pheromone reward factor, Q

A theoretical analysis of the influence of the pheromone reward factor Q on the decision-making policy (10) shows that the parameters t_0 and Q are dependent. A definition of dependent parameters, as well as a proof of the dependence of Q and t_0 , is given in the appendix. The corollary of the proof is that Q can be set to a constant and the optimal value of t_0 can be calibrated to it without compromising the algorithm's performance. Therefore, the adopted expression for Q is

$$Q = C(\Omega^{\max}). \quad (18)$$

Theoretically any value of Q is appropriate, but (18) was selected for convenience as Q is automatically scaled to be within the magnitude of the case study network costs.

B. Penalty factor, PEN

The basic function of the penalty factor PEN is to scale up the physical cost of infeasible networks such that they produce poorer objective function values than the optimum or near optimum solutions. As the optimum solution is generally not known, a conservative approach is to assign a value to PEN such that all infeasible solutions are more expensive than the maximum physical cost $C(\Omega^{\max})$, as the network cost of the optimum solution is clearly less than $C(\Omega^{\max})$. To formulate this, PEN can be defined such that the smallest network cost $NC(\Omega^{\min})$ (i.e. $NC(\Omega^{\min}) = C(\Omega^{\min}) + PC(\Omega^{\min})$, where $C(\Omega^{\min})$ is also given by (17) with use of the $\min\{\cdot\}$ rather than the $\max\{\cdot\}$ function) with an assumed maximum pressure deficit of d is equal to the maximum physical cost $C(\Omega^{\max})$. That is

$$NC(\Omega^{\min}) = C(\Omega^{\min}) + PEN \cdot d = C(\Omega^{\max}), \quad (19)$$

from which PEN can be solved for as

$$PEN = \frac{[C(\Omega^{\max}) - C(\Omega^{\min})]}{d}. \quad (20)$$

This formulation for PEN is referred to as being semi-deterministic, as it is still dependent on the user specified value of d . However, a value that is assigned to d (units in m) is more intuitive than a penalty cost multiplier (units in \$/m), as it deals with a real physical property (i.e pressure deficit). Another way to view d is to consider the following question: what maximum pressure deficit would the user accept to achieve a cost saving of $C(\Omega^{\max}) - C(\Omega^{\min})$?

The parameter d should theoretically be able to be determined from a hydraulic analysis of the Ω^{\min} network. The assumption within the formulation of the hydraulic equations ((5), (6), (7), and (8)) is that the nodal demands are satisfied. For networks with extremely large frictional losses (e.g. large flows through small diameter pipes), the resulting solution to the set of hydraulic equations is typically a set of negative pressure heads of unrealistic magnitudes for all or some of the nodes. As an example, for the Ω^{\min} of the HP the nodal pressure heads ranged from -907 m at node 2 to -17635 m at node 13. Considering that the vapor pressure of water is just over -10 m, these negative pressures for water are unachievable and therefore this solution is not representative of reality. Realistically, in such a network the flow provided at the nodes would be limited such that only realistic headlosses would occur. The corollary of this is that, with the current available hydraulic analysis techniques for WDSs, the actual nodal pressures for networks that are unable to provide realistic pressure whilst satisfying nodal demands cannot be determined. Therefore, d must be user-selected.

In this study, d has been selected to be a small value (0.01 m) such that infeasible solutions are considered to be extremely poor. Reference [27] highlighted an approach called the *minimum penalty rule*. This states that, ideally, the penalty should be kept as low as possible such that the value of the objective function for infeasible solutions is just above the optimum objective function value. This requires PEN to be a function of the magnitude of the pressure violation such that large violations are penalized sufficiently and small violations are not penalized too

much. The advantage of this is that the algorithm would be able to converge on the optimum from both the feasible and infeasible regions as valuable information is gained from searches within the boundaries of the infeasible region. However, for the purposes of this study, $d = 0.01$ m was found to be suitable.

VII. PARAMETRIC STUDY FOR ANT COLONY OPTIMIZATION PARAMETERS

This section provides a summary of parametric studies undertaken for the non-deterministic ACO parameters \mathbf{a} , \mathbf{b} , \mathbf{r} , t_0 , and m and the heuristics derived from these results. Due to the computationally expensive nature of the hydraulic solver process (i.e. this requires the solution to the large system of non-linear equations, given by (5), (6), and (7), performed by iterative techniques such as the Newton-Raphson method or more sophisticated algorithms) and the large number of parameter combinations, five to ten runs were typically performed for the parametric sensitivity analysis simulations. This number of runs was deemed to be sufficient for the observation of behavioral trends with the parameter variations. Extended simulations were performed once the parameter guidelines were determined, the results of which are given in section VIII.

A. Decision policy control parameters \mathbf{a} and \mathbf{b}

Due to the joint influence of \mathbf{a} and \mathbf{b} on the decision policy, these parameters were considered together within the parametric studies. Based on preliminary analyses, values of $0 = \mathbf{a} = 5$ and $0 = \mathbf{b} = 2.5$ were used for more detailed sensitivity analyses.

Fig. 2 gives a summary of the sensitivity analysis results of \mathbf{a} and \mathbf{b} for the NYTP (note: I_{max} is the maximum number of iterations). The results are presented in terms of the means of two main properties of an algorithm's performance, referred to here as 'best-cost' and 'search-time'. Best-cost is the lowest network cost that the algorithm found during the run, and search-time is the number of evaluations that the algorithm took to first find the best-cost solution. As the behavior exhibited by the surfaces was similar for the other case studies, only the results for the NYTP are given.

As displayed in Fig. 2(a), the best-cost surface formed a "valley" shape centered on the range $1 = \mathbf{a} = 2$ and $0 = \mathbf{b} = 1$ with the lowest point occurring at $\mathbf{a} = 1$ and $\mathbf{b} = 0.5$. It can be seen that solution quality was very sensitive to \mathbf{b} (more so than \mathbf{a}). For $\mathbf{b} = 0$ (for this parameter value the searching is effectively independent of the local costs

of the pipe options), the algorithm performed generally only slightly worse than for $\mathbf{b} = 0.5$ and 1.0 , the former being the best parameter value for all values of \mathbf{a} . However, in 2(b) it is seen that a lack of consideration of the local costs of the pipe options greatly reduces the efficiency of the algorithm.

For $\mathbf{b} > 1.5$ only poor solutions were found. On one level, this is a non-intuitive result, as it could be considered that higher values of \mathbf{b} should encourage the selection of better (lower cost) solutions. However it was found that for high values of \mathbf{b} , the search was repeatedly pushed into the infeasible region (i.e. low-cost infeasible solutions were constantly selected) meaning that the algorithm had to gradually find the optimal region based on information from the fewer feasible solutions found. This behavior is also reflected in the search-times (Fig. 2(b)). For example, for $\mathbf{b} = 1.5$ the algorithm searched for a longer time before finding the best-cost solution.

The algorithm's inability to explore the search space for high values of \mathbf{b} is illustrated by the fact that for $\mathbf{b} > 1.5$ the algorithm converged to the same solution for each run (as indicated by the plateau in Fig. 2(a)), indicating a drastically skewed probability distribution due to the high \mathbf{b} values. These solutions were also found within the first iteration, as indicated by the plateau in Fig. 2(b).

As expected, the algorithm performed worst for $\mathbf{a} = 0$, as this parameter setting is equivalent to the ants ignoring the pheromone trails when selecting pipe options. From Fig. 2(a) and (b) it is seen that as \mathbf{a} increased, $AS_{i\text{-best}}$ found the best-cost solution earlier in the run, and increasingly poorer quality solutions were found for $\mathbf{a} > 1$. This behavior is indicative of an increase in the convergence pressure in the algorithm's search, resulting in poorer exploration of alternative regions in the search space. The value of \mathbf{a} dictates the emphasis placed on the relative difference in the pheromone intensities between all options in the probability determination (i.e. (10)). As \mathbf{a} increased, the difference was emphasized more and consequently pheromone additions had a greater influence. The result of this was that the probability of reselection of an option that was updated was proportionally larger. Consequently, the algorithm's ability to consider non-updated options decreased, resulting in premature convergence.

No feasible solutions were found for $\mathbf{a} = 1.5$ and $\mathbf{b} = 1$ for the HP. This may be attributed to the fact that the known-optimum solution contains many pipes that are set to the largest or near largest diameter size [13], and consequently, as the optimum solution lies on the boundary between the feasible and infeasible region [9], it can be deduced that there are few feasible solutions that contain small pipe diameters. Therefore, if the local cost of a diameter option is

overly influential in the searching process, as is the case for high values of \mathbf{b} , the search will be guided into the infeasible region of the search-space.

From the investigations, \mathbf{a} appears to be more robust than \mathbf{b} , which was recognized as being a highly sensitive parameter. Even though $\mathbf{b} = 0$ produced a moderately better performance for the HP (a reduction in the mean best-cost of about 1%), the consistently best combination of parameters with respect to all case studies was $\mathbf{a} = 1.0$ and $\mathbf{b} = 0.5$. Thus \mathbf{b} plays a less important role in that it provides a bias towards the lower cost options and the main factor driving the search is the learned information, as represented by the heuristically evolving pheromone intensities.

B. Pheromone Initialization, \mathbf{t}_0

Based on a preliminary analysis of \mathbf{t}_0 for the three case studies, values of $10 = \mathbf{t}_0 = 300$ were tested. Fig. 3 displays the results for the NYTP2. The top set of three curves give the maximum, mean, and minimum best-cost values for each parameter value. The bottom set of three curves gives the maximum, mean, and minimum search-times, expressed in units of evaluation number / 10^3 . It is advantageous to consider all three statistics as they present a more rounded description of the algorithm's performance.

As is seen in Fig. 3, \mathbf{t}_0 is a relatively robust parameter, where the maximum variation in best-costs for the entire range of \mathbf{t}_0 tested is 2.2 %. The best-cost curves indicate an increasingly improved performance for $\mathbf{t}_0 \leq 200$. Despite other parameter values achieving lower cost solutions, $\mathbf{t}_0 = 200$ was considered to be the best parameter setting as it induced a more robust performance. This is indicated, primarily, by the fact that the lowest mean best-cost was achieved for this parameter value, and also the fact that a tighter bounding of the maximum best-cost curve occurs at this point. From the search-time curves, a definite trend of an increased search-time for \mathbf{t}_0 increasing from 0 to 100 is seen, followed by a plateau for $\mathbf{t}_0 > 100$. This agrees with an intuitive consideration of (10), (11), and (12) in that higher values of \mathbf{t}_0 attenuate the influence of the pheromone additions resulting in slower convergence. Despite this increased search-time, better solutions were generally not found (particularly for the HP). This implies that there is an optimum value of \mathbf{t}_0 for which appropriate importance is given to the pheromone updates.

Despite the relative insensitivity of the algorithm to \mathbf{t}_0 , the results showed that the best value of \mathbf{t}_0 is not case study

independent. This finding is intuitive when one considers that the action of \mathbf{t}_0 is to assign appropriate importance to the pheromone additions. Thus, in agreement with (13), it was considered that \mathbf{t}_0 should be a function of an estimation of the potential pheromone additions. Based on maintaining a simple relationship between \mathbf{t}_0 and an estimation of the potential pheromone addition, the following heuristic was derived for the application of $\text{AS}_{i\text{-best}}$ to the WDSP

$$\mathbf{t}_0 = \frac{Q}{C(\Omega_n^*)} \sqrt{n \cdot NO_{\text{avg}}}, \quad (21)$$

where NO_{avg} is the average number of options for the n decision variables and $Q/C(\Omega_n^*)$ is the pheromone addition for design Ω_n^* where $C(\Omega_n^*)$ is the physical cost of a near optimum design Ω_n^* (i.e. the known-optimum or some other low cost solution). Equation (21) varies from that given in (13) in that the number of decision variables n has a positive exponent and NO_{avg} is also included as a variable. However, the general relationship of \mathbf{t}_0 being proportional to the pheromone addition of a near optimal solution is maintained (note the similarity between the expression in (21) and the relationship between Q and \mathbf{t}_0 derived in the appendix).

From Table 3 it is seen that the heuristic values of \mathbf{t}_0 approximate nearly exactly the best values of \mathbf{t}_0 for the HP and the NYTP2 (i.e. the near-heuristic values are the best values). For the NYTP, the heuristic value was some distance from the best value of \mathbf{t}_0 . However, for this case study the performance of the near-heuristic value was still very good, with the mean best-cost varying just over 0.5% from that obtained when the best value of \mathbf{t}_0 was used.

C. Pheromone Persistence, \mathbf{r}

A summary of the sensitivity analysis results of \mathbf{r} for the NYTP2 is given in Fig. 4. The top set of three curves presents the minimum, mean, and maximum values of the best-cost for each value of \mathbf{r} , and the bottom set of three curves presents similar results for the search-times. It is seen from this figure that as \mathbf{r} increases the mean best-cost tends to decrease with a generally tighter bounding by the maximum and minimum curves, and the mean search-time increases with a generally looser bounding. These observations indicate that for higher values of \mathbf{r} the algorithm's ability to find least cost solutions is improved due to increases in the algorithm's ability to explore, resulting from a

decrease in convergence pressure. These results also agree with an intuitive consideration of (11), as higher values of r will enable the pheromone intensities of non-updated edges to persist for longer thereby allowing their probabilities of selection, given by (10), to maintain non-trivial values. Similar results were observed for the other case studies.

The results for $r = 0.97, 0.98,$ and 0.99 (note that only these values are considered in more detail as, on inspection of Fig. 4, it is clear that the performance is better for $r > 1$) are shown in Table 4. It can be seen that $r = 0.98$ achieves the lowest mean best-cost for the NYTP and NYTP2. A value of $r = 0.97$ results in the best performance for the HP, with a mean best-cost just marginally lower than that obtained using $r = 0.98$. However, despite the better mean, $r = 0.98$ achieved lower maximum and minimum best-cost values. It is interesting to note that $r = 0.99$ did not find any feasible solutions for any of the HP runs. This indicates that, for this case study, the stronger degradation of the pheromone trails (and hence the lowering of the probability of selection) for the poor options is important for the algorithm to find the feasible region within the search space.

Based on good results in all the case studies, $r = 0.98$ was found to be the most reliable value for the pheromone persistence parameter.

D. Number of Ants, m

Fig. 5 gives a summary of the sensitivity analysis results for a range of m values for the NYTP. The top set of three curves gives the maximum, mean, and minimum values of best-cost, and the bottom set of three curves gives the corresponding values of search-time. From the top set of curves it is seen that for $m = 20$ the known-optimum solution of \$38.638 million was found for at least one of the 10 runs (as indicated by the “min best-cost” curve). As m increases the mean best-cost decreases thus indicating more reliable and robust algorithmic performance. This robustness is also seen in the tighter bounding of the mean best-cost by the maximum and minimum best-cost values. The trade-off for this robustness is illustrated in the bottom set of three curves by the increase in search-time (or decrease in efficiency) as m increases. A corollary of these behavioral observations is that the best value of m is the smallest value that gives a suitably robust performance. A further point to note is that the bounding of the mean search-time curve by the maximum and minimum curves is tighter for lower values of m , indicating that the convergence pressure is stronger, as for each run the algorithm converges within similar times.

The expression of $m = n$, given in [20], tended to result in relatively poor performance for all WDS case studies considered, as too few ants were employed. Values of $10 = m = 200$ were tested for the NYTP and NYTP2 networks and $10 = m = 100$ for the HP, where the maximum number of evaluations $I_{max} \times m$ was kept at a constant. Based on the results the following heuristic for m was determined

$$m = n\sqrt{NO_{avg}}. \quad (22)$$

This provides a deterministic method of scaling up the expression given in [20] and relates m not only to the number of decision variables within the problem (i.e. n) but also to the average number of options at each decision point, NO_{avg} . Equation (22) yields values of $m = 84, 83.3, \text{ and } 168$ for the NYTP, HP, and NYTP2, respectively.

A comparison of the best results obtained using the value of m with those obtained using the near-heuristic value of m within the tested parameter series is shown in Table 5. From this table it is seen that even though (22) underestimates the best value of m for the NYTP, the best-cost statistics of the near-heuristic value are very close to the minimum, mean, and maximum best-cost values, deviating 0.0%, 0.05%, and 0.51%, respectively, from best value statistics. Equation (22) provides a conservative overestimate of m for the HP and the NYTP2. From Table 5 it is seen that the performance of the algorithm with the near-heuristic value of m for these two case studies compares well to that obtained using the best value. For both case studies, the near-heuristic value of m results in a lower minimum best-cost compared with that obtained using the best value of m and the mean and maximum best-costs vary only by just over 1% for the HP and under 0.2% for the NYTP2.

Based on these results, (22) is considered to provide a very good estimate of m for the three case studies considered.

VIII. ASSESSMENT OF PERFORMANCE OF HEURISTIC PARAMETER SETTINGS

Based on the parameter setting recommendations presented in the previous section, extended simulations were performed for the three case studies involving 100 runs. Results from these simulations, along with results from other algorithms presented in the literature, are given in Table 6. In addition to the standard GA [11], these other algorithms include: the Improved GA [10] (GA_{imp}), which is a GA that uses grey coding combined with creep mutation and a variable power scaling of the fitness function; ACOA [15], which is a version of ACO that possesses

a similar updating scheme to $AS_{i\text{-best}}$ but has a different penalty function and does not use the parameter heuristics; the Fast Messy GA [13] (GA_{fm}), which uses variable-length strings, a threshold genetic selection and messy operators of cut and splice. It is important to note that other authors [11]-[14] have presented cheaper solutions for the NYTP and the HP that were found to be infeasible by EPANET 2 (see [15] for the NYTP and section V for the HP hydraulic analysis of these designs), and as such are not included in Table 6.

From Table 6 it is seen that $AS_{i\text{-best}}$ performed extremely well for the NYTP, with a mean best-cost deviating only 0.54% from the known-optimum (\$38.638 million). Out of the 100 runs performed, $AS_{i\text{-best}}$ found the known-optimum solution 41 times. Consequently, $AS_{i\text{-best}}$ with the heuristic parameter settings is more robust than the ACO formulation used by [17] with calibrated parameter values. Both $AS_{i\text{-best}}$ and ACOA performed better than GA_{imp} in terms of solution quality and efficiency. A good performance for the NYTP2 was also observed, as $AS_{i\text{-best}}$ was able to find the known-optimum once and achieved a mean best-cost deviating only 1.33% from this.

Despite its relatively good performance for the two other case studies, $AS_{i\text{-best}}$ was unable to find the known-optimum for the HP. The best solution found was \$6.367 million (2.99% deviation from the known-optimum) with a mean best-cost deviating 10.68%. To give a comparison of the $AS_{i\text{-best}}$, GA and GA_{fm} solutions for the HP, from Table 7 it is seen that only 11 of the 34 pipes differ between the $AS_{i\text{-best}}$ and the GA_{fm} design, and nine of these pipes only differ by one diameter size. This relative closeness in solutions indicates that $AS_{i\text{-best}}$ was searching in the neighborhood of the known-optimum solution. Of the 11 different pipes, 7 are smaller than the optimum design. This fact highlights an issue in the management of ACO's bias towards selecting lower cost options, resulting from the use of the desirability values in the decision probabilities in (10). To accommodate the smaller diameters in $AS_{i\text{-best}}$'s solution, pipes 16 and 18 were forced to be much larger, inducing the majority of the cost difference.

An important point to note is that $AS_{i\text{-best}}$ has a far smaller search-time than the other algorithms; 53.9% of GA_{fm} 's search-time and 6.1% of the GA's search-time. Similar results reporting ACO's shorter search-times are also reported by [17] and [18]. This is clearly an advantage with problems like the NYTP where $AS_{i\text{-best}}$'s solution quality remains high. However for problems like the HP, this shorter search-time combined with a poorer solution quality indicates that $AS_{i\text{-best}}$ is not able to explore for a long enough period of time, and converges too quickly for this type of problem.

IX. SUMMARY AND CONCLUSIONS

In this paper, a formulation of ACO for WDS optimization has been presented, namely the iteration-best ant system ($AS_{i\text{-best}}$). ACO has many advantages for WDS optimization, however one major disadvantage is the many user-defined parameters that govern the algorithm's search behavior.

In order to overcome these limitations, parameter guidelines for ACO applied to the WDSP have been developed. From simple theoretical analyses, deterministic expressions have been developed for two of the parameters; the pheromone addition factor Q and the penalty factor PEN . For the remainder of the parameters (the decision control parameters \mathbf{a} and \mathbf{b} , the pheromone persistence factor \mathbf{r} , the initial pheromone value \mathbf{t}_0 , and the number of ants m) guidelines have been developed based on a detailed parametric study using three case studies. The guidelines are summarized in Table 8.

Using the derived parameter guidelines, $AS_{i\text{-best}}$ has been seen to perform extremely competitively when compared to other optimization algorithms. For two of the three case studies, $AS_{i\text{-best}}$ found the known-optimum solution. The algorithm's performance was also extremely robust, as the mean cost found for 100 different runs varied only slightly from the optimum (i.e. less than 1.5%). Additionally, ACO was found to be a much faster and more efficient algorithm than GAs when applied to WDS optimization.

The advantage of the parameter guidelines that have been established is that they provide guidance for the future application of ACO to the WDSP. Because of their success with the case studies used, there is scope for their applicability to other, possibly real-world, problems. In addition, recent developments in the use of EAs show potential for application to WDS optimization. Examples of these are: an elitist compact GA (CGA) [34], a CGA that retains the elitist solution, indefinitely or for a fixed generation period, and uses it to evolve the CGAs probability vector; the society and civilization algorithm [35], which is new algorithm for constrained problems that mimics the interaction between societies (i.e. populations of solutions distributed throughout the solution space) and the distribution of information to individuals (i.e. the manner in which individual solutions are altered); and the Lévy evolutionary programming (EP) algorithm [36], which uses the Lévy distribution (a generalized form of the Cauchy distribution) to govern mutation to encourage broader searching through the solution space due to the infinite second moment property of this distribution.

APPENDIX: PROOF OF DEPENDENCE OF Q AND t_0

Dependence between two parameters is defined here as the ability to express either one of the parameters as some function of the other parameter without loss of generality of the algorithmic process². This means that one of the dependent parameters can assume any value and the other parameter can be calibrated to it without compromising the algorithm's performance. That is, as long as one of the parameters is calibrated, the algorithm's performance is independent of the value that the other dependent parameter takes.

Proposition: Q and t_0 are dependent parameters.

Proof: To prove that Q and t_0 are dependent parameters, two steps are taken: (i) a general expression of the algorithmic process is derived as a function of the parameters Q and t_0 ; and (ii) from this it is shown that t_0 can be expressed as some function of Q without loss of generality in the algorithmic process.

The algorithmic process of ACO can be seen as an evolution in time of the probability functions at each decision point. This can be justified by considering that the sole aim of ACO is primarily to alter the probability functions to make the selection of the global optimum more probable. Consequently, step (i) involves deriving an expression for $p_{i,j}(t)$ as a function of Q and t_0 . To do this, firstly an expression of $t_{i,j}(t)$ as a function of Q and t_0 is required.

From (11), $t_{i,j}(t)$ can be expressed as a function of $t_{i,j}(t-2)$, $\Delta t_{i,j}(t-2)$, and $\Delta t_{i,j}(t-1)$ by

$$\begin{aligned}
 t_{i,j}(t) &= \mathbf{r}t_{i,j}(t-1) + \Delta t_{i,j}(t-1) \\
 &= \mathbf{r}(\mathbf{r}t_{i,j}(t-2) + \Delta t_{i,j}(t-2)) + \Delta t_{i,j}(t-1) \\
 &= \mathbf{r}^2 t_{i,j}(t-2) + \mathbf{r}\Delta t_{i,j}(t-2) + \Delta t_{i,j}(t-1)
 \end{aligned} \tag{23}$$

In a similar recursive pattern, $t_{i,j}(t)$ can be expressed as a function of the pheromone value in the first iteration and all the pheromone additions up to iteration $t-1$. Recognizing that $t_{i,j}(1) = t_0$, by recursion the following expression is obtained

² "Without loss of generality of the algorithmic process" is defined as actions on the parameters that do not confine the generality of the process. For example as \mathbf{r} is not a dependent parameter if it is set to 1 no manipulation of the other parameters can restore the general nature of the process.

$$\mathbf{t}_{i,j}(t) = \mathbf{r}^{t-1} \mathbf{t}_0 + \sum_{l=1}^{t-1} \mathbf{r}^{t-1-l} \Delta \mathbf{t}_{i,j}(l). \quad (24)$$

To include the Q term, (12) is substituted into (24) to obtain the following expression

$$\mathbf{t}_{i,j}(t) = \mathbf{r}^{t-1} \mathbf{t}_0 + \sum_{l=1}^{t-1} \mathbf{r}^{t-1-l} \mathbf{d}_{i,j}(l) \frac{Q}{f(S_{best}(l))}, \quad (25)$$

where $\mathbf{d}_{i,j}(t)$ is a delta function given by

$$\mathbf{d}_{i,j}(t) = \begin{cases} 1 & \text{if edge } (i,j) \in S_{best}(t) \\ 0 & \text{otherwise} \end{cases}. \quad (26)$$

This means that edge (i,j) receives the pheromone addition of $Q/f(S_{best}(t))$ if it is an element of $S_{best}(t)$ (note: (i) this is just a convenient way of expressing (12) and (ii) this proof holds for any definition of $\Delta \mathbf{t}_{i,j}(t)$ that is linearly proportional to Q) Substituting (25) into (10), an expression for the evolution of the probability distributions based on the initial pheromone value and the pheromone additions from iteration 1 to $t-1$ is achieved. This can be written as

$$p_{i,j}(t) = \frac{\left[\mathbf{r}^{t-1} \mathbf{t}_0 + \sum_{l=1}^{t-1} \mathbf{r}^{t-1-l} \mathbf{d}_{i,j}(l) \frac{Q}{f(S_{best}(l))} \right]^a [\mathbf{h}_{i,j}]^b}{\sum_{k \in \mathbf{q}_i} \left[\mathbf{r}^{t-1} \mathbf{t}_0 + \sum_{l=1}^{t-1} \mathbf{r}^{t-1-l} \mathbf{d}_{i,k}(l) \frac{Q}{f(S_{best}(l))} \right]^a [\mathbf{h}_{i,k}]^b}. \quad (27)$$

Equation (27) represents a complete expression of the algorithmic process (i.e. an expression for $p_{i,j}(t)$ for all t) based on Q and \mathbf{t}_0 , as required by step (i) above. To achieve step (ii), dividing (27) by Q^a/Q^a and taking out $[\mathbf{r}^{t-1}]^a$ as a common factor yields

$$p_{i,j}(t) = \frac{\left[\frac{t_0}{Q} + \sum_{l=1}^{t-1} \frac{d_{i,j}(l)}{r^l f(S_{best}(l))} \right]^a [h_{i,j}]^b}{\sum_{k \in q_i} \left[\frac{t_0}{Q} + \sum_{l=1}^{t-1} \frac{d_{i,k}(l)}{r^l f(S_{best}(l))} \right]^a [h_{i,k}]^b}. \quad (28)$$

The parameters t_0 and Q appear only together as a ratio in (28), and as (28) represents a complete expression of the algorithmic process it can also be seen that t_0 and Q can be replaced by another parameter (say \mathbf{x} , where $\mathbf{x} = t_0 / Q$) without loss of generality in the process. A result of this is that t_0 can be expressed as a function of Q (i.e. $t_0 = \mathbf{x} Q$) without loss of generality. This implies that Q and t_0 are dependent parameters. ?

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BIOGRAPHIES

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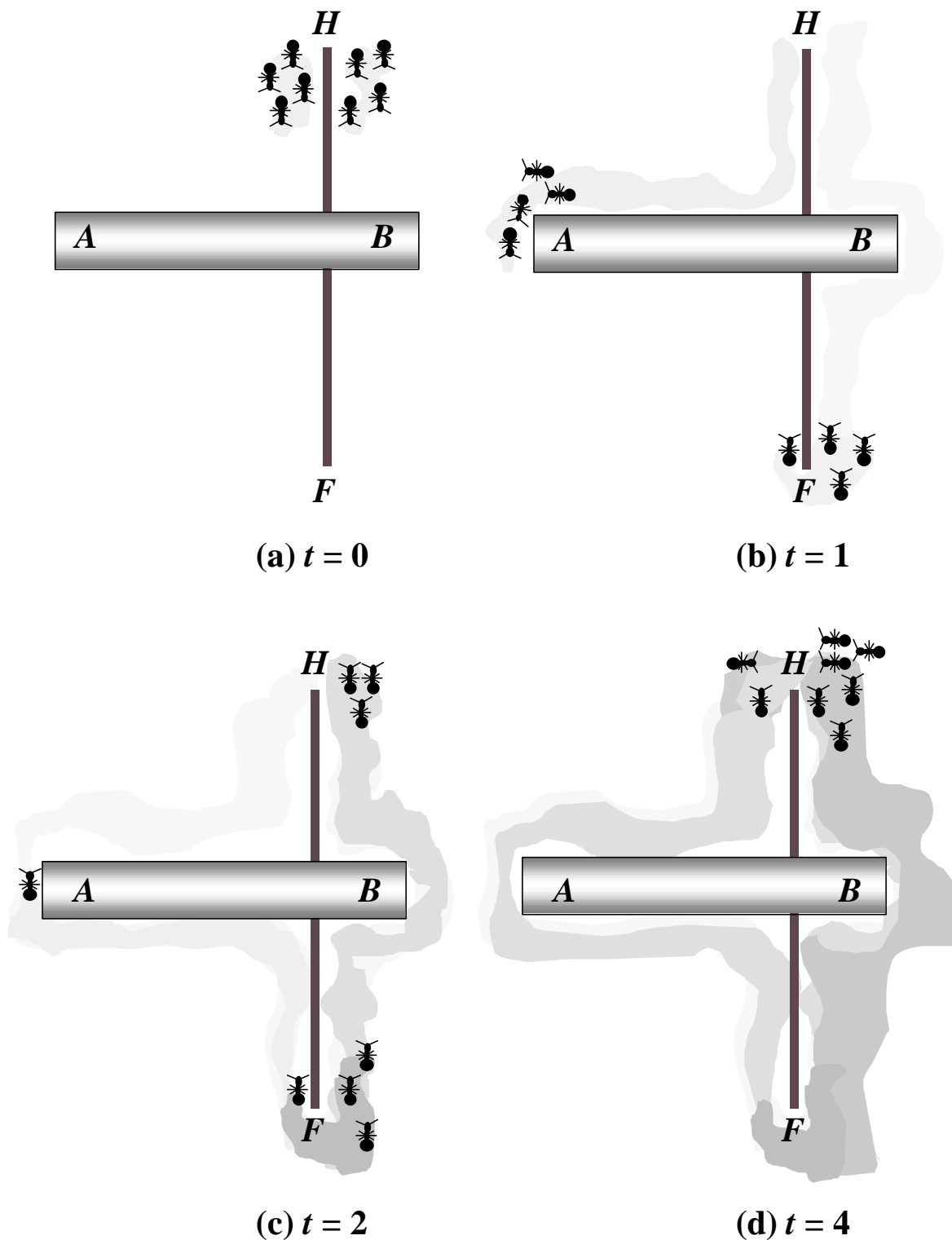
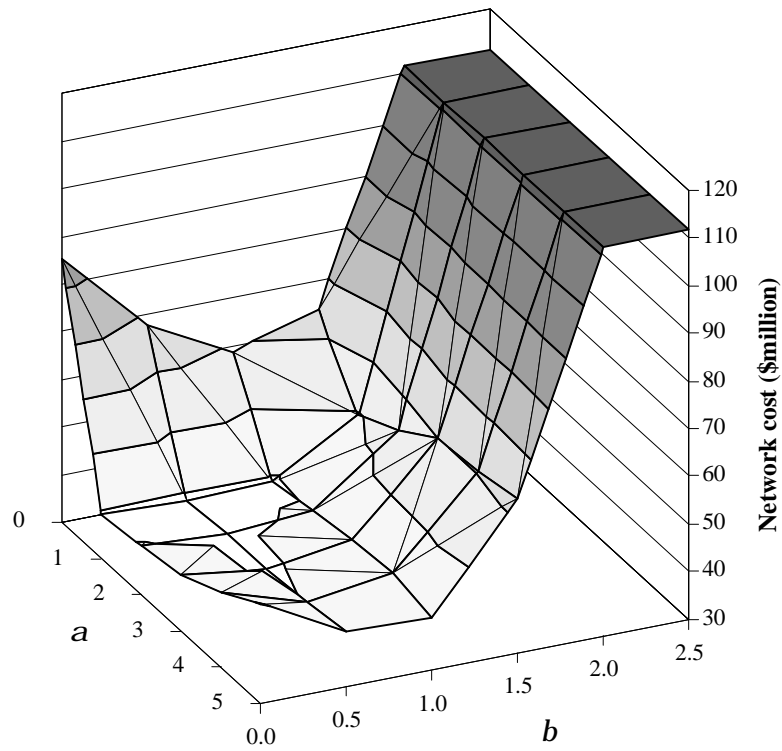
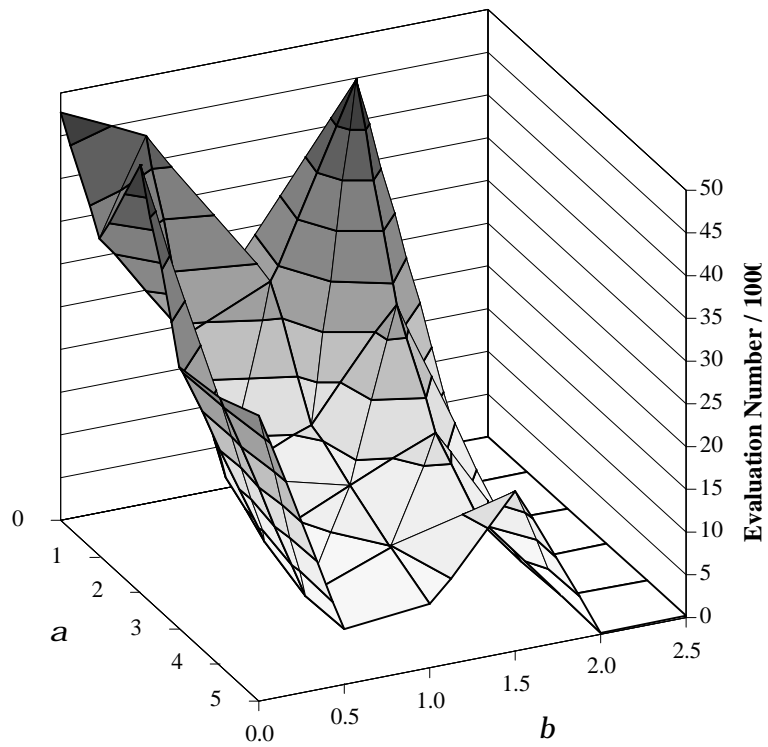


Figure 1 Example of evolution of pheromone trails.



(a) Mean best-cost



(b) Mean search-time

Figure 2 Sensitivity analysis results for a and b for the New York Tunnels Problem, $m = 100$, $t_0 = 100$, $r = 0.98$, and $I_{max} = 1000$. Performance measures for each parameter value are averaged from five runs

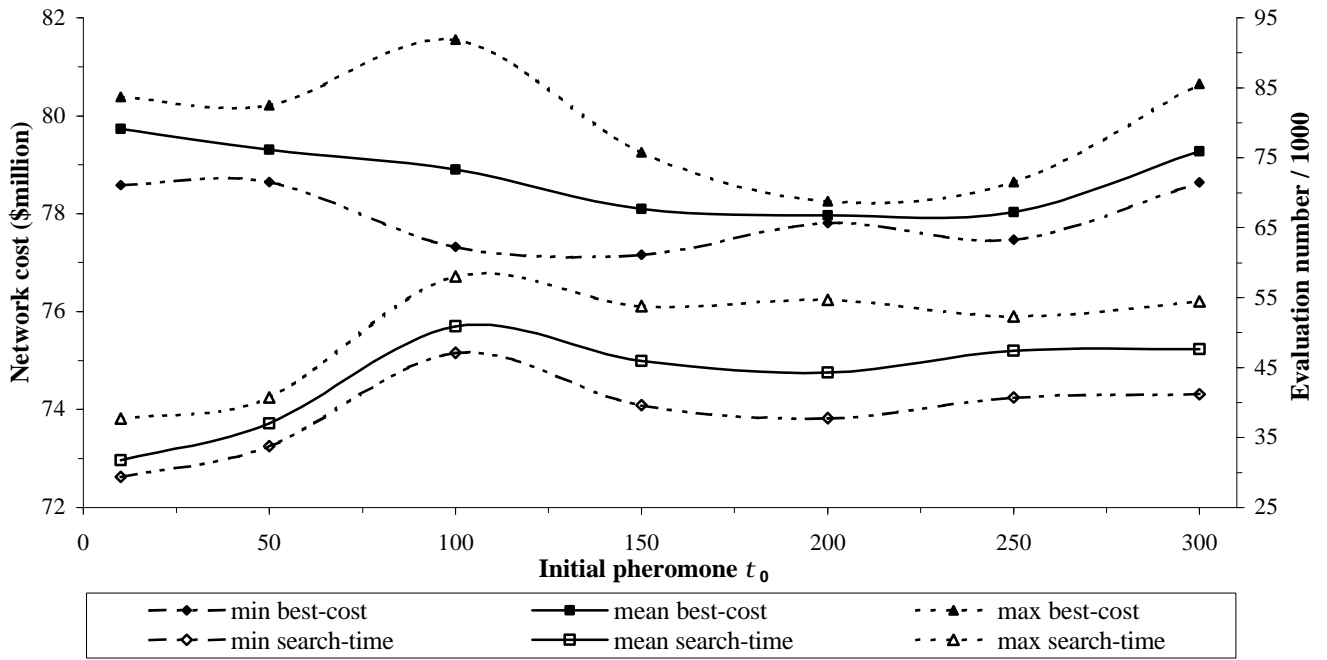


Figure 3 Summary of sensitivity analysis results for the initial pheromone value t_0 varied for the Doubled New York Tunnels Problem where $a = 1.0$, $b = 0.5$, $m = 100$, $r = 0.98$, and $I_{max} = 1000$. Performance statistics for each parameter value are based on results from five runs.

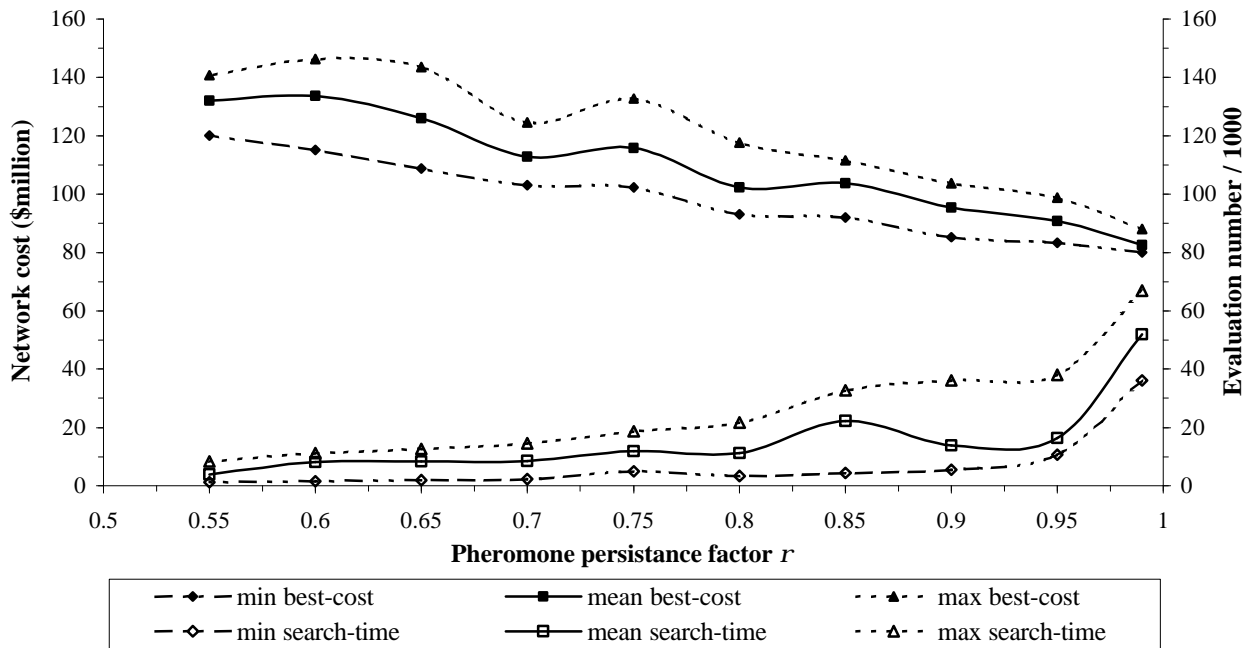


Figure 4 Summary of sensitivity analysis results for r varied for the Doubled New York Tunnels Problem where $a = 2.0$, $b = 0.5$, $t_0 = 300$, $r = 0.98$, $m = 100$, and $I_{max} = 1000$. Performance statistics for each parameter value are based on results from ten runs.

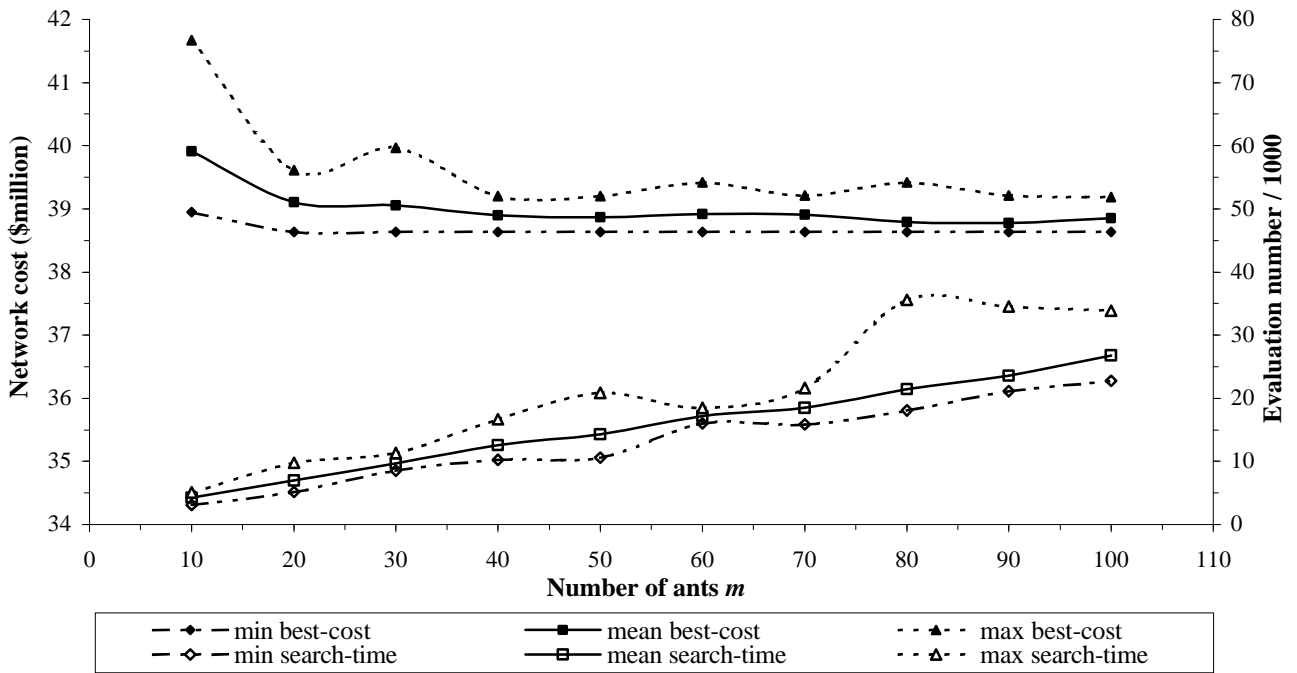


Figure 5 Summary of sensitivity analysis results for the number of ants m varied for the New York Tunnels Problem where $a = 1.0$, $b = 0.5$, $t_0 = 140$, $r = 0.98$, and $I_{max} \times m \sim 100\ 000$. Performance statistics for each parameter value are based on results from ten runs.

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Table 1 Summary of the conversion from the general ACO problem formulation to the Water Distribution System Problem.

General ACO problem formulation		WDSP equivalent	
Element	Symbol	Element	Symbol
Path	S	Design	Ω
Edge	(i,j)	Option	$option_{i,j}$
Set of edges available from decision point i	\mathbf{q}_i	Set of options available for pipe i	$\left(\begin{matrix} option_{i,1}, \dots \\ \dots, option_{i,n_{o_i}} \end{matrix} \right)$
Objective function	$f(S)$	Network cost	$NC(\Omega)$

Table 2 Nodal heads of the critical nodes for the Hanoi Problem. Hydraulic analysis was performed using EPANET2 with a maximum number of iterations = 5000 and accuracy = 0.00001.

Nodal heads for critical nodes (m), (difference from minimum allowable head, m)								
Node	Savic & Walters (1997), GA No.1 ^a		Savic & Walters (1997), GA No.2 ^a		Cunha & Sousa (1999) ^b		Wu et al. (2001) ^c	
13	29.83	(-0.17)	34.18	(+4.18)	29.76	(-0.24)	31.76	(+1.76)
16	30.30	(+0.30)	34.31	(+4.31)	29.94	(-0.06)	32.05	(+2.05)
27	30.19	(+0.19)	33.10	(+3.10)	29.75	(-0.25)	32.00	(+2.00)
29	30.87	(+0.87)	31.84	(+1.84)	29.84	(-0.16)	32.29	(+2.29)
30	29.86	(-0.14)	30.97	(+0.97)	30.10	(+0.10)	31.85	(+1.85)
Cost (\$million)	6.073		6.195		6.065		6.182	
Feasibility	infeasible		feasible		infeasible		feasible	

^a ref. [11]^b ref. [14]^c ref. [13]

Table 3 Comparison of the best-cost statistics for the best value and the near-heuristic value of initial pheromone t_0 , with $a = 1.0$, $b = 0.5$, $m = 100$, $r = 0.98$, $I_{max} = 1000$. Performance statistics for each parameter value are based on results from five runs.

Case study	Best value of t_0 in tested parameter series			Heuristic for t_0 given by (21) ^a	Near-heuristic value of t_0 in tested parameter series							
	t_0	Best-cost (\$million)			t_0	Best-cost (\$million) (% deviation from best value statistics)						
		minimum	mean			maximum	minimum	mean	maximum			
NYTP	100	38.638	38.638	38.638	139.51	140	38.638	(0.0)	38.8732	(0.61)	39.187	(1.42)
HP	25	6.427	6.7786	7.156	25.87	25	6.427	(0.0)	6.7786	(0.0)	7.156	(0.0)
NYTP2	200	77.808	77.9662	78.256	197.31	200	77.808	(0.0)	77.9662	(0.0)	78.256	(0.0)

^a $C(\Omega_n^*)$ was taken as \$38.638 million (from [17]), \$6.056 million (from [13]), and \$77.285 million for the NYTP, HP, and NYTP2, respectively.

Table 4 Sensitivity analysis summary for pheromone persistence factor r with $a = 1.0$, $b = 0.5$, $m = 100$, $I_{max} = 1,000$ for the NYTP and HP and 10,000 for the NYTP2 and $t_0 = 100, 50$, and 200 for the NYTP, HP, and NYTP2, respectively. Performance statistics for each parameter value are based on results from five runs and the statistics are ordered as (minimum), mean, and [maximum]. The lowest mean best-cost values are bolded. NFS means no feasible solution was found.

Case study	Best-cost (\$million), Search-time (evaluation number)					
	$r = 0.99$		$r = 0.98$		$r = 0.97$	
NYTP	(38.638)	(31,655)	(38.638)	(19,328)	(38.638)	(16,462)
	38.700	36,749	38.638	23,964	38.818	17,509
	[38.949]	[44,263]	[38.638]	[29,596]	[39.062]	[18,886]
HP	(NFS)	(6,295)	(6.637)	(75,385)	(6.652)	(50,692)
	NFS	17,330	6.770	80,425	6.757	58,454
	[NFS]	[29,543]	[6.839]	[95,175]	[6.948]	[77,793]
NYTP2	(77.434)	(85,685)	(77.808)	(37,769)	(79.600)	(30,177)
	78.213	97,759	77.966	44,282	80.128	31,938
	[79.254]	[106,669]	[80.678]	[54,702]	[80.678]	[35,324]

Table 5 Comparison of the best-cost statistics for the best value and the near-heuristic value of the number of ants m with $a = 1.0$, $b = 0.5$, $r = 0.98$, $t_0 = 140$, 50 , and 200 for the NYTP, HP and NYTP2 respectively, and $I_{max} \times m \approx 100,000$ for the NYTP and the HP, and $1,000,000$ for the NYTP2. Performance statistics for each parameter value are based on results from ten runs for the NYTP and five runs for the HP and NYTP2.

Case study	Best value of m in tested parameter series			Heuristic for m given by (22)	Near-heuristic value of m in tested parameter series							
	m	Best-cost (\$million)			m	Best-cost (\$million) (% deviation from best value statistics)						
		minimum	mean			maximum	minimum	mean	maximum			
NYTP	100	38.638	38.774	39.216	84	80	38.638	(0.00)	38.794	(0.05)	39.415	(0.51)
HP	40	6.492	6.706	6.900	83.28	80	6.354	(-2.13)	6.779	(1.08)	6.970	(1.01)
NYTP2	100	77.808	77.966	78.256	168	160	77.474	(-0.43)	78.082	(0.15)	78.364	(0.14)

Table 6 Performance of AS_{i-best} with heuristic parameter settings for different case studies. $I_{max} \times m \approx 100,000, 200,000,$ and $500,000$ for the New York Tunnels Problem, the Hanoi Problem, and the Doubled New York Tunnels Problem respectively. Performance statistics for AS_{i-best} are based on results from 100 runs. NA means information was not available.

Case Study	Algorithm	Best-cost statistics (\$million) (% Deviation from known optimum)					Search-time statistics (evaluation number)			
		minimum		mean		maximum	minimum	mean	maximum	
NYTP	AS _{i-best}	38.638	(0.00)	38.849	(0.54)	39.492	(2.21)	16,664	22,052	33,610
	GA _{imp} ^a	38.796	(0.41)	NA		NA		96,750	NA	NA
	ACOA ^b	38.638	(0.00)	NA		NA		7,014	13,928	23,045
HP	AS _{i-best}	6.367	(2.99)	6.842	(10.68)	7.474	(20.90)	60,433	67,136	110,561
	GA ^c	6.195	(0.21)	NA		NA		$\sim 10^6$	NA	NA
	GA _{fm} ^d	6.182	(0.00)	NA		NA		113,626	NA	NA
NYTP2	AS _{i-best}	77.275	(0.00)	78.302	(1.33)	79.922	(3.43)	60,376	75,760	141,388

^a Ref. [10]

^b Ref. [17]

^c Ref. [11]

^d Ref. [13]

Table 7 Comparison of $AS_{i\text{-best}}$ design to current best designs for the Hanoi Problem. Diameters differing from GA_{fm} solution are in italics. Smaller diameters are also bolded.

Link no.	GA_{fm}^a	GA^B	$AS_{i\text{-best}}$
1	40	40	40
2	40	40	40
3	40	40	40
4	40	40	40
5	40	40	40
6	40	40	40
7	40	40	40
8	40	40	40
9	40	40	30
10	30	30	30
11	24	<i>30</i>	24
12	24	24	24
13	16	16	12
14	12	<i>16</i>	12
15	12	12	12
16	12	<i>16</i>	<i>24</i>
17	20	20	20
18	24	24	<i>40</i>
19	24	24	24
20	40	40	40
21	20	20	20
22	12	12	12
23	40	40	40
24	30	30	30
25	30	30	24
26	24	20	20
27	12	12	12
28	12	12	12
29	16	16	20
30	16	16	16
31	12	12	<i>16</i>
32	16	12	12
33	16	16	12
34	24	20	20

^a Ref. [13]^b Ref. [11]

Table 8 Summary of heuristics for ACO parameters for the Water Distribution System Problem.

Parameter	Description	Value / expression
\mathbf{a}	Decision control parameter for pheromone values	1.0
\mathbf{b}	Decision control parameter for desirability values	0.5
t_0	Initial pheromone value	$Q\sqrt{n \cdot NO_{avg}} / C(\Omega_n^*)$
\mathbf{r}	Pheromone persistence factor	0.98
m	Number of ants	$n\sqrt{NO_{avg}}$
Q	Pheromone reward factor	$C(\Omega^{\max})$
PEN	Penalty factor	$(C(\Omega^{\max}) - C(\Omega^{\min})) / d$