Spin-3/2 baryons in lattice QCD

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We present first results for masses of spin-3/2 baryons in lattice QCD, using a novel fat-link clover fermion action in which only the irrelevant operators are constructed using fat links. In the isospin-1/2 sector, we observe, after appropriate spin and parity projection, a strong signal for the $J^P = \frac{3}{2}^-$ state, and find good agreement between the $\frac{1}{2}^+$ mass and earlier nucleon mass simulations with a spin-1/2 interpolating field. For the isospin-3/2 Δ states, clear mass splittings are observed between the various $\frac{1}{2}^{\pm}$ and $\frac{3}{2}^{\pm}$ channels, with the calculated level orderings in good agreement with those observed empirically.

1. INTRODUCTION

Recent advances in computing power and the development of improved actions have enabled simulations of the spectrum of excited states of the nucleon in lattice QCD [1,2,3]. The lattice studies complement the high precision measurements of the N^* spectrum under way at Jefferson Lab [4]. In a recent paper [5], results were presented for the excited nucleon and spin-1/2 hyperon spectra using the Fat-Link Irrelevant Clover (FLIC) fermion action [6] with an $\mathcal{O}(a^2)$ improved gluon action. Here we extend the analysis of Ref. [5] to the spin-3/2 sector, and present results in both the isospin-1/2 and 3/2 channels.

In the isospin-3/2 sector, our results for the $\Delta(\frac{3}{2}^+)$ state agree well with earlier simulations [7] using Wilson fermions. We find a clear signal for the P-wave $\Delta(\frac{3}{2}^-)$ parity partner of the Δ ground state, and a discernible signal for the $\Delta(\frac{1}{2}^+)$ states. In particular, the $\Delta(\frac{1}{2}^-)$ state is found to have a mass ~ 350 –400 MeV above the $\Delta(\frac{3}{2}^+)$, with the $\Delta(\frac{3}{2}^-)$ slightly heavier. The $\Delta(\frac{1}{2}^+)$ state is found to lie ~ 200 –300 MeV above these. This level ordering is consistent with experiment.

In the spin-3/2 nucleon sector, there is good agreement for the projected $\frac{1}{2}^+$ state with earlier nucleon mass calculations [5] using standard spin-1/2 nucleon interpolating fields. Furthermore, we find a good signal for the $N(\frac{3}{2}^-)$ state, with a mass splitting of \sim 700–1000 MeV with the nucleon ground state.

2. SPIN- $\frac{3}{2}$ BARYONS ON THE LATTICE

The standard isospin-1/2, spin-3/2, charge +1 interpolating field is given by $\chi_{\mu}^{N^+}(x) = \epsilon^{abc} \left(u^{Ta}(x) \; C \gamma_5 \gamma_{\mu} \; d^b(x)\right) \gamma_5 u^c(x)$, which transforms as a pseudovector under parity, in accord with a positive parity Rarita-Schwinger spinor. The quark field operators u and d act at Euclidean space-time point x, C is the charge conjugation matrix, a, b and c are color labels, and the superscript T denotes the transpose. The charge neutral interpolating field is obtained by interchanging $u \leftrightarrow d$.

The commonly used interpolating field for the Δ^{++} resonance is given by [8] $\chi_{\mu}^{\Delta^{++}}(x) = \epsilon^{abc} \left(u^{Ta}(x) C \gamma_{\mu} u^{b}(x)\right) u^{c}(x)$. Since the spin-3/2 Rarita-Schwinger spinor-vector is a tensor product of a spin-1 vector and a spinor, the spin-3/2 interpolating field contains spin-1/2 contributions. To project a spin-3/2 state one needs

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to use a spin-3/2 projection operator [9]. Following spin projection, the correlation function for a given spin s, $G^s_{\mu\nu}$, still contains positive and negative parity states. In an analogous procedure to that used in Ref. [5], where a fixed boundary condition is used in the time direction, positive and negative parity states are obtained by taking the trace of $G^s_{\mu\nu}$ with the operators $\Gamma_{\pm} = \frac{1}{2} (1 \pm \gamma_4)$, respectively.

3. RESULTS

Lattice simulations are performed on a $16^3 \times 32$ lattice at $\beta = 4.60$, corresponding to a lattice spacing of a = 0.122(2) fm set by the string tension [10] with $\sqrt{\sigma} = 440$ MeV. We consider 392 configurations generated on the Orion cluster at the CSSM, U. Adelaide. A mean-field improved plaquette plus rectangle gauge action is used. For the quark fields, a Fat-Link Irrelevant Clover (FLIC) [6] action is implemented. We employ a highly improved definition of $F_{\mu\nu}$ [6,11] leaving errors of $\mathcal{O}(a^6)$. Mean-field improvement of the tree-level clover coefficient with fat links represents a small correction and proves to be quite adequate [6]. A fixed boundary condition in the time direction is used for the fermions by setting $U_t(\vec{x}, N_t) = 0 \ \forall \ \vec{x}$ in the hopping terms of the fermion action, with periodic boundary conditions imposed in the spatial directions. Gaugeinvariant gaussian smearing in the spatial dimensions is applied at the fermion source (placed at time slice t = 3) to increase the overlap of the interpolating operators with the ground states.

In Fig. 1 the results for the spin-projected $\Delta(\frac{3}{2}^+)$ (triangles) and $\Delta(\frac{3}{2}^-)$ (diamonds) masses are shown as a function of the pseudoscalar meson mass, m_π^2 . The trend of the $\Delta(\frac{3}{2}^+)$ data points with decreasing m_q is consistent with the physical mass of the $\Delta(1232)$, with some nonlinearity in m_π^2 expected near the chiral limit. The mass of the $\Delta(\frac{3}{2}^-)$ lies some 500 MeV above that of its parity partner.

When performing a spin projection to extract the $\Delta(\frac{1}{2}^{\pm})$ states, a discernible, although noisy, signal is detected. The $\chi_{\mu}^{\Delta^{++}}$ interpolating field has small overlap with spin-1/2 states, however,

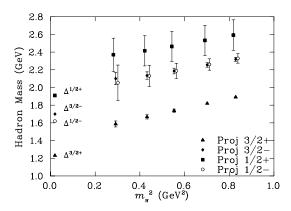


Figure 1. Masses of the spin-projected $\Delta(\frac{3}{2}^{\pm})$ and $\Delta(\frac{1}{2}^{\pm})$ states. The empirical masses are indicated along the ordinate.

with ~ 400 configurations we are able to extract masses for the spin-1/2 states at sufficiently early times. The $\Delta(\frac{1}{2}^+)$ (squares) and $\Delta(\frac{1}{2}^-)$ (open circles) are also displayed in Fig. 1. The lowest excitation of the ground state, namely the $\Delta(\frac{1}{2}^-)$, has a mass ~ 350 –400 MeV above the $\Delta(\frac{3}{2}^+)$, with the $\Delta(\frac{3}{2}^-)$ slightly heavier. The $\Delta(\frac{1}{2}^+)$ state is found to lie ~ 200 –300 MeV above these, although the signal becomes weak at smaller quark masses. This level ordering is consistent with that observed in the empirical mass spectrum.

In the isospin-1/2 sector, large statistical fluctuations make it difficult to obtain a clear signal, even with 392 configurations. A reasonable value of $\chi^2/N_{\rm DF}$ is obtained for time slices t=7–8. These results are shown as a function of m_π^2 in Fig. 2. Parity projecting to extract the $N(\frac{3}{2}^+)$ state, we find that the correlation function changes sign and has a large negative contribution in the range t=7–11. This behavior is an artifact associated with the quenched decay of the excited state into $N+\eta'$, and is further explored in Ref. [12].

The $N(\frac{1}{2}^+)$ channel displays the interplay of a quenched decay channel and the ground state contribution. A strong P-wave coupling of the $N(\frac{1}{2}^+)$ to $N\eta'$ forces the correlation function to be negative at small times, which then turns positive at larger times when the ground state contri-

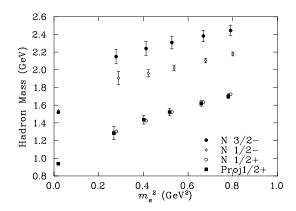


Figure 2. Masses of the spin-projected $N(\frac{3}{2}^-)$ and $N(\frac{1}{2}^+)$ states, compared with the nucleon and $N(\frac{1}{2}^-)$ masses from Ref. [5].

bution begins to dominate the correlation function. A good $\chi^2/N_{\rm DF}$ value is obtained for a fit to time slices t=10–14. This suggests that the first excited state has strong coupling to the η' , which implies a gluon-rich structure for the Roper resonance [13].

The extracted masses of the $N(\frac{3}{2}^-)$ and $N(\frac{1}{2}^+)$ states are displayed in Fig. 2 as a function of m_π^2 . Earlier results using the standard spin-1/2 interpolating field [5,6] are also shown in Fig. 2 for reference. There is excellent agreement between the spin-projected $\frac{1}{2}^+$ state obtained from the spin-3/2 interpolating field and the earlier $\frac{1}{2}^+$ results. A clear mass splitting is also seen between the $N(\frac{3}{2}^-)$ and $N(\frac{1}{2}^+)$ states obtained from the spin-3/2 interpolating field, with a mass difference of 700–1000 MeV.

4. CONCLUSION

First results for the spectrum of spin-3/2 baryons in the isospin-1/2 and 3/2 channels are reported, using a FLIC fermion action and an $\mathcal{O}(a^2)$ mean-field improved gauge action. Good agreement is found with earlier calculations for the Δ ground state, and clear mass splittings between the ground state and its parity partner are observed. A signal is also obtained for the $\Delta(\frac{1}{2}^{\pm})$ states, with the level ordering consistent with the

observed empirical mass spectrum.

For isospin-1/2 baryons, clear signals are obtained for both the $N(\frac{3}{2}^-)$ and spin-projected $N(\frac{1}{2}^+)$ states from a spin-3/2 interpolating field. The $\frac{1}{2}^+$ state in particular is in good agreement with earlier simulations of the nucleon mass using standard spin-1/2 interpolating fields.

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