

Scaling Behavior of the Landau Gauge Overlap Quark Propagator * †

J. B. Zhang^a, F. D. R. Bonnet^{ab}, P. O. Bowman^a, D. B. Leinweber^a, A. G. Williams^a

^aCSSM Lattice Collaboration,
Special Research Center for the Subatomic Structure of Matter (CSSM) and Department of Physics,
University of Adelaide 5005, Australia

^bDepartment of Physics, University of Regina, Regina, SK, S4S 0A2, Canada

The properties of the momentum space quark propagator in Landau gauge are examined for the overlap quark action in quenched lattice QCD. Numerical calculations are done on three lattices with different lattice spacings and similar physical volumes to explore the approach of the quark propagator towards the continuum limit. We have calculated the nonperturbative momentum-dependent wavefunction renormalization function $Z(p^2)$ and the nonperturbative mass function $M(p^2)$ for a variety of bare quark masses and extrapolate to the chiral limit. We find the behavior of $Z(p^2)$ and $M(p^2)$ are in good agreement for the two finer lattices in the chiral limit. The quark condensate is also calculated.

1. INTRODUCTION

There have been several studies of the momentum space quark propagator [1,2,3,4,5] using different gauge fixing and fermion actions. Here we focus on Landau gauge fixing and the overlap-fermion action and extend previous work [5] to three lattices with different lattice spacings a and very similar physical volumes. This allows us to probe the scaling behavior and the continuum limit of the quark propagator in Landau gauge.

2. QUARK PROPAGATOR ON THE LATTICE

In the continuum the renormalized Euclidean-space quark propagator must have the form

$$S(\zeta; p) = \frac{1}{i\not{p}A(\zeta; p^2) + B(\zeta; p^2)} = \frac{Z(\zeta; p^2)}{i\not{p} + M(p^2)}, \quad (1)$$

where $Z(\zeta; p^2)$ is the wavefunction renormalization function, $M(p^2)$ is the nonperturbative mass function, and ζ is the renormalization point.

On the lattice the bare quark propagator can be written as

$$S^{\text{bare}}(p) \equiv -i \left(\sum_{\mu} C_{\mu}(p) \gamma_{\mu} \right) + \mathcal{B}(p). \quad (2)$$

We use periodic boundary conditions in the spatial directions and anti-periodic in the time direction. The discrete momentum values for a lattice of size $N_i^3 \times N_t$, with $n_i = 1, \dots, N_i$ and $n_t = 1, \dots, N_t$, are

$$p_i = \frac{2\pi}{N_i a} \left(n_i - \frac{N_i}{2} \right), \quad p_t = \frac{2\pi}{N_t a} \left(N_t - \frac{1}{2} - \frac{N_t}{2} \right).$$

We can perform a spinor and color trace to identify

$$C_{\mu}(p) = \frac{i}{12} \text{tr}[\gamma_{\mu} S^{\text{bare}}(p)], \quad \mathcal{B}(p) = \frac{1}{12} \text{tr}[S^{\text{bare}}(p)].$$

The general approach to tree-level correction[2] utilizes the fact that QCD is asymptotically free and so it is the difference of bare quantities from their tree-level form on the lattice that contains the best estimate of the nonperturbative information. For the overlap fermion, the tree-level correction is nothing but to identify appropriate kinematic lattice momentum q . We can identify the appropriate kinematic lattice momentum q directly from the definition of the tree-level quark propagator numerically,

$$q_{\mu} \equiv C_{\mu}^{(0)}(p) = \frac{C_{\mu}^{(0)}(p)}{(\mathcal{C}^{(0)}(p))^2 + (\mathcal{B}^{(0)}(p))^2}. \quad (3)$$

We can also obtain the kinematic lattice momentum q analytically [4].

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Having identified the lattice momentum q , we can now define the bare lattice propagator as

$$\begin{aligned} S^{\text{bare}}(p) &\equiv \frac{1}{i\not{q}A(p) + B(p)} = \frac{Z(p)}{i\not{q} + M(p)} \\ &= Z_2(\zeta; a)S(\zeta; p), \end{aligned} \quad (4)$$

where $S(\zeta; p)$ is the lattice version of the renormalized propagator in Eq. (1), and $Z_2(\zeta; a)$ is the quark wave-function renormalization constant chosen so as to ensure $Z(\zeta; \zeta^2) = 1$.

The overlap fermion formalism [6,7] realizes an exact chiral symmetry on the lattice and is automatically $\mathcal{O}(a)$ improved. The massive overlap operator can be written as as [8]

$$D(\eta) = \frac{1}{2} [1 + \eta + (1 - \eta)\gamma_5\epsilon(H)], \quad (5)$$

where ρ is the Wilson mass with negative sign, and the quark mass parameter $\eta \equiv m_0/2\rho$. Written according to bare quark mass m_0 , we have

$$D(m_0) = \frac{1}{2\rho} \left[\rho + \frac{m_0}{2} + (\rho - \frac{m_0}{2})\gamma_5\epsilon(H) \right], \quad (6)$$

and the overlap quark propagator is given by

$$S^{\text{bare}}(m_0) \equiv \tilde{D}_c^{-1}(\eta), \quad (7)$$

where

$$\tilde{D}_c^{-1}(\eta) \equiv \frac{1}{2\rho}\tilde{D}^{-1}(\eta), \quad (8)$$

$$\tilde{D}^{-1}(\eta) \equiv \frac{1}{1-\eta} [D^{-1}(\eta) - 1]. \quad (9)$$

3. NUMERICAL RESULTS

Here we work on three lattices with different lattice spacing a and very similar physical volumes using a tadpole-improved plaquette plus rectangle gauge action. For each lattice size, 50 configurations are used. Lattice parameters are summarized in Table 1.

In the calculations, $\kappa = 0.19163$ was used for all three lattices, which gives $\rho a = (8 - 1/\kappa)/2 = 1.391$. We calculate for 10 quark masses on each lattice by using a shifted Conjugate Gradient solver. The 14th order Zolotarev rational approximation is used to evaluate the matrix sign function $\epsilon(H_w)$. The ten bare quark masses we

Table 1

Lattice parameters.

Action	Volume	β	a (fm)	u_0
Improved	$16^3 \times 32$	4.80	0.093	0.89650
Improved	$12^3 \times 24$	4.60	0.125	0.88888
Improved	$8^3 \times 16$	4.286	0.194	0.87209

use in our calculation are $m_0 = 2\rho\eta = 106, 124, 142, 177, 212, 266, 354, 442, 531, \text{ and } 620$ MeV respectively.

The detailed results will be presented elsewhere. Here we focus on the comparison of the results on these three lattices. All data has been cylinder cut [9] and extrapolated to the chiral limit using a simple linear extrapolation. The mass function $M(p)$ in the chiral limit for the three lattices is plotted in Fig. 1 and the renormalization function $Z(p)$ of the three lattices is plotted in Fig. 2. We can see that when the mass function $M(p)$ is plotted against the discrete lattice momentum p the results of the three lattices are in good agreement, while for the renormalization function $Z(p)$, good agreement is reached on the three lattices if it is plotted against the kinematical lattice momentum q . The overall agreement between the two finer lattices is good.

In the chiral and continuum limits, the asymptotic quark mass function has the form

$$\begin{aligned} M(p^2) \stackrel{p^2 \rightarrow \infty}{=} & -\frac{4\pi^2 d_M}{3} \frac{\langle \bar{\psi}\psi \rangle}{[\ln(\mu^2/\Lambda_{\text{QCD}}^2)]^{d_M}} \\ & \times \frac{[\ln(p^2/\Lambda_{\text{QCD}}^2)]^{d_M-1}}{p^2} \end{aligned} \quad (10)$$

where the anomalous dimension of the quark mass is $d_M = 12/(33 - 2N_f)$. Using the momentum p , in the fitting range $ap \subset (1.3, 2.5)$, on the $16^3 \times 32$ lattice, the resulting value for the quark condensate is

$$\langle \bar{\psi}\psi \rangle = -(288 \pm 24 \text{MeV})^3. \quad (11)$$

This is in excellent agreement with the value $\langle \bar{\psi}\psi \rangle = -(268 \pm 27 \text{MeV})^3$ extracted from the Asqtad action using the same method [10].

4. SUMMARY

In this report, we have considered tadpole-improved quenched lattice configurations, and the

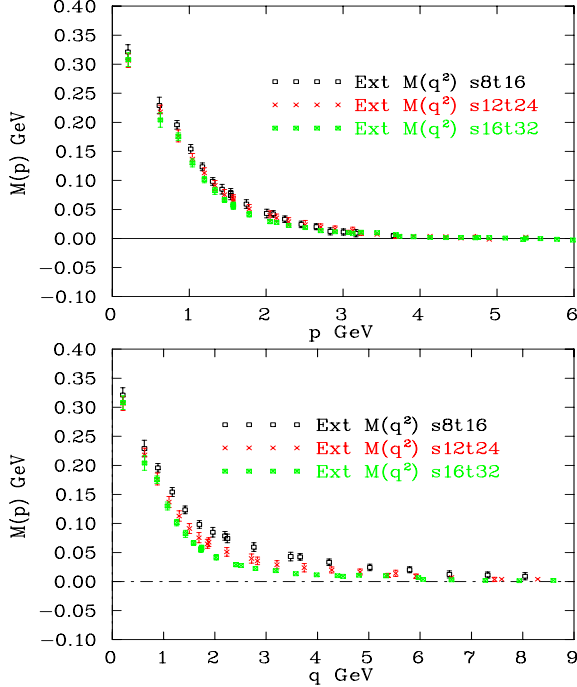


Figure 1. Comparison of the mass function $M(p)$ of three lattices in the chiral limit. The upper graph is plotted against the discrete lattice momentum p and the lower graph is plotted against the kinematical lattice momentum q .

overlap fermion operator with the Wilson fermion kernel. The momentum space quark propagator has been calculated in Landau gauge on three lattices with different lattice spacing a and similar physical volumes to explore the scaling property. The continuum limit for $Z(p)$ is most rapidly approached when it is plotted against the kinematical lattice momentum q , whereas the quark mass function, $M(p)$, should be plotted against the discrete lattice momentum p . The good agreement between the two finer lattices suggests that we are close to the continuum limit.

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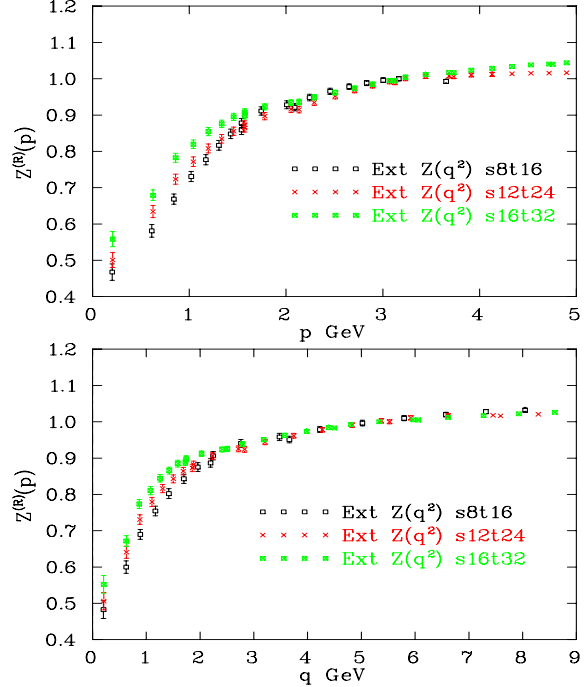


Figure 2. Comparison the renormalization function $Z(p) = Z(\zeta; p)$ for renormalization point $\zeta = 5.31$ GeV for the q -scale and $\zeta = 3.15$ GeV for the p -scale for the three lattices in the chiral limit. The upper graph is plotted against the discrete lattice momentum p and the lower graph is plotted against the kinematical lattice momentum q .

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