VECTOR MESON MODELS OF STRONGLY INTERACTING SYSTEMS

THESIS FOR DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS AND MATHEMATICAL PHYSICS

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October 1996

Abstract

The consequences of current conservation for vector meson models are examined. As an example of this, the $\rho - \omega$ mixing model for the isospin violation seen in the pion electromagnetic form factor is studied in detail. Assuming current conservation, we predict a strong momentum dependence for vector mixing. As this result also applies to photon–vector meson mixing, in contradiction to traditional photon–hadron models, we describe an equivalent model which includes a momentum dependent coupling of the photon to vector mesons. To ensure that the information obtained previously from the pion form-factor is consistent with conserved current picture, we redo the fit to data and find a considerable model dependence to the quantities of interest that is *not* a consequence of momentum dependence.

Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief contains no material previously published or written by another person, except where due reference has been made in the text.

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Acknowledgements

I would like to thank my supervisors, Tony Thomas and Tony Williams, for their guidance and close attention to my work. I also extend thanks to the staff at Adelaide University who have helped me during my PhD candidature, Rod Crewther, Lindsay Dodd, Jim McCarthy and Bruce Pearce. In addition, my work has benefitted from discussions with Maurice Benayoun, Reg Cahill, Susan Gardner, Kim Maltman, Andy Rawlinson, Craig Roberts and Peter Tandy. I also thank the hospitality of the Physics departments of Argonne National Laboratory, Indiana University, Kent State University, Los Alamos National Laboratory, Ohio State University, TRIUMF, the Technical University of Munich and the University of Maryland.

Thanks also to my family for their support and the Australian Government for financial assistance.

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Chapter 1

Introduction

Before proceeding with our discussion of $\rho-\omega$ mixing, it will be helpful to introduce the concept of Vector Meson Dominance (VMD), the model within which $\rho-\omega$ mixing is usually described. To motivate this, it is useful to provide the historical development of the subject, which will then lead naturally into the consideration of $\rho-\omega$ mixing and its theoretical description. Where possible, the connection between the original hadronic models and our present understanding will be discussed.

The physics of hadrons was a topic of intense study long before the gauge field theory Quantum Chromodynamics (QCD), now believed to describe it completely, was invented [1]. Hadronic physics was described using a variety of models and incorporating approximate symmetries. It is a testimony to the insight behind these models (and the inherent difficulties in solving non-perturbative QCD) that they still play an important role in our understanding.

One particularly important aspect of hadronic physics which concerns us here is the interaction between the photon and hadronic matter [2]. This has to date been remarkably well described using the vector meson dominance (VMD) model. This assumes that the hadronic components of the vacuum polarisation of the photon consist exclusively of the known vector mesons. This is certainly an approximation, but in the resonance region around the vector meson masses, it appears to be a particularly good one. As vector mesons are believed to be bound states of quark-antiquark pairs, it is tempting to try to establish a connection between the old language of VMD and the Standard Model. In the Standard Model, quarks, being charged, couple to the photon and so the strong sector contribution to the photon propagator arises, in a manner analogous to the electron-positron loops in QED, as shown in Fig. 1.1.

The diagram contains dressed quark propagators and the proper (i.e., one-particle irreducible) photon-quark vertex (the shaded circles include one-particle-reducible parts, while the empty circles are one-particle-irreducible [3]). This diagram contains *all* strong interaction contributions to the photon self-energy. In QED, because the fine structure constant ($\alpha \simeq 1/137$) is small, we can approximate the photon self-energy reasonably well using bare propagators and vertices without worrying about higher-order dressing. However, in QCD, the dressing of these quark propagators and the quark-photon proper vertex cannot be so readily dismissed as being of higher order in a perturbative expansion (although for the heavier quarks, higher order effects can be ignored as a consequence of asymptotic freedom [4]). Recent work [5], using numerical solutions to the non-perturbative Dyson-Schwinger equations of QCD [3], has succeeded in producing a spectrum of quark-antiquark bound states, but the mass distribution does not yet correspond to the observed spectrum.

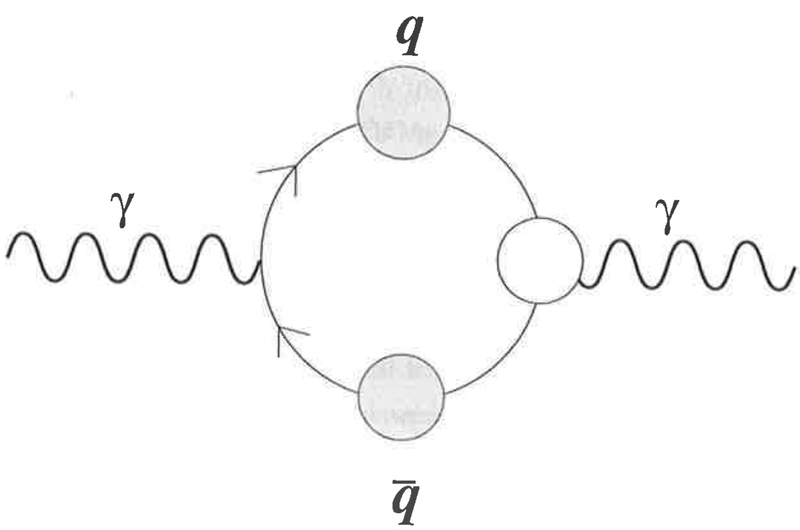

Figure 1.1: One-particle-irreducible QCD contribution to the photon propagator. This is the complete QCD contribution to the photon self-energy.

1.1 Historical development of VMD

The seeds of VMD were sown by Nambu [6] in 1957 when he suggested that the charge distribution of the proton and neutron, as determined by electron scattering [7], could be accounted for by a heavy neutral vector meson contributing to the nucleon form factor. This isospin-zero field is now called the ω.

The anomalous magnetic moment of the nucleon was believed to be dominated by a two-pion state [8]. The pion form-factor, $F_\pi(q^2)$, (to be discussed later in some detail) was taken to be unity in these initial calculations —i.e., the pions were treated as point-like objects. In 1959 Frazer and Fulco [9] attempted to show that the both the magnetic explained by the inclusion of a strong pion-pion interaction. After an investigation of analytic structure (following studies of the electromagnetic properties of the nucleon using the dispersion relation method [8,10]), it was seen that the pion form-factor had to satisfy

$$F_\pi(q^2) = 1 + \frac{q^2}{\pi}\int_{4m_\pi^2} dr \frac{\text{Im } F_\pi(r)}{r(r - q^2 - i\epsilon)}, \tag{1.1}$$

and that to be consistent with data a suitable peak in the pion form-factor was required. They believed that this could result from a strong pion-pion interaction. The analytic structure of the partial wave amplitude in the physical region could be approximated as a pole of appropriate position and residue (a successful approximation in nucleon-nucleon scattering). An analysis determined that the residue should be positive, raising the possibility of a resonance, which we now know as the ρ^0.

It was Sakurai who proposed a theory of the strong interaction mediated by vector mesons [11] based on the non-Abelian field theory of Yang and Mills [12]. He was deeply troubled by the problem of the masses of the mesons in such a theory, as they would destroy the local (flavour) gauge invariance. He published his work with this matter unresolved in the hope that it would stimulate further interest in the field.

Kroll, Lee and Zumino did pursue the idea of reproducing VMD from field theory [13]. Within the simplest VMD model the hadronic contribution to the polarisation of the photon takes the form of a propagating vector meson (see Fig. 1.2). This is now a model for the QCD contribution to the polarisation process depicted in Fig. 1.1 and arises

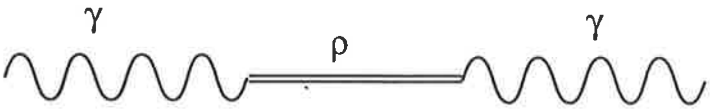

Figure 1.2: A simple VMD-picture representation of the hadronic contribution to the photon propagator. The heavier vector mesons are included in generalised VMD models.

from the assumption that the hadronic electromagnetic (EM) current operator, j_μ^{EM}, is proportional to the field operators of the vector mesons (multiplied by their mass squared). This is referred to as the the field-current identity (FCI). One can then incorporate this idea into an effective Lagrangian, giving a precise formulation of VMD in terms of a local, Lagrangian field theory. One starts with the FCI for the neutral ρ-meson

$$[j_\mu^{\text{EM}}(x)]_{I=1} = \frac{m_\rho^2}{g_\rho}\rho_\mu^0(x), \tag{1.2}$$

and then generalises [14] to an isovector field, $\vec{\rho}(x)$, of which $\rho^0(x)$ is the third component [i.e., $\rho^0(x) \equiv \rho^3(x)$]. Eq. (1.2) implies that the field $\vec{\rho}(x)$ is divergenceless under the strong interaction, which is just the usual Proca condition (this, however, need not be true for all effective Lagrangians)

$$\partial_\mu \vec{\rho}^{\,\mu} = 0, \tag{1.3}$$

for a massive vector field coupling to a conserved current. The resulting Lagrangian for the hadronic sector is the same as the (flavour) Yang-Mills Lagrangian [12], but also has a mass term which destroys local gauge invariance. Although gauge invariance is necessary for renormalisability[1], Kroll *et al.* were unconcerned by this; stating that the non-zero value for the mass made it possible to connect the field conservation equation, Eq. (1.3), with the equation of motion of the field. The case of a global SU(2) massive vector field (the ρ-field), interacting with a triplet pion field and coupled to a conserved current, is treated in detail by Lurie [16].

1.2 Gauge invariance and VMD

Sakurai's analysis of VMD [17] takes place in the context of a local gauge theory. Although a mass term in the Lagrangian breaks (flavour) gauge symmetry, Sakurai viewed the generation of interactions by minimal substitution in the Lagrangian to be interesting enough to ignore this problem. Lurie [16] has discussed the ρ, π, N system using coupling to conserved currents which reproduces Sakurai's results. As it only assumes that the Lagrangian is invariant under *global* SU(2), the appearance of mass terms causes no difficulty. One can then examine how to include the photon in this system. Lurie's primary concern was to have the ρ couple to a conserved current, and he did this by constructing a Lagrangian whose equation of motion had the Noether current associated with the global SU(2) symmetry appearing on the right hand side. In doing this, he arrived at the standard non-Abelian Lagrangian (given on p. 700 of Ref. [18]), which is where we begin our discussion.

We begin with the Lagrangian (while Sakurai and Lurie worked in a Euclidean metric, we follow the conventions of Bjorken and Drell [19])

$$\mathcal{L}_{\rm full} = -\frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\,\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^{\,\mu} + \frac{1}{2}D_\mu\vec{\pi}\cdot D^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi}, \tag{1.4}$$

[1] In general there are only two cases in which a massive vector field is renormalisable, see Ref. [15], p. 61:

a) a gauge theory with mass generated by spontaneous symmetry breaking;

b) a theory with a massive vector boson coupled to a conserved current and without additional self-interactions.

where

$$\vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu - g\vec{\rho}_\mu \times \vec{\rho}_\nu, \tag{1.5}$$

and where[2]

$$D_\mu \vec{\pi} = (\partial_\mu - ig\vec{\rho}_\mu \cdot \vec{T})\vec{\pi}, \tag{1.6}$$
$$= \partial_\mu \vec{\pi} - g\vec{\rho} \times \vec{\pi}. \tag{1.7}$$

This Lagrangian is symmetric under the transformation

$$\vec{\phi} \to \vec{\phi} + \vec{\phi} \times \vec{\epsilon}, \tag{1.8}$$

where $\vec{\phi}$ represents the isovector fields of the $\vec{\rho}$ and $\vec{\pi}$. The generation of interactions from minimal substitution is used by Sakurai and Lurie to motivate universality (i.e., the coupling constant of the ρ introduced via the covariant derivative, D_μ, is the same for all particles). However, as a slight violation to this rule is seen experimentally, we shall distinguish between g and the constant g_ρ appearing in Eq. (1.2), which Sakurai equates in order to satisfy a constraint on the pion form-factor (to be discussed later).

From Eq. (1.7) it follows that

$$\frac{1}{2}D_\mu \vec{\pi} \cdot D^\mu \vec{\pi} = \frac{1}{2}\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - g\vec{\rho}_\mu \cdot (\vec{\pi} \times \partial^\mu \vec{\pi}) + \frac{1}{2}g^2(\vec{\rho}_\mu \times \vec{\pi})^2. \tag{1.9}$$

After some algebra we obtain the equation of motion for the ρ field

$$\partial_\nu \vec{\rho}^{\,\nu\mu} + m_\rho^2 \vec{\rho}^{\,\mu} = g\vec{J}^\mu_{\text{Noether}} \tag{1.10}$$

where the Noether current is

$$\vec{J}^\mu_{\text{Noether}} = -\frac{\partial \mathcal{L}}{\partial(\partial_\mu \vec{\rho}_\nu)} \times \vec{\rho}_\nu - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \vec{\pi})} \times \vec{\pi} \tag{1.11}$$

giving

$$\vec{J}^\mu_{\text{Noether}} = \vec{\rho}^{\,\mu\nu} \times \vec{\rho}_\nu + \vec{\pi} \times \partial^\mu \vec{\pi} + g(\vec{\rho}^{\,\mu} \times \vec{\pi}) \times \vec{\pi}. \tag{1.12}$$

As the Noether current is necessarily conserved, Eq. (1.10) tells us that the field is divergenceless, as in Eq. (1.3), i.e., acting with ∂_μ on each side of Eq. (1.10) and noting that $\vec{\rho}^{\nu\mu} = -\vec{\rho}^{\mu\nu}$ and $\partial_\mu \vec{J}^\mu_{\text{Noether}} = 0$ implies that $\partial_\mu \vec{\rho}^\mu = 0$. Transferring the non-Abelian part of the field strength tensor (the cross product in Eq. (1.5)) to the RHS of Eq. (1.10) gives us,

$$\partial_\nu(\partial^\nu \vec{\rho}^{\,\mu} - \partial^\mu \vec{\rho}^{\,\nu}) + m_\rho^2 \vec{\rho}^{\,\mu} = g(\vec{J}^\mu_{\text{Noether}} + \partial_\nu(\vec{\rho}^{\,\nu} \times \vec{\rho}^{\,\mu})). \tag{1.13}$$

[2]We use hermitian T's given by the algebra $[T^a, T^b] = -ic^{abc}T^c$ and normalised by $\text{Tr}(T^aT^b) = \delta_{ab}/2$. Thus, in the adjoint representation, $(T^c)_{ba} = -ic^{cab}$. For SU(2), $c^{abc} = \epsilon_{ijk}$.

Again using the fact that the ρ field is divergenceless, we can rewrite the equation of motion in the inverse propagator form

$$(\partial^2 + m_\rho^2)\vec{\rho}^{\,\mu} = g\vec{J}^\mu, \tag{1.14}$$

where $\vec{J}_\mu$ is also a divergenceless current given by

$$\begin{aligned} \vec{J}^\mu &= \vec{J}^\mu_{\text{Noether}} + \partial_\nu(\vec{\rho}^{\,\nu} \times \vec{\rho}^{\,\mu}) \\ &= \vec{J}^\mu_{\text{Noether}} + \vec{\rho}^{\,\nu} \times \partial_\nu \vec{\rho}^{\,\mu}. \end{aligned} \tag{1.15}$$

That J^μ is also conserved can be seen by taking the divergence of Eq. (1.15); the RHS vanishes due to the Proca condition, Eq. (1.3). and the fact that $\partial_\mu\vec{\rho}^{\,\nu} \times \partial_\nu\vec{\rho}^{\,\mu} = 0$ from symmetry under $\mu \leftrightarrow \nu$. As Lurie notes, the presence of the ρ field itself in $\vec{J}^\mu_{\text{Noether}}$ prevents us from writing the interaction part of the Lagrangian in the simple $\vec{\rho}_\mu \cdot \vec{J}^\mu$ fashion (which is possible for the fermion-vector interaction). A similar situation for scalar electrodynamics is discussed by Itzykson and Zuber [18] (p. 31–33).

Our task now is to include electromagnetism in this model, and to do this we shall allow Eq. (1.2) to guide us. Using the FCI, Eq. (1.2), we replace the electromagnetic current by the ρ meson term. We then invert Eq. (1.14) to replace the ρ by the *hadronic* current, using $\partial_\mu \longrightarrow iq_\mu$, to obtain a corresponding matrix element relation for the electromagnetic interaction[3]

$$\begin{aligned} \langle B|\, ej_\mu^{\text{EM}}\, |A\rangle &= e\,\langle B|\, \frac{m_\rho^2}{g_\rho}\rho_\mu^3\, |A\rangle \\ &= e\frac{m_\rho^2}{g_\rho}\,\langle B|\, \frac{-gJ_\mu^3}{q^2 - m_\rho^2}\, |A\rangle \qquad (1.16) \\ &= \left(\frac{-iem_\rho^2}{g_\rho}\right)\left(\frac{-i}{q^2 - m_\rho^2}\right)\langle B|\, gJ_\mu^3\, |A\rangle\,. \qquad (1.17) \end{aligned}$$

Thus the photon appears to couple to the hadronic field via a ρ meson, to which it couples with strength em_ρ^2/g_ρ. (This model is illustrated in Fig. 4.2b, below.)

Before proceeding, we shall make, as Sakurai does, the simplifying assumption that one can neglect the ρ self-interaction (from now on we shall refer only to the $\rho^0 \equiv \rho^3$). As the ρ^0 decays almost entirely via the two-pion channel, this is a physically reasonable approximation. Therefore we ignore the parts of the current given by Eq. (1.15) involving ρ terms, and concern ourselves only with the piece of the current that looks like

$$J_\pi^\mu \equiv (\vec{\pi} \times \partial^\mu \vec{\pi})_0, \tag{1.18}$$

[3] We take e to be positive, $e = |e|$.

to which, for notational brevity, we shall refer from now on simply as J^μ. Changing from a Cartesian to a charge basis, we can re-write Eq. (1.18) as

$$J_\mu = i(\pi^- \partial_\mu \pi^+ - \pi^+ \partial_\mu \pi^-). \tag{1.19}$$

We can then write the simple linear coupling term in the Lagrangian, and from now on we shall write g as $g_{\rho\pi\pi}$

$$\mathcal{L}_{\rho\pi} = -g_{\rho\pi\pi} \rho_\mu J^\mu. \tag{1.20}$$

1.3 The electromagnetic form-factor of the pion

One problem in which VMD found particular success was the description of the electromagnetic form-factor of the pion [20]. As this has played such a crucial role in our understanding of $\rho-\omega$ mixing it is useful to outline what is means by it and how the theoretical predictions are compared with experimental data.

We are concerned with the s-channel process depicted in Fig. 1.3, in which an electron-positron pair annihilate, forming a photon which then decays to two pions. We define

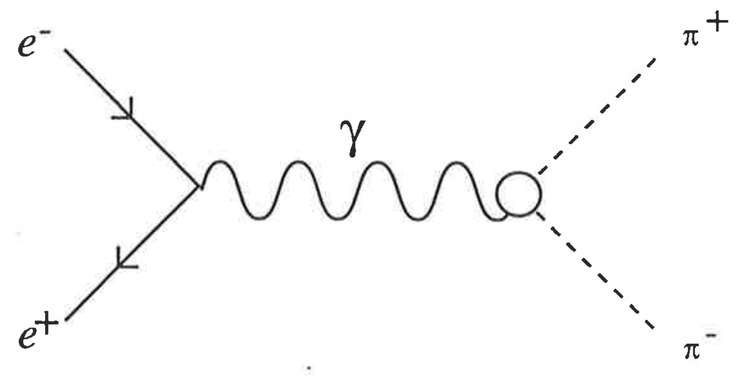

Figure 1.3: Electron-positron pair annihilating to form a photon which then decays to a pion pair.

the form-factor, $F_\pi(s)$, by Eq. (4.44). The form-factor represents all possible strong interactions occurring within the circle in Fig. 1.3, which we model using VMD.

In the time-like region, $F_\pi(q^2)$ is measured experimentally in the reaction $e^+e^- \to \pi^+\pi^-$, which, to lowest order in e^2, is given by the process shown in Fig. 1.3. The momenta of the electron and positron are p_1 and p_2 respectively, and p_3 and p_4 are the momenta of the π^+ and π^-. The differential cross-section is given by [21]

$$\frac{d\sigma}{d\Omega} = \frac{\vec{p}_3^{\,2}}{|\vec{p}_3|(p_3^0 + p_4^0) - p_3^0 \hat{p}_2 \cdot (\vec{p}_1 + \vec{p}_2)} \frac{\frac{1}{4}\Sigma_{\text{pols}}|\mathcal{M}_{fi}|^2}{64\pi^2\sqrt{(p_1 \cdot p_2)^2 - m_e^4}}, \tag{1.21}$$

where $\hat{p}$ is the unit vector in the direction of $\vec{p}$. We are thus interested in calculating the Feynman amplitude, $\mathcal{M}_{fi}$, for this process. The lepton and photon parts of the diagram are completely standard. The interesting part of the diagram concerns the coupling of the photon to the pion pair represented by Fig. 1.3. The form of this part of the diagram, $\mathcal{M}_{\gamma\to\pi^+\pi^-}$, shall be discussed in detail later (see Eq. (4.44)). In full, the amplitude is

$$\mathcal{M}_{fi} = \overline{v}(2)ie\gamma^\mu u(1)iD_{\mu\nu}(q)eF_\pi(q^2)(p_4 - p_3)^\nu, \tag{1.22}$$

with the photon propagator being given by

$$iD_{\mu\nu}(q) = \frac{(-i)}{q^2}\left[g_{\mu\nu} + (\xi - 1)\frac{q_\mu q_\nu}{q^2}\right]. \tag{1.23}$$

Particular choices of ξ correspond to particular covariant gauge choices. The second term in Eq. (1.23) vanishes because the photon couples to conserved currents and so the result is gauge invariant as expected.

In the centre of mass frame (in which we shall define $|\vec{p}| \equiv p$) we have $E^2 - p^2 = m_e^2$, $E^2 - p'^2 = m_\pi^2$, and $\vec{p}\cdot\vec{p}\,' = -pp'\cos\theta$. Using $\sqrt{s} = 2E$ the differential cross-section becomes

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{s^2}\sqrt{\frac{s-4m_\pi^2}{s^2-4sm_e^2}}\frac{(E^4 - E^2m_\pi^2 - ((E^4 - E^2(m_\pi^2 + m_e^2) + m_\pi^2 m_e^2)\cos^2\theta))|F_\pi(s)|^2}{\sqrt{s}\,64\pi^2}. \tag{1.24}$$

Since $m_e^2 << m_\pi^2$ and we are considering the energy region $E > m_\pi$, we can simplify the above formula to

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{s^2}\frac{(s-4m_\pi^2)^{1/2}}{s\sqrt{s}}\frac{1}{8\pi^2}(E^4 - E^2m_\pi^2)(1-\cos^2\theta)|F_\pi(s)|^2.$$

From this we obtain the total cross-section

$$\sigma = \frac{\alpha^2\pi}{3}\frac{(s-4m_\pi^2)^{3/2}}{s^{5/2}}|F_\pi(s)|^2, \tag{1.25}$$

where $\alpha \equiv e^2/4\pi \simeq 1/137$.

While VMD is a successful low energy model for the photon hadron interaction, certain elements of it seem somewhat naive. For instance, no attention is paid to the momentum dependence of the real part of the mass, but (as the propagator must be real below threshold, when the particle cannot decay) momentum dependence must be present and can be introduced for the width. One simple possibility for the momentum dependent width being $m_\rho\Gamma_\rho\theta(q^2 - 4m_\pi^2)$. The neglect of momentum dependence is especially relevant for the use of vector mesons as mediators of the NN interaction. This application traditionally uses simple perturbative propagators (with no width) in the spacelike q^2

region. The fact that the vector resonance contribution is suppressed away from the mass pole, allowing one to use simple Breit–Wigner propagators very effectively, means we have little experimental information about the momentum dependence of these contributions. As we are using vector mesons in an approximation to *non-perturbative* QCD these issues deserve much deeper study.

As $\rho-\omega$ mixing is based upon VMD, these underlying factors will be germane to this study and one must keep this in mind.

1.4 Summary

We have seen how a $\rho - \pi$ Lagrangian, with a *global* isospin symmetry, in which the ρ couples to a conserved current and satisfies the Proca condition, can be constructed. This required the use of minimal substitution to create a covariant derivative which introduces universality ($g = g_{\rho\rho\rho} = g_{\rho\pi\pi}$), extending this system to other hadrons, such as nucleons, will similarly give $g_{\rho NN} = g$. Electromagnetism was then included through the use of the field current identity for on-shell matrix elements, leading to the vector meson dominance model.

Chapter 2

Standard treatment of $\rho - \omega$ mixing

To make sense of the present work and the purpose behind it, it is necessary to have an understanding of the treatment of $\rho - \omega$ mixing through the previous twenty years. This chapter begins by providing a very brief description of the first theoretical prediction of $\rho - \omega$ mixing in the early sixties. We then move forward ten years to what we shall refer to as the "Standard Picture" (SP) of $\rho - \omega$ mixing which was essentially unquestioned from the mid-seventies to the end of the eighties. My own work (covered in Chapters 4, 5 and 6) has largely been a careful re-examination and refinement of this SP. I briefly discuss the earlier work done in $\rho - \omega$ mixing, in the period from the mid-sixties to the mid seventies in Appendix C, where we will see some intersting parallels with my own research.

2.1 The theoretical beginning of $\rho - \omega$ mixing

We have seen in the previous chapter that the pion EM form-factor could be modelled using vector meson dominance (VMD), in which the photon couples to a ρ meson which then decays to the two pion final state, enhancing the interaction in the ρ resonance region, $e^+e^- \to \pi^+\pi^-$. A similar model can be used for $e^+e^- \to \pi^+\pi^0\pi^-$. The vector meson we associate with this process is the isospin zero ω meson, which has a similar mass to the ρ, but a much smaller width. The strong interaction was believed to preserve G parity and hence would not allow the ω $(I = 0)$ to decay to a pion pair $(I = 1)$. Thus one should only see the ρ resonance in the pion form-factor. In 1961, though, Glashow suggested [22] that EM effects could mix the two states of pure isospin, ρ_I and ω_I, on the grounds that they were very close in mass and differed only by isospin (a quantum number broken by electromagnetism). This would result in the mass (or "physical") eigenstates, ρ and ω, being superpositions of the two initial fields. The most obvious possibility for the mixing, although it is only a very small effect, would be via the process shown in Fig. 2.1. Glashow commented that other EM mixing processes such as $\rho_I \to \gamma + \pi^0 \to \omega_I$ would also be possible. At the time, however, there was no experimental evidence to support the mixing of the two isospin states.

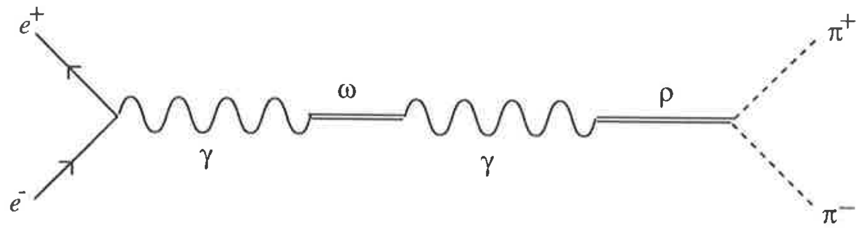

Figure 2.1: Electromagnetic contribution to the ω-resonance of $e^+e^- \to \pi^+\pi^-$.

Symmetry breaking, though, was then a new and active field of research. In 1964 Coleman and Glashow [23] produced a model for the breaking of unitary symmetry (the symmetry scheme of Gell-Mann and Ne'eman based on SU(3) for the hadronic octet [24]). They postulated the existence of a unitary octet of scalar mesons, composed of an isotopic singlet (η'), triplet (π') and two doublets (K', $\bar{K}'$). This octet would allow the possibility of scalar tadpoles, a class of diagrams that vanish for all other (non-scalar) particles. These tadpoles would vanish in the limit of exact unitary symmetry. The diagrams can be broken into two parts connected only by a scalar meson line - the tadpole part and the SU(3) invariant part. Coleman and Glashow assumed that symmetry violating processes are dominated by these tadpoles, but claimed that their explanation for symmetry breaking did not necessarily depend on the existence of the scalar octet. As a concluding remark they described a calculation of $\rho-\omega$ mixing assuming tadpole dominance. The vector mesons are assumed to comprise a unitary singlet and octet which mix. The ρ^0 is then connected to both the unitary singlet and unitary octet part of the physical ω via $\pi^{0\prime}$ tadpole diagrams. This allows one to express the mixing amplitude in terms of vector meson multiplet mass differences, an idea pursued recently [25].

2.2 The experimental discovery of $\rho-\omega$ mixing

As more data was collected (for the reaction $e^+e^- \to \pi^+\pi^-$ and other related reactions such as $\pi^+ + p \to \pi^+\pi^- + \Delta^{++}$) and the resolution of the resonance curve improved, it became clear that there was a kink around the mass of the ω-meson in the otherwise smooth curve observed [26]. As an illustration, Fig. 2.2 shows a graph of the modern somewhat sparser, the theoretical techniques for modelling it are essentially the same.

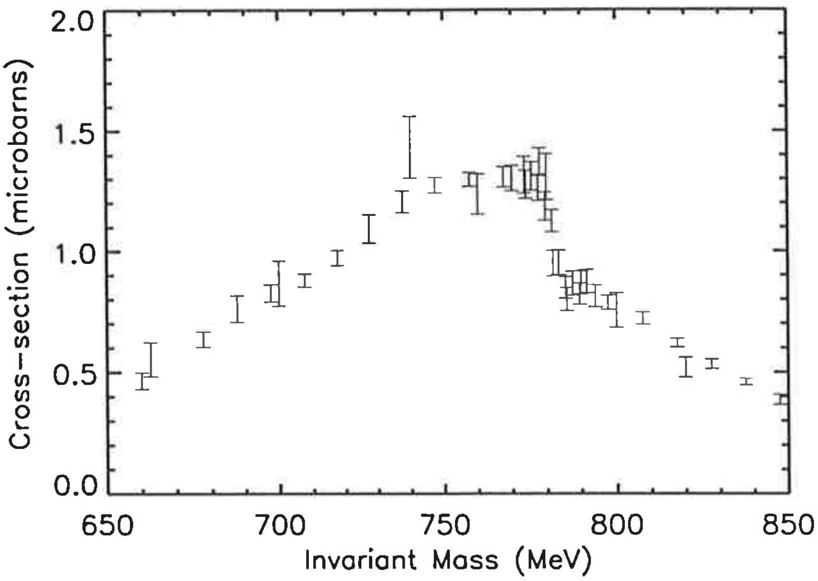

Figure 2.2: The cross-section for the reaction $e^+e^- \rightarrow \pi^+\pi^-$ from the data of Ref. [27] in the $\rho-\omega$ resonance region.

2.3 $\rho-\omega$ mixing and $F_\pi(q^2)$

The standard way to include ρ–ω mixing in the VMD description of $F_\pi(q^2)$ is by using a matrix propagator for the vector meson. From this we obtain the parametrisation for isospin breaking discussed in Ref. [28]. Using a matrix notation, the Feynman amplitude for the process $\gamma \rightarrow \pi\pi$, proceeding via vector mesons, can be written in the form

$$i\mathcal{M}^{\mu}_{\gamma\rightarrow\pi\pi} = \left(\, i\mathcal{M}^{\nu}_{\rho_I\rightarrow\pi\pi} \quad i\mathcal{M}^{\nu}_{\omega_I\rightarrow\pi\pi} \, \right) iD_{\nu\mu} \begin{pmatrix} i\mathcal{M}_{\gamma\rightarrow\rho_I} \\ i\mathcal{M}_{\gamma\rightarrow\omega_I} \end{pmatrix} . \tag{2.1}$$

The propagator matrix for the isospin pure ρ_I and ω_I (which will become mixed to form the physical states, which are *not* eignestates of isospin) is $D_{\nu\mu}$ and is discussed in detail in the section surrounding Eq. (4.22). The other Feynman amplitudes are derived from either the VMD1 or VMD2 Lagrangian (Eqs. (4.31) and (4.32)). Since we will here always couple the vector mesons to conserved currents, the terms proportional to $q_\mu q_\nu$ in the propagator (Eq. (4.22)) can be neglected. It should be carefully noted that models which do not have coupling to conserved currents will need to explicitly retain these terms. If we assume that the pure isospin state ω_I does not couple to two pions ($\mathcal{M}^{\nu}_{\omega_I\rightarrow\pi\pi} = 0$, this point is addressed below in Sec. 2.4) then to lowest order in the mixing, Eq. (2.1) is just

$$\mathcal{M}^{\mu}_{\gamma\to\pi\pi} = \left(\mathcal{M}^{\mu}_{\rho_I\to\pi\pi} \quad 0 \right) \left(\begin{array}{cc} 1/s_\rho & \Pi_{\rho\omega}/s_\rho s_\omega \\ \Pi_{\rho\omega}/s_\rho s_\omega & 1/s_\omega \end{array} \right) \left(\begin{array}{c} \mathcal{M}_{\gamma\to\rho_I} \\ \mathcal{M}_{\gamma\to\omega_I} \end{array} \right), \tag{2.2}$$

where $s_V = q^2 - m_V^2$ and m_V^2 (for $V = \rho, \omega$) are the complex pole locations with real part $\hat{m}_V^2$ and imaginary part $-\hat{m}_V\Gamma_V$. Expanding this just gives

$$\mathcal{M}^{\mu}_{\gamma\to\pi\pi} = \mathcal{M}^{\mu}_{\rho_I\to\pi\pi}\frac{1}{s_\rho}\mathcal{M}_{\gamma\to\rho_I} + \mathcal{M}^{\mu}_{\rho_I\to\pi\pi}\frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\omega}\mathcal{M}_{\gamma\to\omega_I} \tag{2.3}$$

which we recognise as the sum of the two diagrams shown in Fig. 2.3.

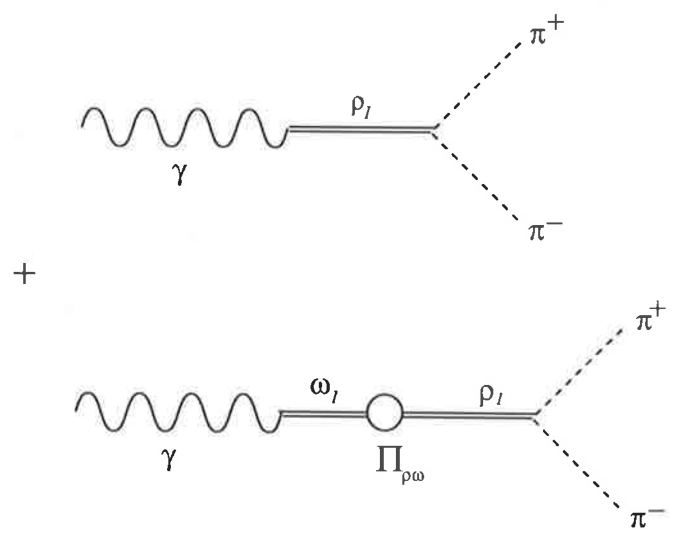

Figure 2.3: The contribution of $\rho-\omega$ mixing to the pion form-factor.

The couplings that enter this expression, through $\mathcal{M}^{\mu}_{\rho_I\to\pi\pi}$, $\mathcal{M}_{\gamma\to\rho_I}$ and $\mathcal{M}_{\gamma\to\omega_I}$, always involve the unphysical pure isospin states ρ_I and ω_I. However, we can re-express Eq. (2.3) in terms of the physical states by first diagonalising the vector meson propagator. To do this we introduce an orthogonal diagonalising matrix

$$C = \left(\begin{array}{cc} 1 & \epsilon \\ -\epsilon & 1 \end{array} \right) \tag{2.4}$$

where, to lowest order in the mixing,

$$\epsilon = \frac{\Pi_{\rho\omega}}{s_\rho - s_\omega}. \tag{2.5}$$

We now insert identities into Eq. (2.2) and obtain

$$\mathcal{M}^{\mu}_{\gamma\to\pi\pi} = \left(\mathcal{M}^{\mu}_{\rho_I\to\pi\pi} \quad 0 \right) CC^{-1} \left(\begin{array}{cc} 1/s_\rho & \Pi_{\rho\omega}/s_\rho s_\omega \\ \Pi_{\rho\omega}/s_\rho s_\omega & 1/s_\omega \end{array} \right) CC^{-1} \left(\begin{array}{c} \mathcal{M}_{\gamma\to\rho_I} \\ \mathcal{M}_{\gamma\to\omega_I} \end{array} \right)$$

$$= \begin{pmatrix} \mathcal{M}^\mu_{\rho\to\pi\pi} & \mathcal{M}^\mu_{\omega\to\pi\pi} \end{pmatrix} \begin{pmatrix} 1/s_\rho & 0 \\ 0 & 1/s_\omega \end{pmatrix} \begin{pmatrix} \mathcal{M}_{\gamma\to\rho} \\ \mathcal{M}_{\gamma\to\omega} \end{pmatrix} \tag{2.6}$$

where we have identified the physical amplitudes as (to first order in isospin violation)

$$\mathcal{M}^\mu_{\rho\to\pi\pi} = \mathcal{M}^\mu_{\rho_I\to\pi\pi}, \tag{2.7}$$

$$\mathcal{M}^\mu_{\omega\to\pi\pi} = \mathcal{M}^\mu_{\omega_I\to\pi\pi} + \epsilon\mathcal{M}^\mu_{\rho_I\to\pi\pi}, \tag{2.8}$$

$$\mathcal{M}_{\gamma\to\rho} = \mathcal{M}_{\gamma\to\rho_I} - \epsilon\mathcal{M}_{\gamma\to\omega_I}, \tag{2.9}$$

$$\mathcal{M}_{\gamma\to\omega} = \mathcal{M}_{\gamma\to\omega_I} + \epsilon\mathcal{M}_{\gamma\to\rho_I}. \tag{2.10}$$

Expanding Eq. (2.6), we find

$$\begin{aligned} \mathcal{M}^\mu_{\gamma\to\pi\pi} &= \mathcal{M}^\mu_{\rho\to\pi\pi}\frac{1}{s_\rho}\mathcal{M}_{\gamma\to\rho} + \mathcal{M}^\mu_{\omega\to\pi\pi}\frac{1}{s_\omega}\mathcal{M}_{\gamma\to\omega} \\ &= \mathcal{M}^\mu_{\rho\to\pi\pi}\frac{1}{s_\rho}\mathcal{M}_{\gamma\to\rho} + \mathcal{M}^\mu_{\rho\to\pi\pi}\frac{\Pi_{\rho\omega}}{s_\rho - s_\omega}\frac{1}{s_\omega}\mathcal{M}_{\gamma\to\omega}. \end{aligned} \tag{2.11}$$

At first glance there might appear to be a slight discrepancy between Eqs. (2.3) and (2.11). The source of this is the definition used for the coupling of the vector meson to the photon. The first, Eq. (2.3), uses couplings to pure isospin states, the second, Eq. (2.11) uses "physical" couplings (i.e., couplings to the mass eigenstates) which introduce a leptonic contribution to the Orsay phase [26] between the ω contribution and the ρ contribution. We assume $\mathcal{M}_{\gamma\to\rho_I} \simeq 3\mathcal{M}_{\gamma\to\omega_I}$, which is exact in the limit of exact SU(6) spin-flavour symmetry, and define the leptonic phase θ by

$$\frac{\mathcal{M}_{\gamma\to\omega}}{\mathcal{M}_{\gamma\to\rho}} = \frac{1}{3}e^{i\theta} \tag{2.12}$$

then, to order ϵ,

$$\tan\theta = -\frac{10\Pi_{\rho\omega}}{3m_\rho\Gamma_\rho}. \tag{2.13}$$

This gives $\theta = 5.7^\circ$ for $\Pi_{\rho\omega} = -4520$, as obtained by Coon *et al.* [28]. This small so-called "leptonic" contribution to the Orsay phase is the principal manifestation of diagonalising the $\rho-\omega$ propagator.

2.4 Intrinsic ω_I decay in $e^+e^- \to \pi^+\pi^-$

We present here the argument introduced by Gourdin, Stodolsky and Renard [29] that the isospin violation of the intrinsic decay $\omega_I \to 2\pi$, though of the same order in isospin violation as that due to $\rho-\omega$ mixing, gives no contribution to the pion form-factor. The original suggestion was rather terse, but it was elaborated on later by Renard [30] and we thus refer to it here as the "Renard argument".

The coupling of the physical ω to the two pion state can be expressed as (from Eq. (2.11))

$$g_{\omega\pi\pi} = g_{\omega_I\pi\pi} + \epsilon g_{\rho_I\pi\pi}, \tag{2.14}$$

where ϵ is given by Eq. (2.5). Neglecting the small mass difference of the two mesons and the decay width of the ω allows us to approximate ϵ, given in Eq. (2.5), by

$$\epsilon = -i\left(\frac{\mathrm{Re}\,\Pi_{\rho\omega} + i\mathrm{Im}\,\Pi_{\rho\omega}}{m_\rho\Gamma_\rho}\right). \tag{2.15}$$

We shall return in Chapter 6 to re-examine the implications of the approximations leading to Eq. (2.15). If $\mathcal{M}_{\omega_I\pi\pi} \neq 0$, then we would have a contribution to $\rho-\omega$ mixing shown in Fig. 2.4. We can determine the contribution to $\Pi_{\rho\omega}$ from $\rho \to \pi\pi \to \omega$. To do this,

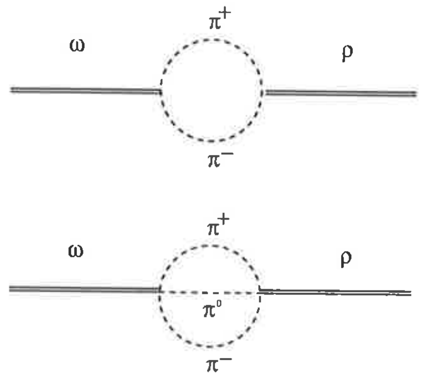

Figure 2.4: Physical intermediate states contributing to $\rho-\omega$ mixing.

however, it is first useful to consider the analogous case for the simpler $\rho\pi$ system. The self energy of the ρ, $\Pi_{\rho\rho}$, has a contribution from a virtual pion loop as in Fig. 2.5. Because the ρ decays predominantly (the quoted branching ratio is $\sim 100\%$ [31]) to the

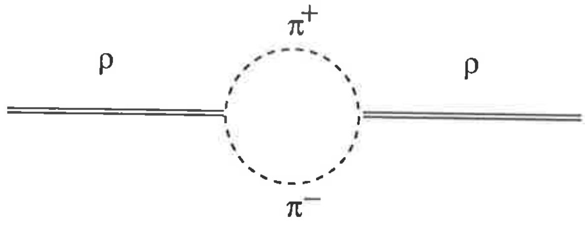

Figure 2.5: Contribution of a pion loop to the ρ self-energy.

two pion state it is assumed that this loop will dominate the dressing of the propagator.

$$\simeq \frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho}, \tag{2.17}$$

where m_0 is the bare mass, m_ρ the renormalised mass and Γ_ρ the width of the ρ-meson. The width of the ρ, Γ_ρ, is defined by the imaginary part of the polarisation function, $\Pi_{\rho\rho}(q^2)$, at the ρ mass point, $q^2 = m_\rho^2$,

$$\text{Im}\ \Pi_{\rho\rho}(m_\rho^2) \equiv -m_\rho\Gamma_\rho. \tag{2.18}$$

Similarly we can determine the imaginary part of the one-loop diagram shown in Fig. 2.4, which contributes to Im $\Pi_{\rho\omega}$. One now assumes that the contribution to $\rho-\omega$ mixing from a pion loop and the ρ polarisation are related by (comparing Figs. 2.4 and 2.5)

$$\Pi_{\rho\omega}^{\pi\pi} = \frac{g_{\omega_I\pi\pi}}{g_{\rho_I\pi\pi}}\Pi_{\rho\rho}. \tag{2.19}$$

Due to the strength of the $\rho \to \pi\pi$ decay, the pion loop contribution can also be assumed to dominate the imaginary part of the total $\rho-\omega$ mixing, and we then have

$$\begin{aligned} \text{Im}\ \Pi_{\rho\omega} &= \frac{g_{\omega_I\pi\pi}}{g_{\rho_I\pi\pi}}\text{Im}\ \Pi_{\rho\rho} \\ &= -\frac{g_{\omega_I\pi\pi}}{g_{\rho_I\pi\pi}}m_\rho\Gamma_\rho. \end{aligned} \tag{2.20}$$

Substituting Eq. (2.20) into Eq. (2.15) and then substituting the result into Eq. (2.14) we have

$$g_{\omega\pi\pi} = g_{\omega_I\pi\pi} - i\frac{\text{Re}\ \Pi_{\rho\omega}}{m_\rho\Gamma_\rho}g_{\rho_I\pi\pi} + \frac{\text{Im}\ \Pi_{\rho\omega}}{m_\rho\Gamma_\rho}g_{\rho_I\pi\pi}, \tag{2.21}$$

giving us

$$g_{\omega\pi\pi} = g_{\omega_I\pi\pi} - i\frac{\text{Re}\ \Pi_{\rho\omega}}{m_\rho\Gamma_\rho}g_{\rho_I\pi\pi} - g_{\omega_I\pi\pi}. \tag{2.22}$$

As can be seen the contributions from the isospin violating $g_{\omega_I\pi\pi}$ coupling cancel each other.

So, in summary: We allowed G-parity violation through $\omega_I \to \pi\pi$ (in the same form as $\rho_I \to \pi\pi$), which contributed to the mixing parameter, ϵ, through the process depicted in Fig. 2.4. We then found that the imaginary part of the single pion loop actually cancelled the decay of the ω_I in the process $\omega \to \pi\pi$. Hence the decay of the the ω_I can be ignored. We shall critically re-examine this argument in Chapter 6.

2.5 The use of $\rho-\omega$ mixing in nuclear physics

Isospin violation in nuclear interactions has certain obvious contributions arising from the unequal masses and charges of the proton and neutron. We can make allowances for

2.5 The use of ρ–ω mixing in nuclear physics

Isospin violation in nuclear interactions has certain obvious contributions arising from the unequal masses and charges of the proton and neutron. We can make allowances for these. However, on top of these there remains, at the 1% level, an isospin violation from the strong interaction itself. The reader might notice the similarity of this situation to the previous statement that EM effects alone cannot account for the ρ–ω mixing seen in the pion form-factor. Following Henley and Miller [32] it is usual to consider the individual classes of this isospin violation.

Class	Contribution	Name
I	1 and $\vec{t}(1)\cdot\vec{t}(2)$	charge-independent
II	$t_z(1)t_z(2)-\vec{t}(1)\cdot\vec{t}(2)$	charge dependent
III	$t_z(1)+t_z(2)$	CSV
IV	$[\vec{t}(1)\times\vec{t}(2)]_3$	CSV

Table 2.1: The classification scheme of Henley and Miller [32] for isospin dependent forces in nuclear interaction. Class III and IV forces violate charge symmetry.

It is the Class III force that is of interest to us here. Charge symmetry is the rotation of hadrons in isospin space by π radians about the I_2 axis. Thus, for the nucleons this corresponds to the transformation

$$\begin{aligned} p &\rightarrow n \\ n &\rightarrow -p. \end{aligned}$$

Experimental results suggest a strong interaction contribution to Charge Symmetry Violation (CSV). The two most commonly quoted examples of this are the nn, (Coulomb corrected) pp and np scattering lengths [33] and the mass difference of mirror nuclei, the Okamoto Nolen Schiffer (ONS) anomaly [34]. Making the EM and $n-p$ mass difference allowances one finds a remaining $\simeq$ 70 keV ^{3}He–^{3}H binding energy difference which is compatible with the -1.5 fm difference between the scattering lengths a_{nn} and a_{pp}.

The standard explanation for this small strong interaction CSV has relied on ρ–ω mixing. One of the more modern expressions of this is that due to Coon and Barrett [35], which provides fresh experimental input to the earlier analysis [28]. I shall now provide an outline of the standard use of ρ–ω mixing in nuclear physics (we shall call this standard use, ρ–ω CSV). This leads naturally into a discussion of work questioning the validity of

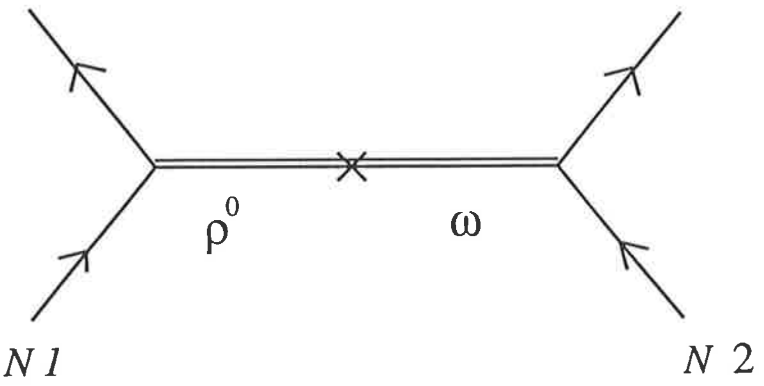

Figure 2.6: $\rho-\omega$ mixing contribution to the CSV NN interaction.

CSV amplitude [35] (where q_μ is the momentum four vector of the exchanged meson)

$$T_{NN}^{\rho\omega} = -\frac{H_\mu(\omega N_1 N_1) H^\mu(\rho N_2 N_2) \Pi_{\rho\omega}}{(q^2 - m_\rho^2)(q^2 - m_\omega^2)} + (1 \leftrightarrow 2). \tag{2.23}$$

The meson-nucleon vertices are given by

$$\begin{aligned} H_\mu(\rho NN) &= \frac{1}{2} g_\rho \bar{N} \tau_3 (\gamma_\mu + \kappa_V i \frac{\sigma_{\mu\nu} q^\nu}{2M_N}) N \\ H_\mu(\omega NN) &= \frac{1}{2} g_\omega \bar{N} (\gamma_\mu + \kappa_S i \frac{\sigma_{\mu\nu} q^\nu}{2M_N}) N \end{aligned} \tag{2.24}$$

the factor of 1/2 being a result of using the vector meson universality couplings g_ρ and g_ω (see the section surrounding Eq. (4.60)). From this one can construct a CSV contribution to a potential for use in nuclear models. It is easily seen that Eq. (2.24) on the whole uses the well-known quantities of the more familiar charge symmetry conserving (CSC) NN interaction. However, there is one quantity lacking in the CSC interaction, the mixing amplitude $\Pi_{\rho\omega}$. This is obtained from the pion form-factor data, and typically takes a value of $\simeq -4000$ MeV^2 (e.g., Ref. [35] quotes an extracted value of $\Pi_{\rho\omega} = -4520 \pm 600$ MeV^2). Using this in Eq. (2.23) produces a substantial fraction of the CSV observed in the nucleon scattering length and the mirror nuclei binding energies (at least for the lightest nuclei).

In 1991 Goldman, Henderson and Thomas (GHT) [36] raised an interesting point in connection with $\rho-\omega$ CSV. The value for $\Pi_{\rho\omega}$ obtained from the pion form-factor is extracted for timelike q^2 in the vector meson resonance region, while the four-momentum of the vector mesons exchanged between on-shell nucleons is necessarily spacelike. Thus if $\Pi_{\rho\omega}$ were to have any momentum dependence it could significantly affect the nuclear potential described above. A quark loop model for $\rho-\omega$ mixing performed by GHT (to be described in greater detail later) gave a strongly momentum-dependent mixing, which would drastically alter the potential, and ruin the fit to data.

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The GHT paper initiated a great deal of work in the study both of isospin violation and meson mixing, which had largely accepted the standard $\rho-\omega$ CSV approach up until then. The finding of significant momentum-dependence in most models for $\rho-\omega$ mixing has stimulated investigations into other possible mechanisms for CSV [37–40].

Chapter 3

Behaviour of $\rho-\omega$ mixing

The GHT paper [36] proposed a simple quark model to generate $\rho-\omega$ mixing in order to investigate the momentum dependence of such a process. Since then, other authors have conducted further studies using various approaches to examine momentum dependence. Unfortunately, it is very difficult to make model-independent statements about such a process. This is not helped by the freedom associated with the vector mesons themselves, which one can take as being convenient parametrisations of medium–energy strong processes. In principle, one is always allowed to make a field redefinition, which if done appropriately, will not change the overall calculation of a given physical process, but can lead to an altered form of the interaction.

Experimentally, one sees these vector mesons as resonances in strong interaction processes whose complex S-matrix poles can be usefully represented by Breit-Wigner propagators for the vector particle. For the case of $e^+e^- \to \pi^+\pi^-$ which is relevant to $\rho-\omega$ mixing, we have the pion form-factor with the dominant ρ pole, P_ρ, and the suppressed ω pole, P_ω,

$$F_\pi \propto P_\rho + Ae^{i\phi}P_\omega + \text{background}, \tag{3.1}$$

where

$$P_V \equiv \frac{1}{q^2 - \hat{m}_V^2 + i\hat{m}_V\Gamma_V}$$

and $\hat{m}_V$, Γ_V, A and ϕ are real. The complex prefactor $Ae^{i\phi}$ is, like the pole positions, a purely experimental quantity. Any acceptable model, therefore, must reproduce the experimentally extracted values of A and ϕ. The matrix formalism, discussed in Chapter 2, allows for a surprisingly good prediction for the Orsay phase, ϕ, from the "extracted" (its meaning is dependent on the use of the mixed matrix propagator analysis) parameter $\Pi_{\rho\omega}$. The assumptions inherent in this meson-mixing model are critically examined in Chapter 6. What is presently lacking is a detailed prediction for $Ae^{i\phi}$ from some higher principle such as a suitably detailed effective hadronic model. This would, naturally, be of considerable interest.

3.1 Models for $\rho-\omega$ mixing

The GHT model attempted to bring together quark and meson degrees of freedom [36]. Let us briefly review this model. Essentially, the isospin violating mixing between the ρ and ω mesons takes place via a quark loop and arises from the CSV mass difference between the up and down quarks. In this calculation the quarks were taken to couple to the vector mesons in the same way as nucleons (see Eq. (2.24)), through isospin matrices. The quark propagator in the $u-d$ sector with four-momentum p_μ, can then be written as a 2×2 isospin matrix with the rows and columns labelled by u or d. We may then introduce the mass difference via

$$S(p) = \frac{1}{\not{p} - (\bar{m} + \delta m \tau_3)}, \tag{3.2}$$

where $\bar{m} = (m_u + m_d)/2$ and $\delta m = (m_d - m_u)$. The reader will notice that the use of these free Dirac propagators for the quarks will produce an unphysical quark-production threshold in the quark loop. This is a shortcoming of such a simple treatment. While GHT considered this to be an unsatisfactory feature, it was still sufficient for an initial investigation. One can argue that a judicious choice of quark masses can simulate the physical production thresholds (in this instance the two pion threshold). This reasoning was used to justify Dirac propagators in a calculation of the pion self-energy from a quark loop, which, handled carefully, could reproduce a physical cut beginning at $q^2 = (m_\rho + m_\pi)^2$ [41]. We shall see shortly how the matter of production thresholds from loop diagrams was handled by subsequent authors. The propagator in Eq. (3.2) was expanded to first order in the small parameter, δm. This introduces a factor of τ_3 to the numerator which, together with the τ_3 at the $\rho - q - \bar{q}$ vertex yields a non-vanishing trace over quark flavours (when δm is zero, isospin is conserved and, hence, there is no $\rho-\omega$ mixing).

The other feature of the calculation is the use of a form-factor at the quark–meson vertex. Their particular choice was dependent only on the loop momentum, k,

$$\frac{M^2}{M^2 - k^2},$$

where M "describes the vector meson (quark) structure" and was given a value of about 1500 MeV which minimises its direct effect. The independence of the form-factor on the meson four-momentum (q_μ) was chosen purely for simplicity. This is justified by saying that since one wishes to investigate the q^2 dependence of the mixing it is better to avoid any *ad-hoc* introduction of q^2 dependence into the vertex.

The result of this calculation was to show that $\Pi_{\rho\omega}$ is strongly momentum dependent in this model. In addition, this brings into question the CSV potential generated by the Standard Picture diagram given in Fig. 2.6. This was a radical departure from the standard thinking. GHT suggest two things following this. Firstly, further investigation of the momentum-dependence of ρ–ω mixing should be done and, secondly, some inclusion of the quark structure of the nucleons might be warranted (as this model has examined the quark structure of the mesons). With respect to this second point a contemporary paper is mentioned [42] which examined nuclear CSV through QCD-induced corrections to QED processes (such as one photon exchange and one photon loop graphs). The point of GHT was that while it was a simple and somewhat flawed calculation it did highlight the importance of examining the question of the momentum dependence of $\rho-\omega$ mixing.

In time both suggestions were taken up by other authors. The first of the subsequent calculations to examine the momentum dependence of $\rho-\omega$ mixing was carried out by Piekarewicz and Williams (PW) [43]. It was an adaptation of the GHT model, in which the isospin violating mixing is generated not by quark loops, but by nucleon loops. This has the advantage of only requiring parameters that are relatively well known, such as nucleon masses and meson-nucleon couplings.

Technically, the PW approach was simpler than the GHT calculation which had included the effect of isospin violation by writing the quark masses as $m_{d,u} = \bar{m} \pm \delta m$, and then expanding in the isospin violating δm. They find the total amplitude is the difference between a proton loop and a neutron loop. The nucleon mass difference plays the same role as the $u - d$ mass difference. The divergences of the individual graphs (treated using dimensional regularisation) cancel allowing PW to use pointlike meson-nucleon vertices, as opposed to the form-factors of the quark-meson vertices used by GHT. This avoided introducing violations of current conservation, which we will see later is an important consideration. The GHT node is *not* at $q^2 = 0$. This calculation, with virtually no free parameters, yields a prediction for $\Pi_{\rho\omega}$ in the resonance region of roughly the same sign and magnitude as that obtained by Coon and Barrett [35] from the pion form factor data [27]. Whether this fortuitous agreement has any deep origins is as yet unclear. Like GHT, PW find significant momentum dependence, but with a node exactly at $q^2 = 0$, i.e., $\Pi_{\rho\omega}(0) = 0$.

The quark loop calculation of Krein, Thomas and Williams (KTW) [44] sought to address the problem of quark confinement. Following studies of non-perturbative QCD [45], KTW use propagators with no poles on the complex plane (entire functions), and as such, the quarks are never on mass-shell (real or complex). This is one means to implement confinement. Like GHT they assume that the quark-meson vertex is independent of the meson four momentum squared. For an assumed quark mass difference of 4 MeV the calculation was able to fit the lone data point reasonably well, though this fit turns out to be sensitive to the mass difference. Once again a large momentum dependence with a node *near* $q^2 = 0$ is seen.

More complicated still is the calculation of Mitchell, Tandy, Roberts and Cahill (MTRC) [46]. This uses a QCD based model in which the vector mesons appear as composite $\bar{q}q$ bound states. The propagator for the quarks is obtained from a model Dyson-Schwinger equation, and like the KTW model, is confining. The $\rho-\omega$ mixing once again takes place through a quark loop. The paper largely involves itself with a detailed technical description of the model and its use in the isospin breaking calculation. They conclude that the quark loop mechanism, by itself, generates an insufficient CSV potential for the NN system and suggest an additional pion loop contribution could be examined. This could easily be accommodated into the model.

Friedrich and Reinhardt [47] utilise the bosonised Nambu–Jona-Lasinio (NJL) model in their study of $\rho-\omega$ mixing. They reproduce a strong momentum dependence, with a node in the mixing at $q^2 = 0$. Gao, Shakin and Sun (SGS) [48] examine $\rho-\omega$ mixing using an extension to the NJL model that includes quark confinement (which is not an issue for the bosonised NJL model). SGS make use of the current correlator (see below) in their work.

In summary, all models predicted a strong momentum dependence in $\Pi_{\rho\omega}$ with a node near or at $q^2 = 0$.

3.2 QCD Sum Rules

QCD is not yet understood well enough to directly assist us in studying $\rho-\omega$ mixing because of its non-perturbative nature in the medium energy world. However, its non-Abelian structure means that the coupling constant, g_S, decreases with rising q^2 until perturbative calculations can be done — this property is known as asymptotic freedom [4]. How might we obtain some insight into $\rho-\omega$ mixing from this perturbative region? A technique, QCD Sum Rules (SR), has been developed that utilises what is known from high energy calculations and combines this with hadron phenomenology in an attempt to describe low energy physics. We shall now discuss the use of this technique in $\rho-\omega$ mixing. Unfortunately, as we shall see, it is not a simple matter [49]. We therefore do not give a detailed exposition of this technique, or its use, but will instead concentrate on the conclusions that have been drawn from it for $\rho-\omega$ mixing.

What one seeks to do with the SR is to start at the short distance physics, where quark-gluon interactions can be treated reliably and to extrapolate to larger distances such that the non-perturbative effects appear as corrections (though there is currently some discussion as to how reliable perturbative QCD is, even at high energies [50]).

Consider a two point correlation function of various currents, J^a_μ and J^b_μ,

$$C^{ab}_{\mu\nu}(q^2) = i \int d^4x \, e^{iq\cdot x} \langle 0|T(J^a_\mu(x) J^b_\nu(0))|0\rangle . \tag{3.3}$$

This can be expressed by general dispersion relations in terms of observable cross-sections. Alternatively this correlator is given at large, spacelike q^2 (i.e., large Q^2, where $Q^2 \equiv -q^2$) by a Wilson operator expansion in terms of vacuum to vacuum matrix elements. Equating these two expressions for the correlator (one in terms of dispersion relations and the other involving a Wilson expansion) gives the sum rule. As $Q^2 \to \infty$ the only non-vanishing operator is the identity, (as QCD is non-interacting due to asymptotic freedom). However, as Q^2 drops, we probe larger and larger distances. The fundamental assumption of the SR technique is that terms of the form $(1/Q^2)^n$ with increasing integer n come into play as we go to lower Q^2.

Interestingly, the original papers on SR by Shifman, Vainshtein and Zakharov (SVZ) [49] used ρ–ω mixing as an example. The correlator (see Eq. (3.3)) of the electromagnetic isospin zero and isospin one currents (which they identified as J^ω_μ and J^ρ_μ respectively) was considered. This was rewritten as

$$C^{\rho\omega}_{\mu\nu}(q^2) = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) C^{\rho\omega}(q^2) \tag{3.4}$$

where

$$J^\rho_\nu = \frac{1}{2}(\bar{u}\gamma_\nu u - \bar{d}\gamma_\nu d) \tag{3.5}$$

$$J^\omega_\nu = \frac{1}{6}(\bar{u}\gamma_\nu u + \bar{d}\gamma_\nu d). \tag{3.6}$$

Note that in the limit of exact isospin symmetry in which u and d are equal, this correlator vanishes.

SVZ identified two places for isospin breaking — the operator expansion coefficients and the matrix elements between the vacuum. The relevance of this is that the expansion coefficients can be found explicitly, whereas the matrix elements require additional assumptions or independent experimental data. The expansion in powers of $1/Q^2$ is then associated with various diagrams. The first term, for instance, corresponds to single photon exchange (as shown in Fig. 2.1).

The use of SR is particularly reliant on techniques to ensure that the sum converges quickly and that we only need to add up the first few terms to obtain a good approximation to the exact result (otherwise this technique would be unreliable). To do this, one often looks not at the actual quantity of interest (in our present case the correlator in Eq. (3.4)), but at a transform of the quantity. These transforms ensure that the series converges more quickly. The Borel transform of the correlator is given by,

$$
\begin{aligned}
12\tilde{C}^{\rho\omega}(M^2) &= \frac{\alpha}{16\pi^3} + \frac{(m_u - m_d)}{M^4}\langle 0|\bar{u}u + \bar{d}d|0\rangle \left(1 - \frac{(m_u + m_d)}{2\Delta}\right) \\
&\quad - \frac{224\pi}{81}\alpha_s(\mu)\frac{\langle 0|\bar{q}q|0\rangle^2}{M^6}\left(\frac{(m_d - m_u)}{\Delta} + \frac{\alpha}{8\alpha_S(M)}\right),
\end{aligned}
\tag{3.7}
$$

where Δ is a parameter later determined to be approximately 200 MeV. The electromagnetic ($O(\alpha)$) and strong ($O(m_u - m_d)$) contributions to isospin violation are quite distinct. SVZ conclude that the first electromagnetic (EM) term is small, despite its leading position in the $1/Q^2$ power series. They also discuss a cancellation between the ρ and ω contributions to this correlator, which we shall discuss further below.

This first SR analysis of $\rho-\omega$ mixing concentrated on its prediction for the quark mass difference (comparing it to previous extractions of this difference). In their analysis they thus concentrated on the region around $q^2 = m_\rho^2$ which was assumed to be saturated by the nearby ρ and ω poles. Hatsuda *et al.* [51] used the correlator to investigate the q^2-dependence of the mixing, following GHT [36]. To do this they equated the correlator of Eq. (3.4) with the vector meson propagator $D_{\mu\nu}^{\rho\omega} = i\langle 0|T(\rho_\mu\omega_\nu)|0\rangle$ to arrive at

$$
\frac{\Pi_{\rho\omega}}{(q^2 - m_\rho^2)(q^2 - m_\omega^2)} \equiv C^{\rho\omega}(q^2). \tag{3.8}
$$

This calculation ignored the vector meson widths, which for the ρ is rather significant. From their analysis they concluded that there is a significant momentum dependence to $\Pi_{\rho\omega}$, in support of GHT. They also found a node in the mixing amplitude near $q^2 = 0$.

Maltman [52] argued that this analysis was slightly flawed. Firstly, the identification in Eq. (3.8) of the current correlator with the meson propagator is only valid if the hadronic currents themselves are used as the interpolating fields for the vector mesons. This is due to an argument that is becoming increasingly familiar in this field, i.e. quantities such as propagators, when off-shell, are dependent on the choice of interpolating field (unlike the current correlator) and that only the *total* S-matrix elements are physical quantities (and hence are not dependent on interpolating field choice). As such, this analysis can say little that is interpolation field field independent about the off-shell behaviour of $\Pi_{\rho\omega}$. Secondly, they assume that all isospin violation in this vector meson system is due to $\rho-\omega$ mixing. Maltman allows for the possibility that the isospin pure interpolating fields ρ_I and ω_I can themselves couple to the J^ω and J^ρ hadronic currents respectively. This would allow for the intrinsic decay of the ω_I to two pions (which would be ignored in the Hatsuda *et al.* analysis where $\langle 0|J^\rho|\omega_I\rangle = 0$) to contribute to $e^+e^- \to \pi^+\pi^-$. The resulting calculation indicates that the intrinsic decay of the ω_I is non-negligible in the

$\rho-\omega$ interference region because the correlator result underpredicts the isospin violation seen in experiment. Thirdly, the possible contribution of the ϕ and higher resonance poles is ignored. Naively, this is not expected to be as strong as the ρ and ω contributions, but as noted by SVZ [49], the stronger ρ and ω terms partially cancel, so the higher resonances cannot be ignored. The ϕ contribution turns out to be important in Maltman's analysis.

The SR technique was again used to study $\rho-\omega$ mixing by Iqbal *et al.* [53,54]. Their first paper [53] concentrated on extending the Hatsuda *et al.* analysis by including the meson widths, which had been neglected in the propagator of Eq. (3.8). This has a significant effect on the q^2 behaviour of $\Pi_{\rho\omega}(q^2)$ in their analysis. A more elaborate calculation followed [54]. It is shown here that the difference between the ρ and ω widths is of importance, as they obtain the "no width" results of Hatsuda *et al.* in the artificial case where the widths are set equal.

3.3 Chiral Perturbation Theory

Chiral Perturbation Theory (ChPT), which seeks to reproduce the symmetries of QCD in a model independent, low–energy, effective meson theory should provide a useful tool in examining isospin violating systems. Unfortunately, it is unreliable at energies around the lowest vector meson masses. Despite this, its low energy predictions for isospin violation should be of interest.

ChPT will be discussed in greater detail in Chapter 6, but I shall present a brief outline of it here to facilitate the discussion of its application to $\rho-\omega$ mixing. It provides an effective theory for the pseudoscalar octet organised as a perturbative series in q^2 and chiral symmetry violation (which allows for a convenient use of Feynman diagrams, with the familiar loop structure). As such it relies upon small q^2, thus making it unsuitable for the vector meson pole region. This is further complicated by higher order terms sometimes giving very large contributions and thus truncation at a given order should be taken with great care.

The construction of the ChPT Lagrangian, however, allows for an easy extraction of the hadronic currents. Maltman therefore used this [52] to compare his QCD SR analysis of the current correlator (Eq. (3.3)) to the ChPT prediction. Initially, he went to order q^4 (the one-loop result). This gave a markedly different result near $q^2 = 0$, from which he concluded that the series must be very slowly convergent, and so the next order would need to be considered. The two-loop result [55] provides large corrections to both the magnitude and q^2 dependence of the former calculation but it is likely that even the two-loop expression for the correlator is not well converged — an example of the limitations of ChPT.

Urech also used ChPT to approach $\rho-\omega$ mixing [56] though in a very different way to Maltman. As mentioned, vector mesons are generally too heavy to be accommodated in ChPT. The standard line of reasoning is that their effects are felt through the low energy constants of the $\mathcal{O}(q^4)$ Lagrangian (see Chapter 5). This idea was examined by constructing a chiral model that included the vector mesons, which could then be compared with ChPT [57]. It was found that the vector meson contributions did indeed seem to saturate the low energy constants. Following the techniques of this paper, Urech derived an expression for the off-diagonal element of the matrix propagator. As in the QCD SR result (Eq. (3.7), the result consists of a strong piece proportional to the quark mass difference and an EM piece from one-photon exchange. Urech makes no comment on any momentum dependence though.

3.4 Discussion

We have seen that it is very difficult to draw unambiguous conclusions from the various calculations for ρ–ω mixing. This difficulty stems from the fact that there is no definitive way to treat vector mesons. What is common to all of the calculations done for $\rho-\omega$ mixing, though, is a strong momentum dependence of the mixing amplitude with a node at or near the point $q^2 = 0$. In the next chapter this behaviour is shown to be unavoidable for a large class of models on general grounds.

Chapter 4

Effective models and momentum dependence

The various calculations for $\rho-\omega$ mixing of the previous chapter all predicted a considerable momentum dependence. Furthermore, they all gave rise to a mixing which decreased as q^2 decreased until it vanished at or near $q^2 = 0$. This prompted a study by our group at Adelaide into the general behaviour one could expect from the process [58]. Our aim was to make as general a statement as possible about $\rho-\omega$ mixing. The conclusion (the analysis is presented in the next section) was that for a wide class of models the mixing should indeed vanish at $q^2 = 0$ (the node theorem). This observation in turn prompted questions about vector meson dominance (VMD), where one would expect the $\rho-\gamma$ coupling to also obey this theorem for the same class of models. This then led to a revisiting of the formulation of VMD, since this naively appeared to be in contradiction to the standard formulation of VMD where the coupling is constant.

4.1 The node theorem

We begin by considering an effective Lagrangian model (e.g., $\mathcal{L}(\vec{\rho}, \omega, \vec{\pi}, \bar{\psi}, \psi, \cdots)$), where there are no explicit mass mixing terms in the bare Lagrangian. Examples of mass mixing terms are $m^2_{\rho\omega}\rho^0_\mu\omega^\mu$ or $\sigma\rho^0_\mu\omega^\mu$ with σ some scalar field; the second is of the type described by Coleman and Glashow in their tadpole mixing scheme (see Section 2.1). We also assume the vector mesons have a local coupling to conserved currents which satisfy the usual vector current commutation relations and that the effective Lagrangian is renormalised in a way which preserves these features. As the ρ and ω couple to the EM current, which is necessarily conserved, this does not seem like an unrealistic assumption. The boson-exchange model of Ref. [43] where, e.g., $J^\mu_\omega = g_\omega\bar{N}\gamma^\mu N$, is one simple example of such a model.

Consider the dressing of the bare vector propagator, $D^0_{\mu\nu}$, which is given by

$$iD_{\mu\nu} = iD^0_{\mu\nu} + iD^0_{\mu\alpha} iC^{\alpha\beta} iD^0_{\beta\nu}, \tag{4.1}$$

where $D_{\mu\nu}$ is the dressed propagator. It follows that the mixing tensor (analogous to the full self-energy function for a single vector boson such as the ρ [59])

$$C^{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x} \langle 0|\, T(J^\mu_\rho(x) J^\nu_\omega(0))\, |0\rangle\,. \tag{4.2}$$

is transverse (we shall prove this below). We recognise Eq. (4.2) as being the current correlator of Eq. (3.3) and note that this was taken to have a transverse form in the sum rules treatment with little comment. Here, the operator J^μ_ω is the operator appearing in the equation of motion for the field operator ω given in Eq. (1.14). Note that when J^μ_ω is a conserved current (i.e. $\partial_\mu J^\mu_\omega = 0$), the Proca condition $\partial_\mu \omega^\mu = 0$ follows from taking the divergence of Eq. (1.14) (see, e.g., Lurie, pp. 186–190 [16], or other field theory texts [19,60]). The operator J^μ_ρ is similarly defined.

Having defined $C^{\mu\nu}$ through Eqs. (4.1) and (4.2) we wish to study the one-particle-irreducible self-energy or polarisation, $\Pi^{\mu\nu}(q)$ (defined through Eq. (4.3) below),

$$iD_{\mu\nu} = iD^0_{\mu\nu} + iD^0_{\mu\alpha} i\Pi^{\alpha\beta} iD_{\beta\nu}. \tag{4.3}$$

The starting point for our argument is that $C^{\mu\nu}(q)$ is transverse, so let us briefly recall the proof of this. As shown, for example, by Itzykson and Zuber (pp. 217–224) [18], provided we use covariant time-ordering the divergence of $C^{\mu\nu}$ leads to a naive commutator of the appropriate currents

$$\begin{aligned} q_\mu C^{\mu\nu}(q) &= -\int d^4x\, e^{iq\cdot x} \partial_\mu \left\{ \theta(x^0) \langle 0|\, J^\mu_\rho(x) J^\nu_\omega(0)\, |0\rangle \right. \\ &\quad \left. + \theta(-x^0) \langle 0|\, J^\nu_\omega(0) J^\mu_\rho(x)\, |0\rangle \right\} \qquad (4.4) \\ &= -\int d^3x\, e^{i\vec{q}\cdot\vec{x}} \langle 0|\, [J^0_\rho(0,\vec{x}), J^\nu_\omega(0)]\, |0\rangle_{\text{naive}}\,. \qquad (4.5) \end{aligned}$$

Covariant time-ordering follows from the use of a suitable renormalisation scheme, which preserves current conservation. That is, there is a cancellation between the seagull and Schwinger terms. Thus, for any model in which the isovector- and isoscalar- vector currents satisfy the same commutation relations as QCD we find

$$q_\mu C^{\mu\nu}(q) = 0. \tag{4.6}$$

Thus, by Lorentz invariance, as there is only one available four-vector, the tensor must be of the form given in Eq. (4.7)

$$C^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) C(q^2)\,. \tag{4.7}$$

Having established that $C^{\mu\nu}(q)$ is transverse, we now turn our attention to $\Pi_{\mu\nu}$. For simplicity consider first the case of a single vector meson (e.g. a ρ or ω) without coupling to other channels. For such a system one can readily see (by comparing Eqs. (4.1) and (4.3) that $C^{\mu\nu}$ and the one-particle irreducible self-energy, $\Pi^{\mu\nu}$, are related via

$$\Pi^{\mu\alpha} D_{\alpha\nu} = C^{\mu\alpha} D^0_{\alpha\nu} \tag{4.8}$$

(where D and D^0 are defined below). As the bare and dressed vector meson propagators (D^0 amd D) are not, in general, transverse, then $C^{\mu\nu}$ being transverse implies $\Pi^{\mu\nu}$ is also transverse. Hence

$$\Pi^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \Pi(q^2)\,. \tag{4.9}$$

We are now in a position to establish the behaviour of the scalar function, $\Pi(q^2)$. In a general theory of massive vector bosons coupled to a conserved current, the bare propagator has the form (compared to Eq. (1.23) for the photon)

$$D^0_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2}\right) \frac{1}{q^2 - m^2} \tag{4.10}$$

whence

$$(D^0)^{-1}_{\mu\nu} = (m^2 - q^2) g_{\mu\nu} + q_\mu q_\nu. \tag{4.11}$$

Multiplying Eq. (4.3) by D^{-1} on the right and $(D^0)^{-1}$ on the left we have

$$\begin{aligned} D^{-1}_{\mu\nu} &= (D^0)^{-1}_{\mu\nu} + \Pi_{\mu\nu} \\ &= (m^2 - q^2 + \Pi(q^2)) g_{\mu\nu} + \left(1 - \frac{\Pi(q^2)}{q^2}\right) q_\mu q_\nu. \end{aligned} \tag{4.12}$$

Thus the full propagator has the form

$$D_{\mu\nu}(q) = \frac{-g_{\mu\nu} + (1 - \Pi(q^2)/q^2)\,(q_\mu q_\nu/m^2)}{q^2 - m^2 - \Pi(q^2)}. \tag{4.13}$$

Having established this form for the propagator, we wish to compare it with the spectral representation [16] of the vector field (V_μ) propagator which we obtain by first re-writing it as

$$\begin{aligned} D_{\mu\nu}(q^2) &= \int d^4x e^{-iq\cdot x} \langle 0|T(V_\mu(x) V_\nu(0))|0\rangle \\ &= \int_{s_0} dr \frac{\sigma(r)}{q^2 - r} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{r}\right), \end{aligned} \tag{4.14}$$

where $\sigma(r)$ is the spectral density of the vector states. Defining

$$D(q^2) \equiv \int_{s_0} dr \frac{\sigma(r)}{q^2 - r} \tag{4.15}$$

we can rewrite Eq. (4.14) in Renard form [30]

$$D_{\mu\nu}(q^2) = D(q^2)g_{\mu\nu} + \frac{1}{q^2}(D(0) - D(q^2))q_\mu q_\nu. \tag{4.16}$$

By comparing the coefficients of $g_{\mu\nu}$ in Eqs. (4.13) and (4.16) we deduce

$$D(q^2) = \frac{-1}{q^2 - m^2 - \Pi(q^2)}, \tag{4.17}$$

while from the coefficients of $q_\mu q_\nu$ we have

$$\begin{aligned} \frac{1}{m^2}\frac{(1 - \Pi(q^2)/q^2)}{(q^2 - m^2 - \Pi(q^2))} &= \frac{1}{q^2}(D(0) - D(q^2)) \\ &= \frac{1}{q^2}\frac{q^2 + \Pi(0) - \Pi(q^2)}{(m^2 + \Pi(0))(q^2 - m^2 - \Pi(q^2))}. \end{aligned} \tag{4.18}$$

Cancelling $(q^2 - m^2 - \Pi(q^2))$ from each side of Eq. (4.18) we obtain after a few lines of algebra

$$\frac{\Pi(0)}{q^2}(q^2 - m^2 - \Pi(q^2)) = 0, \quad \forall q^2. \tag{4.19}$$

Since Eq. (4.19) is true for *arbitrary* q^2 it then immediately follows that

$$\Pi(0) = 0. \tag{4.20}$$

This is an important constraint on the self-energy function, namely that $\Pi(q^2)$ should vanish as $q^2 \to 0$ at least as fast as q^2. Note that for the ρ self energy there will in general be counterterms for the mass (which are currently included in m). The physical mass is given by the pole location, i.e. $m^2_{\text{phys}} = m^2 + \Pi(m^2_{\text{phys}})$.

While the preceding discussion dealt with the single channel case, for $\rho - \omega$ mixing we are concerned with two coupled channels. Our calculations therefore involve matrices. As we now demonstrate, this does not change our conclusion.

The matrix analogue of Eq. (4.12) is

$$D^{-1}_{\mu\nu} = \begin{pmatrix} m^2_\rho g_{\mu\nu} + (\Pi_{\rho\rho}(q^2) - q^2)T_{\mu\nu} & \Pi_{\rho\omega}(q^2)T_{\mu\nu} \\ \Pi_{\rho\omega}(q^2)T_{\mu\nu} & m^2_\omega g_{\mu\nu} + (\Pi_{\omega\omega}(q^2) - q^2)T_{\mu\nu} \end{pmatrix}, \tag{4.21}$$

where we have defined $T_{\mu\nu} \equiv g_{\mu\nu} - (q_\mu q_\nu/q^2)$ for brevity. By obtaining the inverse of this we have the two-channel propagator

$$D_{\mu\nu} = \frac{1}{\alpha}\begin{pmatrix} s_\omega g_{\mu\nu} + a(\rho,\omega)q_\mu q_\nu & \Pi_{\rho\omega}(q^2)T_{\mu\nu} \\ \Pi_{\rho\omega}(q^2)T_{\mu\nu} & s_\rho g_{\mu\nu} + a(\omega,\rho)q_\mu q_\nu \end{pmatrix}, \tag{4.22}$$

where

$$s_\omega \equiv q^2 - \Pi_{\omega\omega}(q^2) - m_\omega^2 \quad (4.23)$$

$$s_\rho \equiv q^2 - \Pi_{\rho\rho}(q^2) - m_\rho^2 \quad (4.24)$$

$$a(\rho,\omega) \equiv \frac{1}{q^2 m_\rho^2}\{\Pi_{\rho\omega}^2(q^2) - [q^2 - \Pi_{\rho\rho}(q^2)]s_\omega\} \quad (4.25)$$

$$\alpha \equiv \Pi_{\rho\omega}^2(q^2) - s_\rho s_\omega. \quad (4.26)$$

In the uncoupled case $[\Pi_{\rho\omega}(q^2) = 0]$ Eq. (4.22) clearly reverts to the appropriate form of the one particle propagator, Eq. (4.13), as desired.

We can now make the comparison between Eq. (4.22) and the Renard form [30] of the propagator, as given by Eq. (4.16). The transversality of the off-diagonal terms of the propagator, requires that $\Pi_{\rho\omega}(0) = 0$. Note that the physical ρ^0 and ω masses, which arise from locating the poles in the diagonalised propagator matrix $D^{\mu\nu}$, no longer correspond to exact isospin eigenstates (as predicted by Glashow [22]).

In conclusion, it is important to review what has and has not been established. There is as yet no unique way to derive an effective field theory including vector mesons from QCD. Our result, that $\Pi_{\rho\omega}(0)$ should vanish, applies to those effective theories in which: (i) the vector mesons have local couplings to conserved currents which satisfy the same commutation relations as QCD [i.e., Eq. (4.5) is zero] and (ii) there is no explicit mass-mixing term in the bare Lagrangian. This includes a broad range of commonly used, phenomenological theories. It does not include the model treatment of Ref. [46] for example, where the mesons are bi-local objects in a truncated effective action. However, it is interesting to note that a node near $q^2 = 0$ was found in this model in any case. The presence of an explicit mass-mixing term in the bare Lagrangian will shift the mixing amplitude by a constant (i.e., by $m_{\rho\omega}^2$ for a Lagrangian term like $\frac{1}{2}m_{\rho\omega}^2\rho_\mu\omega^\mu$).

4.2 The consequences of momentum dependence for VMD

Following this, and the preceding examinations of ρ–ω mixing, it would appear that a rather convincing case has been made for its momentum dependence in general and we have shown that it is unavoidable in a large class of theories. This might, it was argued, seriously damage the standard picture of nuclear CSV described in Section 2.5. Leaving aside, for one moment, any questions that one might have about the use of vector mesons as mediators of the strong interaction in nuclear physics (CSV or no), this prompted a considerable amount of activity. Alternative mechanisms for CSV have been proposed [37–39]. Indeed, as the vector mesons are off-shell, the individual mechanisms should not be examined in isolation, because they are dependent on the choice of interpolating fields for the vector mesons and are not physical quantities. It has been argued that one could find a set of interpolating fields for the rho and omega such that *all* nuclear CSV occurs through a constant ρ–ω mixing with the CSV vertex contributions vanishing [61]. However this possibility has been questioned on the grounds of unitarity and analyticity [62]. This entire argument is symptomatic of the lack of a methodic field theory treatment of the strong interaction in this energy range for the NN interaction in particular, despite the somewhat hopeful descriptions of it being "well understood." It is clear that much work is still required in this area.

The appeal of the Standard Picture (see Section 2.5), though, provided a considerable incentive to demonstrate that there must be something "wrong" with momentum dependent $\rho-\omega$ mixing. So, what has actually been said about $\rho-\omega$ mixing? The mixing of vector particles (which couple to conserved currents and without explicit mass mixing terms) should vanish at $q^2 = 0$. But then this should also apply to the mixing between the photon and the ρ for such models. The same models which have been used to examine the question of $\rho-\omega$ mixing should then also be able to be applied to studies of $\rho-\gamma$ mixing. They can then be compared with the successful phenomenology of vector meson dominance (VMD, see Chapter 1). However, VMD has *traditionally* assumed the coupling of the photon to the ρ was independent of q^2. Thus, a momentum dependence for the $\rho-\gamma$ coupling (a direct consequence of the node theorem of the previous section) would naively appear to ruin photon-hadron phenomenology, and therefore there must be something wrong with it. The first person to raise this question was Miller [63] and it has been discussed subsequently [25, 64].

What is the resolution of this apparent contradiction? Consider the well known constraint on the pion form-factor, $F_\pi(0) = 1$. In the infinite wavelength limit, the photon sees only the charge of the pion. Essentially, at $q^2 = 0$ the photon interacts with a point-like pion, there is no need to involve coupling through the ρ meson. In simple physical terms *the photon decouples from the vector meson exactly as the node theorem would predict.* In the traditional VMD picture the parameters must be carefully constrained so as to maintain the condition $F_\pi(0) = 1$. Surely then, one could build a VMD based model of the photon-hadron interaction by adding the non-resonant photon-pion contact piece to a q^2 dependent vector meson contribution.

The more deeply one considers the traditional VMD treatment with its constant coupling of the photon to the vector meson, the more physically troublesome it seems. For instance, in dressing the photon propagator, such a contribution would shift the pole away from $q^2 = 0$, giving the photon a mass, which must then be returned to zero by the choice of an appropriate counterterm in the Lagrangian [65]. It is now appropriate to return to

Sakurai's original work on VMD [11,17].

4.3 Sakurai and the two representations of VMD

Sakurai was troubled by this photon mass problem. His concern was to ensure that adding electromagnetism to a strong interaction model based on vector mesons gave a (flavour) gauge invariant theory. The naive $\gamma-\rho$ vertex prescription usually seen in discussions of VMD, i.e.,

$$-\frac{em_\rho^2}{g_\rho},$$

as motivated by Eq. (1.17), suggests a coupling term in the effective Lagrangian of the form

$$\mathcal{L}_{\text{eff}} = -\frac{em_\rho^2}{g_\rho}\rho_\mu^3 A^\mu. \tag{4.27}$$

This is suggested by the substitution of the field current identity (Eq. (1.2)) into the interaction piece of the electromagnetic Lagrangian, $-ej_\mu^{\text{EM}}A^\mu$. However electromagnetism cannot be incorporated into the rho-pion Lagrangian, Eq. (1.4), simply by adding Eq. (4.27) and a kinetic term for the photon. This would result in the photon acquiring an *imaginary* mass [11] when its propagator is dressed in the manner of Fig. 4.1 using $\rho-\gamma$ vertices determined by Eq. (4.27). In the traditional VMD treatment a mass counterterm must then be introduced for the photon to ensure that it remains massless in the renormalised theory.

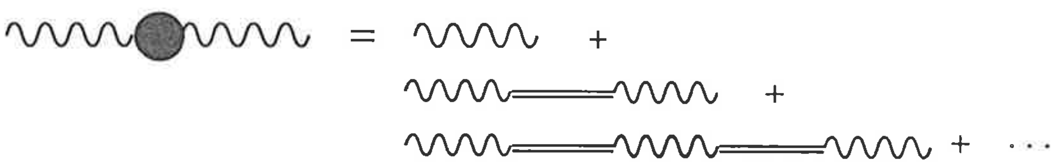

Figure 4.1: VMD dressing of the photon propagator by a series of intermediate ρ propagators.

We can find a term, though, that emulates Eq. (4.27) while ensuring that the photon remains massless in a more natural way. Such a term is

$$\mathcal{L}_{\gamma\rho} = -\frac{e}{2g_\rho}F_{\mu\nu}\rho^{\mu\nu}. \tag{4.28}$$

It is helpful to re-express this in momentum space which can be done using integration by parts to transform $\partial_\mu A_\nu \partial^\mu \rho^\nu$ to $-\partial_\mu \partial^\mu A_\nu \rho^\nu$ and then using $\partial_\mu \to iq_\mu$ giving

$$F_{\mu\nu}\rho^{\mu\nu} \to 2q^2 A_\mu \rho^\mu. \tag{4.29}$$

The other term in $F_{\mu\nu}\rho^{\mu\nu}$ can be discarded because it contains a piece that can be written as $q_\mu \rho^\mu$ and thus vanishes as the ρ field is divergenceless when coupling to conserved currents. However, the interaction Lagrangian of Eq. (4.28) is not sufficient as it would decouple the photon from the ρ (and hence then from hadronic matter) at $q^2 = 0$. What is needed is another term which directly couples the photon to hadronic matter. This is the usual QED type of photon-matter interaction

$$- eA_\mu J^\mu, \tag{4.30}$$

where J_μ is the hadronic current to which the ρ couples. The pion component (for example) of this current is given in Eq. (1.18). Thus the interaction between the photon and hadronic matter is of exactly the same form as that between the ρ and hadronic matter (though suppressed by a factor of $e/g_{\rho\pi\pi}$). This term is all that is present at $q^2 = 0$ where the influence of the ρ-meson in the photon-pion interaction vanishes.

To summarise, the photon and vector meson part of the Lagrangian discussed immediately above is

$$\mathcal{L}_{\rm VMD1} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2 \rho_\mu \rho^\mu - g_{\rho\pi\pi}\rho_\mu J^\mu - eA_\mu J^\mu - \frac{e}{2g_\rho}F_{\mu\nu}\rho^{\mu\nu}. \tag{4.31}$$

We shall refer to this as the *first representation* of VMD, and denote this as VMD1. We note that this representation has a direct photon—matter coupling as well as a photon—ρ coupling which vanishes at $q^2 = 0$.

The alternative formulation of VMD, has survived to become the standard representation. Its Lagrangian is given by

$$\mathcal{L}_{\rm VMD2} = -\frac{1}{4}(F'_{\mu\nu})^2 - \frac{1}{4}(\rho'_{\mu\nu})^2 + \frac{1}{2}m_\rho^2(\rho'_\mu)^2 - g_{\rho\pi\pi}\rho'_\mu J^\mu - \frac{e' m_\rho^2}{g_\rho}\rho'_\mu A'^\mu + \frac{1}{2}\left(\frac{e'}{g_\rho}\right)^2 m_\rho^2 (A'_\mu)^2. \tag{4.32}$$

Note the last term which is a photon mass counterterm to restore the masslessness of the photon. In the limit of exact universality ($g_\rho = g_{\rho\pi\pi}$) the two representations become equivalent and one can transform between them using

$$\rho'_\mu = \rho_\mu + \frac{e}{g_\rho}A_\mu, \tag{4.33}$$

$$A'_\mu = A_\mu\sqrt{1 - \left(\frac{e}{g_\rho}\right)^2}, \tag{4.34}$$

$$e' = e\sqrt{1 - \left(\frac{e}{g_\rho}\right)^2}. \tag{4.35}$$

Substituting for ρ'_μ, A'_μ and e' in Eq. (4.32) gives Eq. (4.31) $+O((e/g_\rho)^3)$. We shall refer to Eq. (4.32) as the *second representation* of VMD, which we will denote as VMD2. The appearance of a photon mass term at first seems slightly troublesome. However, when dressing the photon in the manner of Fig. 4.1, we see that the propagator has the correct form as $q^2 \to 0$. The dressed propagator is given by

$$iD(q^2) = \frac{-i}{q^2 - \frac{e^2 m_\rho^2}{g_\rho^2}} + \frac{-i}{q^2 - \frac{e^2 m_\rho^2}{g_\rho^2}} \frac{-iem_\rho^2}{g_\rho} \frac{-i}{q^2 - m_\rho^2} \frac{-iem_\rho^2}{g_\rho} \frac{-i}{q^2 - \frac{e^2 m_\rho^2}{g_\rho^2}} + \cdots \tag{4.36}$$

Summing this using the general operator identity

$$\frac{1}{A - B} = \frac{1}{A} + \frac{1}{A} B \frac{1}{A} + \frac{1}{A} B \frac{1}{A} B \frac{1}{A} + \cdots \tag{4.37}$$

we obtain ($m \equiv m_\rho$)

$$\begin{aligned} iD(q^2) &= -i \left[q^2 - \frac{e^2 m^2}{g_\rho^2} - \frac{e^2 m^4}{g_\rho^2 (q^2 - m^2)} \right]^{-1} \\ &= -i \left[q^2 - \frac{e^2 m^2}{g_\rho^2} + \frac{e^2 m^2}{g_\rho^2 (1 - q^2/m^2)} \right]^{-1} && (4.38) \\ &\to \frac{-i}{q^2 (1 + e^2/g_\rho^2)} && (4.39) \end{aligned}$$

as $q^2 \to 0$. This therefore results in a redefinition of the coupling constant

$$e^2 \to e^2 (1 - e^2/g_\rho^2), \tag{4.40}$$

and interestingly the photon propagator is significantly modified away from $q^2 = 0$. Both forms of VMD were discussed by Sakurai [17].

The use of the two models can be compared by describing the process $\gamma \to \pi^+ \pi^-$. The relevant terms in the Lagrangian can be identified for each case. From $\mathcal{L}_{\mathrm{VMD1}}$ (Eq. (4.31)) and $\mathcal{L}_{\mathrm{VMD2}}$ (Eq. (4.32)) we have, respectively,

$$\begin{aligned} \mathcal{L}_1 &= -\frac{e}{2g_\rho} F_{\mu\nu} \rho^{\mu\nu} - eJ_\mu A^\mu - g_{\rho\pi\pi} \rho^\mu J_\mu, && (4.41) \\ \mathcal{L}_2 &= -\frac{em_\rho^2}{g_\rho} \rho_\mu A^\mu - g_{\rho\pi\pi} \rho_\mu J^\mu. && (4.42) \end{aligned}$$

If the photon coupled to the pions directly, then to lowest order the Feynman amplitude for this process would be (as in scalar electrodynamics [18])

$$\mathcal{M}^\mu_{\gamma \to \pi^+ \pi^-} = \langle \pi^+ \pi^- | eJ^\mu | 0 \rangle = -e(p^+ - p^-)^\mu, \tag{4.43}$$

where J_μ is given in Eq. (1.19). However, in the presence of the vector meson interactions of Eqs. (4.41) and (4.42), the total amplitude is modified. The pion form factor, $F_\pi(q^2)$, which represents the contribution from the intermediate steps connecting the photon to the pions, is defined by the relation

$$\mathcal{M}^{\mu}_{\gamma\to\pi^+\pi^-} = -e(p^+ - p^-)^{\mu} F_{\pi}(q^2), \tag{4.44}$$

where now $\mathcal{M}^{\mu}_{\gamma\to\pi^+\pi^-}$ is the full amplitude including all possible processes. The form-factor is the multiplicative deviation from a pointlike behaviour of the coupling of the photon to the pion field. We discuss $F_{\pi}(q^2)$ in detail later.

To lowest order, we have for $\mathcal{L}_1$ (see Eq. (4.29))

$$F_{\pi}(q^2) = \left[1 - \frac{q^2}{q^2 - m_{\rho}^2}\frac{g_{\rho\pi\pi}}{g_{\rho}}\right], \tag{4.45}$$

and for $\mathcal{L}_2$

$$F_{\pi}(q^2) = -\frac{m_{\rho}^2}{q^2 - m_{\rho}^2}\frac{g_{\rho\pi\pi}}{g_{\rho}}. \tag{4.46}$$

In the limit of zero momentum transfer, the photon can resolve none of the structure and "sees" only the charge of the pions, and hence we must have

$$F_{\pi}(0) = 1. \tag{4.47}$$

The reader may notice that Eq. (4.47) is automatically satisfied by the dispersion relation of Frazer and Fulco, Eq. (1.1) and by VMD1 (Eq. (4.45)) but must be imposed on the VMD2 result (Eq. (4.46)) by appropriately choosing parameters, (i.e., at the simplest level we can set $g_{\rho\pi\pi} = g_{\rho}$). This is the basis of Sakurai's argument for universality mentioned earlier, i.e., that the photon couples to the ρ as in Eq. (4.42) and that therefore $g_{\rho\pi\pi}$ must equal g_{ρ}. This is a direct consequence of assuming *complete* ρ dominance of the form-factor (i.e., VMD2). The other implications of exact universality, namely that

$$g_{\rho\pi\pi} = g_{\rho NN} = \ldots = g_{\rho} \tag{4.48}$$

result from the assumption that the interactions are all generated from the gauge principle (i.e., by minimal substitution for the covariant derivative given in Eq. (1.6)).

As Sakurai pointed out, the two representations of VMD are equivalent in the limit of exact universality (as we would expect from Eqs. (4.33–4.35)). Without universality *only* VMD1 automatically maintains the condition $F_{\pi}(0) = 1$. In the VMD2 approach in the absence of exact universality to maintain this condition the fine tuning of parameters is required. Due to the popularity of the second interpretation, though, $F_{\pi}(0) = 1$ is simply viewed as a constraint on various introduced parameters [70]. We illustrate the difference between the two representations in Fig. 4.2.

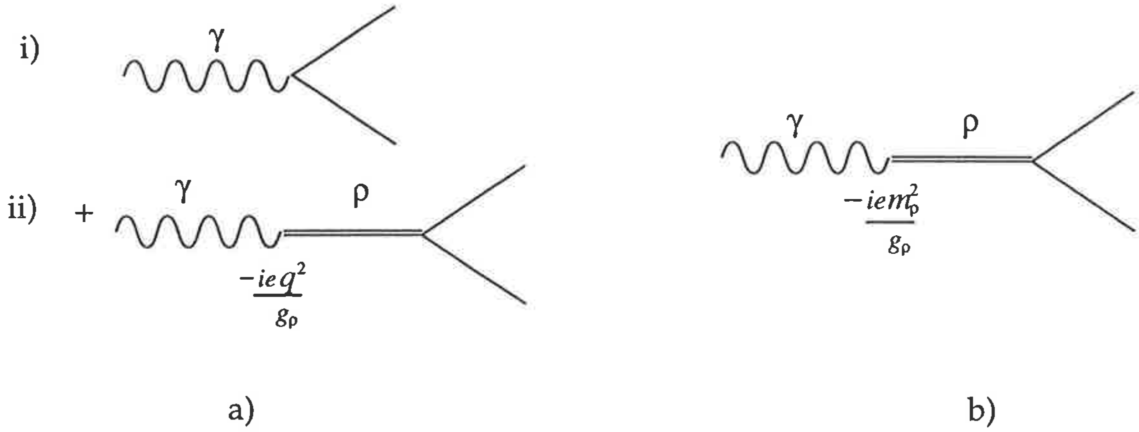

Figure 4.2: Contributions to the pion form-factor in the two representations of vector meson dominance a) VMD1 b) VMD2.

4.4 The use of VMD1

So, interestingly, the question raised by Miller [63] had already been answered thirty years ago. VMD can be reformulated so that the $\rho-\gamma$ coupling is momentum dependent and vanishes at $q^2 = 0$. However, this fact was not commonly known, so we gave an example of its use [66], which was then extended to the incorporation of a typical model used for $\rho-\omega$ mixing into a $\rho-\gamma$ mixing model [67]. The purpose of this first section is to show that one can fit the measured pion form-factor with a $\gamma - \rho$ coupling that vanishes at $q^2 = 0$, i.e. using VMD1 (see Ref. [66]).

In the process of fitting the data, we extracted a revised value of the $\rho-\omega$ mixing amplitude at the ω pole, $\Pi_{\rho\omega}(m_\omega^2)$. At the time the widely quoted value of for $\Pi_{\rho\omega}$ [35], had been obtained from the branching ratio formula for the ω,

$$B(\omega \rightarrow \pi\pi) = \Gamma(\omega \rightarrow \pi\pi)/\Gamma(\omega),$$

derived from a $\rho-\omega$ mixing analysis where

$$\Gamma(\omega \rightarrow \pi\pi) = |\Pi_{\rho\omega}/im_\rho\Gamma_\rho|^2\,\Gamma(\rho \rightarrow \pi\pi).$$

Using the branching ratio determined in 1985 by the Novosibirsk group [27], $B(\omega \rightarrow \pi\pi) = (2.3 \pm 0.4 \pm 0.2)\%$, Coon and Barrett obtained $\Pi_{\rho\omega} = -4520 \pm 600\text{MeV}^2$. A better, more direct method would be to extract $\Pi_{\rho\omega}$ from a fit to the cross-section of the reaction $e^+e^- \rightarrow \pi^+\pi^-$ using

$$\sigma(q^2) = \frac{\alpha^2\pi}{3}\frac{(q^2 - 4m_\pi^2)^{3/2}}{s^{5/2}}|F_\pi(q^2)|^2, \qquad (4.49)$$

and the form-factor determined by VMD1 (Eq. (4.45)).

So far, we have not introduced any effects of isospin violation into our VMD1 system, and hence the ω (which cannot otherwise couple to a $\pi^+\pi^-$ state) does not appear. To do this, we introduce the isospin violation in the standard way and combine the VMD1 form-factor of Eq. (4.45) and the mixed state contribution of Eq. (2.11),

$$F_\pi(q^2) = 1 - \frac{q^2 g_{\rho\pi\pi}}{g_\rho[q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)]} - \frac{q^2 \epsilon g_{\rho\pi\pi}}{g_\omega[q^2 - m_\omega^2 + im_\omega\Gamma_\omega]} \tag{4.50}$$

where,

$$\epsilon = \frac{\Pi_{\rho\omega}}{s_\rho - s_\omega} = \frac{\Pi_{\rho\omega}}{m_\omega^2 - m_\rho^2 - i(m_\omega\Gamma_\omega - m_\rho\Gamma_\rho(q^2))}. \tag{4.51}$$

The ω decay formula of Coon and Barrett can now be seen to follow from Eq. (4.50) with an approximation for ϵ (namely that Γ_ω is very small and that $m_\rho^2 = m_\omega^2$). Because the width of the ω is very small we can safely neglect any momentum dependence in it, and simply use $\Gamma_\omega(m_\omega^2)$ [68,69].

All parameters except $\Pi_{\rho\omega}$ are fixed by various data as discussed below. The results of fitting this remaining parameter to the data are shown in Fig. 4.3 with the resonance region shown in close-up in Fig. 4.4. The mass and width of the ω are as given by the Particle Data Group (PDG) [31], $m_\omega = 781.94 \pm 0.12$ MeV and $\Gamma_\omega = 8.43 \pm 0.10$ MeV. There has recently been considerable interest in the value of the ρ parameters, m_ρ and Γ_ρ with studies showing that the optimal values [69, 70] may differ slightly from those given by the PDG. The value of $\Pi_{\rho\omega}$ is not sensitive to the masses and widths, and we obtained a good fit with $m_\rho = 772$ MeV and Γ_ρ=149 MeV, which are close to the PDG values.

The values of the coupling constants are, however, quite important for an extraction of $\Pi_{\rho\omega}$. We obtained g_ρ and $g_{\rho\pi\pi}$ from $\Gamma(\rho \to e^+e^-) \sim 6.8$ MeV and $\Gamma(\rho \to \pi\pi) \sim 149$ MeV, namely $g_{\rho\pi\pi}^2/4\pi \sim 2.9$, $g_\rho^2/4\pi \sim 2.0$. This shows, for example, that universality is not strictly obeyed (as mentioned previously). VMD1 and VMD2 naively differ at order $g_{\rho\pi\pi}/g_\rho \simeq 1.2$ before any separate fine tuning of parameters is carried out.

Historically the ratio g_ω/g_ρ was believed to be around 3 [71], a figure supported in a recent QCD-based analysis [72]. Empirically though, the ratio can be determined [70] from leptonic partial decay rates [31] giving

$$\frac{g_\omega}{g_\rho} = \sqrt{\frac{m_\omega\Gamma(\rho \to e^+e^-)}{m_\rho\Gamma(\omega \to e^+e^-)}} = 3.5 \pm 0.18. \tag{4.52}$$

Using these parameters we obtained a best fit around the resonance region shown in Fig. 4 (χ^2/d.o.f. = 14.1/25) with $\Pi_{\rho\omega} = -3800$ MeV^2. In this analysis there are two principle sources of error in the value of $\Pi_{\rho\omega}$. The first is a statistical uncertainty of 310 MeV^2 for the fit to data, and the second, of approximately 200 MeV^2, is due to the error quoted in Eq. (4.52). Adding these in quadrature gives us a final value for the *total* mixing amplitude, to be compared with the value -4520 ± 600 MeV^2 obtained by Coon and Barrett [35]. We find

$$\Pi_{\rho\omega} = -3800 \pm 370 \text{ MeV}^2. \tag{4.53}$$

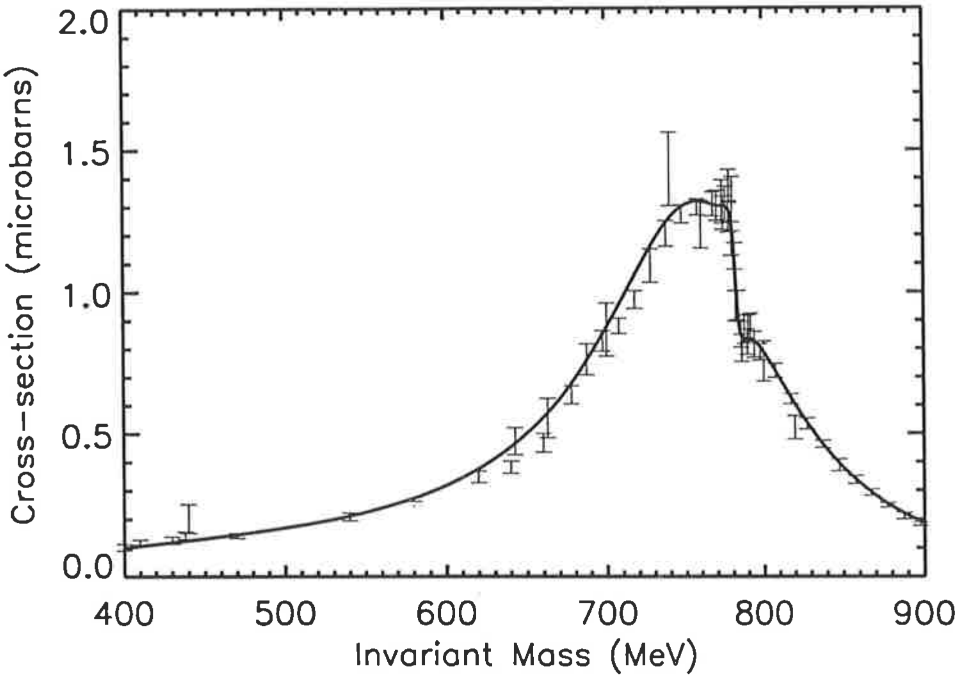

Figure 4.3: Cross-section of $e^+e^- \to \pi^+\pi^-$ plotted as a function of the energy in the centre of mass. The experimental data is from Refs. [17,21].

It is now clear that a momentum dependent $\rho-\gamma$ coupling, together with a direct coupling of the photon to hadronic matter, yields an entirely adequate model of the pion form-factor. In fact, this picture is basically suggested by attempts to examine the $\rho-\gamma$ coupling via a quark loop. Model calculations typically find that the loop is momentum-dependent, and vanishes at $q^2 = 0$ (unless gauge invariance is spoiled by form-factors, or something of this nature). However, coupling the photon to quarks in the loop implies that the photon must also couple to the quarks in hadronic matter. Thus, in general we might expect a direct photon-hadron coupling (independent of the ρ-meson), and this leads us to consider VMD1 as the more natural representation of vector meson dominance. It should now be clear that the appropriate representation of vector meson dominance to be used in combination with mixing amplitudes that vanish at $q^2 = 0$ is VMD1. To use VMD2 in conjunction with such vector mixing amplitudes is clearly inconsistent. As long as one is clear on this point, there are no dire consequences for momentum dependence in $\rho-\omega$ mixing.

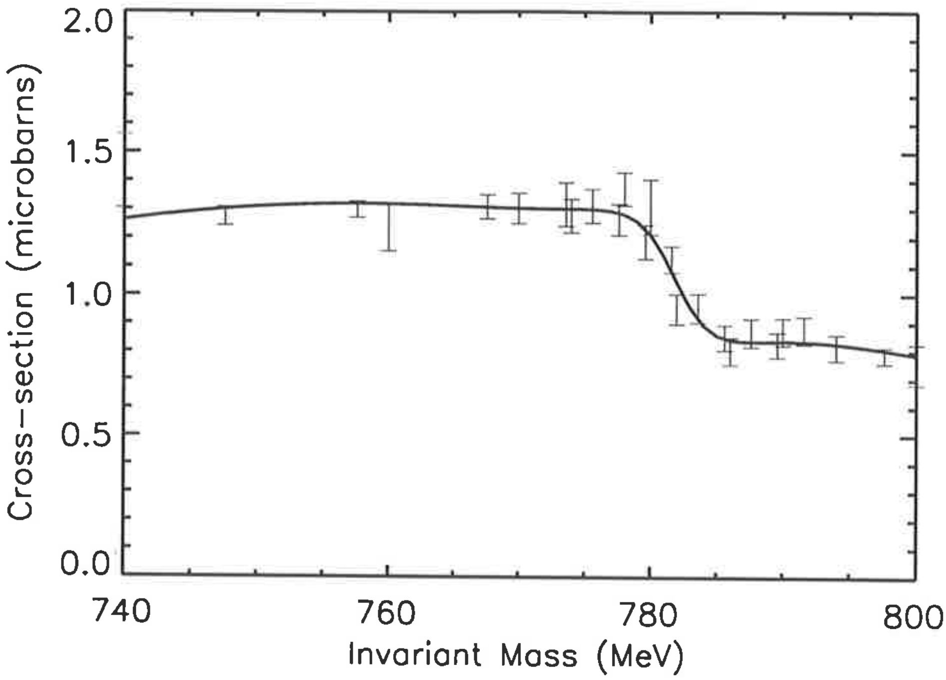

Figure 4.4: Cross-section for $e^+e^- \to \pi^+\pi^-$ in the region around the resonance where ρ—ω mixing is most noticeable. The experimental data is from Refs. [17,21].

4.5 A VMD1-like model

One can now expand upon the previous section, by examining models for the photon-hadron interaction. We shall define a "VMD1-like" model to be one in which the photon couples to the hadronic field both directly and via a q^2-dependent coupling (with a node at $q^2 = 0$) to vector mesons. A VMD1-like model may differ from pure VMD1 as the coupling of the photon to the ρ (generated by some microscopic process) will not generally be linear in q^2 for q^2 sufficiently far from zero. Hence g_ρ, which is a constant in VMD1 (and VMD2 as they share the same g_ρ up to universality [11, 68]), may acquire some momentum dependence in a VMD1-like model; the test for the phenomenological validity of the model is then that this momentum dependence for g_ρ is not too strong. For example, we can easily determine the coupling of the photon to the pion field via the ρ meson for a VMD1-like model to establish the connection between the ρ—γ mixing amplitude, $\Pi_{\rho\gamma}(q^2)$, of the model with g_ρ of VMD1. We note the appearance in Eq. (4.54) of the $\rho-\gamma$ mixing term, $\Pi^{\mu\nu}_{\rho\gamma}(q^2)$, which can be determined from Feynman rules, and which will, in general, be q^2-dependent. Such an analysis gives for any VMD1-like model

$$-i\mathcal{M}^\mu(q^2) \equiv -ie(p^+ - p^-)_\sigma[D_\gamma(q^2)]^{\mu\sigma}F_\pi(q^2)$$

$$= i[D_\gamma(q^2)]^{\mu\sigma} i[\Gamma_{\gamma\pi}(q^2)]_\sigma + i[D_\gamma(q^2)]^{\mu\sigma} i[\Pi_{\gamma\rho}(q^2)]_{\sigma\tau} i[D_\rho(q^2)]^{\tau\nu} i[\Gamma_{\rho\pi}(q^2)]_\nu$$
$$= -ie(p^+ - p^-)_\sigma i[D_\gamma(q^2)]^{\mu\sigma} \left[1 + \frac{\Pi_{\rho\gamma}(q^2)}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} \frac{g_{\rho\pi\pi}}{e}\right], \quad (4.54)$$

where D, Π and Γ denote propagators, one-particle irreducible mixing amplitudes and proper vertices respectively. Here p^+ and p^- are the outgoing momenta of the π^+ and π^- respectively. For this model to reproduce the phenomenologically successful VMD, and hence provide a good fit to the data (assuming exact universality), $\Pi_{\rho\gamma}(q^2)$ and g_ρ must be related by (comparing Eqs. (4.45) and (4.54))

$$\Pi_{\rho\gamma}^{\rm VMD1}(q^2) = -\frac{q^2 e}{g_\rho(q^2)}. \quad (4.55)$$

Thus Eq. (4.55) is the central equation of this examination, since vector-meson mixing models (e.g., $\rho-\omega$ mixing) can also be used to calculate $\rho-\gamma$ mixing and then confronted with traditional VMD phenomenology.

The results quoted in the review by Bauer et al. [73] for ρ meson parameters are summarised in Tables I and XXXII of that reference. They list a range of values which vary depending on the details of the fit to the ρ mass (m_ρ) and width (Γ_ρ). Within the context of the traditional VMD (i.e., VMD2) framework they extract $g_\rho^2(q^2 = 0)/4\pi$ from ρ^0 photoproduction ($\gamma p \to \rho^0 p$) and $g_\rho^2(q^2 = m_\rho^2)/4\pi$ from $\rho^0 \to e^+e^-$. The three sets of results quoted are (in an obvious shorthand notation):

$$\Gamma_\rho = 135,\ 145,\ 155\,\text{MeV}, \quad (4.56)$$
$$m_\rho = 767,\ 774,\ 776\,\text{MeV}, \quad (4.57)$$
$$g_\rho^2(q^2 = 0)/4\pi = 2.43 \pm 0.10,\ 2.27 \pm 0.23,\ 2.18 \pm 0.22, \quad (4.58)$$
$$g_\rho^2(q^2 = m_\rho^2)/4\pi = 2.21 \pm 0.017,\ 2.20 \pm 0.06,\ 2.11 \pm 0.06 \quad (4.59)$$

respectively. We see that g_ρ is a free parameter of the traditional VMD model (VMD2) which is adjusted to fit the available cross section data. The central feature of the VMD2 model is that it presumes a constant value for its coupling constant g_ρ. We note in passing that the universality condition is

$$g_\rho \sim g_{\rho\pi\pi} \sim g_{\rho NN}^{\rm univ} \sim g_{\rho\rho\rho} \quad (4.60)$$

and where experimentally we find [73, 74] for each of these $g^2/4\pi \sim 2$. For example, the values of $g_{\rho\pi\pi}$ corresponding to the above three sets of results are

$$g_{\rho\pi\pi}^2(q^2 = m_\rho^2)/4\pi = 2.61, 2.77, 2.95 \quad (4.61)$$

and are extracted from $\rho^0 \to \pi^+\pi^-$. It should be noted that the ρNN interaction Lagrangian is here defined as in Refs. [43, 75] with no factor of two [11, 74] and hence

$g_{\rho NN} = g_{\rho NN}^{\rm univ}/2$. As a typically used value is $g_{\rho NN}^2/(4\pi) = 0.41$ we see that universality is not accurate to better than 40% in g_ρ^2, which corresponds to $\simeq 20\%$ in g_ρ.

The results of the VMD2 analysis [73] are approximately consistent with g_ρ being a constant and so Eq. (4.55) tells us $\Pi_{\rho\gamma}$ in VMD1-like models should be roughly linear in q^2.

We shall now present our calculation [67] of the process within the context of the model used by Piekarawicz and Williams (PW) discussed earlier [43]. They considered $\rho-\omega$ mixing as being generated by a nucleon loop within the Walecka model. The ρ coupling is not a simple, vector coupling, but rather [76]

$$\Gamma^\mu_{\rho NN} = g_{\rho NN}\gamma^\mu + i\frac{f_{\rho NN}}{2M}\sigma_{\mu\nu}q^\nu, \tag{4.62}$$

where $C_\rho \equiv f_{\rho NN}/g_{\rho NN} = 6.1$ and M is the nucleon mass. With the introduction of tensor coupling the model is no longer renormalisable, but to one loop order we can introduce some appropriate renormalisation prescription. As the mixings are transverse, we write $\Pi_{\mu\nu}(q^2) = (g_{\mu\nu} - q_\mu q_\nu/q^2)\Pi(q^2)$ (see Eq. (4.9). The photon couples to charge, like a vector and so, unlike the PW calculation, we have only a proton loop to consider. Here we can safely neglect the coupling of the photon to the nucleon magnetic moment and so there is no neutron loop contribution nor any tensor-tensor contribution to the proton loop. This sets up two kinds of mixing, vector-vector $\Pi^{\mu\nu}_{\rm vv}$ and vector-tensor $\Pi^{\mu\nu}_{\rm vt}$, where (using dimensional regularisation with the associated scale, μ)

$$\Pi_{\rm vv}(q^2) = -q^2\frac{eg_{\rho NN}}{2\pi^2}\left[\frac{1}{3\epsilon} - \frac{\gamma}{6} - \int_0^1 dx\; x(1-x)\ln\left(\frac{M^2 - x(1-x)q^2}{\mu^2}\right)\right] \tag{4.63}$$

$$\Pi_{\rm vt}(q^2) = -q^2\frac{eg_{\rho NN}}{8\pi^2}\left[\frac{1}{\epsilon} - \gamma - \int_0^1 dx\; \ln\left(\frac{M^2 - x(1-x)q^2}{\mu^2}\right)\right]. \tag{4.64}$$

Note that these functions vanish at $q^2 = 0$, as expected from the node theorem since we have coupling to conserved currents. To remove the divergence and scale-dependence we add a counter-term

$$\mathcal{L}_{CT} = e\frac{g_{\rho NN}C_T}{2\pi^2}\rho_{\mu\nu}F^{\mu\nu}$$

to the Lagrangian in a minimal way so as to renormalise the model to one loop. This will contribute $-iC_T g_{\rho NN}eq^2/\pi^2$ to the ρ–γ vertex, which will add to the contribution $i\Pi$ generated by the nucleon loop. The counter-term will contain pieces proportional to $1/\epsilon$, γ and $\ln\mu^2$ to cancel the similar terms in Eqs. (4.63) and (4.64), and a constant piece, β, which will be chosen to fit the extracted value for $g_\rho(0)$. The counter-term can be written as

$$C_T = -\frac{1}{\epsilon}\left(\frac{1}{6} + \frac{C_\rho}{8}\right) + \gamma\left(\frac{1}{12} + \frac{C_\rho}{8}\right) - \left(\frac{1}{12} + \frac{C_\rho}{8}\right)\ln\mu^2 + \beta, \tag{4.65}$$

which gives us the renormalised mixing,

$$\Pi_{\rho\gamma}(q^2) = q^2\frac{eg_{\rho NN}}{\pi^2}\left[\frac{1}{2}\left(\frac{5}{18}+\frac{2M^2}{3q^2}-\frac{8M^4+2M^2q^2-q^4}{3q^3\sqrt{4M^2-q^2}}\arctan\sqrt{\frac{q^2}{4M^2-q^2}}\right.\right.$$
$$\left.\left. - \frac{\ln M^2}{6}\right)+\frac{C_\rho}{8}\left(-2+2\sqrt{\frac{4M^2-q^2}{q^2}}\arctan\sqrt{\frac{q^2}{4M^2-q^2}}+\ln M^2\right)-\beta\right].$$

We find that the choice $\beta = 8.32$ in our counter-term approximately reproduces the extracted value of $g_\rho(0)$ at $q^2 = 0$.

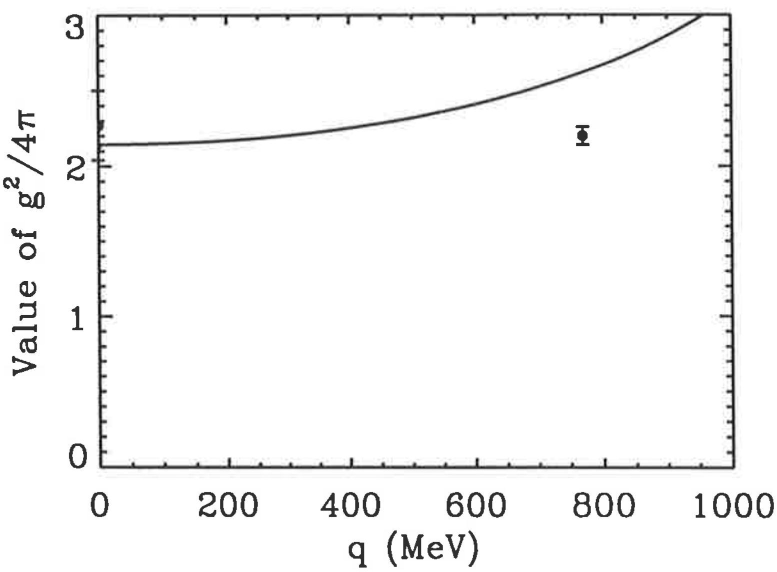

Figure 4.5: The PW model prediction for the mixing amplitude is related to the traditional VMD coupling $g_\rho(q^2)$ using the central result of Eq. (4.55). The resulting behaviour of $g_\rho^2/4\pi$ versus $q \equiv \sqrt{q^2}$ is then plotted in the timelike region for this model. Shown for comparison are a typical pair of results (2.27 ± 0.23 at $q = 0$ and 2.20 ± 0.06 at $q = m_\rho$, see text) taken from a traditional VMD based analysis of cross section data in Ref. [73].

The results for $g_\rho(q^2)$ for the PW model are shown in Fig. 4.5. Despite this model having a node in the ρ–γ mixing at $q^2 = 0$ the resulting q^2 dependence of g_ρ is small. As can be seen from this plot, we obtain values of $g_\rho^2(0)/(4\pi) = 2.14$ and $g_\rho^2(m_\rho^2)/(4\pi) = 2.6$ compared to the experimental averages 2.3 and 2.17 respectively.

It should be remembered that Eq. (4.55) is only as reliable as universality, which is itself violated at a level of 30-40% . Based on this important observation, we can conclude then that the PW model provides a result consistent with the spread of extracted results given in Ref. [73]. It should be noted that any VMD1-like model which predicts a significantly in Ref. [73]. It should be noted that any VMD1-like model which predicts a significantly greater deviation from linearity with q^2 over this momentum region will fail to reproduce phenomenology because of Eq. (4.55).

4.6 Discussion

In summary, we have provided a general proof that in a wide class of models, those in which the vector mesons couple to conserved currents and there is no explicit mass mixing, the mixing amplitude vanishes at $q^2 = 0$. As ρ–γ mixing would also be subject to this theorem in such models, we have explicitly shown in Eq. (4.55) that the vanishing of vector-vector mixing at $q^2 = 0$ is completely consistent with the standard phenomenology of vector meson dominance provided one uses the appropriate formulation, namely VMD1. We have, in addition, applied the same type of model used in a study of $\rho-\omega$ mixing to extract the momentum dependence of $\rho-\gamma$ mixing and have compared the result to the VMD2 based analysis of the experimental data. We see that the phenomenological constraints of VMD can provide a useful independent test of VMD-like models of vector mixing.

Chapter 5

Isospin violation in $F_{\pi}(q^2)$ with ChPT

As mentioned earlier, Chiral Perturbation Theory (ChPT) seeks to produce, in a model independent way, a low energy meson theory from the symmetries of the initial QCD Lagrangian. The principle symmetry used in this construction is chiral symmetry (ChS). ChS is the simultaneous requirement of isospin symmetry and helicity conservation, i.e. $SU(2)_L \otimes SU(2)_R$. Having $m_u = m_d \neq 0$ violates helicity conservation but not isospin symmetry. In the real world we have $m_u \neq m_d \neq 0$ and both symmetries are broken by quark masses. Since m_u and m_d are small on the hadronic scale the violation of chiral symmetry is small. Isospin is also explicitly violated by electromagnetic and weak interactions. The systematic nature of ChPT is then able to give us a model independent method for examining isospin breaking in its regime of applicability. Despite the potential usefulness of this there is still relatively little in the literature on this matter [52, 55, 56, 77, 78].

The pion form-factor, $F_{\pi}(q^2)$, was one of the first quantities calculated using ChPT [79]. It has recently been generalised to off-shell pions [80]. However, these one-loop treatments assume $m_u = m_d$. In this chapter, we extend this to the case with $m_u \neq m_d$, following the work of Maltman [52, 55, 77]. The previous calculations of $F_{\pi}(q^2)$ are briefly reviewed and the isospin-violating calculation is then discussed in detail. It should be noted that the calculation is quite complicated, as it simultaneously involves three small parameters, q^2, α and $m_u - m_d$. We shall work to first order in each of these, which is sufficient for a starting point.

5.1 An introduction to ChPT

Although there are many excellent reviews of ChPT [81], to keep this chapter relatively self contained, I present a short summary of the theory. This will also be useful in showing how we set up the calculation. The basic idea is to take the known symmetries of QCD and reproduce them in a low-energy meson theory. Thus we start with the QCD Lagrangian given by[1]

$$\mathcal{L}^{\rm QCD} = \sum_f \bar{\psi}(x)(i\not{D} - m_f)\psi(x) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}. \tag{5.1}$$

In the chiral limit the quark masses are zero and the fermion fields, ψ, can be split into left and right handed helicity components,

$$\psi_{L,R} = (1 \pm \gamma_5)\psi. \tag{5.2}$$

These transform *independently* under the chiral transformation,

$$\psi_{L,R} \rightarrow e^{i\alpha\gamma_5}\psi_{L,R}, \tag{5.3}$$

leaving Eq. (5.1) unchanged. Massless QCD is then said to be *chirally symmetric.* These transformations can then be generalised to separate left and right handed transformations rather than just the single $e^{i\alpha\gamma_5}$ transforming both fields. In this case we have

$$\psi_{L,R} \rightarrow U^{L,R}\psi_{L,R} \tag{5.4}$$

where U^L and U^R are unitary $N^f \times N^f$ matrices, N^f being the number of flavours. One normally only considers the up, down and strange quarks, for reasons that will become apparent later. The heavier quark flavours play no dynamical role in the region of interest and do not need to be explicitly included. If strange quarks are included the flavour symmetry changes from $SU(2)_{\rm flavour}$ to $SU(3)_{\rm flavour}$ and the chiral symmetry group is then $SU(3)_L \otimes SU(3)_R$.

Now, of course, the quarks do have mass, but it is only small and so we can say that chiral symmetry is an *approximate* symmetry of QCD, and we expect it to have some relevance to the way the theory works, and provide a guide in our construction of a meson theory. To construct this meson theory, we consider the QCD generating functional,

$$\exp[iW[l_\mu, r_\mu, s, p]] = \int [\mathcal{D}\psi][\mathcal{D}\bar{\psi}][\mathcal{D}G^a_\mu] \exp\left[i\int d^4x \mathcal{L}_{\rm QCD}(l_\mu, r_\mu, s, p)\right]. \tag{5.5}$$

[1]To work with this, one needs to remove the unphysical gauge degree of freedom which is usually accomplished by adding a gauge fixing term to Eq. (5.1), however this is not important for our discussion and will be omitted.

The sources for the left and right handed vector currents are given by

$$l_\mu \equiv l_\mu^a \lambda^a/2, \quad r_\mu \equiv r_\mu^a \lambda^a/2, \tag{5.6}$$

where the λ^a are the Gell-Mann matrices that make up the generators of SU(3). The sources for the scalar and pseudoscalar "currents" are given by

$$s = s_0 + s^a \lambda^a/2, \quad p = p_0 + p^a \lambda^a/2. \tag{5.7}$$

If we define $\mathcal{L}_{\rm QCD}^0$ to be the massless QCD Lagrangian (Eq. (5.1) with $m_f = 0$) the full QCD Lagrangian, now with sources, can be written,

$$\mathcal{L}_{\rm QCD}(l_\mu, r_\mu, s, p) = \mathcal{L}_{\rm QCD}^0 - \bar{q}_L \gamma^\mu l_\mu q_L - \bar{q}_R \gamma^\mu r_\mu q_R - \bar{q}(s - i\gamma_5 p)q. \tag{5.8}$$

Defining vector and axial vector current sources through

$$l_\mu^a = v_\mu^a - a_\mu^a, \quad r_\mu^a = v_\mu^2 + a_\mu^a, \tag{5.9}$$

we can rewrite Eq. (5.8) in terms of these vector and axial vector sources

$$\mathcal{L}_{\rm QCD}(v_\mu, a_\mu, s, p) = \mathcal{L}_{\rm QCD}^0 - \bar{q}(\not{v} + \not{a}\gamma_5)q - \bar{q}(s - i\gamma_5 p)q. \tag{5.10}$$

The role of the sources is an important one and any low energy theory attempting to emulate QCD must be expressible in a form involving such sources. We will want as many symmetries as possible of the QCD Lagrangian to be manifest in the effective theory (by construction).

As it happens, *if* v_μ and a_μ transform as gauge fields, and the scalar and pseudoscalar fields transform like

$$(s + ip)(x) \to R(x)(s + ip)L^\dagger(x) \tag{5.11}$$

$$(s - ip)(x) \to L(x)(s - ip)R^\dagger(x) \tag{5.12}$$

then $\mathcal{L}_{\rm QCD}$ has a larger *local* $SU(3)_L \otimes SU(3)_R$ (or $SU(3)_V \otimes SU(3)_A$) symmetry as opposed to the *global* $SU(3)_L \otimes SU(3)_R$ chiral symmetry for 3 flavours. Thus we would insist that the effective theory also be invariant under such a transformation. This point is crucial. We have demonstrated that a low energy theory based on QCD should at zeroth order be invariant under local $SU(3)_V \otimes SU(3)_A$ *if* Eqs. (5.11) and (5.12) hold. We know, though, that chiral symmetry is *broken* in QCD by the quark masses, and this is done by identifying s with the quark mass matrix,

$$s = m \equiv \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \tag{5.13}$$

which, being a constant, does not transform in the manner of Eqs. (5.11) and (5.12), thus breaking the $SU(3)_V \otimes SU(3)_A$ symmetry. Setting s to be the quark mass matrix in our low energy theory will therefore break chiral symmetry in exactly the same way as it is broken in QCD.

Importantly for our work, the mass matrix in Eq. (5.13) breaks more than just chiral symmetry, it also breaks isospin symmetry (in the strong interaction itself) if $m_u \neq m_d$.

Knowing the symmetry structure, we can now construct the meson theory with these features. The Lagrangian is written as a series in powers of q^2,

$$\mathcal{L} = \sum_{n=1} \mathcal{L}_{2n}, \tag{5.14}$$

where the $\mathcal{L}_{2n}$ are the pieces of the Lagrangian at order n in the chiral series. At each order one writes down all the terms allowable by the symmetry, with coefficients that are then fixed by comparison to experiment. Thus, as it has an infinite number of terms, the theory is non-renormalisable (one cannot change a finite number of parameters and remove all divergences), but to a given order in the chiral series, it produces finite answers (as the next order in the series contains the necessary counter-terms).

The lowest order term in the chiral Lagrangian (Eq. (5.14)) is,

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U\rangle, \tag{5.15}$$

where,

$$\begin{aligned} U &= e^{i\pi/F}, \\ \pi \equiv \pi^a \lambda^a &= \begin{pmatrix} \pi^3 + \pi^8/\sqrt{3} & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^3 + \pi^8/\sqrt{3} & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -2\pi_8/\sqrt{3} \end{pmatrix}, \end{aligned} \tag{5.16}$$

F is a constant with dimensions of mass and $\langle A\rangle$ denotes the trace of matrix A. The covariant derivative is

$$D_\mu U = \partial_\mu U + i[v_\mu, U] - i\{a_\mu, U\}, \tag{5.17}$$

and the field, χ is given by,

$$\chi = 2B_0(s - ip). \tag{5.18}$$

The lowest order part of the Lagrangian, Eq. (5.15), is what we need to give the kinetic and mass terms for (say) the pion field, when we set s in Eq. (5.18) to the quark mass matrix of Eq. (5.13). We simply expand the exponential of U in terms of the pion field to give

$$\frac{F^2}{4}2B_0\langle m(U^\dagger + U)\rangle = B_0\left(-\langle m\pi^2\rangle + \frac{1}{6F^2}\langle m\pi^4\rangle + \cdots\right). \tag{5.19}$$

Making the appropriate identifications gives us the following relations between the quark masses and the meson masses [81]

$$\begin{aligned} m^2_{\pi^\pm} &= (m_u+m_d)B_0, \quad m^2_{\pi^0} = (m_u+m_d)B_0 - \delta + O(\delta^2) \\ m^2_{K^\pm} &= (m_u+m_s)B_0, \qquad\quad m^2_{K^0} = (m_d+m_s)B_0 \end{aligned} \tag{5.20}$$

where the second-order CSV parameter, δ, is given by

$$\delta = \frac{B_0}{4}\frac{(m_u-m_d)^2}{(m_s-m_u-m_d)}. \tag{5.21}$$

One can easily see from this, that if the quark masses vanish, then so do the meson masses.

Our calculation will be to one loop order. In ChPT the number of loops corresponds to a particular chiral order, in this case q^4 which can be seen from a consideration of momentum contributions arising from vertices, propagators and loop integration. This means that we only require $\mathcal{L}_2$ and $\mathcal{L}_4$; the former will provide the interaction pieces to make up the loop (and possible lowest order tree in reactions) and the latter will provide the tree-level counterterms required to cancel any divergences arising from the loop.

The next order piece of the Lagrangian is given by,

$$\begin{aligned} \mathcal{L}_4 &= L_1\langle D_\mu U D^\mu U^\dagger\rangle^2 + L_2\langle D_\mu U D_\nu U^\dagger\rangle\langle D^\mu U D^\nu U^\dagger\rangle + L_3\langle D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger\rangle \\ &\quad + L_4\langle D_\mu U D^\mu U^\dagger\rangle\langle \chi U^\dagger + U\chi^\dagger\rangle + L_5\langle D_\mu U D^\mu U^\dagger(\chi U^\dagger + U\chi^\dagger)\rangle \\ &\quad + L_6\langle \chi U^\dagger + U\chi^\dagger\rangle^2 + L_7\langle \chi U^\dagger - U\chi^\dagger\rangle^2 + L_8\langle \chi U^\dagger\chi U^\dagger + U\chi^\dagger U\chi^\dagger\rangle \\ &\quad + iL_9\langle L_{\mu\nu} U R^{\mu\nu} U^\dagger\rangle + H_1\langle R_{\mu\nu}R^{\mu\nu} + L_{\mu\nu}L^{\mu\nu}\rangle + H_2\langle \chi^\dagger\chi\rangle. \end{aligned} \tag{5.22}$$

This constitutes the complete set of terms allowed by the symmetry of order q^4 in momentum (remembering χ as defined in Eq. (5.18) contributes as order q^2.)

5.2 The standard ChPT treatment of $F_\pi(q^2)$

To obtain $F_\pi(q^2)$ we need some way to incorporate the photon into ChPT. To do this, the covariant derivative of Eq. (5.17) is rewritten as

$$D_\mu U = \partial_\mu U + ie[A_\mu, U], \tag{5.23}$$

where we have constructed the matrix A_μ from the four-vector function, $A_\mu(x)$, multiplied by the charge matrix, Q

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} = \frac{e}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right) = Q^3 + Q^8. \tag{5.24}$$

In this way we can make the identification

$$\begin{aligned} A_\mu &= A^3_\mu + A^8_\mu, \\ &= A_\mu(x)(Q^3 + Q^8), \end{aligned} \tag{5.25}$$

where $A_\mu(x)$ is a four vector, and the other quantities are matrices. This is merely a simplification, we have removed the axial current sources and all but the third and eighth components of the vector current sources. The isospin conserving ($m_u = m_d$) treatment was first performed by Gasser and Leutwyler [79] and then extended to the case where the pions are off-shell by Rudy *et al.* [80]. I shall not present the details of the calculation (as this will be done for the $m_u \neq m_d$ case) but merely quote the result

$$F_\pi^{u=d}(q^2) = 1 + q^2\left[\frac{2L}{F^2} + \frac{1}{192F^2\pi^2}A\right], \tag{5.26}$$

where

$$\begin{aligned} A &= 2\ln(m_\rho^2/m_\pi^2) + \ln(m_\rho^2/m_K^2) - B, \\ B &= 1 + 2(1 - 4m_\pi^2/q^2)H(q^2/m_\pi^2) + (1 - 4m_k^2/q^2)H(q^2/m_K^2). \end{aligned}$$

L is the finite part of the low energy constant (see Eq. (5.22)

$$L \equiv L_9^r(m_\rho) = (6.9 \pm 0.7) \times 10^{-3}, \tag{5.27}$$

$F = f_\pi$ is the pion decay constant, 92.4 MeV and

$$\begin{aligned} H(q^2/m_\pi^2) &= -2 + 2\sqrt{\frac{4m_\pi^2}{q^2} - 1}\,\mathrm{arccot}\sqrt{\frac{4m_\pi^2}{q^2} - 1}, \quad 0 < q^2 < 4m_\pi^2 \\ &= -2 + \sqrt{1 - \frac{4m_\pi^2}{q^2}}\left(\ln\left|\frac{\sqrt{1 - \frac{4m_\pi^2}{q^2}} + 1}{\sqrt{1 - \frac{4m_\pi^2}{q^2}} - 1}\right| + i\pi\right), \quad q^2 > 4m_\pi^2. \end{aligned}$$

5.3 The $(m_u - m_d)$ contribution

We are now in a position to examine contributions to the $F_\pi(q^2)$ resulting from the quark mass difference. The basic procedure will be to look at the A^8_μ coupling to the two pion final state. No attempt has been made to examine the $A^3_\mu \to \pi^+\pi^-$ for the case of unequal quark masses, and although any such contribution will be small compared with the isospin conserving contribution, it could be of comparable size to that for $A^8_\mu \to \pi^+\pi^-$. Thus, our procedure now is to search the Lagrangian for terms linear in A_μ, and then choose the special case that only A^8_μ is nonzero. Now by looking at the effect of the covariant derivative, Eq. (5.23), in the Lagrangian Eqs. (5.15) and (5.22) we can deduce the interaction piece, $A_\mu J^\mu$. This will, of course, appear inside a trace where the property $\mathrm{Tr}(\lambda^a\lambda^b) = \delta^{ab}/2$ will ensure that we find $A^8 \cdot J^8$.

Before obtaining J^8_μ from the Lagrangian, it is useful to have in mind a picture of the terms in the Lagrangian that will be relevant to us. The possible graphs are shown in Fig. 5.1. We see can see from this that the pieces of J^8_μ we are looking for are firstly the tree-level piece from $\mathcal{L}_2$, $\pi^+\pi^-$ (corresponding to Fig 5.1.a). For M any meson in Eq. (5.16) we need terms of the form $\bar{M}M$ from J^8_μ, to give the vertex (i) in Fig 5.1.b, and $\bar{M}M\pi^+\pi^-$ (ii) from the kinetic and mass pieces of $\mathcal{L}_2$. Fig 5.1.c is generated by a term in J^8_μ of the form $\bar{M}M\pi^+\pi^-$. Our final piece comes from $\mathcal{L}_4$, which (as this is already $O(q^4)$) can only be a tree level term of the form discussed above for Fig 5.1(a).

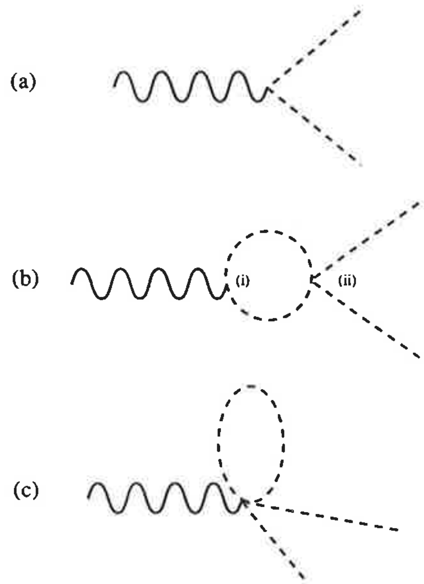

Figure 5.1: The chiral contributions to $\gamma \to \pi^+\pi^-$.

We can now begin to calculate J^8_μ by making a few helpful simplifications in $\mathcal{L}$. Keeping in mind that $\partial_\mu(U^\dagger U) = 0$, this allows us to send

$$\frac{F^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle \to \frac{i}{2}F^2\langle(\partial_\mu U U^\dagger - U^\dagger\partial_\mu U)A^\mu\rangle. \tag{5.28}$$

We can now expand U (an exponential) in powers of π, to give

$$\begin{aligned}\partial_\mu U U^\dagger - U^\dagger\partial_\mu U &= \frac{1}{F^2}(\partial_\mu\pi\pi - \pi\partial_\mu\pi) + \frac{1}{6F^4}(\pi^3\partial_\mu\pi - \partial_\mu\pi\pi^3) \\ &+\frac{1}{4F^4}(\partial_\mu\pi^2\pi^2 - \pi^2\partial_\mu\pi^2) + \frac{1}{6F^4}(\pi\partial_\mu\pi^3 - \partial_\mu\pi^3\pi),\end{aligned} \tag{5.29}$$

where π is the matrix given in Eq. (5.16). Setting A^3_μ to zero and keeping only A^8_μ allows us to obtain from Eq. (5.28) the terms we want from the current. The current, J^8_μ, is given in full in the appendix, Eq. (B.1). However, only a few terms in the current are relevant to our calculation. These terms contributing to the photon-hadron vertex in Fig. 5.1 are

$$\begin{aligned}
J^8_\mu &= i\frac{\sqrt{3}}{2}(\partial_\mu K^0\bar{K}^0 - \partial_\mu \bar{K}^0 K^0 + \partial_\mu K^+K^- - \partial_\mu K^- K^+) + \frac{i\sqrt{3}}{4F^2}(\partial_\mu\pi^-\pi^+K^+K^- \\
&\quad -\partial_\mu\pi^+\pi^-K^+K^- + \partial_\mu K^-K^+\pi^+\pi^- - \partial_\mu K^+K^-\pi^+\pi^- + \partial_\mu\pi^+\pi^-K^0\bar{K}^0 \\
&\quad -\partial_\mu\pi^-\pi^+K^0\bar{K}^0 + \partial_\mu\bar{K}^0K^0\pi^+\pi^- - \partial_\mu K^0\bar{K}^0\pi^+\pi^-). \qquad (5.30)
\end{aligned}$$

We notice in Eq. (5.30) that there is no tree-level contribution (Fig. 5.1(a)) coming from $\mathcal{L}_2$. To calculate the vertices in Fig. 5.1(b) we require the π^4 parts of the kinetic and mass terms of $\mathcal{L}_2$. This is given by (we assume here summation over the Lorentz indices of the partial derivative)

$$\begin{aligned}
\mathcal{L}^{\mathrm{KE}}_{2(\pi^4)} &= \frac{1}{6F^2}(2\partial\pi^+\pi^-\partial K^+K^- - \partial\pi^-\partial K^+K^- - \pi^+\pi^-\partial K^+\partial K^- - \partial\pi^+\partial\pi^-K^+K^- \\
&\quad -\partial\pi^+\pi^-K^+\partial K^- + 2\pi^+\partial\pi^-K^+\partial K^- + 2\partial\pi^+\pi^-K^0\partial\bar{K}^0 - \partial\pi^+\pi^-\partial K^0\bar{K}^0 \\
&\quad -\pi^+\pi^-\partial K^0\partial\bar{K}^0 - \partial\pi^+\partial\pi^-K^0\bar{K}^0 + 2\pi^+\partial\pi^-\partial K^0\bar{K}^0 - \pi^+\partial\pi^-K^0\partial\bar{K}^0) \\
\mathcal{L}^{\mathrm{mass}}_{2(\pi^4)} &= \frac{B_0}{6F^2}[(2m_u + m_d + m_s)\pi^+\pi^-K^+K^- + (m_u + 2m_d + m_s)\pi^+\pi^-K^0\bar{K}^0].
\end{aligned}$$

This takes care of the contributions from $\mathcal{L}_2$. We must now go to $\mathcal{L}_4$, given in Eq. (5.22). As it turns out, this has no coupling of A^8_μ to the two pion final state. We might have expected a contribution from $\mathcal{L}_4$ to Fig. 5.1(a). Usually in ChPT this is responsible for removing the divergences (as well as the unphysical dependence on the scale, μ) associated with the loops of Fig. 5.1(b) and (c). Thus, the loop graphs themselves must combine to give a finite answer.

We are now in a position to construct the Feynman amplitudes associated with the graphs of Fig. 5.1, remembering that $q \equiv -i\partial$. The problem is now completely standard (a good discussion of the relevant loop integrals can be found in, for example Ref. [82]). We obtain the amplitude for $A^8 \to \pi^+\pi^-$, $\mathcal{M}_\mu$ defining the associated form-factor by

$$\mathcal{M}_\mu = (p_+ - p_-)_\mu F^8_\pi(q^2). \qquad (5.31)$$

The calculation of the amplitude is described in detail in the appendix, so we merely present the result for the form-factor here

$$F^8_\pi(q^2) = \frac{\sqrt{3}}{4F^2}\left[\frac{i}{96\pi^2}q^2\ln\frac{m^2_{K^\pm}}{m^2_{K^0}} - \frac{i}{960\pi^2}q^4\left(\frac{1}{m^2_{K^\pm}} - \frac{1}{m^2_{K^0}}\right)\right]. \qquad (5.32)$$

Using Eq. (5.20) we can rewrite this form-factor in terms of the quark masses,

$$F^8_\pi(q^2) = \frac{\sqrt{3}}{4F^2}\left[\frac{i}{96\pi^2}q^2\ln\frac{m_u + m_s}{m_d + m_s} - \frac{i}{960\pi^2}q^4\left(\frac{m_d - m_u}{B_0(m_u + m_s)(m_d + m_s)}\right)\right]. \qquad (5.33)$$

It is then easily seen that the contribution to the pion form-factor from A^8_μ vanishes when $m_u = m_d$ and hence the contribution examined previously (Eq. (5.26)) is that due to A^3 only.

5.4 Discussion

Setting $q^2 = 4m_\pi^2$ in Eq. (5.32) reveals a very small ($O(10^{-4})$) correction to the leading order expression $F_\pi^3(q^2)$ as given in Eq.(5.26). The first response to this might be to assume that A_μ^8 contributes little to the pion form-factor in the low q^2 region relevant to ChPT. While this is possibly quite true, that is not to say that the higher order contributions might not be *larger* than those of Eq. (5.32). Indeed, there is evidence to suggest that this could be the case [55]. Basically, the low energy constants of Eq. (5.22) are the result of "integrating out" the heavy resonances in an extended Lagrangian that includes the vector mesons as well as the pseudoscalar octet. Thus, in any calculation where the low energy constants are absent, such as this one, the effects of the vector resonances are not included. As the isospin violation in $F_\pi(q^2)$ is largely due to the ω we would expect these constants to play a leading role. We can compare this with the case of the decay $\eta \to \pi^0\gamma\gamma$ where the one loop ChPT prediction [83] is approximately 170 times smaller than the experimental result. The $\mathcal{O}(q^6)$ contributions then bring the ChPT result into satisfactory accord with experiment. Maltman finds a similar situation in his calculation for the mixed current correlator $\langle 0|V_\mu^3 V_\nu^8|0\rangle$. Isospin violation is most visible in the pion form-factor data around the ω pole where we determine that the ω contributes with a strength $\sim 3\%$ that of the ρ. Although one cannot probe the resonance pole region using ChPT, it would thus be very interesting to see a similar two loop study of the pion form-factor including isospin violating effects.

Chapter 6

New analysis of the pion form-factor

There are a number of features of the standard extraction of the value for $\Pi_{\rho\omega}$ (discussed in Chapter 2) that deserve closer scrutiny. Foremost of these is that this value is traditionally calculated [35] by comparing a formula for the decay $\omega \to \pi^+\pi^-$ with the branching ratio quoted for this decay [27]. We believe that a fit to the pion form-factor data is a much more direct method and as such should produce a more reliable result. An initial attempt has been made at this (see Section 4.4). However, the purpose of that previous exercise was mainly to illustrate the use of VMD1, rather than to carefully extract the value of $\Pi_{\rho\omega}$. In this chapter we shall take an existing, and very precise, fit to the pion form-factor and match the results from that to our model to obtain information about $\rho-\omega$ mixing.

Upon establishing this procedure, there are two further issues to address. Firstly, we should develop a general scheme where momentum dependence in $\Pi_{\rho\omega}$ can be accomodated. The second point of contention is the accuracy of the Renard argument (see Section 2.4). Although the basic mechanism is physically reasonable, the cancellation is clearly not exact (this will be fully explained in Section 6.3), and so our aim is to see if there is any residual competition between $\omega_I \to \pi^+\pi^-$ and $\Pi_{\rho\omega}$. This study, published recently [84], is presented below.

6.1 An S-matrix approach

The cross-section for $e^+e^- \to \pi^+\pi^-$ in the $\rho-\omega$ resonance region displays a narrow interference shoulder resulting from the superposition of narrow, resonant ω and broad, resonant ρ exchange amplitudes [27]. The strength of the ω "interference" amplitude has generally been taken to provide a measurement of $\rho-\omega$ mixing [35, 66].

To obtain properties of unstable particles which are process-independent and physically meaningful, one determines the locations of the resonance poles in the amplitude under consideration, and makes expansions about these pole locations [85]. The (complex) pole locations are properties of the S-matrix and hence *independent of the choice of interpolating fields*, and the separate terms in the Laurent expansion about the pole position have well-defined physical meaning [85]. The importance of such an "S-matrix" formalism for characterising resonance properties has been stressed recently by a number of authors in the context of providing gauge- and process-independent definitions of the Z^0 mass and width in the Standard Model [86,87].

For our purposes this means that: (1) the "physical" $\{\rho, \omega\}$ fields are to be identified as those combinations of the $\{\rho_I, \omega_I\}$ fields containing the corresponding S-matrix poles and (2) to analyse $e^+e^- \to \pi^+\pi^-$ one should include both resonant terms involving the complex ρ and ω pole locations and "background" (i.e. non-resonant) terms. In quoting experimental results we will, therefore, restrict ourselves to analyses which satisfy these requirements (as closely as possible). To our knowledge, only one such exists: the fifth fit of Ref. [70] which is performed explicitly in the S-matrix formalism, though without an s-dependent background. As stressed in Ref. [70], using the S-matrix formalism, one finds a somewhat lower real part for the (complex) ρ pole position ($\hat{m}_\rho = 757.00 \pm 0.59$, $\Gamma_\rho = 143.41 \pm 1.27$ MeV) than is obtained in conventional, non-S-matrix formalism treatments. Therefore, for comparison we will also employ the results of the second fit of the more conventional (but non-S-matrix) formalism of Ref. [69], which employs an s-dependent background, an s-dependent ρ width, and imposes the (likely too large) Particle Data Group value for the ρ mass by hand.

6.2 Mixing Formalism

Let us turn to the question of $\rho-\omega$ mixing in the presence of a q^2-dependent off-diagonal element of the self-energy matrix. We shall work consistently to first order in isospin breaking (generically, $\mathcal{O}(\epsilon)$), which will mean to first order in $\Pi_{\rho\omega}$.

As we consider only vector mesons coupled to conserved currents, we can replace $D_{\mu\nu}(q^2)$ by $-g_{\mu\nu}D(q^2)$. We will refer to $D(q^2)$ as the "scalar propagator". We assume that the isospin-pure fields, ρ_I and ω_I, have already been renormalised – i.e., that the relevant counterterms have been absorbed into the complex mass and wavefunction renormalisations. Taking the full expression for the dressed propagator and keeping terms to $\mathcal{O}(\epsilon)$, one finds

$$D^I(q^2) = \begin{pmatrix} D^I_{\rho\rho} & D^I_{\rho\omega} \\ D^I_{\rho\omega} & D^I_{\omega\omega} \end{pmatrix} = \begin{pmatrix} (q^2 - m_\rho^2 - \Pi^R_{\rho\rho}(q^2))^{-1} & D^I_{\rho\omega}(q^2) \\ D^I_{\rho\omega}(q^2) & (q^2 - m_\omega^2 - \Pi^R_{\omega\omega}(q^2))^{-1} \end{pmatrix}, \tag{6.1}$$

where the *renormalised* self-energies are defined here such that

$$\Pi^R_{kk}(q^2) \to 0 \quad \text{as} \quad q^2 \to m_k^2. \tag{6.2}$$

We then have $\Pi^R_{kk}(q^2) = \mathcal{O}[(q^2 - m_k^2)^2]$, not to be confused with the unrenormalised $\Pi_{kk}(q^2)$ of Section 4.1, subject to the node condition. From the complex pole positions, m_k^2, we define the (real) mass ($\hat{m}_k$) and width (Γ_k) via,

$$m_k^2 \equiv \hat{m}_k^2 - i\,\hat{m}_k\Gamma_k. \tag{6.3}$$

To $\mathcal{O}(\epsilon)$, $D^I_{\rho\omega}(q^2)$ is then (see Eq. (4.22)

$$\begin{aligned} D^I_{\rho\omega}(q^2) &= \frac{\Pi_{\rho\omega}(q^2)}{(q^2 - m_\rho^2 - \Pi^R_{\rho\rho}(q^2))(q^2 - m_\omega^2 - \Pi^R_{\omega\omega}(q^2))} \\ &= D^I_{\rho\rho}(q^2)\Pi_{\rho\omega}(q^2)D^I_{\omega\omega}(q^2), \end{aligned} \tag{6.4}$$

which contains both a broad ρ resonance and narrow ω resonance piece.

As explained above, the physical ρ and ω fields are defined to be those combinations of the ρ_I and ω_I for which *only* the diagonal elements of the propagator matrix contain poles, in the ρ, ω basis. This definition is, in fact, implicit in the standard interpretation of the $e^+e^- \to \pi^+\pi^-$ experiment, which associates the broad resonant part of the full amplitude with the ρ and the narrow resonant part with the ω. Using different linear combinations of ρ_I, ω_I, (call them ρ', ω') than those given above (ρ, ω), one would find also narrow resonant structure in the off-diagonal element of the vector meson propagator in the $\{\rho', \omega'\}$ basis, preventing, for example, the association of the narrow resonant behaviour with the ω' pole term alone. Despite this, the Bernicha *et al.* paper [70] on whose data fit we rely do not use this physical basis. For most of their paper, they use the isospin pure basis, which contains the propagator in Eq. (6.4). Their analysis would therefore be of little use to us were it not for their "freezing" the q^2 in the $D^I_{\rho\rho}(q^2)$ propagator in one of a number of fits, which gives an expression of our desired form (given in Eq. (6.26)). It is therefore of note to point out that the more standard Ref. [69] does use the physical basis, in which only two poles occur (but includes momentum dependent widths).

We define the transformation between the physical and isospin pure bases by (to $\mathcal{O}(\epsilon)$)

$$\rho = \rho_I - \epsilon_1\,\omega_I, \qquad \omega = \omega_I + \epsilon_2\,\rho_I \tag{6.5}$$

where we have allowed for two distinct mixing parameters, ϵ_1 and ϵ_2. With

$$D^{\mu\nu}_{\rho\omega}(x-y) \equiv -i\langle 0|T(\rho^\mu(x)\omega^\nu(y))|0\rangle, \tag{6.6}$$

one then has for the scalar propagator, to $\mathcal{O}(\epsilon)$,

$$D_{\rho\omega}(q^2) = D^I_{\rho\omega}(q^2) - \epsilon_1 D^I_{\omega\omega}(q^2) + \epsilon_2 D^I_{\rho\rho}(q^2). \tag{6.7}$$

The condition that $D_{\rho\omega}(q^2)$ contains no ρ or ω pole then fixes $\epsilon_{1,2}$ by demanding that the numerator of Eq. (6.7) vanish at these poles. Hence we find

$$\epsilon_1 = \frac{\Pi_{\rho\omega}(m_\omega^2)}{m_\omega^2 - m_\rho^2 - \Pi_{\rho\rho}^R(m_\omega^2)}, \quad \epsilon_2 = \frac{\Pi_{\rho\omega}(m_\rho^2)}{m_\omega^2 - m_\rho^2 + \Pi_{\omega\omega}^R(m_\rho^2)} . \tag{6.8}$$

To $\mathcal{O}(\epsilon)$ the diagonal propagator elements are unchanged, $D_{\rho\rho} = D_{\rho\rho}^I$ (and similarly for the ω). We now have an expression for the physical vector meson propagator in terms of the isospin pure propagator elements.

There are a number of points to note about Eqs. (6.5) and (6.8). When $\Pi^{\rho\omega}(q^2)$ is q^2-dependent, we thus see explicitly that $\epsilon_1 \neq \epsilon_2$; the relation between the isospin-pure and physical bases is not a simple rotation as usually assumed. This is a universal feature of q^2-dependent mixing in field theory. However, as the ρ and ω poles are nearby, the single parameter, ϵ which is usually used to describe the transformation in Eq. (6.5) reducing it to a simple rotation, is a good approximation.

As an illustration, let us consider the case where this momentum dependence is purely linear (which is, admittedly a rather restrictive assumption) so that

$$\Pi_{\rho\omega}(q^2) = \Pi_{\rho\omega}^0 + q^2\Pi_{\rho\omega}^1 \tag{6.9}$$

then to first order in isospin breaking, $\Pi_{\rho\omega}^1$ is just the off-diagonal element of the wave-function renormalisation matrix, Z. The results in Eqs. (6.5) and (6.8) then reproduce those that follow from the more familiar formalism of first defining renormalised fields, ϕ_a^r $(a = \rho, \omega)$, via

$$\phi_a^r = (Z^{-1/2})_{ab}\phi_b \tag{6.10}$$

(where $\phi_1 = \rho_I$ and $\phi_2 = \omega_I$) and then identifying the physical, renormalised fields by rotating of the $\{\phi_a^r\}$ basis to diagonalise the (symmetric) meson mass-squared matrix and hence also the propagator. Put simply, in the special case that the mixing is linear, the physical propagator is diagonal and the effect of the mixing survives only through the wave-function renormalisation of the physical fields. We shall, however, keep our discussion completely general.

Recall that $\Pi_{\rho\rho}^R(q^2)$ and $\Pi_{\omega\omega}^R(q^2)$ vanish by definition as $q^2 \to m_\rho^2$ and m_ω^2 (respectively) at least as fast as $(q^2 - m_{\rho,\omega}^2)^2$ where m_ρ^2 and m_ω^2 are complex (see Eq. (6.3)). The usual assumption is that these two quantities are zero in the vicinity of the resonance region, which leads to the standard Breit-Wigner form for the vector meson propagators. $\Pi_{\rho\rho}^R(q^2)$ and $\Pi_{\omega\omega}^R(q^2)$ are, of course, momentum-dependent in general since the vector propagators must be real below the $\pi\pi$ and $\pi\gamma$ thresholds. Note that, from Eqs. (6.7) and (6.8), any deviation from the Breit-Wigner form and/or any non-linearity in the q^2-dependence of $\Pi_{\rho\omega}(q^2)$ will produce a non-zero off-diagonal element of the vector propagator *even in the* *physical basis.* This means that a background (non-resonant) term is completely unavoidable even in the traditional VMD framework, where all contributions are associated with vector meson exchange. In general this background will be q^2-dependent. Finally, even in the vicinity of the ρ and ω poles, where it should be reasonable to set $\Pi^R_{\rho\rho}(q^2)$ and $\Pi^R_{\omega\omega}(q^2)$ to zero, the ρ_I admixture into the physical ω is governed, not by $\Pi^{\rho\omega}(m^2_\omega)$ as usually assumed, but by $\Pi^{\rho\omega}(m^2_\rho)$.

We will see that these findings related to the momentum dependence of self-energies, while important in principle, do relatively little to change the quantitative analysis of the data. Isospin violation is such a small effect that it is only observable at the ω pole, and so the non-resonant off-diagonal piece is unlikely to play much of a role in any fit, other than as part of a general background. Our lack of knowledge of the momentum dependent behaviour of $\Pi_{\rho\omega}$ and the fact that the real parts of the complex meson poles are quite close together means that determining that the mixing is extracted at one pole rather than the other will have little practical significance. Despite this, an understanding of the principle of defining the physical fields in terms of the poles in the matrix propagator is useful and provides a relatively transparent framework for understanding what is seen in the data.

6.3 Contributions to the pion form-factor

The time-like EM pion form-factor is given, in the interference region, by

$$F_\pi(q^2) = \left[g_{\omega\pi\pi} D_{\omega\omega} \frac{f_{\omega\gamma}}{e} + g_{\rho\pi\pi} D_{\rho\rho} \frac{f_{\rho\gamma}}{e} + g_{\rho\pi\pi} D_{\rho\omega} \frac{f_{\omega\gamma}}{e}\right] + \text{background}, \tag{6.11}$$

where $g_{\omega\pi\pi}$ is the coupling of the *physical* ω to the two pion final state and $f_{\rho\gamma}$ and $f_{\omega\gamma}$ are the electromagnetic ρ and ω couplings. The third piece of Eq. (6.11), $g_{\rho\pi\pi} D_{\rho\omega} f_{\omega\gamma}$, results from the non-vanishing of the off-diagonal element of the *physical* meson propagator and, being non-resonant, can be absorbed into the background, as can any deviations from the Breit-Wigner form for the ρ and ω propagators. Since the variation of q^2 over the interference region is tiny, we can presumably also safely neglect any q^2-dependence of $f_{\rho\gamma}$, $f_{\omega\gamma}$, $g_{\rho\pi\pi}$ and $g_{\omega\pi\pi}$ (this is a standard assumption in VMD). The photon-meson coupling, $f_{V\gamma}$, is related to the "universality coupling", g_V, of traditional VMD treatments by $f_{V\gamma} = -e\hat{m}^2_V/g_V$ (see the discussion surrounding Eq. (4.48) on page 38). As we have assumed a renormalisation at *complex* points on the q^2 plane, one might need to carefully examine the use of *real* coupling constants as we might expect this scheme to deliver complex renormalisation constants [88]. We shall not address this issue here, but note it for future consideration.

We now focus on the resonant ω exchange contribution, whose magnitude and phase, relative to the resonant ρ exchange, are extracted experimentally. We have

$$g_{\omega\pi\pi} = \langle\pi\pi|\omega_I + \epsilon_2\rho_I\rangle = g_{\omega_I\pi\pi} + \epsilon_2 g_{\rho_I\pi\pi}, \tag{6.12}$$

where ϵ_2 is given in Eq. (6.8) which we shall rewrite as

$$\epsilon_2 = -i\frac{z\Pi_{\rho\omega}(m_\rho^2)}{\hat{m}_\rho\Gamma_\rho}, \tag{6.13}$$

to introduce a new quantity

$$z \equiv \left[1 - \frac{\hat{m}_\omega\Gamma_\omega}{\hat{m}_\rho\Gamma_\rho} - i\left(\frac{\hat{m}_\omega^2 - \hat{m}_\rho^2}{\hat{m}_\rho\Gamma_\rho}\right)\right]^{-1} \tag{6.14}$$

that will be helpful in our analysis. Note that $z \approx 1$ but equals 1 *only* if we neglect the ω width and $\rho - \omega$ mass difference. This brings us to the Renard argument [30], mentioned in Section 2.4. Since, in general, $g_{\omega_I\pi\pi} \neq 0$, $\Pi_{\rho\omega}(q^2)$ could contain a contribution from the intermediate $\pi\pi$ state which, because essentially the entire ρ width is due to the $\pi\pi$ mode, is given by

$$\begin{aligned} \Pi_{\rho\omega}^{2\pi}(m_\rho^2) &= \frac{g_{\omega_I\pi\pi}}{g_{\rho_I\pi\pi}}\Pi_{\rho\rho}^{2\pi}(m_\rho^2) \\ &= G(\mathrm{Re}\,\Pi_{\rho\rho}^{2\pi}(m_\rho^2) - i\hat{m}_\rho\Gamma_\rho), \end{aligned} \tag{6.15}$$

where

$$G = \frac{g_{\omega_I\pi\pi}}{g_{\rho_I\pi\pi}} \tag{6.16}$$

is the ratio of the ρ_I and ω_I couplings to $\pi\pi$. In arriving at Eq. (6.15) we have used the facts that (1) the imaginary part of the ρ self-energy at resonance ($q^2 = m_\rho^2$) is, by definition, $-\hat{m}_\rho\Gamma_\rho$, and (2) $g_{\rho\pi\pi} = g_{\rho_I\pi\pi}$ to $\mathcal{O}(\epsilon)$. Defining $\tilde{\Pi}_{\rho\omega}$ through

$$\Pi_{\rho\omega} = \tilde{\Pi}_{\rho\omega} - iG\hat{m}\Gamma_\rho, \tag{6.17}$$

we rewrite ϵ_2 as

$$\epsilon_2 = z\frac{-i}{\hat{m}_\rho\Gamma_\rho}[\tilde{\Pi}_{\rho\omega}(m_\rho^2) - iG\hat{m}_\rho\Gamma_\rho] \tag{6.18}$$

and define a new quantity,

$$\tilde{\epsilon}_2 = (-iz/\hat{m}_\rho\Gamma_\rho)\tilde{\Pi}_{\rho\omega}(m_\rho^2). \tag{6.19}$$

This allows us to rewrite Eq. (6.12) as

$$g_{\omega\pi\pi} = g_{\omega_I\pi\pi}(1 - z) + \tilde{\epsilon}_2 g_{\rho_I\pi\pi}. \tag{6.20}$$

We shall also define, for convenience,

$$\tilde{T} \equiv \frac{\tilde{\Pi}_{\rho\omega}(m_\rho^2)}{\hat{m}_\rho \Gamma_\rho}. \tag{6.21}$$

The standard Renard analysis (see Section 2.4) approximates z by 1 (see Eq. (6.14)). The contribution to $\omega \to \pi\pi$ from the intrinsic ω_I decay is then exactly cancelled in Eq. (6.20). Using the (preferred) experimental analysis of Ref. [70], however, we find

$$z = 0.9324 + 0.3511\, i\ . \tag{6.22}$$

(For comparison, the analysis of Ref. [69] gives $1.023+0.2038i$). Because of the substantial imaginary part, the intrinsic decay cannot be neglected in $e^+e^- \to \pi^+\pi^-$.

Substituting the results above into Eq. (6.11), we find

$$F_\pi(q^2) = \frac{f_{\rho\gamma}}{e} g_{\rho_I\pi\pi} \left[|r_{\text{ex}}| e^{i\phi_{e^+e^-}} \left((1-z)G - iz\tilde{T} \right) P_\omega + P_\rho \right] + \text{ background}, \tag{6.23}$$

where we have expanded the propagators $D_{\rho\rho,\omega\omega}$ of Eq. (6.11) in Laurent series about the simple Breit-Wigner poles $P_{\rho,\omega} \equiv 1/(p^2 - m_{\rho,\omega}^2)$. The ratio of the physical couplings to the photon is given by

$$r_{\text{ex}} \equiv \frac{f_{\omega\gamma}}{f_{\rho\gamma}} = |r_{\text{ex}}| e^{i\phi_{e^+e^-}}, \tag{6.24}$$

with $\phi_{e^+e^-}$ the "leptonic phase" (to be discussed in more detail below, see also Eq. (2.13)). This analysis essentially follows that in Section 2.3. We emphasise that the present work represents a logical refinement of the standard analysis. As such, our conclusions are really implicit in the earlier work, but have not been realised due to various approximations. Experimentally,

$$|r_{\text{ex}}| = \left[\frac{\hat{m}_\omega^3 \Gamma(\omega \to e^+e^-)}{\hat{m}_\rho^3 \Gamma(\rho \to e^+e^-)} \right]^{1/2} = 0.30 \pm 0.01 \tag{6.25}$$

using the values found in Ref. [70]. The form of $F_\pi(q^2)$ in Eq. (6.23) is what is required for comparison with experimental data, for which one has [70]

$$F_\pi \propto P_\rho + Ae^{i\phi} P_\omega; \quad A = -0.0109 \pm 0.0011; \quad \phi = (116.7 \pm 5.8)^\circ. \tag{6.26}$$

As will be demonstrated, the uncertainty in the Orsay phase, ϕ, makes a precise extraction of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ impossible. Indeed, the two contributions to the ω exchange amplitude (i.e., multiplying P_ω) either have nearly the same phase or they differ in phase by close to π (depending on the relative signs of G and $\tilde{T}$). In either case, a large range of combinations of G and $\tilde{T}$, all producing nearly the same overall phase, will produce the same value of A. The experimental data can thus place only rather weak constraints on the relative size of the two contributions, as we will see more quantitatively below.

Let us write $r_{\rm ex}$, the ratio of electromagnetic couplings given in Eq. (6.24), in terms of the corresponding isospin-pure ratio, $r_I = f_{\omega_I\gamma}/f_{\rho_I\gamma}$. In the limit of *exact* isospin, in which G would be zero and $\rho-\omega$ mixing not observed, this ratio would be 1/3 [72]. However, as isospin is slightly broken we expect r_I to be slightly different from 1/3, the way G is not exactly zero. However, for simplicity we demand that r_I be real [1] and this will place constraints on $\phi_{e^+e^-}$ (given in Eq. (6.24)). In analogy to Eq. (6.15), we have

$$f_{\omega\gamma} = f_{\omega_I\gamma} + \epsilon_2 f_{\rho_I\gamma}, \tag{6.27}$$

$$f_{\rho\gamma} = f_{\rho_I\gamma} - \epsilon_1 f_{\omega_I\gamma}, \tag{6.28}$$

and one finds

$$r_{\rm ex} = (r_I + \epsilon_2)/(1 - \epsilon_1 r_I). \tag{6.29}$$

To $\mathcal{O}(\epsilon)$ the constraint that r_I be real now means that we can determine the leptonic phase. Rearranging Eq. (6.29) to obtain an expression for r_I, and then demanding that the imaginary part of this vanish, we obtain

$$\sin\phi_{e^+e^-} = \frac{{\rm Im}(\epsilon_2) + |r_{\rm ex}|^2 {\rm Im}(\epsilon_1)}{|r_{\rm ex}|}. \tag{6.30}$$

Ignoring the small difference in ϵ_1 and ϵ_2 (since $r_{\rm ex}^2$ is small, see Eq. (6.8)) we obtain

$$\sin\phi_{e^+e^-} = \frac{(1 + |r_{\rm ex}|^2){\rm Im}\epsilon_2}{|r_{\rm ex}|}. \tag{6.31}$$

In order to simplify the discussion of our main point, which is the effect of including the intrinsic decay on the experimental analysis, let us now make the usual assumption that the imaginary part of $\Pi_{\rho\omega}$ is dominated by $\pi\pi$ intermediate states. (Note, however, that, because the argument is complex, there may also be a small imaginary part of $\Pi_{\rho\omega}$ even in the absence of real intermediate states; to illustrate this consider the model of Ref. [46], with confined quark propagators, where the phase of the quark loop contribution to $\Pi_{\rho\omega}(m_\rho^2)$ at complex $q^2 = m_\rho^2$ is about -13° [89], despite the model having, for this contribution, no available intermediate states.) Making this $\pi\pi$ dominance assumption, $\tilde{\Pi}_{\rho\omega}$ (and thus $\tilde{T}$) becomes pure real and the imaginary part of $\Pi_{\rho\omega}(m_\rho^2)$ reduces to $-G\hat{m}_\rho\Gamma_\rho$. Using Eqs. (6.18) and (6.31) the leptonic phase becomes

$$\sin\phi_{e^+e^-} = -\left(\frac{1 + |r_{\rm ex}|^2}{|r_{\rm ex}|}\right)(\tilde{T}\,{\rm Re}\,z + G\,{\rm Im}\,z) \tag{6.32}$$

which is completely fixed by G and $\tilde{\Pi}_{\rho\omega}$. We see then that in the small angle limit where $\tan\theta = \sin\theta$, Eq. (6.32) reduces to the standard lepton phase given in Eq. (2.13), upon

[1] As we have renormalised at *complex* pole positions, however, this assumption deserves further investigation [88].

substituting for $\tilde{T}$ (see Eq. (6.21)) and setting $r_{\rm ex} = 1/3$. For each possible $\tilde{\Pi}_{\rho\omega}$, only one solution for G gives both the correct experimental magnitude for the ω pole pre-factor (A) and has a phase lying in the second quadrant, as required by experiment. Knowing $\tilde{\Pi}_{\rho\omega}$ and G, Eq. (6.32) allows us to compute the total phase, ϕ. Those pairs $(\tilde{\Pi}_{\rho\omega}, G)$ producing the experimentally allowed (A, ϕ) constitute our full solution set. This is shown, explicitly, below.

So the problem now reduces to that of two unknowns, $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ and G which combine in a non-linear way to give a theoretical prediction for the experimentally determined A and the Orsay phase, ϕ. Comparing Eq. (6.23) with (6.26)

$$|r_{\rm ex}|e^{i\phi_{e^+e^-}}\left((1-z)G - iz\tilde{T}\right) = Ae^{i\phi}. \tag{6.33}$$

We now wish to find G in terms of $\tilde{\Pi}_{\rho\omega}$. To do this we take the modulus squared of both sides (to eliminate phases). This sets up a quadratic equation $aG^2 + bG + c = 0$, where

$$\begin{aligned} a &= (1 - {\rm Re}\, z)^2 + ({\rm Im}\, z)^2 && (6.34)\\ b &= 2[(1 - {\rm Re}\, z){\rm Im}\, z\, \tilde{T} + {\rm Im}\, z\; {\rm Re}\, z\, \tilde{T}] && (6.35)\\ c &= [({\rm Im}\, z)^2 + ({\rm Re}\, z)^2]\,\tilde{T}^2 - A^2/|r_{\rm ex}|^2. && (6.36) \end{aligned}$$

We can therefore solve for G in terms of $\tilde{T}$. Our other constraint is that of the Orsay phase in terms of the leptonic and hadronic phases, $\phi = \phi_{e^+e^-} + \phi_{\rm had}$. The hadronic phase is obtained from Eq. (6.33) giving

$$\tan\phi_{\rm had} = -\frac{{\rm Im}\, z\; G + {\rm Re}\, z\; \tilde{T}}{(1 - {\rm Re}\, z)G + {\rm Im}\, z\; \tilde{T}}. \tag{6.37}$$

The Orsay phase is now obtained by inverting Eqs. (6.32) and (6.37). This then gives us ϕ as a function of $\tilde{T}$, which we can now compare to the experimental value for the Orsay phase.

6.4 Numerical results

The results of the above analysis are presented in Fig. 6.1, where we have used as input the results of Ref. [70]. The spread in G values reflects the experimental error in A. We see that, barring theoretical input on the precise size of G, experimental data is incapable of providing even reasonably precise constraints on the individual magnitudes of G and $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$. The reason for this situation has been explained above. If we fix A at its central value, the experimental phase alone would restrict $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ to the range $(-1090\ {\rm MeV}^2, -5980\ {\rm MeV}^2)$. Including the experimental error on A extends, for example, the phase constraint range to $(-840\ \text{MeV}^2, -6240\ \text{MeV}^2)$. For comparison, artificially setting $G = 0$ produces $\tilde{\Pi}_{\rho\omega}(m_\rho^2) = -3960\ \text{MeV}^2$. One may repeat the above analysis using the input parameters of Ref. [69] (where, however, the ρ pole position is presumably too high by about 10 MeV [70]). For the central A value, the experimentally allowed range of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ is $(-3720\ \text{MeV}^2, -5080\ \text{MeV}^2)$. The large uncertainty in the extracted values of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ and G is thus not an artefact of the particular fit of Ref. [70]. The small $(\pm 600\ \text{MeV}^2)$ error usually quoted for $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ [35], and associated with the experimental error in the determination of A, thus represents a highly inaccurate statement of the true uncertainty in the extraction of this quantity from the experimental data. It is important to stress that no further information on $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ is obtainable from the $e^+e^- \to \pi^+\pi^-$ data without additional theoretical input.

Note, for example, that, in the model of Ref. [46], as currently parametrised, the sign of G is determined to be positive, and the magnitude to be $\simeq 0.02$. Such a value of G, however, coupled with the phase correction mentioned above, would fail to satisfy the experimental phase constraint. This shows that, despite the weakness of the experimental constraints for the magnitudes of G and $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$, the experimental results are, nonetheless, still capable of providing non-trivial constraints for models of the mixing.

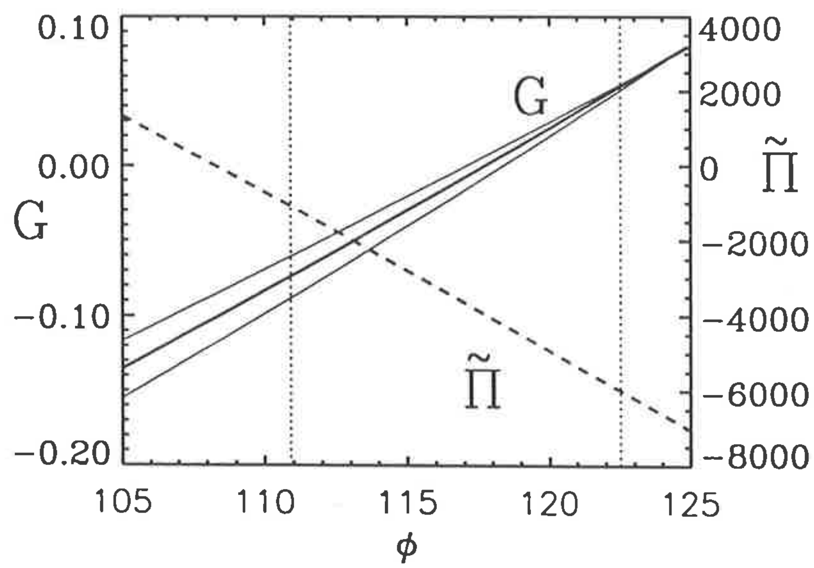

Figure 6.1: The allowed values of $G = g_{\omega_I\pi\pi}/g_{\rho_I\pi\pi}$ and $\tilde{\Pi}(m_\rho^2)$ (in MeV^2) are plotted as a function of the Orsay phase, ϕ. The vertical (dotted) lines indicate the experimental uncertainty in ϕ $(= 116.7 \pm 5.8)^\circ$ and the uncertainty in the amplitude A (0.0109 ± 0.0011) (see text) gives rise to the spread of possible values of G at each value of ϕ.

6.5 A word on transformations between bases

It is now useful to generalise the procedure in quantum field theory for a transformation between bases, such as the one we have used here to arrive at our final form for the pion form-factor. In this analysis, an important point becomes clear.

Consider a two-channel vector boson system in the isospin-pure (I) basis, which we shall denote (suppressing all Lorentz indices) by the column vector

$$V^I \equiv \begin{pmatrix} V_1^I \\ V_2^I \end{pmatrix}. \tag{6.38}$$

The propagator for this two channel system is a 2×2 matrix given by (see Eq. (6.6))

$$D_{ij}^I = \langle 0|T(V_i^I V_j^I)|0\rangle \tag{6.39}$$

which we can write in matrix from as

$$D^I = \langle 0|T(V^I (V^I)^T)|0\rangle \tag{6.40}$$

where the row vector $(V^I)^T$ is the transpose of Eq. (6.38).

We now wish to consider the process $A \to B$ which is mediated by the vector bosons V^I. To do this, we need to determine the vertex functions between the V^I and the initial and final states A and B. Once again we have column vectors (we give the VA vertex as an example)

$$\begin{aligned} \Gamma_{AV}^I &= \langle 0|T(V^I A)|0\rangle & (6.41) \\ &= \langle 0|T \begin{pmatrix} V_1^I A \\ V_2^I A \end{pmatrix} |0\rangle. & (6.42) \end{aligned}$$

If we assume, as we did for the pion form-factor calculation, that these vertices are pointlike, the vertex functions are given simply by coupling constants. Using the notation f to denote the coupling of the vector boson to the initial state, and g to the final state, we would have

$$\begin{aligned} (\Gamma_{AV}^I)^T &= (f_1^I \quad f_2^I) & (6.43) \\ \Gamma_{VB}^I &= \begin{pmatrix} g_1^I \\ g_2^I \end{pmatrix}. & (6.44) \end{aligned}$$

We now have all the pieces to construct the amplitude for the one boson exchange process. This is given by

$$T^I = (\Gamma_{AV}^I)^T D^I \Gamma_{VB}^I. \tag{6.45}$$

Suppose now that we wish to make a linear transformation to a different basis, as we did for the pion form-factor (see Eq. (6.5)). In that case we wanted a basis, the "physical basis", in which the propagator would have poles only on its diagonal. We can express this in a matrix equation

$$V^I \to V = CV^I. \tag{6.46}$$

Here C is a 2×2 matrix and hence the transpose of V will be given by

$$V^T = (V^I)^T C^T. \tag{6.47}$$

As the propagator and the vertex functions are constructed from V^I these will also be subject to a transformation when we change to the new basis. For the propagator we have

$$\begin{aligned} D^I \to D &= \langle 0|T(CV^I(V^I)^T C^T)|0\rangle \\ &= CD^I C^T. \end{aligned} \tag{6.48}$$

Similarly the vertex functions transform like (see the transformation for the $\omega - \pi\pi$ vertex, Eq. (6.12))

$$\begin{aligned} \Gamma^I_{VA} \to \Gamma_{VA} &= \langle 0|T(CV^I A)|0\rangle = C\Gamma^I_{VA} \\ \Gamma^I_{BV} \to \Gamma_{BV} &= \langle 0|T(B(V^I)^T C^T)|0\rangle = \Gamma^I_{BV} C^T \end{aligned} \tag{6.49}$$

Therefore, recalling Eq. (6.45), the amplitude transforms like

$$\begin{aligned} T^I \to T &= (\Gamma^I_{AV})^T C^T C D^I C^T C \Gamma^I_{VB} \\ &= \Gamma_{AV} D \Gamma_{VB}. \end{aligned} \tag{6.50}$$

We easily see from this that the amplitudes as determined in the two bases are only equal if the transformation between them is orthogonal, i.e., $C^T C = I$. However, the transformation between the $\{\rho_I, \omega_I\}$ and $\{\rho, \omega\}$ that we have used (see Eqs. (6.5) and (6.8)) bases is not orthogonal in the general case (i.e., when $\epsilon_1 \neq \epsilon_2$ as occurs when there was momentum dependence present). Thus the choice of physical basis is one of significance, rather than mere convenience.

6.6 Conclusion

In general, there is a contribution to the $\rho - \omega$ interference in $e^+e^- \to \pi^+\pi^-$ from the intrinsic $\omega_I \to \pi\pi$ coupling. Given the current level of accuracy of the experimentally extracted Orsay phase, we cannot extract any value for $\rho - \omega$ mixing which is even reasonably precise in the absence of additional theoretical input. It is important to stress that this conclusion, and the central result of Eq. (6.23), do not depend in the least on the possible q^2-dependence of $\Pi_{\rho\omega}(q^2)$ nor on the use of the S-matrix formalism: even for constant $\Pi_{\rho\omega}$ and a more traditional Breit-Wigner analysis where quantities are defined at real mass points (rather than complex poles) one would still have a significant imaginary part of z (Eq. (6.14)) and hence a residual contribution from the direct coupling which, being nearly parallel to that associated with ρ–ω mixing, would lead also to the conclusion stated above. It is an entirely straightforward matter for the reader to reproduce the steps leading to Eq. (6.23) when there is no momentum dependence and the traditional Breit-Wigner form for the resonances is used. We see that the central result is that z is not exactly 1 (i.e., when we do not neglect the ρ–ω mass difference and the ω width) and that this leads to significant quantitative uncertainties in the extraction of the ρ–ω mixing amplitude.

A significant improvement in the determination of the experimental phase would allow one to simultaneously extract the mixing and the isospin-breaking ratio, G (Eq. (6.16)). However, both these quantities are constructs of the matrix model we have used (i.e., that traditionally used) and this must be considered when discussing them. They are not model independent like the ω pole prefactor, $Ae^{i\phi}$ (see Eq. (6.26).

We note that (1) even if G were zero, the data would provide the value of the mixing amplitude at m_ρ^2 and not m_ω^2, (2) since it is the complex S-matrix pole positions of the ρ and ω which govern the mixing parameters $\epsilon_{1,2}$, only an analysis utilising the S-matrix formalism can provide reliable input for these pole positions, and hence for the analysis of the isospin-breaking interference in $e^+e^- \to \pi^+\pi^-$, (3) the simultaneous use of the experimental magnitude and phase can provide non-trivial constraints on models of the vector meson mixing process and (4) for q^2 dependent mixing, the transformation between the isospin pure and physical bases is no longer a simple rotation.

Due to the similarities of the vector propagator structure and dependence on the up down mass difference one might expect that ρ–ω mixing and γ–Z^0 mixing studies could draw upon each other. The diagonalisation to mass eigenstates used for the ρ–ω system would possibly be able to be used in the $\gamma - Z^0$ system (though the node condition, $\Pi_{\gamma Z}(0) = 0$, ensures there is no off-diagonal photon term [90]), although the two poles for this system are very far apart (unlike the $\rho - \omega$ case). The momentum dependent behaviour of $\Pi_{\gamma Z^0}(q^2)$ can be calculated perturbatively and therefore we can examine the behaviour of the non-resonant off-diagonal pieces of the physical propagator, which we cannot in the ρ–ω case.

Chapter 7

Conclusion

QCD is almost universally accepted as the theory for the strong interaction. However, in the low to medium energy region it leads to strong interactions and we cannot use perturbation theory for calculations in the same way that we can for the Electro-Weak theory. For this reason we still need effective models for hadronic systems, many of which predate the invention of QCD. If QCD is the correct theory, then the profound achievement of a solution to it, far from making such models obsolete, will enable us to improve them as we calculate the parameters of appropriate effective Lagrangians from first principles. As quarks and gluons are not the degrees of freedom we observe directly in Nature, effective hadronic models of the strong interaction will remain with us in much the same way as Dirac's equation for the electron did not replace Chemistry. It is the importance of being able to develop a clear and systematic understanding of these models that has motivated the present work.

In the Standard Model (which combines QCD with the Electro-Weak sector) the photon couples to conserved quark currents. In effective Lagrangian models of the photon–hadron interaction, the photon couples to vector mesons and may or may not also couple directly to the matter fields. We saw that there were two versions of the Vector Meson Dominance (VMD) model, one with direct photon-matter coupling (VMD1) and one without (VMD2). With QCD in mind, we concentrated on models in which the vector mesons couple to conserved currents. Our initial study was of the ρ–ω system. We found that for this class of models, $\rho-\omega$ mixing would necessarily vanish at $q^2 = 0$ (the node theorem) and hence conclude that in general q^2 dependence can be expected although it may be small in some models.

Naturally, this constraint would also apply to the mixing between the photon and vector mesons in these "conserved current" models. VMD, though, had traditionally been used with a *constant* coupling of the photon to vector mesons (we referred to this as VMD2), not one that vanished at $q^2 = 0$. It was thus wrongly argued by some that this would decouple the photon from hadronic matter at $q^2 = 0$, thereby ruining the phenomenology. To clarify this confusion, we discussed a representation of VMD in which the coupling of the photon to the vector meson *does* satisfy the node theorem (VMD1) and discussed the field transformations from one effective Lagrangian to the other. In changing from the traditional VMD2 to VMD1 a direct coupling of the photon to hadronic matter is generated thereby ensuring the photon does not decouple from matter at $q^2 = 0$. This raises a central point of this thesis — when performing phenomenological calculations using effective models it is crucial to apply the one effective Lagrangian to the entire calculation of the physical observable. Mixing inconsistent models can lead to meaningless results as the above example clearly shows.

In building a model with couplings to conserved currents, we predicted a momentum dependence for $\rho-\omega$ mixing. However, the experimental determination of this quantity had assumed that it was a constant. We therefore performed a fit to the data for the pion form-factor, with a parametrisation that would take account of momentum dependence. Although we found that the implications of a momentum dependence was a small effect, we discovered through our careful re-analysis that the value obtained for the mixing is very model-dependent (i.e., very dependent on the size of the intrinsic $\omega_I \to \pi\pi$ decay). This highlights the importance of discussing the "extraction" of $\Pi_{\rho\omega}$ in terms of the particular model used, and its inherent assumptions. We feel that this point has perhaps been overlooked in the past.

We now come to prediction and avenues for further work. It has recently been suggested that $\rho-\omega$ mixing could provide an enhancement of CP violation processes in B meson decays. Considering the model dependent nature of the various parameters relevant to $\rho-\omega$ mixing, one would need to take care in applying the information obtained from the pion form-factor data to B decay. Turning to a more familiar application, the implications for CSV in the NN interaction of a node in the mixing of the ρ and ω are very profound indeed. However, keeping in mind our own admonition, a comprehensive treatment of the NN interaction within a single effective Lagrangian framework remains to be done.

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Appendix A

Field current identity

The following is an analysis by Hatsuda [91] of the Field Current Identity (FCI) introduced in Eq. (1.2), that proves it can only be strictly realised on mass-shell. Consider the action for the NJL model [92,93] with covariant derivative $D_\mu = \partial_\mu - iA_\mu$

$$S = \int d^4x \bar{\psi}(i\not{D} - M)\psi - \frac{1}{g^2}(\bar{\psi}\gamma_\mu\psi)^2 + \bar{\eta}\psi + \bar{\psi}\eta + K_\mu\bar{\psi}\gamma_\mu\psi, \tag{A.1}$$

where K_μ is the current source, g is a coupling constant with dimensions of mass and A_μ here is some external four vector electromagnetic field. From this we construct the generating functional

$$Z[\eta, \bar{\eta}, A_\mu, K_\mu] = \int [D\psi D\bar{\psi}] \exp iS. \tag{A.2}$$

To make this relevant to a meson theory, we need to bosonise the fermion field. This can be done by introducing a new field, ρ_μ, with the gaussian weighting

$$\int [D\rho_\mu] \exp i \int d^4x (g\rho_\mu - \bar{\psi}\gamma_\mu\psi/g)^2.$$

This gives us

$$Z = \int [D\psi D\bar{\psi} D\rho_\mu] \exp i \left(\bar{\psi}(i\not{\partial} + \not{A} + \not{K} - 2\not{\rho} - M)\psi + g^2\rho^2 + \bar{\eta}\psi + \bar{\psi}\eta\right). \tag{A.3}$$

Integrating over the fermion fields leads us to

$$Z = \int [D\rho_\mu] \det(i\not{\partial} + \not{A} + \not{K} - 2\not{\rho} - M) \exp i \left(g^2\rho^2 - \bar{\eta}(i\not{\partial} + \not{A} + \not{K} - 2\not{\rho} - M)^{-1}\eta\right). \tag{A.4}$$

Now if we only want ρ in the determinant we make the change of variables

$$A_\mu + K_\mu - 2\rho_\mu \to -\rho_\mu \tag{A.5}$$

to give

$$Z = \int [D\rho_\mu] \det(i\not{\partial} - \not{\rho} - M) \exp i \left(\frac{g^2}{4}(A + K + \rho)^2 - \bar{\eta}(i\not{\partial} - \not{\rho} - M)^{-1}\eta\right) \tag{A.6}$$

$$\sim \int [D\rho_\mu] \exp i \left(-\frac{1}{4}(F_{\mu\nu}(\rho))^2 + \frac{g^2}{4}(A + K + \rho)^2 - \bar{\eta}(i\not{\partial} - \not{\rho} - M)^{-1}\eta\right). \tag{A.7}$$

The first term is the kinetic piece for the ρ field (which comes from an expansion of the determinant [93,94] and neglecting terms higher order in derivatives), the second generates the mass and mixing terms and the third is the interaction piece for the ρ and quarks. We can easily see that this corresponds to VMD2 – the photon-ρ mixing is independent of momentum, and all coupling of the photon to the quark field is mediated by the ρ-meson. Making a field redefinition gives us

$$Z = \int [D\rho_\mu] \exp i \left(-\frac{1}{4}(F_{\mu\nu}(\rho - A - K))^2 + \frac{g^2}{4}\rho^2 - \bar{\eta}(i\partial\!\!\!/ - \rho\!\!\!/ + A\!\!\!/ + K\!\!\!/ - M)^{-1}\eta \right). \quad (A.8)$$

We recognise this as akin to VMD1, with a q^2 dependent mixing between the photon and the ρ and a direct coupling of the photon to the quark field.

We can consider, however, a second bosonisation procedure using the δ-function

$$\int [D\rho] \delta(g^2 \rho_\mu - \bar{\psi}\gamma_\mu\psi) = \int [D\rho D\lambda] \exp i \int d^4x (g^2\rho_\mu - \bar{\psi}\gamma_\mu\psi)\lambda^\mu. \quad (A.9)$$

This δ function strictly imposes the field current identity by demanding that $\rho_\mu \propto J_\mu^{\mathrm{EM}}$ *at the operator level.* So, inserting the δ function in Z (see Eqs. (A.1) and (A.2))

$$Z = \int [D\psi D\bar{\psi} D\rho D\lambda] \exp i \left(\bar{\psi}(i\partial\!\!\!/ + A\!\!\!/ + K\!\!\!/ - \lambda\!\!\!/ - M)\psi + g^2\lambda\cdot\rho - g^2\rho^2 + \bar{\eta}\psi + \bar{\psi}\eta \right), \quad (A.10)$$

where we have replaced the fermion four-point term of Eq. (A.1) by a quadratic ρ term, using the δ function of Eq. (A.9). We can now once again integrate over the fermion fields and expand the determinant to obtain

$$Z \sim \int [D\rho D\lambda] \exp i \left(-\frac{F_{\mu\nu}^2(A + K - \lambda)}{4} + g^2(\lambda\cdot\rho - \rho^2) - \bar{\eta}(i\partial\!\!\!/ + A\!\!\!/ + K\!\!\!/ - \lambda\!\!\!/ - M)^{-1}\eta \right). \quad (A.11)$$

So now making the change of variables

$$A + K - \lambda \to \lambda \quad (A.12)$$

we have

$$Z = \int [D\rho D\lambda] \exp i \left(-\frac{1}{4}F_{\mu\nu}^2(\lambda) + g^2(A + K + \lambda)\cdot\rho - g^2\rho^2 - \bar{\eta}(i\partial\!\!\!/ - \lambda\!\!\!/ - M)^{-1}\eta \right). \quad (A.13)$$

It is now not possible to perform the λ functional integral to leave a sensible form for the functional integral over the ρ field. i.e., the resulting field theory for ρ would not correspond to any acceptable field theory describing a massive vector particle. To see this we recall the discrete result

$$\int \prod_{i=1}^{n} d\phi_i \exp\left(-\sum_{jk} \frac{1}{2}\phi_j A_{jk}\phi_k + \sum_j \phi_j J_j \right) = (2\pi)^{n/2}\sqrt{\det A^{-1}} \exp\left(\sum_{jk} \frac{1}{2} J_j A_{jk}^{-1} J_k \right). \quad (A.14)$$

We extend this to the continuous case to obtain

$$\int [D\phi] \ \exp \int d^4x d^4y - \frac{1}{2}\phi_\mu(x) M^{\mu\nu}(x,y)\phi_\nu(y) + \phi_\mu(x) J^\mu(x)$$
$$\sim \ \sqrt{\det M^{-1}} \exp \int d^4x d^4y \phi^\mu(x) M^{-1}_{\mu\nu} \phi^\nu(y). \tag{A.15}$$

For Eq. (A.13) the operator M is obtained from the gauge field kinetic term for the λ,

$$\int d^4x \int d^4y \lambda_\mu(x) M^{\mu\nu}(x,y) \lambda_\nu(y) = \int d^4x - \frac{1}{4} F_{\mu\nu}(\lambda) F^{\mu\nu}(\lambda)(y) \tag{A.16}$$

to give us

$$M_{\mu\nu}(x,y) = -\frac{1}{2}(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\delta^4(x-y). \tag{A.17}$$

This operator $M_{\mu\nu}$ has zero modes, namely gradient fields. As such it has zero eigenvalues and hence its determinant does not exist. Therefore we cannot use Eq. (A.15). Furthermore, the inverse of M would definitely not have the form to produce a kinetic term for the ρ_μ field. We conclude from this that it is not possible to strictly impose the field current identity in a massive vector meson theory. This is our main conclusion.

As an aside, integration of Eq. (A.13) over ρ_μ brings us back to Eq. (A.7) but now with $\lambda = \rho$. Note however, that this does not impose the δ function condition of Eq. (A.9) and so we have gained nothing.

Having established that the FCI cannot be strictly imposed, we turn our attention to the two-point functions for the current, $J_\mu = \bar{\psi}\gamma_\mu\psi$ and the ρ. To construct the first quantity, we consider

$$\langle J_\mu J_\nu \rangle \equiv \frac{\delta}{\delta K_\mu} \frac{\delta}{\delta K_\nu} \ln Z \Big|_{K=0, A=0, \eta=0}. \tag{A.18}$$

The Lagrangian for a massive vector field is given by [18]

$$\mathcal{L} = -\frac{1}{4}F^2(V) + \frac{1}{2}m^2 V^2. \tag{A.19}$$

So we now consider the action in Eq. (A.7). Comparing the coefficient of the quadratic ρ term with the vector meson mass term in Eq. (A.19), we replace the parameter g (which has dimensions of mass) by $\sqrt{2}m$ to give

$$S = \int d^4x \left[-\frac{1}{4}F(\rho)^2 + \frac{1}{2}m^2(A + K + \rho)^2 - \bar{\eta}(i\partial\!\!\!/ - A\!\!\!/ - M)^{-1}\eta \right]. \tag{A.20}$$

Using this action in Eq. (A.18) we obtain

$$\begin{aligned} \langle J_\mu J_\nu \rangle &= (m^2 \int [D\rho](ig_{\mu\nu} - m^2 \rho_\mu \rho_\nu) e^{iS})/Z \\ &= m^2(ig_{\mu\nu} - m^2 \langle \rho_\mu \rho_\nu \rangle). \end{aligned} \tag{A.21}$$

The second term in Eq. (A.21) is simply the massive vector meson propagator

$$\langle \rho_\mu \rho_\nu \rangle = i \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2} \right) \frac{1}{q^2 - m^2}. \tag{A.22}$$

We therefore have

$$\begin{aligned} \langle J_\mu J_\nu \rangle &= im^2 \left(g_{\mu\nu} + \frac{m^2 g_{\mu\nu} - q_\mu q_\nu}{q^2 - m^2} \right) \\ &= im^2 \left(\frac{q^2 g_{\mu\nu} - q_\mu q_\nu}{q^2 - m^2} \right). \end{aligned} \tag{A.23}$$

As expected, this is transverse. We also note that as $q^2 \to 0$ the piece proportional to $g_{\mu\nu}$ vanishes (the $q_\mu q_\nu$ does not figure in any interactions as we assume coupling to conserved currents). As stated the ρ field two-point function is just the massive vector meson propagator (Eq. (A.22)) and does *not* vanish at $q^2 = 0$. However, at $q^2 = m^2$, both functions in Eqs. (A.22) and (A.23) become singular and differ only by an irrelevant finite piece. It is interesting to note that for a particle with a finite width (that is, an unstable particle such as the ρ meson) even this equality breaks down since the vector propagator is no longer singular on the *real* axis. We can conclude from this that although the FCI cannot be strictly imposed in general, it can be realised on-mass-shell for stable particles

Appendix B

Appendix

B.1 Chiral Perturbation Theory expressions

The full expression for the current, J_μ is given by

$$
\begin{aligned}
J^8_\mu = & (i/2\sqrt{3}\partial K^0\bar{K^0} + i/2\sqrt{3}\partial K^+K^- - i/2\sqrt{3}\partial K^-K^+ \\
& -(i\partial K^0\bar{K^0}K^-K^+)/(\sqrt{3}F^2) - (i\partial K^+K^{-2}K^+)/(\sqrt{3}F^2) \\
& +(i\partial K^-K^-K^{+2})/(\sqrt{3}F^2) - i/2\sqrt{3}\partial\bar{K^0}K^0 \\
& -(i\partial K^0\bar{K^0}^2K^0)/(\sqrt{3}F^2) - (i\partial K^+\bar{K^0}K^-K^0)/(\sqrt{3}F^2) \\
& +(i\partial K^-\bar{K^0}K^+K^0)/(\sqrt{3}F^2) + (i\partial\bar{K^0}K^-K^+K^0)/(\sqrt{3}F^2) \\
& +(i\partial\bar{K^0}\bar{K^0}K^{0^2})/(\sqrt{3}F^2) \\
& +(i/2\sqrt{3/2}\partial\pi_3\bar{K^0}K^+\pi^-)/F^2 - (i/4\sqrt{3}\partial\pi^+K^-K^+\pi^-)/F^2 \\
& +(i/4\sqrt{3}\partial\pi^+\bar{K^0}K^0\pi^-)/F^2 + (i/4\sqrt{3}\partial\pi^-K^-K^+\pi^+)/F^2 \\
& -(i/4\sqrt{3}\partial\pi^-\bar{K^0}K^0\pi^+)/F^2 - (i/2\sqrt{3/2}\partial\pi_3K^-K^0\pi^+)/F^2 \\
& -(i/4\partial K^0\bar{K^0}\pi^-\pi^+)/(\sqrt{3}F^2) - (i/4\partial K^+K^-\pi^-\pi^+)/(\sqrt{3}F^2) \\
& +(i/4\partial K^-K^+\pi^-\pi^+)/(\sqrt{3}F^2) + (i/4\partial\bar{K^0}K^0\pi^-\pi^+)/(\sqrt{3}F^2) \\
& -(i/2\sqrt{3/2}\partial\pi^-\bar{K^0}K^+\pi_3)/F^2 + (i/2\sqrt{3/2}\partial\pi^+K^-K^0\pi_3)/F^2 \\
& -(i/8\partial K^0\bar{K^0}\pi_3^2)/(\sqrt{3}F^2) - (i/8\partial K^+K^-\pi_3^2)/(\sqrt{3}F^2) \\
& +(i/8\partial K^-K^+\pi_3^2)/(\sqrt{3}F^2) \\
& +(i/8\partial\bar{K^0}K^0\pi_3^2)/(\sqrt{3}F^2) - (i/2\partial K^+\bar{K^0}\pi^-\pi_8)/(\sqrt{2}F^2) \\
& +(i/2\partial\bar{K^0}K^+\pi^-\pi_8)/(\sqrt{2}F^2) - (i/2\partial K^0K^-\pi^+\pi_8)/(\sqrt{2}F^2) \\
& +(i/2\partial K^-K^0\pi^+\pi_8)/(\sqrt{2}F^2) + (i/4\partial K^0\bar{K^0}\pi_3\pi_8)/F^2 - \\
& (i/4\partial K^+K^-\pi_3\pi_8)/F^2 + (i/4\partial K^-K^+\pi_3\pi_8)/F^2 - (i/4\partial\bar{K^0}K^0\pi_3\pi_8)/F^2 \\
& -(i/8\sqrt{3}\partial K^0\bar{K^0}\pi_8^2)/F^2
\end{aligned}
$$

$$-(i/8\sqrt{3}\partial K^+K^-\pi_8^2)/F^2 + (i/8\sqrt{3}\partial K^-K^+\pi_8^2)/F^2$$
$$+(i/8\sqrt{3}\partial \bar{K^0}K^0\pi_8^2)/F^2) \quad \text{(B.1)}$$

B.2 Useful Integrals

The following integrals are treated in some detail in Refs. [82,95]. However, Golowich and Kambor expand the expressions in powers of q^2 as required for ChPT.

Let us define the one-point integral, in $D = 4 - \epsilon$ dimensions

$$\frac{i}{16\pi^2}\mu^{D-4}A(m^2) \equiv \int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2 - m^2}, \quad \text{(B.2)}$$

where μ is an arbitrary mass scale required to to keep the action ($\int d^Dx\mathcal{L}_{\rm int}$) dimensionless. Evaluating $A(m^2)$ gives us

$$A = m^2\left(\Delta - \ln\frac{m^2}{\mu^2} + 1\right) + O(\epsilon) \quad \text{(B.3)}$$

where

$$\Delta = \frac{2}{\epsilon} - \gamma + \ln 4\pi. \quad \text{(B.4)}$$

For convenience we define the quantity

$$\sigma = \frac{i}{16\pi^2}. \quad \text{(B.5)}$$

The higher-point functions are, of course, more complicated, but are related in such a way that one can simplify expressions before calculating them explicitly.

$$\sigma\mu^{D-4}B(q^2, m^2) \equiv \int \frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2 - m^2)((k+q)^2 - m^2)} \quad \text{(B.6)}$$
$$\sigma\mu^{D-4}B_\mu(q^2, m^2) \equiv \int \frac{d^Dk}{(2\pi)^D}\frac{k_\mu}{(k^2 - m^2)((k+q)^2 - m^2)} \quad \text{(B.7)}$$
$$\sigma\mu^{D-4}B_{\mu\nu}(q^2, m^2) \equiv \int \frac{d^Dk}{(2\pi)^D}\frac{k_\mu k_\nu}{(k^2 - m^2)((k+q)^2 - m^2)}. \quad \text{(B.8)}$$

From simple Lorentz covariance, we can rewrite these as,

$$B_\mu(q^2, m^2) = q_\mu B_1(q^2, m^2) \quad \text{(B.9)}$$
$$B_{\mu\nu}(q^2, m^2) = q_\mu q_\nu B_{21}(q^2, m^2) + g_{\mu\nu}B_{22}(q^2, m^2) \quad \text{(B.10)}$$

The functions B_{21} and B_{22} can be written in terms of $A(m^2)$ and $B(q^2, m^2)$ [82]

$$B_{21}(q^2, m^2) = \frac{1}{3q^2}\left[A + (q^2 - m^2)B - m^2 + \frac{q^2}{6}\right] \quad \text{(B.11)}$$
$$B_{22}(q^2, m^2) = \frac{1}{6}\left[A + (2m^2 - \frac{q^2}{2})B + 2m^2 - \frac{q^2}{3}\right]. \quad \text{(B.12)}$$

$A(m^2)$ is given in Eq. (B.3) and $B(q^2, m^2)$ is given by

$$B(q^2, m^2) = \Delta - \int_0^1 dx \ln \frac{x(x-1)q^2 + m^2}{\mu^2}. \tag{B.13}$$

We see from Eq. (B.4) that $B(q^2, m^2)$ is divergent. Not only that but, as Golowich and Kambor [95] point out, it should be expanded in powers of q^2 or otherwise our use of it in ChPT will not be consistent. To do this they define

$$\overline{B}(q^2, m^2) \equiv B(q^2, m^2) - B(0, m^2) \tag{B.14}$$
$$= -\int_0^1 dx \ln\left(1 - x(1-x)\frac{q^2}{m^2}\right) \tag{B.15}$$
$$= \frac{1}{6}\frac{q^2}{m^2} + \frac{1}{60}\frac{q^4}{m^4} + \ldots \tag{B.16}$$

We note now that

$$B(0, m^2) = \frac{\partial}{\partial m^2} A(m^2) = \frac{A(m^2)}{m^2} - 1 \tag{B.17}$$

We therefore rewrite Eqs. (B.11) and (B.12) using

$$B(q^2, m^2) = \overline{B}(q^2, m^2) + \frac{A(m^2)}{m^2} - 1. \tag{B.18}$$

We arrive at [95]

$$B_{21}(q^2, m^2) = \frac{1}{3}\left[\left(1 - \frac{m^2}{q^2}\right)\overline{B} + \frac{A}{m^2} - \frac{5}{6}\right] \tag{B.19}$$

$$B_{22}(q^2, m^2) = -\frac{q^2}{12}\left[\left(1 - \frac{4m^2}{q^2}\right)\overline{B} + \frac{A}{m^2}\left(1 - \frac{6m^2}{q^2}\right) - \frac{1}{3}\right]. \tag{B.20}$$

B.3 Calculation for $A^8 \to \pi^+\pi^-$

Now equipped with various ways to handle the integrals appearing in our calculation, we present the relevant details, which would be a distraction in the main body of the text.

We begin by considering Fig. 5.1. We split the contributions to the amplitude into a_μ, b_μ and c_μ (in an obvious way). The outgoing pions are assigned momenta p^+ and p^- and we let k be the loop momentum in amplitudes b_μ and c_μ. From Eq. (5.30) we know that $a_\mu = 0$ to this order in the chiral series. So we turn to the $O(\pi^4)$ pieces of J^8_μ to determine c_μ which is given by,

$$c_\mu = -\frac{\sqrt{3}}{4F^2}\int_k ((p^+ - p^-)_\mu - 2k_\mu)\frac{1}{k^2 - m^2_{K^+}} - [K^+ \leftrightarrow K^0], \tag{B.21}$$

where we have used an obvious notation for the integral over $d^Dk/(2\pi)^D$. In dimensional regularisation, the pieces proportional to k_μ form an odd function and vanish upon integration leaving,

$$c_\mu = -\frac{\sqrt{3}}{4F^2}(p^+ - p^-)_\mu \int_k \frac{1}{k^2 - m_{K^+}^2} - [K^+ \leftrightarrow K^0]. \tag{B.22}$$

The contribution b_μ is slightly more complicated, as we have to consider two ChPT vertices which we shall call V_μ (a four-vector) and S (a scalar). The loop integral now has two propagators,

$$b_\mu = \int_k V_\mu \frac{1}{(k^2 - m_{K^+}^2)((k+q)^2 - m_{K^+}^2)} S - [K^+ \leftrightarrow K^0]. \tag{B.23}$$

From Eq. (5.30)

$$V_\mu = \frac{\sqrt{3}}{2}(2k_\mu + q_\mu). \tag{B.24}$$

This deserves a moment's consideration. If S had no k dependence then b_μ would be proportional to $q_\mu(2B_1 + B)$ which vanishes as $B_1 = -B/2$ [82]. Therefore the only parts of Eq. (B.23) that will survive are those for which S contains k. We find that these terms are

$$S = -\frac{1}{6F^2}(-3p^+\cdot k + 3p^-\cdot k + q\cdot k + k^2). \tag{B.25}$$

We can now write,

$$b_\mu = \frac{\sqrt{3}}{12F^2}\int_k \frac{(2k_\mu + q_\mu)(3(p^+ - p^-)\cdot k - (q\cdot k + k^2))}{(k^2 - m_{K^+}^2)((k+q)^2 - m_{K^+}^2)} - [K^+ \leftrightarrow K^0]. \tag{B.26}$$

Before attempting to evaluate this, it helps to first consider that, because $B_1 = -B/2$, we can add terms independent of k to the numerator of Eq. (B.26), hence

$$\begin{aligned}
\int_k \frac{(2k_\mu + q_\mu)(q\cdot k + k^2)}{(k^2 - m^2)((k+q)^2 - m^2)} &= \frac{1}{2}\int_k \frac{(2k_\mu + q_\mu)(k^2 - m^2 + (q+k)^2 - m^2)}{(k^2 - m^2)((k+q)^2 - m^2)} \\
&= \frac{1}{2}\int_k \left(\frac{2k_\mu + q_\mu}{(q+k)^2 - m^2} + \frac{2k_\mu + q_\mu}{k^2 - m^2}\right) \\
&= \frac{1}{2}\int_k \left(\frac{k_\mu - q_\mu}{k^2 - m^2} + \frac{q_\mu}{k^2 - m^2}\right) \\
&= 0.
\end{aligned}$$

So the only surviving piece of Eq. (B.26) is (recalling the factor σ defined in Eq. (B.5))

$$\begin{aligned}
b_\mu &= \frac{\sqrt{3}}{12F^2}\int_k \frac{3k^\nu(p^+ - p^-)_\nu(2k_\mu + q_\mu)}{(k^2 - m_{K^+}^2)((k+q)^2 - m_{K^+}^2)} - [K^+ \leftrightarrow K^0] \\
&= \frac{\sqrt{3}}{4F^2}\sigma(p^+ - p^-)^\nu(2B_{\mu\nu}(K^+) + q_\mu B_\nu(K^+)) - [K^+ \leftrightarrow K^0] \\
&= \frac{\sqrt{3}}{2F^2}\sigma(p^+ - p^-)^\nu B_{22}(K^+) - [K^+ \leftrightarrow K^0],
\end{aligned} \tag{B.27}$$

as $q\cdot(p^+ - p^-) = 0$.

We now have the expression for the amplitude,

$$\begin{aligned} \mathcal{M}_\mu &= b_\mu + c_\mu \\ &= \frac{\sqrt{3}}{4F^2}\sigma(p^+ - p^-)_\mu(2B_{22}(K^+) - A(K^+) - [K^+ \leftrightarrow K^0]). \end{aligned} \tag{B.28}$$

We now turn to Eqs. (B.3) and (B.20) to find expressions for $A(m^2)$ and $B_{22}(q^2, m^2)$ respectively. Substituting, we find

$$2B_{22}(K^+) - A(K^+) - [K^+ \leftrightarrow K^0] = \frac{q^2}{6}\ln\frac{m_{K^+}^2}{m_{K^0}^2} - \frac{q^4}{60}\left(\frac{1}{m_{K^+}^2} - \frac{1}{m_{K^0}^2}\right). \tag{B.29}$$

Eqs. (B.28) and (B.29) can then be combined to give us an expression for the form-factor $F_\pi^8(q^2)$, Eq. (5.32).

Appendix C

Historical perspective on $\rho-\omega$ mixing

This appendix is devoted to the early studies of $\rho-\omega$ mixing from 1964 to 1972, the period before the Standard Picture (outlined in Chapter 2) had been established. Ironically, the papers quoted here contain detailed investigations of many of the topics that have been discussed in the recent literature (which never referred to any of these early papers) and I will draw attention to this.

C.1 Historical perspective

Here we shall present the initial theoretical investigations of $\rho-\omega$ mixing and explain how this was then used in nuclear CSV. This turns out to shed an interesting light on the current literature about momentum dependence.

C.1.1 The earliest work on $\rho-\omega$ mixing

In chapter 2 we have discussed the earliest work on $\rho-\omega$ mixing, which amounts to two papers. I shall mention these here again. The first proposal of $\rho-\omega$ mixing was by Glashow. Because of the closeness of the ρ and ω masses he proposed that one could expect mixing due to an electromagnetic process (as EM does not respect isospin) [22]. At the time QCD was unknown and the isospin violation due to $m_u \neq m_d$ could not have been appreciated. This was then used as an example for the tadpole mixing scheme of Coleman and Glashow [23] following a suggestion by Julian Schwinger.

Tadpole mixing is independent of momentum and as such would be classed as "mass" (or "particle") mixing. Coleman and Schnitzer (CS) argued that this is actually unsuitable for the mixing of spin 1 particles [96]. Previous studies had discussed mass mixing within the framework of a Schrödinger equation acting on a space of one particle states. CS sought to discuss the mixing within a field-theoretic context. We shall briefly describe their method below as it is important to subsequent treatments of $\rho-\omega$ mixing.

They defined the n channel propagator by

$$\langle 0|T(A_i(x)A_j(y))|0\rangle = -i\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \left[D(k^2)\right]_{ij}. \tag{C.1}$$

In the case of ρ–ω mixing we are considering only a 2 channel propagator (see, for example, Eq. (6.1)). Eq. (C.1) can then be compared with an alternative matrix definition of the n channel propagator,

$$D(q^2) = \frac{1}{q^2 - H(q^2)}. \tag{C.2}$$

The (matrix) dressing function, $H(q^2)$, could then be approximated by $M_0^2 + \delta m^2 + q^2\delta_Z$, where M_0^2 is diagonal, and the other pieces (δm^2 and δ_Z), with their off-diagonal terms generate the mixing between the states. CS noted that for processes occurring in a limited energy range, for example, pion production around the ρ and ω pole region, mass mixing ($H(q^2) = M^2$ where M^2 is a constant) would be a reasonable approximation. However, mass mixing violates the conservation of the current to which the mesons couple. As vector mesons play an important role in models for hadronic EM form-factors, this would have disastrous consequences (such as altering the proton charge). This problem is removed and current conservation maintained if the correction to the propagator vanishes at zero momentum transfer, in agreement with the node theorem discussed in Chapter 4.

A similar study was undertaken by Harte and Sachs (HS) [97]. Like CS, they set up a propagator matrix with off-diagonal pieces, paying particular attention to the role of the complex nature of the poles for unstable particles such as the ρ and ω. HS describe how to obtain the "physical" meson fields from the pure states of definite G-parity, by "diagonalising" the matrix propagator (though we know from Chapter 6 that in general, the propagator matrix cannot be strictly diagonalised). From this one sees that the transformation between bases is not a simple rotation (as in the first paper by Glashow [22]), though due to the closeness of the masses, one can safely make such an approximation. The procedure used by us for analysing the pion form-factor (see Chapter 6) is the same as used by HS and naturally we also find that the transformation between bases is not the simple rotation usually assumed. Sachs and Willemsen [98] also consider the momentum dependence of $\rho-\omega$ mixing in an analysis of various data ($e^+e^- \to \pi^+\pi^-$ and $\pi^+ + p \to \pi^+ + \pi^- + \Delta^{++}$).

Because of this the reader may thus wonder why recent "suggestions" of momentum dependence have attracted such strong resistance. Surely Coleman and Schnitzer had produced a definitive statement on this in 1964? The answer lies in the the fact that with the end of the 1960's ρ–ω mixing ceased to be a topic in particle physics and was brought in, in a rather pragmatic spirit, to nuclear and intermediate energy physics. Study was thus devoted not so much to the mixing itself, but rather the role it could play in building CSV potentials within the one boson exchange model of the NN interaction. However, if momentum dependence was discussed in such detail in the early papers on $\rho-\omega$ mixing it would seem strange for it to have been neglected when the CSV models were built. It is thus interesting to note that the Coon and Barrett [35] paper mentions that $\Pi_{\rho\omega}$ is extracted at its mass-shell value, although merely as an aside. Going back to the earlier paper [28], however, we find the words,

> The mixing ... will vary with meson momenta if the mixing is "current" rather than "mass" mixing. Since we will employ (the mixing amplitude) only at $q^2 = m_\rho^2$ or m_ω^2, however, the difference between the two mixing schemes is negligible. In any case we use the standard mass mixing scheme, since there is no evidence that the more complicated current mixing scheme is necessary.

This certainly demonstrates a familiarity with the theoretical studies mentioned above. The second sentence, though, is never explained, why would the mixing amplitude be set at the mass-shell points? Clearly this would be kinematically unfeasible. To understand this, we have to study the first papers on the use of $\rho-\omega$ mixing in nuclear physics.

C.1.2 CSV and $\rho-\omega$ mixing

The original paper is that of Downs and Nogami (DN) who considered CSV generated by $\rho-\omega$ mixing [99]. Although DN quoted the paper by Coleman and Schnitzer [96], they did not mention anything about possible consequences of momentum dependence in the mixing. But they did discuss the amplitude for the NN interaction in terms of the *physical* vector meson states as compared with the isospin pure states. Henley and Keliher (HK) elaborated on this treatment and included a full discussion of momentum dependence [100]. It is this paper that provided the basis for the Standard Picture.

HK began with physical states, defined in the normal, diagonalised manner. They then, following Sachs and Willemsen [98], gave the mixing parameter, ϵ, a q^2 dependent piece. This represents a slight misunderstanding, as the parameters of the mixing between bases are invariably chosen to be constants (see, for example, Eq. (6.8)) and it is rather $\Pi_{\rho\omega}(q^2)$ that is momentum dependent. This, however, does not affect the rest of the HK analysis, which is presented below. Let (in general we can consider this to be a truncated Taylor series expansion)

$$\Pi_{\rho\omega}(q^2) = \Pi_{\rho\omega}^{(0)} + \frac{q^2}{m_\rho^2}\Pi_{\rho\omega}^{(1)}, \tag{C.3}$$

and note that the node theorem would imply that $\Pi^{(0)}_{\rho\omega} = 0$ (though this was not considered by HK and is immaterial to their argument). Now, ignoring the widths of the vector meson propagators (I shall use $\hat{m}$ to denote a real valued mass), the isospin pure state contribution to nuclear CSV has an amplitude proportional to

$$A_{CSV} = \frac{1}{q^2 - \hat{m}_\rho^2}\Pi_{\rho\omega}(q^2)\frac{1}{q^2 - \hat{m}_\omega^2} \tag{C.4}$$

which can be rewritten

$$A_{CSV} = \frac{\Pi_{\rho\omega}(q^2)}{\hat{m}_\omega^2 - \hat{m}_\rho^2}\left(\frac{1}{q^2 - \hat{m}_\rho^2} - \frac{1}{q^2 - \hat{m}_\omega^2}\right). \tag{C.5}$$

They then made the standard assumption (static limit) $q^0 = 0$ and took the Fourier transform of Eq. (C.5) to obtain the potential, $V_{\rho\omega} = [\tau_3(1) + \tau_3(2)]\Delta v$, where

$$\Delta v(r) = 4\pi g_\rho g_\omega \int \frac{d^3q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \frac{\Pi_{\rho\omega}(-\boldsymbol{q}^2)}{\hat{m}_\omega^2 - \hat{m}_\rho^2}\left[\frac{1}{\boldsymbol{q}^2 + \hat{m}_\rho^2} - \frac{1}{\boldsymbol{q}^2 + \hat{m}_\omega^2}\right]. \tag{C.6}$$

The other crucial part of their argument comes in rewriting Eq. (C.3)

$$\Pi_{\rho\omega}(-\boldsymbol{q}^2) = (\Pi^{(0)}_{\rho\omega} + \Pi^{(1)}_{\rho\omega}) - \Pi^{(1)}_{\rho\omega}(\boldsymbol{q}^2/\hat{m}_\rho^2 + 1) \tag{C.7}$$

the second term on the right hand side of Eq. (C.7) then gives rise to a $\delta^3(\boldsymbol{r})$ function potential. As this term was expected to be ineffective due to the strong repulsion which acts at short distance between nucleons they concluded that it can be ignored and that the first term is the only one that needs to be considered (which from Eq. (C.3) gives us)

$$\Pi^{(0)}_{\rho\omega} + \Pi^{(1)}_{\rho\omega} = \Pi_{\rho\omega}(\hat{m}_\rho^2). \tag{C.8}$$

Hence they argued that it is the value of the mixing amplitude at the timelike mass-shell that is required for a process occurring at spacelike q^2. Clearly the argument is sustained for a mixing obeying the node theorem, in which case $\Pi^{(0)}_{\rho\omega} = 0$. This explains the quote on page 91 and shows how, with time, the mixing came to be (falsely) assumed to be independent of momentum. Eq. (C.6) can then be used to construct the CSV potential of the standard picture [28] (for simplicity consider only the vector part of the meson-nucleon coupling, though there is also a contribution from the tensor coupling)

$$\Delta v(r) = \frac{g_\rho g_\omega}{4\pi}\frac{\langle\rho|H|\omega\rangle}{\hat{m}_\omega^2 - \hat{m}_\rho^2}\left(\frac{e^{-m_\rho r}}{r} - \frac{e^{-m_\omega r}}{r}\right). \tag{C.9}$$

$\langle\rho|H|\omega\rangle$ is used because this is how it appears in Ref. [28], where it is assumed to be a constant. The analogous expression in HK showed that the arguments of this mixing function would be fixed at the ρ and ω mass points for the $e^{-m_\rho r}$ and $e^{-m_\omega r}$ pieces respectively. The above argument ignores the effects due to the ρNN and ωNN form factors and which are always included in actual NN calculations. The above approximations should be contrasted with a more correct treatment discussed elsewhere [36,44].

To summarise the HK treatment, they argued that the appropriate value to use in nuclear CSV models is $\Pi_{\rho\omega}(m_\rho^2)$ extracted in the pion form factor and the momentum dependence is lost in the short distance nucleon interaction. This conclusion is not supported by a more careful analysis.

Appendix D

Published work

Below is a list of my published papers. Following this I have included reprints (where possible).

1) *Self Energy of the Pion*,
Heath B. O'Connell and Anthony W. Thomas, Phys. Rev. C 49 (1994) 548.

2) *Constraints on the momentum dependence of rho-omega mixing*,
H. O'Connell, B. Pearce, A. Thomas and A. Williams, Phys. Lett. B336 (1994) 1.

3) *Rho-omega mixing, vector meson dominance and the pion electromagnetic form-factor*,
H.B. O'Connell, B.C. Pearce, A.W. Thomas, A.G. Williams, hep-ph/9501251, To appear in "Progress in Particle and Nuclear Physics," ed. A. Faessler (Elsevier).

4) *Rho-omega mixing and the pion electromagnetic form-factor*,
H. O'Connell, B. Pearce, A. Thomas, A. Williams, Phys. Lett. B354 (1995) 14.

5) *Vector Meson Mixing and Charge Symmetry Violation*,
H. O'Connell, A. Williams, G. Krein and M. Bracco, Phys. Lett. B 370 (1996) 12.

6) *Analysis of rho-omega mixing and interference in the pion form-factor*,
K. Maltman, H.B. O'Connell and A.G. Williams, Phys. Lett. B376 (1996) 19.

7) *Recent developments in rho-omega mixing*,
Heath B. O'Connell, to appear in the Australian Journal of Physics.

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PHYSICS LETTERS B

Physics Letters B 336 (1994) 1–5

Constraints on the momentum dependence of rho-omega mixing

H.B. O'Connell [1], B.C. Pearce [2], A.W. Thomas [3], A.G. Williams [4]

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Received 18 May 1994; revised manuscript received 28 July 1994
Editor: C. Mahaux

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Published weekly

Printed in The Netherlands

ELSEVIER

15 September 1994

PHYSICS LETTERS B

Physics Letters B 336 (1994) 1–5

Constraints on the momentum dependence of rho-omega mixing

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Received 18 May 1994; revised manuscript received 28 July 1994
Editor: C. Mahaux

Abstract

Within a broad class of models we show that the amplitude for $\rho^0 - \omega$ mixing must vanish at the transition from time-like to space-like four-momentum. Hence in such models the mixing is either zero everywhere or is necessarily momentum-dependent. This lends support to the conclusions of other studies of rho-omega mixing and calls into question standard assumptions about the role of rho-omega mixing in the theoretical understanding of charge-symmetry breaking in nuclear systems.

Charge symmetry violation (CSV) is a small but well established feature of the strong nucleon-nucleon (NN) force [1–3]. The class III force which differentiates the nn and pp systems is best established through the Okamoto-Nolen-Schiffer anomaly in the binding energies of mirror nuclei [4,5]. In the np system the class IV CSV interaction mixes spin-singlet and spin-triplet states. Despite presenting a difficult experimental challenge this has been seen in high precision measurements at TRIUMF and IUCF [6,7]

Although there is still no universally accepted theoretical description of the short and intermediate range NN force, the one-boson-exchange model provides a conceptually simple, yet quantitatively reliable framework [8]. Within that approach $\rho - \omega$ mixing is a major component of both class III and class IV CSV forces [1,3,9–12]. For on-mass-shell vector mesons, $\rho - \omega$ mixing is observed directly in the measurement of the pion form-factor in the time-like region (that is, in the reaction $e^+e^- \to \pi^+\pi^-$ [13]). The best value of the strong interaction contribution to this amplitude at present is $\langle \rho^0 | H_{\rm str} | \omega \rangle = -(5130 \pm 600)\mathrm{MeV}^2$ (on mass shell) from Hatsuda et al. [14]. (A small, calculable, electromagnetic contribution of $\approx 610\mathrm{MeV}^2$ from $\rho \to \gamma \to \omega$ has been subtracted from the data $(-4520 \pm 600\mathrm{MeV}^2)$ to leave the strong mixing amplitude.) Within QCD this provides an important constraint on the mass differences of the u and d quarks [15].

Of course, with respect to the CSV component of the NN force a significant extension is required. In particular, the exchanged vector meson has a space-like momentum, far from the on-shell point. For roughly twenty years it was customary to assume that the $\rho - \omega$ mixing amplitude was a constant over this range of four-momentum. Only a few years ago Goldman, Henderson, and Thomas (GHT) questioned this assumption

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Elsevier Science B.V.
SSDI 0370-2693(94)00991-0

[16]. Within a simple model they showed that the mixing amplitude had a node near $q^2 = 0$ so that neither the sign nor magnitude in the space-like region was determined by the on-shell value. Since the initial work by GHT a qualitatively similar result has been obtained using many theoretical approaches including mixing via an $N\bar{N}$ loop using the $p - n$ mass difference [17], several $q\bar{q}$ calculations [18,19], and an approach using QCD sum-rules [14,20]. All of these calculations revealed a node at or near $q^2 = 0$, with a consequent change in the sign of the mixing amplitude. The presence of this node in the corresponding coordinate space CSV potential has been stressed in Refs. [14,16,19,21]. Related studies of the $\pi^0 - \eta$ mixing have also been recently made including $N\bar{N}$ [22] and $q\bar{q}$ [23] loops, and chiral perturbation theory [24]. Significant momentum dependence was also observed in these studies.

It is important to note that the only calculation which found a node at exactly $q^2 = 0$ was that of Piekarewicz and Williams [17]. In this work alone was local current conservation guaranteed exactly. We have been led to examine the general constraint on the mixing amplitude at $q^2 = 0$ by this observation as well as by several inquiries from K. Yazaki [25]. Our findings can be summarised very easily. We argue that the mixing amplitude vanishes at $q^2 = 0$ in any effective Lagrangian model [e.g., $\mathcal{L}(\boldsymbol{\rho}, \omega, \boldsymbol{\pi}, \bar{\psi}, \psi, \cdots)$], where there are no explicit mass mixing terms [e.g., $M^2_{\rho\omega}\rho^0\omega$ or $\sigma\rho^0\omega$ with σ some scalar field] in the bare Lagrangian and where the vector mesons have a local coupling to conserved currents which satisfy the usual vector current commutation relations. The boson-exchange model of Ref. [17] where, e.g., $J^\mu_\omega = g_\omega \bar{N}\gamma^\mu N$, is one particular example. It follows that the mixing tensor (analogous to the full self-energy function for a single vector boson such as the ρ [26])

$$C^{\mu\nu}(q) = i \int d^4x \, e^{iq\cdot x} \langle 0| T(J^\mu_\rho(x) J^\nu_\omega(0)) |0\rangle . \tag{1}$$

is transverse. Here, the operator J^μ_ω is the operator appearing in the equation of motion for the field operator ω, i.e., the Proca equation given by $\partial_\nu F^{\mu\nu} - M^2_\omega \omega^\mu = J^\mu_\omega$. Note that when J^μ_ω is a conserved current then $\partial_\mu J^\mu_\omega = 0$, which ensures that the Proca equation leads to the same subsidiary condition as the free field case, $\partial_\mu \omega^\mu = 0$ (see, e.g., Lurie, pp. 186-190, [30]). The operator J^μ_ρ is similarly defined. We see then that $C^{\mu\nu}$ can be written in the form,

$$C^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) C(q^2) . \tag{2}$$

From this it follows that the one-particle-irreducible self-energy or polarisation, $\Pi^{\mu\nu}(q)$ (defined through Eq. (6) below), must also be transverse [26]. The essence of the argument below is that since there are no massless, strongly interacting vector particles $\Pi^{\mu\nu}$ cannot be singular at $q^2 = 0$ and therefore $\Pi(q^2)$ (see Eq. (7) below) must vanish at $q^2 = 0$, as suggested for the pure ρ case [27]. As we have already noted this is something that was approximately true in all models, but guaranteed only in Ref. [17].

Let us briefly recall the proof of the transversality of $C^{\mu\nu}(q)$. As shown, for example, by Itzykson and Zuber (pp. 217-224) [28], provided we use covariant time-ordering the divergence of $C^{\mu\nu}$ leads to a naive commutator of the appropriate currents

$$q_\mu C^{\mu\nu}(q) = - \int d^4x \, e^{iq\cdot x} \partial_\mu \left[\theta(x^0) \langle 0| J^\mu_\rho(x) J^\nu_\omega(0) |0\rangle + \theta(-x^0) \langle 0| J^\nu_\omega(0) J^\mu_\rho(x) |0\rangle \right] \tag{3}$$

$$= - \int d^3x \, e^{iq\cdot x} \langle 0| [J^0_\rho(0, \boldsymbol{x}), J^\nu_\omega(0)] |0\rangle_{\text{naive}} . \tag{4}$$

That is, there is a cancellation between the seagull and Schwinger terms. Thus, for any model in which the isovector- and isoscalar-vector currents satisfy the same commutation relations as QCD we find

$$q_\mu C^{\mu\nu}(q) = 0. \tag{5}$$

Thus, by Lorentz invariance, the tensor must be of the form given in Eq. (2).

For simplicity we consider first the case of a single vector meson (e.g. a ρ or ω) without channel coupling. For such a system one can readily see that since $C^{\mu\nu}$ is transverse the one-particle irreducible self-energy, $\Pi^{\mu\nu}$, defined through [26]

$$\Pi^{\mu\alpha} D_{\alpha\nu} = C^{\mu\alpha} D^0_{\alpha\nu} \tag{6}$$

(where D and D^0 are defined below) is also transverse. Hence

$$\Pi^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi(q^2) . \tag{7}$$

We are now in a position to establish the behaviour of the scalar function, $\Pi(q^2)$. In a general theory of massive vector bosons coupled to a conserved current, the bare propagator has the form

$$D^0_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2} \right) \frac{1}{q^2 - M^2} \tag{8}$$

whence

$$(D^0)^{-1}_{\mu\nu} = (M^2 - q^2) g_{\mu\nu} + q_\mu q_\nu . \tag{9}$$

The polarisation is incorporated in the standard way to give the dressed propagator

$$D^{-1}_{\mu\nu} = (D^0)^{-1}_{\mu\nu} + \Pi_{\mu\nu} = (M^2 - q^2 + \Pi(q^2)) g_{\mu\nu} + \left(1 - \frac{\Pi(q^2)}{q^2} \right) q_\mu q_\nu . \tag{10}$$

Thus the full propagator has the form

$$D_{\mu\nu}(q) = \frac{-g_{\mu\nu} + \left(1 - [\Pi(q^2)/q^2]\right) (q_\mu q_\nu / M^2)}{q^2 - M^2 - \Pi(q^2)} . \tag{11}$$

Having established this form for the propagator, we wish to compare it with the spectral representation of the propagator [28–30],

$$D_{\mu\nu}(q) = -i \int_{r_0}^{\infty} dr \frac{\rho(r)}{q^2 - r} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{r} \right) . \tag{12}$$

Since no massless states exist in the strong-interaction sector we must have $r_0 > 0$. Hence it is a straightforward exercise to show that we can write for some function $F(q^2)$ [29]

$$D_{\mu\nu}(q) = F(q^2) g_{\mu\nu} + \frac{1}{q^2} (F(0) - F(q^2)) q_\mu q_\nu . \tag{13}$$

By comparing the coefficients of $g_{\mu\nu}$ in Eqs. (11) and (13) we deduce

$$F(q^2) = \frac{-1}{q^2 - M^2 - \Pi(q^2)} , \tag{14}$$

while from the coefficients of $q_\mu q_\nu$ we have

$$\frac{(1 - \Pi(q^2)/q^2)}{(q^2 - M^2 - \Pi(q^2)) M^2} = \frac{1}{q^2} (F(0) - F(q^2)) = \frac{1}{q^2} \frac{q^2 + \Pi(0) - \Pi(q^2)}{(M^2 + \Pi(0))(q^2 - M^2 - \Pi(q^2))} , \tag{15}$$

from which we obtain

$$\frac{\Pi(0)}{q^2}(q^2 - M^2 - \Pi(q^2)) = 0\,, \quad \forall q^2 \tag{16}$$

and thus

$$\Pi(0) = 0. \tag{17}$$

This embodies the principal result of the investigation, namely that $\Pi(q^2)$ should vanish as $q^2 \to 0$ at least as fast as q^2.

While the preceding discussion dealt with the single channel case, for $\rho - \omega$ mixing we are concerned with two coupled channels. Our calculations therefore involve matrices. As we now demonstrate, this does not change our conclusion.

The matrix analogue of Eq. (10) is

$$D^{-1}_{\mu\nu} = \begin{pmatrix} M^2_\rho g_{\mu\nu} + (\Pi_{\rho\rho}(q^2) - q^2)T_{\mu\nu} & \Pi_{\rho\omega}(q^2)T_{\mu\nu} \\ \Pi_{\rho\omega}(q^2)T_{\mu\nu} & M^2_\omega g_{\mu\nu} + (\Pi_{\omega\omega}(q^2) - q^2)T_{\mu\nu} \end{pmatrix}, \tag{18}$$

where we have defined $T_{\mu\nu} \equiv g_{\mu\nu} - (q_\mu q_\nu / q^2)$ for brevity. By obtaining the inverse of this we have the two-channel propagator

$$D_{\mu\nu} = \frac{1}{\alpha}\begin{pmatrix} s_\omega g_{\mu\nu} + a(\rho,\omega) q_\mu q_\nu & \Pi_{\rho\omega}(q^2)T_{\mu\nu} \\ \Pi_{\rho\omega}(q^2)T_{\mu\nu} & s_\rho g_{\mu\nu} + a(\omega,\rho) q_\mu q_\nu \end{pmatrix}, \tag{19}$$

where

$$s_\omega \equiv q^2 - \Pi_{\omega\omega}(q^2) - M^2_\omega \tag{20}$$

$$s_\rho \equiv q^2 - \Pi_{\rho\rho}(q^2) - M^2_\rho \tag{21}$$

$$a(\rho,\omega) \equiv \frac{1}{q^2 M^2_\rho}\{\Pi^2_{\rho\omega}(q^2) - [q^2 - \Pi_{\rho\rho}(q^2)]s_\omega\} \tag{22}$$

$$\alpha \equiv \Pi^2_{\rho\omega}(q^2) - s_\rho s_\omega. \tag{23}$$

In the uncoupled case $[\Pi_{\rho\omega}(q^2) = 0]$ Eq. (19) clearly reverts to the appropriate form of the one-particle propagator, Eq. (11), as desired.

We can now make the comparison between Eq. (19) and the Renard form [29] of the propagator, as given by Eq. (13). The transversality of the off-diagonal terms of the propagator demands that $\Pi_{\rho\omega}(0) = 0$. A similar analysis leads one to conclude the same for $\Pi_{\rho\rho}(q^2)$ and $\Pi_{\omega\omega}(q^2)$. Note that the physical ρ^0 and ω masses which arise from locating the poles in the diagonalised propagator matrix $D^{\mu\nu}$ no longer correspond to exact isospin eigenstates. To lowest order in CSV the physical ρ-mass is given by $M^{\rm phys}_\rho = [M^2_\rho + \Pi_{\rho\rho}((M^{\rm phys}_\rho)^2)]^{1/2}$, i.e., the pole in $D^{\mu\nu}_{\rho\rho}$. The physical ω-mass is similarly defined.

In conclusion, it is important to review what has and has not been established. There is no unique way to derive an effective field theory including vector mesons from QCD. Our result that $\Pi_{\rho\omega}(0)$ (as well as $\Pi_{\rho\rho}(0)$ and $\Pi_{\omega\omega}(0)$) should vanish applies to those effective theories in which: (i) the vector mesons have local couplings to conserved currents which satisfy the same commutation relations as QCD [i.e., Eq. (4) is zero] and (ii) there is no explicit mass-mixing term in the bare Lagrangian. This includes a broad range of commonly used, phenomenological theories. It does not include the model treatment of Ref. [18] for example, where the mesons are bi-local objects in a truncated effective action. However, it is interesting to note that a node near $q^2 = 0$ was found in this model in any case. The presence of an explicit mass-mixing term in the bare Lagrangian will shift the mixing amplitude by a constant (i.e., by $M^2_{\rho\omega}$). We believe that such a term will lead to difficulties in matching the effective model onto the known behaviour of QCD in the high-momentum limit, [33].

Finally the fact that $\Pi(q^2)$ is momentum-dependent or vanishes everywhere in this class of models implies that the conventional *assumption* of a non-zero, constant $\rho - \omega$ mixing amplitude remains questionable. This study then lends support to those earlier calculations, which we briefly discussed, where it was concluded that the mixing may play a minor role in the explanation of CSV in nuclear physics. It remains an interesting challenge to find possible alternate mechanisms to describe charge-symmetry violation in the *NN*-interaction [31,32].

One of us (AWT) would like to thank Prof. K. Yazaki for several stimulating discussions. This work was supported by the Australian Research Council.

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Self-energy of the pion

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(Received 26 July 1993)

We reexamine a recent calculation of the effect of dressing on the pion propagator in the one-pion-exchange potential. Our results confirm the qualitative features of the earlier work, namely that the correction can be represented as the exchange of an effective π' meson. However, at a quantitative level this approximation does not work well over a wide range of momentum transfer unless the mass of the π' is made too large to be of significance in nucleon nucleon scattering.

PACS number(s): 21.30.+y, 13.75.Cs, 14.40.Aq

There is now considerable evidence that the form factor for the emission of an off-mass-shell pion by a free nucleon is relatively soft [1–3]. In a dipole parametrization a mass less than 1 GeV is typical. On the other hand, conventional one-boson-exchange potentials (OBEP) typically require a much harder πNN form factor in order to reproduce the experimental phase shifts and deuteron properties [4,5]. Recently it has been proven possible to obtain equally good fits with a soft form factor provided an additional, heavy pion (π') exchange is included [6]. (An interesting alternative has been proposed by the Bochum group [7].)

Clearly it is of considerable interest to establish the physical mechanism behind the additional short-distance pseudoscalar exchange. It need not be a real π' meson, but could be a convenient representation of a more complicated short-distance physics involving quark-gluon or quark-meson exchange [8–10]. Saito's novel suggestion was that the radiative corrections associated with the internal structure of the pion itself might lead to a pion propagator that could be simulated by the exchange of an elementary pion *and* a heavier π' [11]. His suggestion echoed earlier work by Goldman *et al.* on the off-shell variation of the ρ-ω mixing amplitude [12]; see also Ref. [13].

In Saito's work the pion proagator was modeled as the propagator of an elementary π meson coupled to a q-$\bar{q}$ pair. As in Refs. [12,13], the propagators of the q-$\bar{q}$ pair were taken to be free Dirac propagators (with quark mass m). While this introduces an unphysical threshold at $2m$, it is not necessarily a fatal flaw in the spacelike region, where we need the propagator for NN scattering. Indeed there is a physical cut which begins at $(m_\pi + m_\rho)$ and by choosing m to be a typical constituent quark mass ($\sim$ 400 MeV) one might expect to simulate the effect of this cut.

In the model of Saito the renormalized pion propagator is written in the form

$$G(q^2) = \frac{i}{q^2 - \Sigma(q^2) - m_\pi^{(0)2}}, \tag{1}$$

where $\Sigma(q^2)$ is

$$\Sigma(q^2) = i6g^2 D\int \frac{d^D k}{(2\pi)^D} \frac{-k^2 + \frac{1}{4}q^2 + m^2}{[(k+\frac{q}{2})^2 - m^2][(k-\frac{q}{2})^2 - m^2 k]}, \tag{2}$$

and the factor of 6 arises from color and isospin. For $D = 4$ the integral in Eq. (2) is highly singular. In Ref. [11] it was rewritten as a sum of three terms:

$$\Sigma = \Sigma_a + \Sigma_b + \Sigma_c\ , \tag{3}$$

where

$$i\Sigma_a = g^2 \frac{3}{4\pi^4} \int d^4 k \frac{1}{(k+\frac{q}{2})^2 - m^2 + i\epsilon}, \tag{4}$$

$$i\Sigma_b = g^2 \frac{3}{4\pi^4} \int d^4 k \frac{1}{(k-\frac{q}{2})^2 - m^2 + i\epsilon}, \tag{5}$$

$$i\Sigma_c = -g^2 q^2 \frac{3}{4\pi^4} \int d^4 k \frac{1}{[(k+\frac{q}{2})^2 - m^2 + i\epsilon][(k-\frac{q}{2})^2 - m^2 + i\epsilon]}. \tag{6}$$

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Under an appropriate change of variables it appears that Σ_a and Σ_b are both independent of q^2 and can be incorporated into the bare mass term. This argument is not correct for a subtle reason. Both of the integrals in Eqs. (4) and (5) are quadratically divergent and it is known from the study of the axial anomaly that linear shifts in the integration variable are not permitted in this case [14]. There is in fact a surface term proportional to q^2. Fortunately for the analysis of Saito, this would affect only Σ_1 and Σ_2 (the second and third terms in a Taylor expansion about $q^2 = 0$), and it is Σ_2 alone which determines $\Sigma_R(q^2)$; see Eq. (12) below.

We have chosen to evaluate Eq. (2) directly using dimensional regularization [15], rather than relying on the expansion (3). Our result for $\Sigma(q^2)$ is:

$$\Sigma(q^2) = \frac{6g^2}{8\pi^2}\int_0^1 dx\, 2p(q^2)\Gamma(\frac{\epsilon}{2}-1) + 7q^2x(1-x) - m^2 + 2p(q^2)\ln[q^2x(1-x)-m^2] \tag{7}$$

where $p(q^2) = 3q^2x(1-x) - m^2$, m being the fermion mass. We can remove the divergences in this expression by adding counter terms to the Lagrangian, and bearing in mind the conditions we wish to impose on the renormalized self-energy, $\Sigma^R(q^2)$, in order that the pion propagator reproduces the physical properties of the pion in free space, namely

$$\Sigma^R(m_\pi^2) = 0, \quad \frac{\partial}{\partial q^2}\Sigma^R(m_\pi^2) = 0, \tag{8}$$

To ensure this we add the following counterterms to the Lagrangian,

$$\mathcal{L}_{\rm CT} = -\frac{1}{2}\alpha\boldsymbol{\pi}\cdot(\Box + m_\pi^2)\boldsymbol{\pi} + \frac{1}{2!}\beta\boldsymbol{\pi}^2, \tag{9}$$

where

$$\alpha = \frac{\partial}{\partial q^2}\Sigma(m_\pi^2),\ \beta = \Sigma(m_\pi^2). \tag{10}$$

This gives us

$$\Sigma^R(q^2) = \Sigma(q^2) - \beta - (q^2 - m_\pi^2)\alpha\ , \tag{11}$$

and

$$G^R(q^2) = \frac{i}{q^2 - m_\pi^2 - \Sigma^R(q^2)}, \tag{12}$$

where $\Sigma^R(q^2)$ vanishes as $(q^2 - m_\pi^2)^2$ at the physical pion mass.

After some algebra we find that $\Sigma^R(q^2)$ takes the form

$$\Sigma^R(q^2) = \frac{6g^2}{4\pi^2}\int_0^1 dx\left[p(q^2)\ln\left(\frac{q^2x(1-x)-m^2}{m_\pi^2x(1-x)-m^2}\right) - (q^2-m_\pi^2)\left(\frac{1}{2} + \frac{2m^2x(1-x)}{m_\pi^2x(1-x)-m^2}\right)\right], \tag{13}$$

which becomes

$$\begin{aligned}\Sigma^R(q^2) = \frac{6g^2}{4\pi^2}\Bigg\{&-2q^2\left(\frac{m^2}{q^2} - \frac{m^2}{m_\pi^2}\right) - \frac{1}{2}(q^2 - m_\pi^2)\\
&+4q^2\left[\left(\frac{m^2}{q^2}-\frac{1}{4}\right)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{q^2}-1}}\right) - \left(\frac{m^2}{m_\pi^2}-\frac{1}{4}\right)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\\
&+(3q^2-4m^2)\left[\sqrt{\frac{m^2}{q^2}-\frac{1}{4}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{q^2}-1}}\right) - \sqrt{\frac{m^2}{m_\pi^2}-\frac{1}{4}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\\
&-2(q^2-m_\pi^2)\frac{m^2}{m_\pi^2}\left[1-4\left(\frac{m^2}{m_\pi^2}\right)\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\Bigg\},\end{aligned} \tag{14}$$

for $0 < q^2 < 4m^2$. For $q^2 < 0$ we have

$$\begin{aligned}\Sigma^R(q^2) = \frac{6g^2}{4\pi^2}\Bigg\{&-2q^2\left(\frac{m^2}{q^2} - \frac{m^2}{m_\pi^2}\right) - \frac{1}{2}(q^2 - m_\pi^2)\\
&+4q^2\left[\frac{1}{2}\left(\frac{m^2}{|q^2|}+\frac{1}{4}\right)^{\frac{3}{2}}\ln\left(\frac{\sqrt{\frac{4m^2}{|q^2|}+1}-1}{\sqrt{\frac{4m+2}{|q^2|}+1}+1}\right) - \left(\frac{m^2}{m_\pi^2}-\frac{1}{4}\right)^{\frac{3}{2}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\\
&+(3q^2-4m^2)\left[\frac{1}{2}\sqrt{\frac{m^2}{|q^2|}-\frac{1}{4}}\ln\left(\frac{\sqrt{\frac{4m^2}{|q^2|}+1}-1}{\sqrt{\frac{4m^2}{|q^2|}+1}+1}\right) + \sqrt{\frac{m^2}{m_\pi^2}-\frac{1}{4}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\\
&-2(q^2-m_\pi^2)\frac{m^2}{m_\pi^2}\left[1-4\left(\frac{m^2}{m_\pi^2}\right)\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\arctan\left(\frac{1}{\sqrt{\frac{4m^2}{m_\pi^2}-1}}\right)\right]\Bigg\},\end{aligned} \tag{15}$$

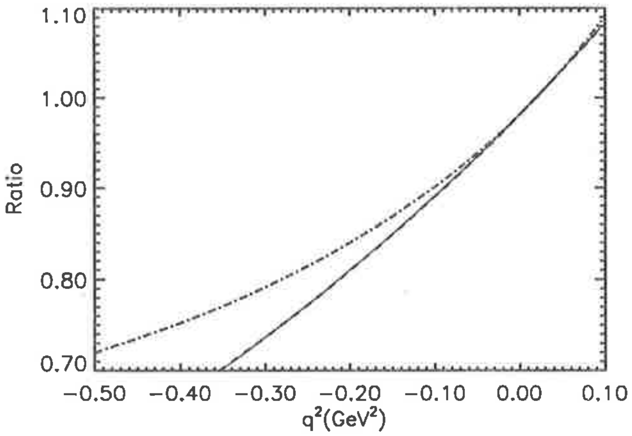

FIG. 1. The ratio of the free to renormalized pion propagators as a function of q^2. The solid curve is our exact result, the dashed curve is a simple parametrization [with mass parameter $\Lambda = 1.295$ GeV; see Eq. (18)], and the dash-dotted curve is the result assuming π plus π' exchange [Eq. (19) with $M = \Lambda$].

An appropriate value for $g_{\pi q}$ can be determined from the pion-nucleon coupling constant $\frac{g_{\pi N}^2}{4\pi} = 14.6$ [16]. An analysis within the consituent quark model yields the following relation

$$g_{\pi q} = \frac{3}{5}\frac{m_q}{m_N} g_{\pi N}. \tag{16}$$

In Fig. 1 we show the ratio (represented by the solid line) of the free to the renormalized pion propagator as a function of q^2. (The quark mass is set at 400 MeV for the reasons explained earlier.) In order to clarify its similarity to the phenomenological introduction of a π' meson we recall that $\Sigma^R(q^2)$ is proportional to $(q^2 - m_\pi^2)^2$. One might then approximate $\Sigma^R(q^2)$ as

$$\Sigma^R(q^2) \sim \frac{c(q^2 - m_\pi^2)^2}{(q^2 - \Lambda^2)}, \tag{17}$$

with c a dimensionless constant and Λ a mass parameter. In this approximation the ratio $R = G(q^2)/G^R(q^2)$ is

$$R = 1 - \frac{c(q^2 - m_\pi^2)}{(q^2 - \Lambda^2)}. \tag{18}$$

The dashed line in Fig. 1 which is almost identical to the solid curve shows the fit obtained for $\Lambda = 1.29$ GeV (with $c = 1.63$).

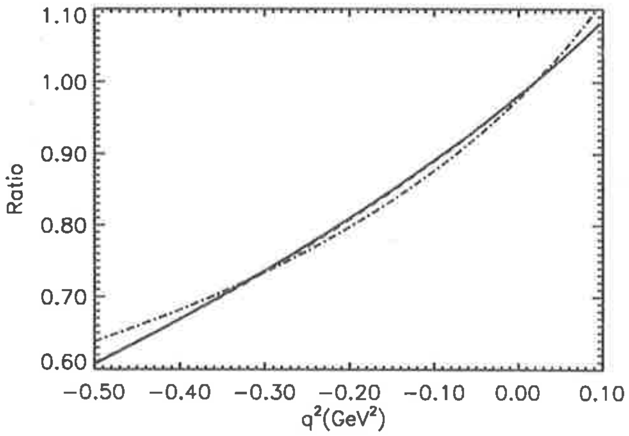

FIG. 2. Same as Fig. 1 but with the parameters in the π' case adjusted to give a best fit [$M = 2.16$ GeV and $C = 5.73$ in Eq. (19)].

If the renormalized pion propagator were to be approximated by the sum of elementary π and π' exchanges (with π' mass M) we should instead find

$$R = \left(1 + C\frac{q^2 - m_\pi^2}{q^2 - M^2}\right)^{-1}, \tag{19}$$

where $C = g_{\pi' N}^2/g_{\pi N}^2$. To first order we would identify $\Lambda = M$ and $c = C$ and the result of this choice is shown as the dot-dashed line in Fig. 1. It clearly is not a good representation of the renormalized propagator. In fact, in order to fit even moderately well over the range of q^2 shown the π' mass must be made considerably larger. Our best fit using Eq. (19) is shown in Fig. 2 where we used a π' mass $M = 2.0$ GeV and $C = 5.73$. (The other two curves are as in Fig. 1.) While the corresponding $\pi' N$ coupling constant is in the range quoted in Ref. [6] the mass is far too large for this π' to play any role in *NN* scattering.

In conclusion, while the very interesting suggestion of Saito has been confirmed qualitatively, we are forced to conclude that this is not the source of the π' meson needed in *NN* scattering.

It is a pleasure to thank Dr. A. A. Rawlinson for many helpful discussions. One of us (A.W.T.) would like to acknowledge the hospitality of J. Speth, K. Holinde, and the theory group at IKP Jülich where this work was completed. This work was supported by the Australian Research Council and the Alexander von Humboldt Foundation.

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O'Connell, H.B., Pearce, B.C., Thomas, A.W., and Williams, A.G., (1997) Rho-omega mixing, vector meson dominance and the pion form-factor.
Progress in Particle and Nuclear Physics, v. 39, pp. 201-252.

Reprinted from

PHYSICS LETTERS B

Physics Letters B 354 (1995) 14–19

Rho–omega mixing and the pion electromagnetic form-factor

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Received 22 March 1995; revised manuscript received 18 May 1995
Editor: C. Mahaux

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amplitude at the ω pole, $\Pi_{\rho\omega}(m_\omega^2)$. The present work improves significantly on an initial analysis reported by us elsewhere [9].

VMD assumes that the dominant role in the interaction of the photon with hadronic matter is played by vector mesons [15,16]. It is an attempt to model non-perturbative interactions determined by QCD, which cannot yet be evaluated in this low-energy regime. The traditional representation of VMD, which we shall refer to as VMD2, is described by a Lagrangian in which the photon couples to hadronic matter *exclusively* through a vector meson, to which it couples with a *fixed* strength proportional to the mass squared of the meson.

For the photon–rho–pion system, the relevant part of the VMD2 Lagrangian is

$$\mathcal{L}_{\mathrm{VMD2}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \tfrac{1}{2}m_\rho^2(\rho_\mu)^2 - g_{\rho\pi\pi}\rho_\mu J_\pi^\mu - \frac{em_\rho^2}{g_\rho}\rho_\mu A^\mu + \frac{1}{2}\left(\frac{e}{g_\rho}\right)^2 m_\rho^2 A_\mu A^\mu, \tag{1}$$

where J_π^μ is the pion current, $(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})_3$, and $F_{\mu\nu}$ and $\rho_{\mu\nu}$ are the EM and ρ field strength tensors (here $e \equiv |e|$). From Eq. (1) one arrives at a pion form-factor of the form

$$F_\pi(q^2) = -\frac{m_\rho^2}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)}\frac{g_{\rho\pi\pi}}{g_\rho}, \tag{2}$$

where conventionally one takes [17–19]

$$\Gamma_\rho(q^2) = \Gamma_\rho\left(\frac{q^2-4m_\pi^2}{m_\rho^2-4m_\pi^2}\right)^{3/2}\frac{m_\rho}{\sqrt{q^2}}. \tag{3}$$

This VMD2 Lagrangian, rederived by Bando et al. [20] from a model based on hidden local gauge symmetry, has some unappealing features. Firstly, the ρ–γ interaction is supposed to be modelling the quark-polarisation of the photon, which necessarily vanishes at $q^2 = 0$ to preserve EM gauge invariance [21], whereas the coupling determined by Eq. (1) is fixed. Hence the VMD2 dressing of the photon propagator shifts the pole away from zero, and a bare photon mass must be introduced into the Lagrangian to counterbalance this and ensure that the dressed photon is massless. Secondly, recent studies [19,22] have used a non-resonant term (i.e., a contribution in which the ρ does not appear), which VMD2 lacks, in fits to $e^+e^- \to \pi^+\pi^-$. Thirdly, the constraint $F_\pi(0) = 1$, which reflects the fact that the photon sees only the charge of the pion at zero momentum transfer, is only realised by Eq. (2) in the limit of universality ($g_\rho = g_{\rho\pi\pi}$), which is seen to be only approximate in nature [23].

For these reasons we prefer the alternative formulation [16] which we shall call VMD1 [9] and which is apparently less widely known. It has the following Lagrangian

$$\mathcal{L}_{\mathrm{VMD1}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \tfrac{1}{2}m_\rho^2\rho_\mu\rho^\mu - g_{\rho\pi\pi}\rho_\mu J_\pi^\mu - eA_\mu J_\pi^\mu - \frac{e}{2g_\rho}F_{\mu\nu}\rho^{\mu\nu}. \tag{4}$$

The key features of this representation are the absence of a photon mass term and the presence of a term $F_{\mu\nu}\rho^{\mu\nu}$, which produces a momentum-dependent γ–ρ coupling of the form [9], $\mathcal{L}_{\gamma\rho} = -e/2g_\rho F_{\mu\nu}\rho^{\mu\nu} \to -e/g_\rho q^2 A_\mu \rho^\mu$. This, of course, decouples the photon from the ρ at $q^2 = 0$, hence keeping the photon massless in a natural way. However, the photon is still able to couple to the hadronic current through the direct coupling $-eA_\mu J_\pi^\mu$, giving us a non-resonant term. We now have a form-factor of the form

$$F_\pi(q^2) = 1 - \frac{q^2 g_{\rho\pi\pi}}{g_\rho[q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)]}. \tag{5}$$

Note that Eq. (5) automatically satisfies $F_\pi(0) = 1$. We emphasise that a simple field redefinition maps these two forms of VMD into one another if universality is taken to be exact and if we work to all orders in perturbation theory. Of course, the predictions will differ somewhat if universality is violated and if the perturbation expansion for the models is truncated in an inconsistent way. We illustrate the difference between the two representations in Fig. 1.

At present the widely quoted value of for $\Pi_{\rho\omega} \equiv \Pi_{\rho\omega}(m_\omega^2)$ [10], is obtained from the branching ratio formula for the ω, $B(\omega\to\pi\pi) = \Gamma(\omega\to\pi\pi)/\Gamma(\omega)$, derived from a ρ-ω mixing analysis where $\Gamma(\omega\to\pi\pi) = |\Pi_{\rho\omega}/im_\rho\Gamma_\rho|^2\Gamma(\rho\to\pi\pi)$. Using the branching ratio determined in 1985 by the Novosibirsk group [17], $B(\omega\to\pi\pi) = (2.3\pm0.4\pm0.2)\%$, Coon and Barrett obtained $\Pi_{\rho\omega} = -4520\pm600$ MeV^2. We aim to extract $\Pi_{\rho\omega}$ from a fit to the cross-section of the reaction $e^+e^-\to\pi^+\pi^-$ using

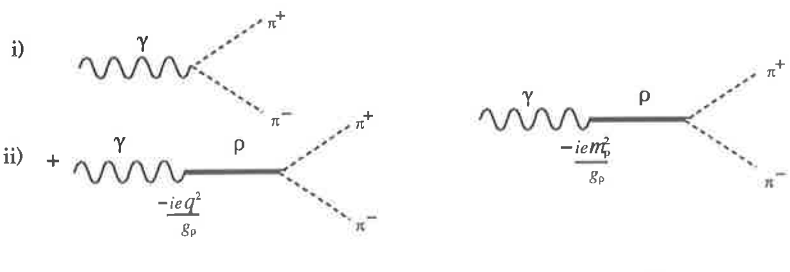

Fig. 1. Contributions to the pion form-factor in the two representations of vector meson dominance a) VMD1 b) VMD2.

$$\sigma(q^2) = \frac{\alpha^2\pi}{3}\frac{(q^2-4m_\pi^2)^{3/2}}{s^{5/2}}|F_\pi(q^2)|^2, \tag{6}$$

and the form-factor determined by VMD1 (Eq. (5)).

So far, we have not introduced any effects of charge symmetry violation (CSV) into our system, and hence the ω (which cannot otherwise couple to a $\pi^+\pi^-$ state) does not appear. In their recent examination of the EM pion and nucleon form-factors using VMD1, Dönges et al. [18] introduced the ω through a covariant derivative in the pion kinetic term. This produces a direct contribution from $\omega \to 2\pi$ without any ρ–ω mixing, but does not provide a good representation of data in the resonance region. We shall use the mixed propagator [7], where the mixing is introduced by an off-diagonal piece, $\Pi_{\rho\omega}$ in the vector meson self-energy. To first order in CSV [i.e., to $\mathcal{O}(\Pi_{\rho\omega})$], the propagator is given by (we ignore pieces proportional to q_μ as we couple to conserved currents)

$$D_{\mu\nu} = -\begin{pmatrix} 1/s_\rho & \Pi_{\rho\omega}/s_\rho s_\omega \\ \Pi_{\rho\omega}/s_\rho s_\omega & 1/s_\omega \end{pmatrix} g_{\mu\nu}, \tag{7}$$

where $s_\rho \equiv q^2 - \Pi_{\rho\rho}(q^2) - m_\rho^2 \equiv q^2 - m_\rho^2 + im_\rho\Gamma_\rho(q^2)$, and similarly for the ω.

In a matrix notation, the Feynman amplitude for the process $\gamma \to \pi\pi$, proceeding via vector mesons, can be written in the form

$$i\mathcal{M}_\mu^{\gamma\to\pi\pi} = \left(i\mathcal{M}^\nu_{\rho_I\to\pi\pi}\; i\mathcal{M}^\nu_{\omega_I\to\pi\pi}\right) \cdot iD_{\nu\mu}\begin{pmatrix} i\mathcal{M}_{\gamma\to\rho_I} \\ i\mathcal{M}_{\gamma\to\omega_I}\end{pmatrix}, \tag{8}$$

where the matrix $D_{\nu\mu}$ is given by Eq. (7) and the other Feynman amplitudes are derived from $\mathcal{L}_{\mathrm{VMD1}}$. We will now make the standard simplification [24] which is that the direct decay of the isospin pure ω to two pions cancels the imaginary piece of the two pion loop contribution to the mixing self-energy. This is based on some reasonable assumptions, but is a point worthy of further study in its own right. Since it is beyond the scope of the present work we do not pursue this further here. Accepting these arguments means that we can neglect the pure isospin state ω_I coupling to two pions ($\mathcal{M}^\nu_{\omega_I\to\pi\pi} = 0$) with the understanding that it is the real part of the mixing amplitude that is being extracted. To lowest order in the mixing, Eq. (8) becomes

$$\mathcal{M}^\mu_{\gamma\to\pi\pi} = \mathcal{M}^\mu_{\rho_I\to\pi\pi}\frac{1}{s_\rho}\mathcal{M}_{\gamma\to\rho_I} + \mathcal{M}^\mu_{\rho_I\to\pi\pi}\frac{1}{s_\rho}\Pi_{\rho\omega}\frac{1}{s_\omega}\mathcal{M}_{\gamma\to\omega_I}, \tag{9}$$

which we recognise as the sum of the two diagrams shown in Fig. 2.

The couplings that enter this expression, through $\mathcal{M}^\mu_{\rho_I\to\pi\pi}$, $\mathcal{M}_{\gamma\to\rho_I}$ and $\mathcal{M}_{\gamma\to\omega_I}$, always involve the unphysical pure isospin states ρ_I and ω_I. However, we can re-express Eq. (9) in terms of the physical states by first diagonalising the vector meson propagator. This leads to the result

$$\begin{aligned}\mathcal{M}^\mu_{\gamma\to\pi\pi} &= \mathcal{M}^\mu_{\rho\to\pi\pi}\frac{1}{s_\rho}\mathcal{M}_{\gamma\to\rho} + \mathcal{M}^\mu_{\omega\to\pi\pi}\frac{1}{s_\omega}\mathcal{M}_{\gamma\to\omega} \\ &= \mathcal{M}^\mu_{\rho\to\pi\pi}\frac{1}{s_\rho}\mathcal{M}_{\gamma\to\rho} + \mathcal{M}^\mu_{\rho\to\pi\pi}\frac{\Pi_{\rho\omega}}{s_\rho - s_\omega}\frac{1}{s_\omega}\mathcal{M}_{\gamma\to\omega},\end{aligned} \tag{10}$$

which is the form usually seen in older works. Although at first glance there seems to be a slight dis-

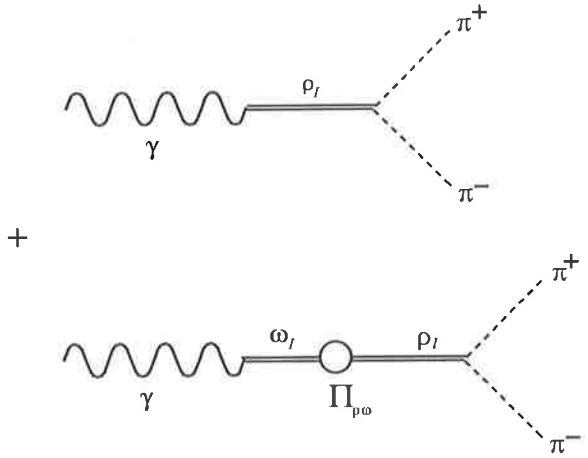

Fig. 2. The contribution of ρ–ω mixing to the pion form-factor from the states of pure isospin. These diagrams are present in addition to those in Fig. 1a).

crepancy between Eqs. (9) and (10) they are equivalent – e.g., see Ref. [9] and the discussion of the Orsay phase by Coon et al. [8].

We are now in a position to write down the CSV form-factor based on the VMD1 form-factor of Eq. (5) and the mixed state contribution of Eq. (10),

$$F_\pi(q^2) = 1 - \frac{q^2 g_{\rho\pi\pi}}{g_\rho[q^2 - m_\rho^2 + i m_\rho \Gamma_\rho(q^2)]} - \frac{q^2 \epsilon g_{\rho\pi\pi}}{g_\omega[q^2 - m_\omega^2 + i m_\omega \Gamma_\omega]}, \qquad (11)$$

where

$$\epsilon = \frac{\Pi_{\rho\omega}}{s_\rho - s_\omega} = \frac{\Pi_{\rho\omega}}{m_\omega^2 - m_\rho^2 - i(m_\omega \Gamma_\omega - m_\rho \Gamma_\rho(q^2))}. \qquad (12)$$

The ω decay formula of Coon and Barrett can now be seen to follow from Eq. (11) with an approximation for ϵ (namely that Γ_ω is very small and that $m_\rho^2 = m_\omega^2$). Because the width of the ω is very small we can safely neglect any momentum dependence in it, and simply use $\Gamma_\omega(m_\omega^2)$ [9,19].

All parameters except $\Pi_{\rho\omega}$ are fixed by various data as discussed below. The results of fitting this remaining parameter to the data are shown in Fig. 3 with the resonance region shown in close-up in Fig. 4. The mass and width of the ω are as given by the Particle Data Group (PDG) [26], $m_\omega = 781.94 \pm 0.12$ MeV

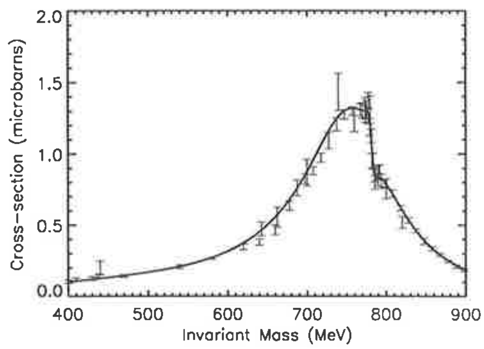

Fig. 3. Cross-section of $e^+e^- \to \pi^+\pi^-$ plotted as a function of the energy in the centre of mass. The experimental data is from Refs. [17,25].

and $\Gamma_\omega = 8.43 \pm 0.10$ MeV. There has recently been considerable interest in the value of the ρ parameters, m_ρ and Γ_ρ with studies showing that the optimal values [19,22] may differ slightly from those given by the PDG. The value of $\Pi_{\rho\omega}$ is not sensitive to the masses and widths, and we have obtained a good fit with $m_\rho = 772$ MeV and $\Gamma_\rho = 149$ MeV, which are close to the PDG values.

The values of the coupling constants are however quite important for an extraction of $\Pi_{\rho\omega}$. We obtain g_ρ and $g_{\rho\pi\pi}$ from $\Gamma(\rho \to e^+e^-) \sim 6.8$ MeV and $\Gamma(\rho \to \pi\pi) \sim 149$ MeV: $g_{\rho\pi\pi}^2/4\pi \sim 2.9$, $g_\rho^2/4\pi \sim 2.0$ which show, for example, that universality is not strictly obeyed (as mentioned previously). VMD1 and

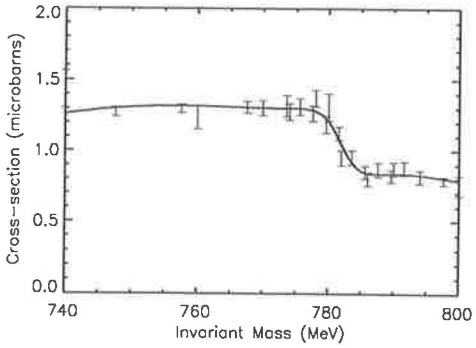

Fig. 4. Cross-section for $e^+e^- \to \pi^+\pi^-$ in the region around the resonance where ρ–ω mixing is most noticeable. The experimental data is from Refs. [17,25].

VMD2 differ at order $g_{\rho\pi\pi}/g_\rho \simeq 1.2$.

Historically the ratio g_ω/g_ρ was believed to be around 3 [27], a figure supported in a recent QCD-based analysis [28]. Empirically though, the ratio can be determined [22] from leptonic partial rates [26] giving

$$\frac{g_\omega}{g_\rho} = \sqrt{\frac{m_\omega \Gamma(\rho \to e^+e^-)}{m_\rho \Gamma(\omega \to e^+e^-)}} = 3.5 \pm 0.18. \qquad (13)$$

Using these parameters we obtain a best fit around the resonance region shown in Fig. 4 (χ^2/d.o.f. = 14.1/25) with $\Pi_{\rho\omega} = -3800$ MeV^2. In this analysis there are two principle sources of error in the value of $\Pi_{\rho\omega}$. The first is a statistical uncertainty of 310 MeV^2 for the fit to data, and the second, of approximately 200 MeV^2, is due to the error quoted in Eq. (13). Adding these in quadrature gives us a final value for the *total* mixing amplitude, to be compared with the value -4520 ± 600 MeV^2 obtained by Coon and Barrett [10]. We find

$$\Pi_{\rho\omega} = -3800 \pm 370 \text{ MeV}^2. \qquad (14)$$

It is now clear that a momentum dependent γ^*–ρ coupling, together with a direct coupling of the photon to hadronic matter, yields an entirely adequate model of the pion form-factor. In fact, this picture is basically suggested by attempts to examine the γ^*–ρ coupling via a quark loop. Model calculations typically find that the loop is momentum-dependent, and vanishes at $q^2 = 0$ (unless gauge invariance is spoiled by form-factors, or something of this nature). However, coupling the photon to quarks in the loop implies that the photon must also couple to the quarks in hadronic matter, thus introducing a direct photon-hadron coupling (independent of the ρ-meson), and leads us to take VMD1 as the preferred representation of vector meson dominance. It should now be clear that the appropriate representation of vector meson dominance to be used in combination with mixing amplitudes that vanish at $q^2 = 0$ is VMD1. To use VMD2 in conjunction with such vector mixing amplitudes is inconsistent. As long as one is clear on this point, there are no dire consequences for momentum dependence in ρ–ω mixing.

One of us (HOC) would like to thank M. Benayoun (Collège de France, Paris) for helpful discussions. This work was supported by the Australian Research Council.

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PHYSICS LETTERS B

7 March 1996

Physics Letters B 370 (1996) 12–16

Vector meson mixing and charge symmetry violation

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Received 25 October 1995; revised manuscript received 13 December 1995

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Received 25 October 1995; revised manuscript received 13 December 1995
Editor: C. Mahaux

Abstract

We discuss the consistency of the traditional vector meson dominance (VMD) model for photons coupling to matter, with the vanishing of vector meson-meson and meson-photon mixing self-energies at $q^2 = 0$. This vanishing of vector mixing has been demonstrated in the context of rho-omega mixing for a large class of effective theories. As a further constraint on such models, we here apply them to a study of photon-meson mixing and VMD. As an example we compare the predicted momentum dependence of one such model with a momentum-dependent version of VMD discussed by Sakurai in the 1960's. We find that it produces a result which is consistent with the traditional VMD phenomenology. We conclude that comparison with VMD phenomenology can provide a useful constraint on such models.

PACS: 12.40.Vv; 13.60.-r; 11.30.Hv; 14.80.Dq
Keywords: Vector meson; Rho-omega mixing; Vector meson dominance; Charge symmetry

The experimental extraction of $\Pi_{\rho\omega}$ (in the pion EM form-factor [1]) is in the timelike q^2 region around the ρ–ω mass, yet it is used to generate charge symmetry violation (CSV) in boson exchange models of the *NN* interaction in the spacelike region [2,3]. The traditional assumption was that the mixing amplitude was independent of q^2.

This assumption was first questioned by Goldman et al. [4] who constructed a model in which the ρ and ω mixed via a quark loop contribution which is non-vanishing if and only if $m_u \neq m_d$. Their conclusion of a significant momentum dependence was subsequently supported by other studies, which included an analogous *NN*-loop calculation [5] using the n–p mass difference and more elaborate quark-loop model calculations [6]. All of these predicted a similar momentum-dependence for $\Pi_{\rho\omega}(q^2)$ with a node near the origin ($q^2 = 0$). At a more formal level, it was subsequently shown that the vector-vector mixings must identically vanish at $q^2 = 0$ in a large class of effective theories [7] where the mixing occurs exclusively through coupling of the vector mesons to conserved currents and where the vector currents commute in the usual way. Recent work in chiral perturbation theory and QCD sum rules has also suggested that such mixing matrix elements must, in general, be expected to be momentum dependent [8].

In response to this, alternative mechanisms involving CSV have been proposed [9]. Indeed, as the vector mesons are off shell, the individual mechanisms

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SSDI 0370-2693(95)01579-5

should not be examined in isolation, because they are dependent on the choice of interpolating fields for the vector mesons and are not physical quantities. It has been argued that one could find a set of interpolating fields for the rho and omega such that *all* nuclear CSV occurs through a constant ρ–ω mixing with the CSV vertex contributions vanishing [10]. However this possibility has been questioned on the grounds of unitarity and analyticity [11].

The same models which have been used to examine the question of ρ–ω mixing can also be applied to studies of ρ–γ mixing. They can then be compared to phenomenology and vector meson dominance (VMD) models, which have *traditionally* assumed the coupling of the photon to the rho was independent of q^2. The first person to raise this question was Miller [12]. The purpose of this letter is to carefully explore the issues raised and compare numerical predictions for such a mixing model with experimental data. As discussed recently [13], the appropriate representation of VMD to use with a momentum dependent photon-rho coupling is VMD1, given by the Lagrangian [14]

$$\mathcal{L}_1 = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \tfrac{1}{2}m_\rho^2\rho_\mu\rho^\mu - g_{\rho\pi\pi}\rho_\mu J^\mu - eA_\mu J^\mu - \frac{e}{2g_\rho}F_{\mu\nu}\rho^{\mu\nu} + \dots , \quad (1)$$

where J_μ is the hadronic current and $F_{\mu\nu}$ and $\rho_{\mu\nu}$ are the EM and ρ field strength tensors respectively (the dots refer to the hadronic part of the Lagrangian). From this we obtain the VMD1 expression for the form-factor for the pion [13]

$$F_\pi^{(1)}(q^2) = 1 - \frac{q^2 g_{\rho\pi\pi}}{g_\rho(q^2 - m_\rho^2 + im_\rho\Gamma_\rho)} . \quad (2)$$

Note that in the limit of exact universality $g_{\rho\pi\pi} = g_\rho$ and we recover the usual VMD2 model prediction for the pion form-factor [13,14]. Recall that in this traditional VMD (i.e., VMD2) model the photon couples to hadrons *only* through first coupling to vector mesons with a *constant* coupling strength, e.g., for the ρ–γ coupling we have $\Pi_{\rho\gamma}^{\mathrm{VMD2}}(q^2) \equiv -m_\rho^2 e/g_\rho$.

We shall define a VMD1-like model to be one in which the photon couples to the hadronic field both directly and via a q^2-dependent coupling (with a node at $q^2 = 0$) to vector mesons. A VMD1-like model may differ from pure VMD1 as the coupling of the photon to the rho (generated by some microscopic process) will not generally be linear in q^2. Hence g_ρ, which is a constant in VMD1 (and VMD2 as they share the same g_ρ [13,14]), may acquire some momentum dependence in a VMD1-like model; the test for the phenomenological validity of the model is then that this momentum dependence for g_ρ is not too strong. For example, we can easily determine the coupling of the photon to the pion field via the rho meson for a for a VMD1-like model. We note the appearance in Eq. (3) of the photon-rho mixing term, $\Pi_{\rho\gamma}^{\mu\nu}(q^2)$, which can be determined from Feynman rules, and which will, in general, be q^2-dependent. Such an analysis gives for any VMD1-like model

$$\begin{aligned} -i\mathcal{M}^\mu(q^2) &\equiv -ie(p^+ - p^-)_\sigma[D_\gamma(q^2)]^{\mu\sigma}F_\pi(q^2) \\ &= i[D_\gamma(q^2)]^{\mu\sigma}i[\Gamma_{\gamma\pi}(q^2)]_\sigma + i[D_\gamma(q^2)]^{\mu\sigma} \\ &\quad \times i[\Pi_{\gamma\rho}(q^2)]_{\sigma\tau}i[D_\rho(q^2)]^{\tau\nu}i[\Gamma_{\rho\pi}(q^2)]_\nu \\ &= -ie(p^+ - p^-)_\sigma i[D_\gamma(q^2)]^{\mu\sigma} \\ &\quad \times \left(1 + \frac{\Pi_{\rho\gamma}(q^2)}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho}\frac{g_{\rho\pi\pi}}{e}\right) , \end{aligned} \quad (3)$$

where D, Π and Γ denote propagators, one-particle irreducible mixing amplitudes and proper vertices, respectively. Here p^+ and p^- are the outgoing momenta of the π^+ and π^-, respectively. For this model to reproduce the phenomenologically successful VMD, and hence provide a good fit to the data (assuming exact universality), $\Pi_{\rho\gamma}(q^2)$ and g_ρ must be related by (comparing Eqs. (2) and (3))

$$\Pi_{\rho\gamma}^{\mathrm{VMD1}}(q^2) = -\frac{q^2 e}{g_\rho(q^2)} . \quad (4)$$

Note that this result then implies that $\Pi_{\rho\gamma}^{\mathrm{VMD1}}(q^2) = (q^2/m_\rho^2)\Pi_{\rho\gamma}^{\mathrm{VMD2}}(q^2)$. Eq. (4) arises from the simple VMD1 picture when universality is assumed and is also consistent with the usual VMD2 picture as explained elsewhere [13,14].

Thus Eq. (4) is the central equation of this work, since vector-meson mixing models (e.g., ρ–ω mixing) can also be used to calculate ρ–γ mixing and then confronted with traditional VMD phenomenology. The results quoted in the review by Bauer et al. [15] are summarized in Tables I and XXXII of that reference. They list a range of values which vary depending on the details of the fit to the ρ mass (m_ρ) and width (Γ_ρ). Within the context of the traditional VMD (i.e., VMD2) framework they extract $g_\rho^2(q^2=0)/4\pi$ from ρ^0 photoproduction ($\gamma p \to \rho^0 p$) and $g_\rho^2(q^2 = m_\rho^2)/4\pi$ from $\rho^0 \to e^+e^-$. The three sets of results quoted are (in an obvious shorthand notation): Γ_ρ = 135, 145, 155 MeV, m_ρ = 767, 774, 776 MeV, $g_\rho^2(q^2 = 0)/4\pi = 2.43\pm0.10, 2.27\pm0.23, 2.18\pm0.22$, $g_\rho^2(q^2 = m_\rho^2)/4\pi = 2.21\pm0.017, 2.20\pm0.06, 2.11\pm0.06$, respectively. We see that g_ρ is a free parameter of the traditional VMD model (VMD2) which is adjusted to fit the available cross section data. The central feature of the VMD2 model is that it presumes a constant value for its coupling constant g_ρ. We note in passing that the universality condition is $g_\rho \sim g_{\rho\pi\pi} \sim g_{\rho NN}^{\rm univ} \sim g_{\rho\rho\rho}$ and where experimentally we find [15,16] for each of these $g^2/4\pi \sim 2$. For example, the values of $g_{\rho\pi\pi}$ corresponding to the above three sets of results are $g_{\rho\pi\pi}^2(q^2 = m_\rho^2)/4\pi$ = 2.61, 2.77, 2.95 and are extracted from $\rho^0 \to \pi^+\pi^-$. It should be noted that the ρNN interaction Lagrangian is here defined as in Refs. [3,5] with no factor of two [14,16] and hence $g_{\rho NN} = g_{\rho NN}^{\rm univ}/2$. As a typically used value is $g_{\rho NN}^2/(4\pi) = 0.41$ we see that universality is not accurate to better than 40% in g_ρ^2, which corresponds to $\simeq 20\%$ in g_ρ.

The results of the VMD2 analysis [15] are approximately consistent with g_ρ being a constant and so we see from Eq. (4) that $\Pi_{\rho\gamma}$ in VMD1-like models should not deviate too strongly from behaviour linear with q^2.

We shall now examine the process within the context of the model used by Piekarewicz and Williams (PW) who considered ρ–ω mixing as being generated by a nucleon loop [5] within the Walecka model. Using nucleon loops as the intermediate states removes the formation of unphysical thresholds in the low q^2 region and allows us to use well-known parameters. The rho-coupling is not a simple, vector coupling, but rather [17]

$$\Gamma_{\rho NN}^\mu = g_{\rho NN}\gamma^\mu + i\frac{f_{\rho NN}}{2M}\sigma_{\mu\nu}q^\nu \,, \tag{5}$$

where $C_\rho \equiv f_{\rho NN}/g_{\rho NN} = 6.1$ and M is the nucleon mass. With the introduction of tensor coupling the model is no longer renormalizable, but to one loop order we can introduce some appropriate renormalization prescription. As the mixings are transverse, we write $\Pi_{\mu\nu}(q^2) = (g_{\mu\nu} - q_\mu q_\nu/q^2)\Pi(q^2)$ [7]. The photon couples to charge, like a vector and so, unlike the PW calculation, we have only a proton loop to consider. Here we can safely neglect the coupling of the photon to the nucleon magnetic moment and so there is no neutron loop contribution nor any tensor-tensor contribution to the proton loop. This sets up two kinds of mixing, vector-vector $\Pi_{\rm vv}^{\mu\nu}$ and vector-tensor $\Pi_{\rm vt}^{\mu\nu}$, where (using dimensional regularization with the associated scale, μ)

$$\Pi_{\rm vv}(q^2) = -q^2\frac{eg_{\rho NN}}{2\pi^2}\left[\frac{1}{3\epsilon} - \frac{\gamma}{6} - \int_0^1 dx\, x(1-x)\ln\left(\frac{M^2 - x(1-x)q^2}{\mu^2}\right)\right], \tag{6}$$

$$\Pi_{\rm vt}(q^2) = -q^2\frac{eg_{\rho NN}}{8\pi^2}\left[\frac{1}{\epsilon} - \gamma - \int_0^1 dx\, \ln\left(\frac{M^2 - x(1-x)q^2}{\mu^2}\right)\right]. \tag{7}$$

Note that these functions vanish at $q^2 = 0$, as expected from the node theorem since we have coupling to conserved currents [7]. To remove the divergence and scale-dependence we add a counter-term

$$\mathcal{L}_{\rm CT} = e\frac{g_{\rho NN}C_T}{2\pi^2}\rho_{\mu\nu}F^{\mu\nu}$$

to the Lagrangian in a minimal way so as to renormalize the model to one loop. This will contribute $-iC_T g_{\rho NN} e q^2/\pi^2$ to the photon-rho vertex, which will add to the contribution $i\Pi$ generated by the nucleon loop. The counter-term will contain pieces proportional to $1/\epsilon$, γ and $\ln\mu^2$ to cancel the similar terms in Eqs. (6) and (7), and a constant piece, β, which will be chosen to fit the extracted value for $g_\rho(0)$. The counter-term is

$$C_T = -\frac{1}{\epsilon}\left(\frac{1}{6} + \frac{C_\rho}{8}\right) + \gamma\left(\frac{1}{12} + \frac{C_\rho}{8}\right) - \left(\frac{1}{12} + \frac{C_\rho}{8}\right)\ln\mu^2 + \beta \,, \tag{8}$$

which gives us the renormalized mixing,

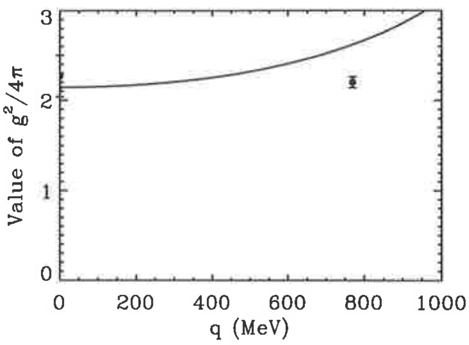

Fig. 1. The PW model prediction for the mixing amplitude is related to the traditional VMD coupling $g_\rho(q^2)$ using the central result of Eq. (4). The resulting behaviour of $g_\rho^2/4\pi$ versus $q \equiv \sqrt{q^2}$ is then plotted in the timelike region for this model. Shown for comparison are a typical pair of results (2.27 ± 0.23 at $q = 0$ and 2.20 ± 0.06 at $q = m_\rho$, see text) taken from a traditional VMD based analysis of cross section data in Ref. [15].

$$\Pi_{\rho\gamma}(q^2) = q^2 \frac{e g_{\rho NN}}{\pi^2} \left[\frac{1}{2} \left(\frac{5}{18} + \frac{2M^2}{3q^2} - \frac{8M^4 + 2M^2 q^2 - q^4}{3q^3\sqrt{4M^2 - q^2}} \arctan \sqrt{\frac{q^2}{4M^2 - q^2}} - \frac{\ln M^2}{6} \right) + \frac{C_\rho}{8} \left(-2 + 2\sqrt{\frac{4M^2 - q^2}{q^2}} \arctan \sqrt{\frac{q^2}{4M^2 - q^2}} + \ln M^2 \right) - \beta \right]. \quad (9)$$

We find that the choice $\beta = 8.32$ in our counter-term approximately reproduces the extracted value of $g_\rho(0)$ at $q^2 = 0$.

The results for $g_\rho(q^2)$ for the PW model are shown in Fig. 1. We see that despite this model having a node in the photon-rho mixing at $q^2 = 0$ the resulting q^2 dependence of g_ρ is small. As can be seen from this plot, we obtain values of $g_\rho^2(0)/(4\pi) = 2.14$ and $g_\rho^2(m_\rho^2)/(4\pi) = 2.6$ compared to the experimental averages 2.3 and 2.17, respectively.

It should be remembered that Eq. (4) is only as reliable as universality, which is itself violated at a level of (30–40)% . Based on this important observation, we can conclude then that the PW model provides a result consistent with the spread of extracted results given in Ref. [15]. It should be noted that any VMD1-like model which predicts a significantly greater deviation from linearity with q^2 will fail to reproduce phenomenology because of Eq. (4).

In summary, we have explicitly shown in Eq. (4) that the vanishing of vector-vector mixing at $q^2 = 0$ is completely consistent with the standard phenomenology of vector meson dominance (VMD). We have, in addition, applied the same type of model used in a study of ρ–ω mixing to extract the momentum dependence of ρ–γ mixing and have compared the result to the VMD2 based analysis of the experimental data. We see that the phenomenological constraints of VMD can provide a useful independent test of VMD-like models of vector mixing and future studies should take adequate account of this. It would, of course, be preferable to reanalyze the data used in Ref. [15] from the outset using VMD1 rather than VMD2, but this more difficult task is left for future investigation.

A.G.W. and H.O.C. would like to thank A.W. Thomas for helpful discussions and a careful reading of the manuscript. AGW acknowledges the hospitality of the IFT, S ao-Paulo, Brazil where part of this work was carried out. This work was supported in part by FAPESP and CNPq (Brazil) and the Australian Research Council.

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Published in Phys. Lett. B376 (1996) 19.
ADP-95-50/T197
hep-ph/9601309

Analysis of rho-omega interference in the pion form-factor

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14 February, 1996

The formalism underlying the analysis of $e^+e^- \to \pi^+\pi^-$ in the $\rho-\omega$ interference region is carefully revisited. We show that the standard neglect of the pure $I=0$ omega, ω_I, "direct" coupling to $\pi\pi$ is not valid, and extract those combinations of the direct coupling and ρ-ω mixing allowed by experiment. The latter is shown to be only very weakly constrained by experiment, and we conclude that data from the $e^+e^- \to \pi^+\pi^-$ interference region *cannot* be used to fix the value of $\rho-\omega$ mixing in a model-independent way unless the errors on the experimental phase can be significantly reduced. Certain other modifications of the usual formalism necessitated by the unavoidable momentum-dependence of $\rho-\omega$ mixing are also discussed.

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Keywords: vector mesons, pion form-factor, mixing, isospin.
PACS: 11.20.Fm, 12.40.Vv, 13.60.Le, 13.65.+i

The cross-section for $e^+e^- \to \pi^+\pi^-$ in the $\rho-\omega$ resonance region displays a narrow interference shoulder resulting from the superposition of narrow resonant ω and broad resonant ρ exchange amplitudes [1]. The strength of the ω "interference" amplitude has generally been taken to provide a measurement of ρ_I-ω_I mixing (where ρ_I, ω_I are the pure isovector ρ and isoscalar ω states) [2,3]. The extracted mixing has then been used to generate ρ_I-ω_I mixing contributions to various few-body observables [4–6], a program which, combined with estimates for other sources of isospin-breaking, produces predictions for few-body isospin breaking in satisfactory accord with experiment [5]. The phenomenological success, for those observables for which ρ_I-ω_I contributions are significant, rests, inextricably, on two assumptions, (1) that the interference amplitude is dominated by ρ_I-ω_I mixing (i.e., negligible "direct" $\omega_I \to \pi\pi$ contribution to the physical ω decay amplitude) and (2) that the resulting mixing amplitude is independent of momentum-squared, so the extracted value can be used unchanged in meson-exchange forces in few-body systems, where $q^2 < 0$.

The neglect of "direct" $\omega_I \to \pi\pi$ coupling (i.e., coupling which does not go via mixing with the ρ_I) can actually be re-interpreted physically, this re-interpretation simultaneously providing the conventional justification for taking the ρ_I-ω_I self-energy, $\Pi^{\rho\omega}$, to be real in modern analyses of $e^+e^- \to \pi^+\pi^-$ [7,8]. As will become clear below, however, corrections to the underlying argument, usually thought to be small, have unexpectedly large effects on the extraction of the $\rho-\omega$ mixing contribution from experimental data.

The assumption of the q^2-independence of $\Pi^{\rho\omega}(q^2)$ is more problematic [9,10]. In general, one knows that a system of, e.g., nucleons, vector mesons and pseudoscalar mesons, can be described by an effective low-energy Lagrangian, constructed so as to be compatible with QCD (e.g., one might think of the effective chiral Lagrangian, $\mathcal{L}_{\rm eff}$, obtainable via the Coleman-Callan-Wess-Zumino construction [11]). Such a Lagrangian, involving terms of arbitrarily high order in derivatives, will produce momentum-dependence in all observables which can in principle become momentum-dependent. This has been seen explicitly for the off-diagonal (mixing) elements of meson propagators by a number of authors, employing various models [12,13], as well as QCD sum rule and Chiral Perturbation Theory (ChPT) techniques [14]. Such q^2-dependence has also been shown to be consistent with the usual vector meson dominance (VMD) framework [15]. The possibility [16] that an alternative choice of interpolating fields might, nonetheless, correspond to the standard assumption of q^2-independence has been shown to be incompatible with the constraints of unitarity and analyticity [17]. It is thus appropriate to revisit and generalize the usual analysis.

As has been known for some time, to obtain properties of unstable particles which are process-independent and physically meaningful, one determines the locations of the resonance poles in the amplitude under consideration, and makes expansions about these pole locations [18]. The (complex) pole locations are properties of the S-matrix and hence *independent of the choice of interpolating fields*, and the separate terms in the Laurent expansion about the pole position have well-defined physical meaning [18]. The importance of such an "S-matrix" formalism for characterizing resonance properties has been stressed recently by a number of authors in the context of providing gauge- and process-independent definitions of the Z^0 mass and width in the Standard Model [19,20]. For our purposes this means that: (1) the "physical" $\{\rho,\ \omega\}$ fields are to be identified as those combinations of the $\{\rho_I,\, \omega_I\}$ fields containing the corresponding S-matrix poles and (2) to analyze $e^+e^- \to \pi^+\pi^-$ one should include both resonant terms involving the complex ρ and ω pole locations (and hence constant widths) and "background" (i.e. non-resonant) terms. In quoting experimental results we will, therefore, restrict ourselves to analyses which, as closely as possible, satisfy these requirements. To our knowledge, only one such exists: the fifth fit of Ref. [21] (performed explicitly in the S-matrix formalism, though without an s-dependence to the background). As stressed in Ref. [21], using the S-matrix formalism, one finds a somewhat lower real part for the (complex) ρ pole position ($\hat{m}_\rho = 757.00 \pm 0.59$, $\Gamma_\rho = 143.41 \pm 1.27$ MeV) than is obtained in conventional, non-S-matrix formalism treatments. For comparison below we will also employ the results of the second fit of the more conventional (but non-S-matrix) formalism of Ref. [22], which employs an s-dependent background, an s-dependent ρ width, and imposes the (likely too large) Particle Data Group value for the ρ mass by hand.

Let us turn to the question of $\rho-\omega$ mixing in the presence of a q^2-dependent off-diagonal element of the self-energy matrix. We shall work consistently to first order in isospin breaking (generically, $\mathcal{O}(\epsilon)$), which will mean to first order in $\Pi_{\rho\omega}$. The dressing of the bare, two-channel meson propagator has been treated in Ref. [10].

As we consider vector mesons coupled to conserved currents, we can replace $D_{\mu\nu}(q^2)$ by $-g_{\mu\nu}D(q^2)$. We refer to $D(q^2)$ as the "scalar propagator". We assume that the isospin-pure fields ρ_I and ω_I have already been renormalized, i.e., that the relevant counterterms have been absorbed into the mass and wavefunction renormalizations. Taking then the full expression for the dressed propagator and keeping terms to $\mathcal{O}(\epsilon)$, one finds

$$D^I(q^2) = \begin{pmatrix} D^I_{\rho\rho} & D^I_{\rho\omega} \\ D^I_{\rho\omega} & D^I_{\omega\omega} \end{pmatrix} = \begin{pmatrix} (q^2 - \Pi_{\rho\rho}(q^2))^{-1} & D^I_{\rho\omega}(q^2) \\ D^I_{\rho\omega}(q^2) & (q^2 - \Pi_{\omega\omega}(q^2))^{-1} \end{pmatrix}, \tag{0.1}$$

where the renormalized self-energies $\Pi_{kk}(q^2) \to m_k^2$ as $q^2 \to m_k^2$. Defining $\Pi^{(0)}_{kk}(q^2) = \Pi_{kk}(q^2) - m_k^2$, we then have $\Pi^{(0)}_{kk}(q^2) = \mathcal{O}[(q^2 - m_k^2)^2]$. From the complex pole positions, m_k^2, we define the (real) mass ($\hat{m}_k$) and width (Γ_k) via, $m_k^2 \equiv \hat{m}_k^2 - i\,\hat{m}_k\Gamma_k$. To $\mathcal{O}(\epsilon)$, $D^I_{\rho\omega}(q^2)$, is then [10]

$$D^I_{\rho\omega}(q^2) = \frac{\Pi_{\rho\omega}(q^2)}{(q^2 - m_\rho^2 - \Pi^{(0)}_{\rho\rho}(q^2))(q^2 - m_\omega^2 - \Pi^{(0)}_{\omega\omega}(q^2))} = D^I_{\rho\rho}(q^2)\Pi_{\rho\omega}(q^2)D^I_{\omega\omega}(q^2), \tag{0.2}$$

which contains both a broad ρ resonance and narrow ω resonance piece.

As explained above, the physical ρ and ω fields are defined to be those combinations of the ρ_I and ω_I for which only the diagonal elements of the propagator matrix contain poles, in the ρ, ω basis. This definition is, in fact, implicit in the standard interpretation of the $e^+e^- \to \pi^+\pi^-$ experiment, which associates the broad resonant part of the full amplitude with the ρ and the narrow resonant part with the ω. Using different linear combinations of ρ_I, ω_I, (call them ρ', ω') than those given above (ρ, ω), one would find also narrow resonant structure in the off-diagonal element of the vector meson propagator in the $\{\rho', \omega'\}$ basis, preventing, for example, the association of the narrow resonant behaviour with the ω' pole term alone.

We define the transformation between the physical and isospin pure bases by (to $\mathcal{O}(\epsilon)$)

$$\rho = \rho_I - \epsilon_1\,\omega_I, \qquad \omega = \omega_I + \epsilon_2\,\rho_I \tag{0.3}$$

where, in general, $\epsilon_1 \neq \epsilon_2$ when the mixing is q^2-dependent. With $D^{\mu\nu}_{\rho\omega}(x-y) \equiv -i\langle 0|T(\rho^\mu(x)\omega^\nu(y))|0\rangle$, one then has for the scalar propagator, to $\mathcal{O}(\epsilon)$,

$$D_{\rho\omega}(q^2) = D^I_{\rho\omega}(q^2) - \epsilon_1 D^I_{\omega\omega}(q^2) + \epsilon_2 D^I_{\rho\rho}(q^2). \tag{0.4}$$

The condition that $D_{\rho\omega}(q^2)$ contain no ρ or ω pole then fixes $\epsilon_{1,2}$ to be

$$\epsilon_1 = \frac{\Pi_{\rho\omega}(m_\omega^2)}{m_\omega^2 - m_\rho^2 - \Pi^{(0)}_{\rho\rho}(m_\omega^2)}, \quad \epsilon_2 = \frac{\Pi_{\rho\omega}(m_\rho^2)}{m_\omega^2 - m_\rho^2 + \Pi^{(0)}_{\omega\omega}(m_\rho^2)}. \tag{0.5}$$

When $\Pi^{\rho\omega}(q^2)$ is q^2-dependent, we thus see explicitly that $\epsilon_1 \neq \epsilon_2$; the relation between the isospin-pure and physical bases is not a simple rotation. This is a universal feature of q^2-dependent mixing in field theory. Recall that $\Pi^{(0)}_{\rho\rho}(q^2)$ and $\Pi^{(0)}_{\omega\omega}(q^2)$ vanish by definition as $q^2 \to m^2_{\rho,\omega}$ at least as fast as $(q^2 - m^2_{\rho,\omega})^2$. The usual assumption is that these two quantities are zero in the vicinity of the resonance region, which leads to the standard Breit-Wigner form for the vector meson propagators. $\Pi^{(0)}_{\rho\rho}(q^2)$ and $\Pi^{(0)}_{\omega\omega}(q^2)$ are, of course, momentum-dependent in general since the vector propagators must be real below the $\pi\pi$ and $\pi\gamma$ thresholds. Note that, from Eqs. (0.4) and (0.5), any deviation from the Breit-Wigner form and/or any non-linearity in the q^2-dependence of $\Pi_{\rho\omega}(q^2)$ will produce a non-zero off-diagonal element of the vector propagator *even in the physical basis.* This means that a background (non-resonant) term is completely unavoidable even in the traditional VMD framework, where all contributions are associated with vector meson exchange. Moreover, in general, this background will be s- (i.e., q^2)-dependent. Finally, even in the vicinity of the ρ and ω poles, where it should be reasonable to set $\Pi^{(0)}_{\rho\rho}(q^2)$ and $\Pi^{(0)}_{\omega\omega}(q^2)$ to zero, the ρ_I admixture into the physical ω is governed, not by $\Pi^{\rho\omega}(m_\omega^2)$ as usually assumed, but by $\Pi^{\rho\omega}(m_\rho^2)$.

The time-like EM pion form-factor is given, in the interference region, by

$$F_\pi(q^2) = \left[g_{\omega\pi\pi} D_{\omega\omega} \frac{f_{\omega\gamma}}{e} + g_{\rho\pi\pi} D_{\rho\rho} \frac{f_{\rho\gamma}}{e} + g_{\rho\pi\pi} D_{\rho\omega} \frac{f_{\omega\gamma}}{e} \right] + \text{background}, \tag{0.6}$$

where $g_{\omega\pi\pi}$ is the coupling of the *physical* omega to the two pion final state and $f_{\rho\gamma}$ and $f_{\omega\gamma}$ are the electromagnetic ρ and ω couplings. The third piece of Eq. (0.6), $g_{\rho\pi\pi} D_{\rho\omega} f_{\omega\gamma}$, results from the non-vanishing of the off-diagonal element of the *physical* meson propagator and, being non-resonant, can be absorbed into the background, for the purposes of our discussion, as can any deviations from the Breit-Wigner form for the ρ and ω propagators. Since the variation of q^2 over the interference region is tiny, we can presumably also safely neglect any q^2-dependence of $f_{\rho\gamma}$, $f_{\omega\gamma}$, $g_{\rho\pi\pi}$ and $g_{\omega\pi\pi}$. $f_{V\gamma}$ is related to the "universality coupling" [15], g_V, of traditional VMD treatments by $f_{V\gamma} = -e\hat{m}^2/g_V$.

We now focus on the resonant ω exchange contribution, whose magnitude and phase, relative to the resonant ρ exchange, are extracted experimentally. We have

$$g_{\omega\pi\pi} = \langle \pi\pi | \omega_I + \epsilon_2 \rho_I \rangle = g_{\omega_I\pi\pi} + \epsilon_2 g_{\rho_I\pi\pi}, \tag{0.7}$$

where ϵ_2 is given in Eq. (0.5) or, equivalently, by $\epsilon_2 = -i\, z\Pi_{\rho\omega}(m_\rho^2)/\hat{m}_\rho\Gamma_\rho$, where

$$z \equiv \left[1 - \frac{\hat{m}_\omega \Gamma_\omega}{\hat{m}_\rho \Gamma_\rho} - i\left(\frac{\hat{m}_\omega^2 - \hat{m}_\rho^2}{\hat{m}_\rho \Gamma_\rho} \right) \right]^{-1}. \tag{0.8}$$

Note that $z \approx 1$ but equals 1 only if we neglect the ω width and $\rho - \omega$ mass difference. This brings us to the Renard argument [7]. Since, in general, $g_{\omega_I\pi\pi} \neq 0$, $\Pi_{\rho\omega}(q^2)$ must contain a contribution from the intermediate $\pi\pi$ state which, because essentially the entire ρ width is due to the $\pi\pi$ mode, is given by

$$\Pi^{2\pi}_{\rho\omega}(m_\rho^2) = \frac{g_{\omega_I\pi\pi}}{g_{\rho_I\pi\pi}} \Pi^{2\pi}_{\rho\rho}(m_\rho^2) = G(\mathrm{Re}\Pi^{2\pi}_{\rho\rho}(m_\rho^2) - i\hat{m}_\rho\Gamma_\rho), \tag{0.9}$$

where $G = g_{\omega_I\pi\pi}/g_{\rho_I\pi\pi}$ is the ratio of the ρ_I and ω_I couplings to $\pi\pi$. In arriving at Eq. 0.9 we have used the facts that (1) the imaginary part of the ρ self-energy at resonance ($q^2 = m_\rho^2$) is, by definition, $-\hat{m}_\rho\Gamma_\rho$, and (2) $g_{\rho\pi\pi} = g_{\rho_I\pi\pi}$ to $\mathcal{O}(\epsilon)$. We have then, defining $\tilde{\Pi}_{\rho\omega}$ by $\Pi_{\rho\omega} = \tilde{\Pi}_{\rho\omega} - iG\hat{m}\Gamma_\rho$,

$$\epsilon_2 = z\frac{-i}{\hat{m}_\rho\Gamma_\rho}[\tilde{\Pi}_{\rho\omega}(m_\rho^2) - iG\hat{m}_\rho\Gamma_\rho] \tag{0.10}$$

and hence

$$g_{\omega\pi\pi} = g_{\omega_I\pi\pi}(1-z) + \tilde{\epsilon}_2 g_{\rho_I\pi\pi}, \tag{0.11}$$

where $\tilde{\epsilon}_2 = (-iz/\hat{m}_\rho\Gamma_\rho)\tilde{\Pi}_{\rho\omega}(m_\rho^2)$. We shall also define, for convenience,

$$\tilde{T} \equiv \tilde{\Pi}_{\rho\omega}(m_\rho^2)/\hat{m}_\rho\Gamma_\rho. \tag{0.12}$$

The standard Renard analysis [7] involves approximating z by 1. The contribution to $\omega \to \pi\pi$ from the intrinsic ω_I decay is then exactly cancelled in Eq. (0.11). Using the (preferred) experimental analysis of Ref. [21], however, we find

$$z = 0.9324 + 0.3511\,i\ . \tag{0.13}$$

(For comparison, the analysis of Ref. [22] gives $1.023+0.2038i$). Because of the substantial imaginary part, the intrinsic decay cannot be neglected in $e^+e^- \to \pi^+\pi^-$.

Substituting the results above into Eq. (0.6), we find

$$F_\pi(q^2) = \frac{f_{\rho\gamma}}{e} g_{\rho_I\pi\pi}\left[|r_{\mathrm{ex}}|e^{i\phi_{e^+e^-}}\left((1-z)G - iz\tilde{T}\right)P_\omega + P_\rho\right] + \text{ background}, \tag{0.14}$$

where we have replaced the propagators $D_{\rho\rho,\omega\omega}$ of Eq. (0.6) with the simple Breit-Wigner pole terms $P_{\rho,\omega} \equiv 1/(p^2 - m^2_{\rho,\omega})$, and where

$$r_{\mathrm{ex}} \equiv \frac{f_{\omega\gamma}}{f_{\rho\gamma}} = |r_{\mathrm{ex}}|e^{i\phi_{e^+e^-}}, \tag{0.15}$$

with $\phi_{e^+e^-}$ the "leptonic phase" (to be discussed in more detail below). Experimentally,

$$|r_{\mathrm{ex}}| = \left[\frac{\hat{m}^3_\omega\Gamma(\omega \to e^+e^-)}{\hat{m}^3_\rho\Gamma(\rho \to e^+e^-)}\right]^{1/2} = 0.30 \pm 0.01 \tag{0.16}$$

using the values found in Ref. [21]. The form of $F_\pi(q^2)$ in Eq. (0.14) is what is required for comparison with experimental data [21], for which one has

$$F_\pi \propto P_\rho + Ae^{i\phi}P_\omega; \quad A = -0.0109 \pm 0.0011; \quad \phi = (116.7 \pm 5.8)^\circ. \tag{0.17}$$

One can now see that the uncertainty in the Orsay phase, ϕ, makes a precise extraction of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ impossible. Indeed, the two contributions to the ω exchange amplitude (i.e., multiplying P_ω) have either nearly the same phase or differ in phase by close to π (depending on the relative signs of G and $\tilde{T}$). In either case, a large range of combinations of G and $\tilde{T}$, all producing nearly the same overall phase, will produce the same value of A. The experimental data can thus place only rather weak constraints on the relative size of the two contributions, as we will see more quantitatively below.

Let us write $r_{\rm ex}$, the ratio of electromagnetic couplings, in terms of the corresponding isospin-pure ratio, $r_I = f_{\omega_I\gamma}/f_{\rho_I\gamma}$. Using $f_{\omega\gamma} = f_{\omega_I\gamma} + \epsilon_2 f_{\rho_I\gamma}$ and $f_{\rho\gamma} = f_{\rho_I\gamma} - \epsilon_1 f_{\omega_I\gamma}$, one finds $r_{\rm ex} = (r_I + \epsilon_2)/(1 - \epsilon_1 r_I)$, where r_I is real. To $\mathcal{O}(\epsilon)$ one then has

$$\sin\phi_{e^+e^-} = \frac{{\rm Im}(\epsilon_2) + |r_{\rm ex}|^2{\rm Im}(\epsilon_1)}{|r_{\rm ex}|}. \tag{0.18}$$

Ignoring the small difference in ϵ_1 and ϵ_2 (since $r_{\rm ex}^2$ is small) we obtain

$$\sin\phi_{e^+e^-} = \frac{(1+|r_{\rm ex}|^2){\rm Im}\epsilon_2}{|r_{\rm ex}|}. \tag{0.19}$$

In order to simplify the discussion of our main point, which is the effect of including the direct coupling on the experimental analysis, let us now make the usual assumption that the imaginary part of $\Pi_{\rho\omega}$ is dominated by $\pi\pi$ intermediate states. (Note, however, that, because the argument is complex, there may be an imaginary part of $\Pi_{\rho\omega}$ even in the absence of real intermediate states; for example, in the model of Ref. [13], with confined quark propagators, the phase of the quark loop contribution to $\Pi_{\rho\omega}(m_\rho^2)$ is about $-13^{\rm o}$ [23], despite the model having, for this contribution, no available intermediate states.) Making this assumption, $\tilde{\Pi}_{\rho\omega}$ (and thus $\tilde{T}$) becomes pure real and the imaginary part of $\Pi_{\rho\omega}(m_\rho^2)$ reduces to $-G\hat{m}_\rho\Gamma_\rho$. Using Eqs. (0.10) and (0.19) the leptonic phase becomes

$$\sin\phi_{e^+e^-} = -\left(\frac{1+|r_{\rm ex}|^2}{|r_{\rm ex}|}\right)(\tilde{T}\,{\rm Re}\,z + G\,{\rm Im}\,z) \tag{0.20}$$

which is completely fixed by G and $\tilde{\Pi}_{\rho\omega}$. For each possible $\tilde{\Pi}_{\rho\omega}$, only one solution for G both gives the correct experimental magnitude for the ω exchange amplitude (A) and has a phase lying in the second quadrant, as required by experiment. Knowing $\tilde{\Pi}_{\rho\omega}$ and G, Eqn. (0.20) allows us to compute the total phase, ϕ. Those pairs $(\tilde{\Pi}_{\rho\omega}, G)$ producing the experimentally allowed (A, ϕ) constitute our full solution set.

The results of the above analysis are presented in Fig. 1, where we have used as input the results of the analysis of Ref. [21], for the reasons explained above. The spread in G values reflects the experimental error in A. We will supplement the experimental constraints by imposing the theoretical prejudice $-0.05 < G < 0.05$. We see that, barring theoretical input on the precise size of G, experimental data is incapable of providing even reasonably precise constraints on the individual magnitudes of G and $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$. The reason for this situation has been explained above. If we fix A at its central value, the experimental phase alone would restrict $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ to the range (-1090 MeV^2, -5980 MeV^2), the G constraint to the range (-2290 MeV^2, -6180 MeV^2). Including the experimental error on A extends, for example, the phase constraint range to (-840 MeV^2, -6240 MeV^2).

For comparison, artificially setting $G = 0$ produces $\tilde{\Pi}_{\rho\omega}(m_\rho^2) = -3960$ MeV^2. One may repeat the above analysis using the input parameters of Ref. [22] (where, however, the ρ pole position is presumably high by about 10 MeV [21]). For the central A value, the experimentally allowed range of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ is (-3720 MeV^2, -5080 MeV^2). The large uncertainty in the extracted values of $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ and G is thus not an artifact of the particular fit of Ref. [21]. The small (± 600 MeV^2) error usually quoted for $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$, and associated with the experimental error in the determination of A, thus represents a highly inaccurate statement of the true uncertainty in the extraction of this quantity from the experimental data. It is important to stress that no further information on $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$ is obtainable from the $e^+e^- \to \pi^+\pi^-$ data without additional theoretical input.

Note that, in the model of Ref. [13], as currently parametrized, the sign of G is determined to be positive, and the magnitude to be $\simeq 0.02$. Such a value of G, however, coupled with the phase correction mentioned above, would fail to satisfy the experimental phase constraint. This shows that, despite the weakness of the experimental constraints for the magnitudes of G and $\tilde{\Pi}_{\rho\omega}(m_\rho^2)$, the experimental results are, nonetheless, still capable of providing non-trivial constraints for models of the mixing.

In conclusion, we have shown that, in general, there is a contribution to the $\rho-\omega$ interference in $e^+e^- \to \pi^+\pi^-$ which arises from the intrinsic $\omega_I \to \pi\pi$ coupling, and that this contribution, given the current level of accuracy of the experimentally extracted Orsay phase, precludes any even reasonably precise extraction of the $\rho-\omega$ mixing in the absence of additional theoretical input. It is important to stress that this conclusion and the central result of Eq. (0.14) do not depend in the least on the possible q^2-dependence of $\Pi_{\rho\omega}(q^2)$ nor on the use of the S-matrix formalism: even for constant $\Pi_{\rho\omega}$ and a more traditional Breit-Wigner analysis one would still have a significant imaginary part of z and hence a residual contribution from the direct coupling which, being nearly parallel to that associated with $\rho-\omega$ mixing, would lead also to the conclusion stated above. Note, however, that a significant improvement in the determination of the experimental phase would allow one to simultaneously extract the self-energy and the isospin-breaking ratio, G. In addition to the main point, just discussed, we also note that (1) even if G were, for some reason, to be zero, the data would provide the value of the mixing amplitude at m_ρ^2 and not m_ω^2, (2) since it is the complex S-matrix pole positions of the ρ and ω which govern the mixing parameters $\epsilon_{1,2}$, only an analysis utilizing the S-matrix formalism can provide reliable input for these pole positions, and hence for the analysis of the isospin-breaking interference in $e^+e^- \to \pi^+\pi^-$ and (3) the simultaneous use of the experimental magnitude and phase can provide non-trivial constraints on models of the vector meson mixing process.

Acknowledgements: We would like to thank M. Benayoun for discussions on the experimental situation and K. Mitchell and P. Tandy for providing information about the phase of the $\rho-\omega$ mixing amplitude off the real axis in the model of Ref. [13] prior to publication. We also acknowledge helpful discussions with A. Thomas.

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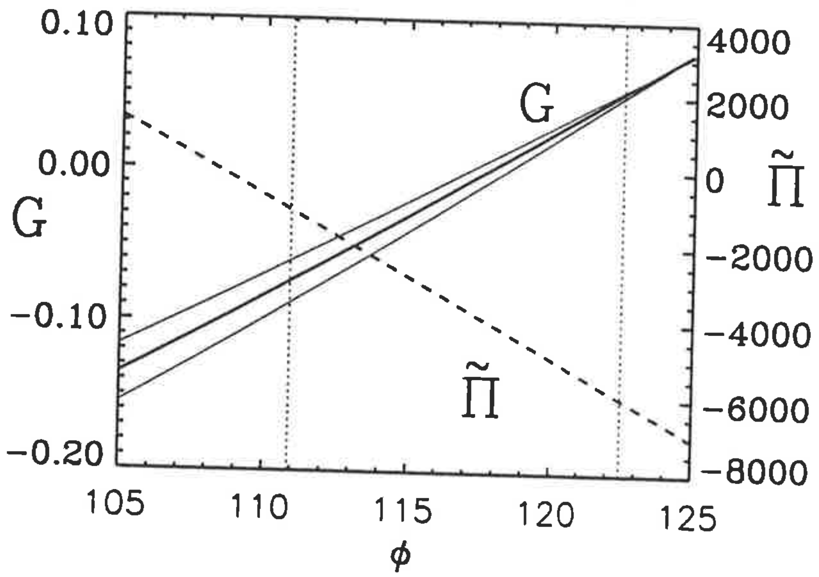

Figure 0.1: The allowed values of $G = g_{\omega_I\pi\pi}/g_{\rho_I\pi\pi}$ and $\tilde{\Pi}(m_\rho^2)$ (in MeV^2) are plotted as a function of the Orsay phase, ϕ. The vertical lines indicate the experimental uncertainty in ϕ $(= 116.7 \pm 5.8)^o$ and the uncertainty in the amplitude A (0.0109 ± 0.0011) (see text) gives rise to the spread of possible values of G at each value of ϕ.

O'Connell, H.B., (1997) Recent developments in rho–omega mixing.
Australian Journal of Physics, v. 50 (1), pp. 255-262.