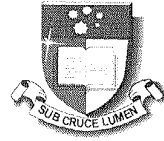
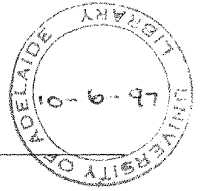


THE UNIVERSITY OF ADELAIDE



# **Modelling and Identification of Dynamic Systems using Modal and Spectral Data**

by

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# ABSTRACT

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## **Modelling and Identification of Dynamic Systems using Modal and Spectral Data**

### **Abstract**

A frequently encountered engineering problem is to determine the physical parameters of vibrating systems from the knowledge of some of their dynamic response characteristics, ie. the natural frequencies (eigenvalues) and mode shapes (eigenvectors). Unlike the inverse eigenvalue approach, which is based on the knowledge of the eigenvalues of the model, we use in this thesis both, eigenvalues and eigenvectors. We show that systems with known connectivity, ie. the finite difference or finite element models, may be reconstructed if small number of eigenvalues and their associated eigenvectors are known. Since these data may be accurately determined from the frequency response function obtained by experimental modal analysis, the proposed numerical algorithms may be effectively used as a practical tool for the solution of the problem.

In the first part of the thesis we used this approach to reconstruct the physical parameters of an axially vibrating rod, modelled using the four-point finite difference scheme. Next, we gave a methodology for the solution of the problem that is valid

for a general discrete model of one-dimensional vibrating systems, while in Chapter 5. we solve the problem for some multi-dimensional models.

In order to validate the practical applicability and behaviour of the proposed methodology, an appropriate experiment is carried out. A model of multi-storey building, which may be accurately modelled as a discrete mass-spring system, is chosen for the testing. The dynamic characteristics of the model, ie. the natural frequencies and their corresponding mode shapes are determined from the frequency response function obtained by an experimental modal analysis equipment. Using certain two eigenpairs corresponding to these data we applied the reconstruction algorithm developed in chapter 3 and determined the mass and stiffness parameters of the model. It is shown that the algorithm produce excellent results using certain eigendata, which indicates a practical use of the proposed methodology.