# NON-LINEAR VIBRATION.

An Analytical and Analogue Investigation of the Effects of Non-Linearity in the Suspension of Modern Road Vehicles .

by

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A thesis submitted to the Faculty of Engineering of the University of Adelaide, to fulfil the requirements for the Degree of Doctor of Philosophy.

Except where specific reference is made to the work of others, this work is original, and has not been submitted to any other university in any form .

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# Chapter 1

# INTRODUCTION

## 1. INTRODUCTION.

The modern road vehicle presents an extremely complicated dynamic system, as it possesses a great number of modes of vibration which can be excited under the action of both transient and repeated loading conditions. Nevertheless, a vehicle can only be regarded as satisfactory if it gives acceptable performance under all reasonable conditions of operation to which it is subjected.

This is particularly true of the suspension system, which is responsible for providing a safe, stable, and comfortable ride for all combinations of live load, road surface, and forward speed that may be encountered. Unfortunately, these criteria are in many ways conflicting, each requiring different parameters of the suspension elements to provide the optimum results. Thus vehicle suspension design is always concerned with the degree of compromise which is desirable with respect to the conditions of the particular application. Furthermore, the development of suspension design has, till the present time, been almost entirely experimental, with successive modifications as these became apparent from actual operating performance. Consequently, although based essentially on linear vibration theory, practical suspensions have invariably been modified to possess some degree of non-linearity in the components of springing and damping, however the trends are by no means consistent, indicating that selection is still primarily a compromise.

Recently, the proven use of magnetic liquids as a controllable force-transmitting medium, suggested that these might be employed in the vehicle suspension damper to relate its characteristic in some way to the operating conditions, so that the overall performance could be optimised.

Thus, this thesis describes an analytical and experimental analogue study of the effects of non-linearity in the damping and springing components of a modern road vehicle suspension. In this respect, the term "non-linear" is used in its broadest meaning, as it implies only that spring forces are not proportional to deflection, and damping forces are not proportional to the magnitude of relative velocity between the elements.

The aim of the project, is to investigate the possibility of using magnetic liquid dampers in such a way, that their performance is controlled by some characteristic of the resultant motions of the vehicle suspension, to be the optimum required for the particular operating conditions, so that the overall performance can be considerably improved.

Thus, although it is the determination of an optimum controlled non-linear damping characteristic which is of prime importance, it is necessary to investigate the interacting effects of the more standard spring non-linearities in order to ensure an optimum solution for the complete suspension.

With this in view, the thesis has been arranged to first present the criteria of performance of the vehicle suspension and the range of operating variables to which the road vehicle can be subjected. The overall performance of the complete vehicle is then reviewed in order to indicate those characteristics which are of greatest influence in satisfying the performance criteria, and the average suspension parameters employed in the modern road vehicle are also included.

Chapter 5, introduces a two mass system which is recognised to be the greatest degree of simplification which is allowable if dynamic responses are to be compared with those of the whole vehicle, and the complete transient and harmonic responses of this system are determined from linear vibration theory, as a basis of comparison for the succeeding non-linear effects.

Then, after introducing the concepts of non-linear damping and springing in the suspension, and reviewing the current literature on the solution of non-linear vibration problems, an analytical method is developed to enable the harmonic response of the simplified system to be calculated for conditions of controlled non-linearity in the magnitude of damper effort.

Then, from these results, an optimum form of controlled damping is suggested.

Chapter 7, gives a detailed description of the mechanical analogue which was designed and constructed especially for this project. The following chapter presents in detail the experimental work conducted on the analogue, thereby proving the validity of the earlier analytical work, and indicating the effects of non-linearity of suspension springing.

Finally, the conclusions are drawn from both the experimental and analytical work, and recommendations made for a composite non-linear control of damping effort based on the relative motion between axle and body, and the recorded axle acceleration, which achieves the desired object of successfully relating damper effort to the prevailing excitation conditions.

# Chapter 2

# CRITERIA OF PERFORMANCE OF THE VEHICLE SUSPENSION

## 2. CRITERIA OF PERFORMANCE OF THE VEHICLE SUSPENSION.

The suspension of modern road vehicles is an extremely important aspect of their design, as it influences every capability other than the actual production of power. Thus, recognizing the basic object of the road vehicle as the transportation of human beings and payload, the suspension is then responsible for producing transportation which is satisfactorily comfortable, safe and efficient to the maximum availability of the supplementary factors of driver competence, brake and engine performance. To do this, it must isolate the vehicle body from road shocks, ensure adequate reaction between the tyres and the road surface so that steering braking and traction can be effectively and efficiently utilized, and it must minimize the force transmission to the body structure and suspension elements so that the vehicle as a whole has a reasonable life.

These requirements are in many ways conflicting, so the basic criteria of performance are investigated under four separate headings to indicate the features of suspension design which must be controlled in order to optimize performance.

### 2.1. PASSENGER COMFORT.

The comfort conditions prevailing in a vehicle depend on the magnitude and quality of the mechanical vibrations and noise to which the passenger is exposed. The subjective nature of passenger response makes it difficult to obtain a quantitative measure of comfort. Experimenters have, however, established the response of human beings to vibration and noise of pure form, and this information is useful in determining the most undesirable qualities of ride motions.

#### 2.1.1 HUMAN REACTION TO VIBRATION.

The human body has developed through the ages under the action of the natural frequencies and amplitudes of walking and running, and presents an extremely complex dynamic system to mechanically forced vibration. The magnitude of any vibration frequency which corresponds to the thresh-hold of discernibility, or discomfort, depends on the region of the body in which sensation is most pronounced. Thus, at low frequencies, discomfort arises from the relative movement of internal organs which causes "sea-sickness" . At higher frequencies muscle fatigue, head vibration, and finally sound intensity determine the limits of acceptable vibration amplitude. Because of this dependence of the source of discomfort on frequency, it is impossible to present a simple relationship between amplitude and frequency and frequency as a limit to the magnitude of allowable vibration. The direction of vibration, the attitude of the body, and the support offered to it, are other factors which have considerable influence on the degree of sensation produced .

The problem of determining vibration limits for acceptable comfort of the human being has attracted numerous experimenters during the last 25 years. One of the greatest problems in such experiments has been the absence of any definite method of assessing comfort. Most experimenters have been forced to use the verbal reports of subjects under test, though several have endeavoured to estimate the fatigue level by physiological and psychological tests. Another difficulty is that much depends on the disposition of the subject prior to test, so satisfactory results can only be determined statistically using an appropriately large range of subjects. Much of the testing has been done on "shake tables", platforms which can be vibrated horizontally or vertically within wide limits of amplitude and frequency. Exhaustive tests of this nature were carried out by Reiher and Meister, in which the subjects were isolated from noise. The results of these tests are given in an article by Wood (1948), from which Fig.2.1. has been constructed. On analysis these results show that the limits of the various comfort zones may be represented by the equation

$af^x$ = Constant. Where a = Amplitude
f = Frequency

the exponent x varying from 1 to 3.

Between f = 2 cycles per sec. to 60 cycles per sec. and amplitude between 0.001 and 3 inches the exponent x = 2. i,e. sensation is caused by acceleration.

For frequencies less than 2 cps, the sensitivity law follows $af^3$ = Constant, and sensation depends on the rate of change of acceleration, i.e. jerk.

At amplitudes less than 0.001 inches af = Constant and sensitivity is proportional to velocity.

These experiments also established that vibrations of greatest magnitude could be tolerated in transverse planes when sitting without back support. Less vibration could be accepted whether standing or sitting if it was in a vertical direction, while the most sensitive position was recumbent when subjected to horizontal vibrations at right angles to the body.

These results agreed reasonably well with those of Jacklin (1936) but he showed that back support, by concentrating the vibrations in the head, resulted in the vertical direction being the least sensitive when sitting.

His results, though for a smaller frequency range, were given in the exponential form

$$K = Ae^{0.6f} \qquad \text{Where } A = \text{Maximum Acceleration}$$
$$f = \text{Frequency cps.}$$

K then defines the comfort index with limits given in the following table for A in feet per sec. per sec.

| Event | Direction | Index K |
|---|---|---|
| Uncomfortable | Vertical | 64.7 |
| Disturbing | " | 31.2 |
| Uncomfortable | Longitudinal | 11.73 |
| Disturbing | " | 4.02 |
| Uncomfortable | Transverse | 8.21 |
| Disturbing | " | 2.35 |

Two German experimenters Helberg and Sperling (Koffman 1957) were concerned with the vibration limits in railway rolling stock, and concluded that a measure of comfort was given by the ride factor

$$F = 3.6 \sqrt[10]{a^3 f^5} \qquad \text{Where } a = \text{Amplitude ins.}$$
$$f = \text{Frequency cps.}$$

This factor has a thresh-hold of discomfort of $F = 2.5$

Janeway(1948) published the results obtained by the Society of Automotive Engineers Riding Comfort Research Committee. These suggested that the frequency spectrum be divided into three ranges, each having a different criteria as a comfort limit.

Thus,

| Frequency Range | Comfort Limit | Factor |
|---|---|---|
| 1-6 cps | Jerk = 40 ft/sec$^3$ | $af^3 = 2$ |
| 6-20 cps | Acceleration = 1.1 ft/sec$^2$ | $af^2 = 1/3$ |
| 20-60 cps | Velocity = .105 in/sec | $af = 1/60$ |

These results were, however , restricted to vertical harmonic motion.

In an effort to collect all the results of experimenters whose conflicting results were largely caused by limited frequency ranges, Postlethwaite (1944) suggested a graphical representation of the limits of mechanical vibration, analagous to the well established auditory response of the human ear. He further defined a unit of vibration sensation , the "trem", as a parallel to the "phon" of sound sensation. His suggested method of presentation is given in Fig.2.2. the thresh-hold of discomfort being achieved at a value of 40 trems.

2.1.2 HUMAN REACTION TO NOISE.

The more general nature of the problem of noise has resulted in a more thorough understanding of the response of the ear to sound. The range of sound pressures to which the human ear responds is approximately 0.0002 to 200 dynes per square centimeter, and for convenience a logarithmic scale of decibels is used for sound level measurement. The reference level is accepted as 0.0002 dynes per square centimeter and sound pressure level is then taken as the ratio of sound pressure to reference pressure expressed in decibels.

It is well established that loudness, the auditory impression of an observer as to the strength of a sound, is its most significant characteristic. The ability of the average observer to accurately compare noises of equal loudness, has led to the use of a standard of equivalent loudness, the "phon" . Thus, the loudness of any noise in phons is numerically equal to the sound pressure level of a 1000cycles per sec. pure reference tone giving the same sensation. The ear is not equally sensitive to all frequencies, so the overall response is presented graphically for pure tones. This response is so consistant that it is included in practically all texts on sound .

The sensation produced by a noise composed of many frequencies may be estimated by separately determining the sound pressure levels of the component frequencies, though the processes by which the ear arrives at loudness judgements for composite sounds is not fully understood.

2.1.3 COMFORT IN THE ROAD VEHICLE.

The experiments previously noted have attempted to discover the characteristics of mechanical vibration and noise which cause discomfort to the human being. To do this, the number of variables has necessarily been kept to a minimum. Thus, vibrations have been pure and uni-directional, the effect of seat cushioning has been neglected, and attempts have been made to eliminate any interaction between noise and vibration effects.

The true comfort conditions in a road vehicle are far more complex than these ideals. In fact, the passenger is subjected to motions simultaneously in all six degrees of freedom, and these vibrations are seldom pure, while panel, road, and wind noises are always present. Fortunately the reaction of the human being to vibration depends largely on the manner in which it is transmitted to him. The universally adopted soft seat thus becomes an important aspect, as it isolates the passenger from high frequency vibration and modifies the character of low frequency effects. It does this by controlling the mean pressure and direction of support offered to the passenger's body, and thereby influences different nerves and muscles. By providing good back support, such seats do,however, tend to concentrate horizontal motions into "neck-whipping" forms which are extremely uncomfortable.

The fact that motions are not truly repetitive tends to relieve the comfort conditions slightly, as discomfort caused by single impulses is not in proportion to their severity because of a relief in monotony. Further features which affect riding comfort are an everchanging panorama, the exhilaration of speed, and such factors which cannot be allowed for in determining an absolute value of comfort level in a particular vehicle.

In an effort to establish more truly the comfort conditions prevailing in road vehicles, Jacklin (1936) carried out exhaustive tests in an automobile equipped with soft seats. Records were taken by accelerometer giving instantaneous readings for motions in the three reference directions when comfort limits as established by the passengers verbal reports were exceeded. He presented his results in a similar manner to those for hard seats, using the exponential index as a limit to comfort, thus;

| Event | Direction | Index | Limit |
|---|---|---|---|
| Disturbing | Vertical | $K_V = A_V e^{.13f_V}$ | 8.5 |
| Uncomfortable | " | | 10.0 |
| Disturbing | Longitudinal | $K_L = A_L e^{.087f_L}$ | 4.0 |
| Uncomfortable | " | | 5.5 |
| Disturbing | Transverse | $K_T = A_T$ | 2.75 |
| Uncomfortable | " | | 3.25 |

The overall comfort index was then obtained by vector addition of the indices for vertical and horizontal vibrations, giving;

$$K_C = \sqrt{K_V^2 + K_L^2 + K_T^2}$$

The thresh-holds for this combined comfort index then being

$K_C$ = 9.80 Disturbing Limit

= 11.9 Uncomfortable Limit

It is obvious that an absolute assessment of comfort in an automobile is meaningless, as it involves numerous factors which cannot be evaluated together with the interaction of effects which even in their simplest form are extremely complicated. The real value of these comfort indices lies in their use to compare the behaviour of various vehicles or suspension parameters under strictly similar conditions of road surface and speed. They do however serve to illustrate the properties of the ride motions which are of most importance, and thus serve as a guide to design.

The overall conclusions which can be drawn regarding passenger comfort in a road vehicle are thus;

(1) Large movements must be of low frequency. The limit to comfort is then determined by the maximum jerk, and depends largely on the characteristics of the cushioning seat.

(2) Any motion which tends to cause neck-whipping must be minimized.

(3) The passenger must be isolated from the high frequency vibrations by well designed soft seats.

(4) The noise level in the cabin must be kept as low as possible.

## 2.2. STABILITY AND SAFETY.

The factor which determines how safe a vehicle is in performing its designed function is the ability of the driver. This ability, naturally depends on numerous factors outside the control of the designer such as experience and fatigue. Nevertheless, it is possible to incorporate desirable inherent stability in any vehicle, and this, by reducing the demands on the driver can greatly improve the safety margin.

However, although inherent stability may be used to accomodate minor variations, the driver is responsible for large corrections. Consequently there is a limit to the degree of inherent stability which can be employed, as the driver must always have some knowledge of the conditions tending to disrupt stability. This is recognized as the "feel" of the vehicle.

There are two types of stability which are of interest, directional stability, and roll stability. It is desirable to maintain these throughout the large range of operating conditions to which the vehicle is subjected.

### 2.2.1. DIRECTIONAL STABILITY.

Directional stability is defined as the ability of a vehicle to continue along a straight path when acted upon by various extraneous forces not under the control of the driver . A stable car is one in which an external force is balanced by a slight change of position of the car without affecting the direction of motion perceptibly. With an unstable car, the external force causes a change in attitude of the vehicle which itself introduces a centrifugal force adding to the original disturbing force. The attitude of the vehicle thus further deteriorates and the driver is forced to make continuous corrections in order to maintain the desired path. The transient nature of the disturbing forces, coupled with the drivers reaction time, makes the instability even more pronounced.

The pneumatic tyre has the sole responsibility of coercing the vehicle to follow a desired path. Thus the basic requirement for directional stability is adequate reaction between the tyre and the road surface. Provided this contact is maintained, other factors of the suspension may be adjusted to give the vehicle good handling characteristics..

The manner by which the pneumatic tyre develops side force to change the direction of motion of the vehicle is illustrated in Fig. 2.3. This shows that the tyre deflects to produce side thrust from the road surface, the small area of tread in contact with the road being pulled to one side as the wheel rolls. A point on the tread approaches the road in its undeflected position, then moves transversely relative to the rim as it takes the load, the wheel actually traversing a path inclined to its own plane by an angle $\phi$. This is termed the "slip angle" of the tyre. The ability of a tyre to produce side force by adopting a slip angle is presented as;

$$\text{Cornering Power} = \frac{\text{Side Force Developed (lb)}}{\text{Slip Angle Required ( degrees)}}$$

The side force produced does not act at the centre of the contact patch, but slightly behind it because of a hysteresis effect and this, coupled with the normal "castor displacement", provides an "aligning torque" which must be exerted by the steering mechanism. This "aligning torque" has the greatest influence on the directional "feel" of the car experienced by the driver.

A second factor which produces side force on a rolling tyre is the camber angle, but this is far less effective than slip angle. Camber thrust also causes a small camber torque, though this is negligible compared to the aligning torque. There are several factors which influence the cornering force, aligning torque, and camber thrust of a tyre. The design factors of inflation pressure, rim width and cord angle are important, but these are restricted by consideration of tyre life. Operating factors of load, traction and braking, have considerable effect though little is known of the latter two. Typical relationships between cornering force, aligning torque, camber thrust and load are shown in Fig. 2.4, ~~2.5~~, ~~2.6~~. after Bull (1939) and Olley (1948).

When established on a level surface, a vehicle, whether stable or not, will continue to run in a straight line unless it is disturbed by either

(a) Lateral gravity forces produced by inclination of the road surface.

(b) Aerodynamic forces with lateral components.

The subsequent behaviour of the vehicle is dependent on the "stability margin", a term adopted from aeronautics practice, defining the horizontal distance from the centre of gravity to the "neutral steer line". This in turn is that line in a vertical fore and aft centreline plane of the vehicle, at any point on which a lateral force produces no yaw as the vehicle proceeds along its track with steering locked.

A positive stability margin exists where the neutral steer line is behind the centre of gravity, a recommended limit being 4 to 6% of the wheelbase. (Lind-Walker, 1950)

When subject to lateral force at the centre of gravity, a positive stability margin results in larger transverse forces being developed at the front tyres. This causes slip angles at the front tyres to be greater than those at the rear, a condition also recognized as "under-steer" . Similarly "over-steer", the condition where rear slip angles exceed those at the front, is present in a vehicle with a negative stability margin.

Unfortunately the neutral steer line is not fixed, its position position depending largely on weight distribution and tyre pressure. As car loading varies, it moves in the same direction as the centre of gravity, but to a lesser extent, while it always retreats from the end having reduced tyre pressure. However, although the degree of inherent directional stability will in general vary with loading condition, it is possible to ensure a positive stability margin throughout the design range.

Directional stability under aerodynamic forces has become quite important in recent years with the advent of ultralight weight cars. The high ratios of frontal area to weight for for these vehicles results in air forces which are a high percentage of their weight. This is true, even though the top speed of such vehicles is relatively low.

The air flow around the car body produces a distribution of pressure, the summation of which is a force having components which affect all six degrees of freedom of the body. Those of greatest interest are drag, side force and yaw. With unstreamlined bodies, the centre of pressure of the wind force is quite close to the centre of gravity of the body, so that directional stability is governed by the stability margin. The small cars, however, require good streamlining to reduce air drag. This results in a centre of pressure which is far forward of the centre of gravity, and can even be ahead of the front axle.

Under such conditions practically all the aerodynamic side force must be balanced at the front tyres which thus develop large slip angles. The condition is thus one of understeer, and the vehicle sets of on a curved path about a centre on the side remote from the disturbing wind force. This tends to increase the angle of attack of the wind and thereby increase the disturb-side force. Thus, to maintain directional stability under suddenly applied aerodynamic forces it is becoming increasingly necessary to provide such vehicles with rear stabilizing fins. These draw the centre of pressure to the rear without materially increasing the drag. For aerodynamic stability, the centre of pressure should be approximately 5% of the wheelbase behind the neutral steer line. (Lay 1953)

The Effect of Directional Stability on Cornering.

Whereas directional stability is concerned with motion in a straight line, the effect on cornering capabilities is important. The behaviour of a car when cornering may be divided into three phases, those of transition - entering and leaving the curve, and that of steady state on a circular path.

During the transition periods, the directional stability of the vehicle is critical. As the duration of cornering phenomena is fairly long, it is possible to assume immediate changes in steering angles at the initiation of the transition. If the vehicle has a negative stability margin, it tends to turn itself into the disturbing force at the centre of gravity, which in this case is the centrifugal force. This means that the vehicle tends to turn about a centre closer than that predicted by the steering angles. It is thus necessary for the driver to reduce the steering angles during the initial transition. This demands a high degree of skill of manipulation on the part of the driver if the vehicle is to follow the desired path. The car having a positive stability margin, however, tends to corner about a centre more remote than that established by the initial steering angles. The driver must then increase the steering angles to maintain the desired path. This is a natural action and produces satisfactory performance, though it does require greater steering effort.

The behaviour when cornering naturally depends on the driver's judgement in selecting a suitable speed. If the driver attempts to corner too fast with an oversteered car i.e. a negative stability margin, the rear slip angles which are greater than those at the front may cause a loss of road adhesion and "breakaway" of the rear wheels. Under such conditions the steering angles of the front wheels have virtually no effect as the vehicle tends to rotate about these wheels. The centrifugal force which initiated the breakaway is thus greatly increased,

and the driver can make little corrective action.
Excessive cornering speed in an under-steered car i.e.a positive stability margin, causes the front tyres to breakaway first. This increases the radius of curvature of the path of the centre of gravity, thus reducing the centrifugal force causing the breakaway. This may allow allow the vehicle to satisfactorily negotiate the corner though in a wider arc than desired.

Under steady state conditions on a circular path, the body develops a definite roll angle. The magnitude of this angle may cause roll steering effects which require changes in the steering angle from two causes;

(a) Load transfer.

(b) Variations in camber and steering angles at the wheels.

During cornering , more load is taken by the outer wheels, and this together with camber changes varies the cornering power of the tyres. Also deflection of the suspension springs may result in steering angles being adopted at the wheels. These roll-steer effects are functions of the suspension geometry, and are slow in building up, so the driver has adequate time to compensate for them.

2.2.2 ROLL STABILITY.

The second factor which determines the overall stability of a vehicle is its stability of position i.e. the ability of the vehicle to keep the resultant force on its centre of gravity within the base of support offered by the wheels. Whereas it is conceivable that such stability would be disrupted on sloped surfaces, it is most likely to be dangerous when cornering under conditions of high roll angle.. Roll stability is a function of the suspension geometry in determining the roll axis and the action of the wheels with spring deflection. ( Eberhorst 1951) The roll axis is the axis about which the body rolls when subject to side force at the centre of gravity. It is fixed by the roll centres of the front and rear suspension. With high roll centres, the roll couple caused by centrifugal force is low and results in small roll angles. Conversely, low roll centre which is a feature of many popular independent suspensions, has high roll couples and roll angles associated with it.. In addition to this, low roll centre causes a greater movement of the body outward relative to the track of the tyres, which may be further emphasized by a decrease in track as shown in Fig2.5 Consequently the resultant force on the centre of gravity falls far closer to the edge of the base of support with suspensions of low roll centre. The importance of roll stability has become apparent since the wide scale use of independent suspensions of this type.

It is possible to provide independent suspensions of high roll centre, though some roll angle is recognized as desirable, for it provides the driver with an indication of the cornering which he is demanding from the tyres. Roll angle thus helps in providing "feel" of the car.

## 2.3. MAINTENANCE AND RUNNING COSTS.

These two cost factors have become of prime importance in establishing the overall performance of road vehicles in both private and commercial fields. Maintenance costs depend almost equally on the power producer and the suspension system, while running costs are influenced mainly by power production and transmission. Secondary effects do appear in the running costs which may be directly attributed to the suspension. Thus the increase in fuel consumption when driving over a rough road was measured by Widney(1936) as 5 to 17% , after experiments on a number of cars. This increase is likely to be caused by a number of factors such as;

(a) The inability of the suspension to keep the wheels in contact with the road under bad conditions resulting in wheelspin and loss of available power.

(b) Considerable energy is dissipated in the "shock absorbers" under rough conditions in an effort to restrict vibratory motions of the suspension elements. This energy must ultimately be supplied by the power unit.

(c) Variations in slip angle caused by deflection of the suspension result in energy dissipation by the tyres. This effect was observed by Bull(1939) who quoted 3 to 4 H.P. consumed by a tyre operating continuously at a 5 degree slip angle.

(d) Vibration of the car body results in direct fuel wastage from spilling and throttle variations.

Maintenance costs that can be directly attributed to the suspension are those due to wear and fatigue of tyres , suspension, and steering members. Structural, transmissiom, and many minor vibration failures can also be caused by the suspension not successfully performing its design tasks.

Tyre wear is primarily a function of suspension geometry. It depends on the changes in tread and camber which can develop when the axle moves relative to the body. Some camber change is useful when cornering to reduce tyre scrub, but this action is not desirable on a straight but bumpy road. It is thus obvious that the geometrical arrangement is necessarily a compromize, however ,in practice neglected maintenance usually far overrides these effects. Tyre life is limited mainly by the fatigue of the walls. It is thus desirable in this respect to have suspension systems which cause the least possible flexure of the walls under arduous road conditions, but it is essential that this be achieved without increasing the force transmission to the body structure and suspension elements as such would naturally increase their wear and fatigue.

Road wear must be included in maintenance costs. It is realized that the wear of a road depends largely on the dynamic characteristics of the suspensions of the vehicles which use it. Suspension vibrations mould the road surface into a form which tends to produce greater vibration. Consequently the wear of both the road surface and the suspension is greatly accelerated.

From this it is obvious that the suspension system plays a major role in determining the maintenance costs, and can also be looked to for an improvement in running costs.

## 2.4 SAFETY OF PAYLOAD.

The commercial vehicle can only be regarded as providing satisfactory operation, if the payload suffers no damage. The performance in this respect reflects on the ability of the suspension in its basic object to isolate the body from road shocks.
The criterion is obviously the maximum force which is transmitted to the payload, and this is determined by the acceleration of the vehicle body. In all cases this should be less than gravity to reduce the possibility of the payload and body separating, as the shock loads developed when they resume contact may be extremely high. Prevention of separation also reduces the possibility of losing the payload under the action of aerodynamic forces.

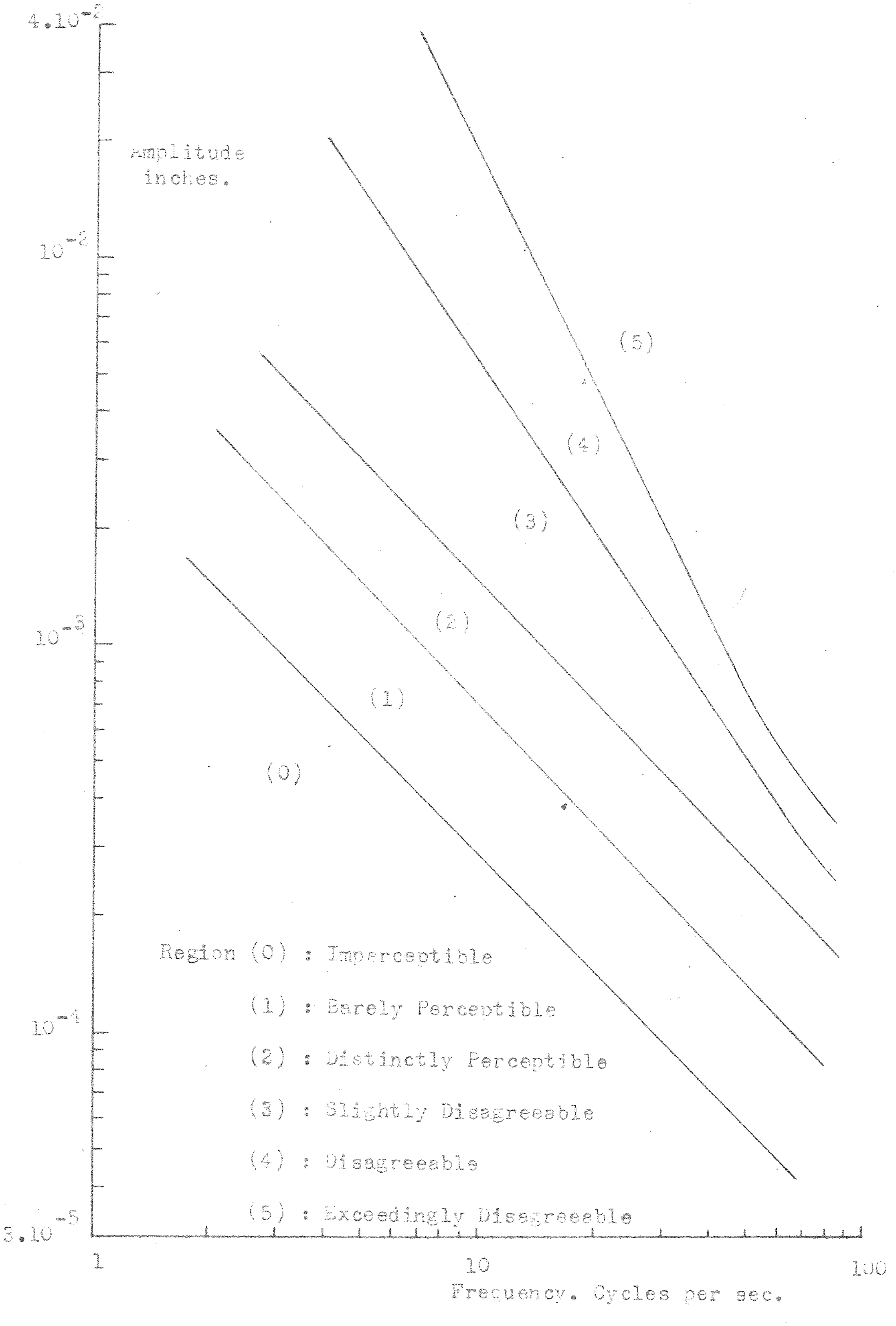



Fig. 2.1. RESPONSE OF HUMAN BEINGS TO VIBRATION - REIHER & MEISTER

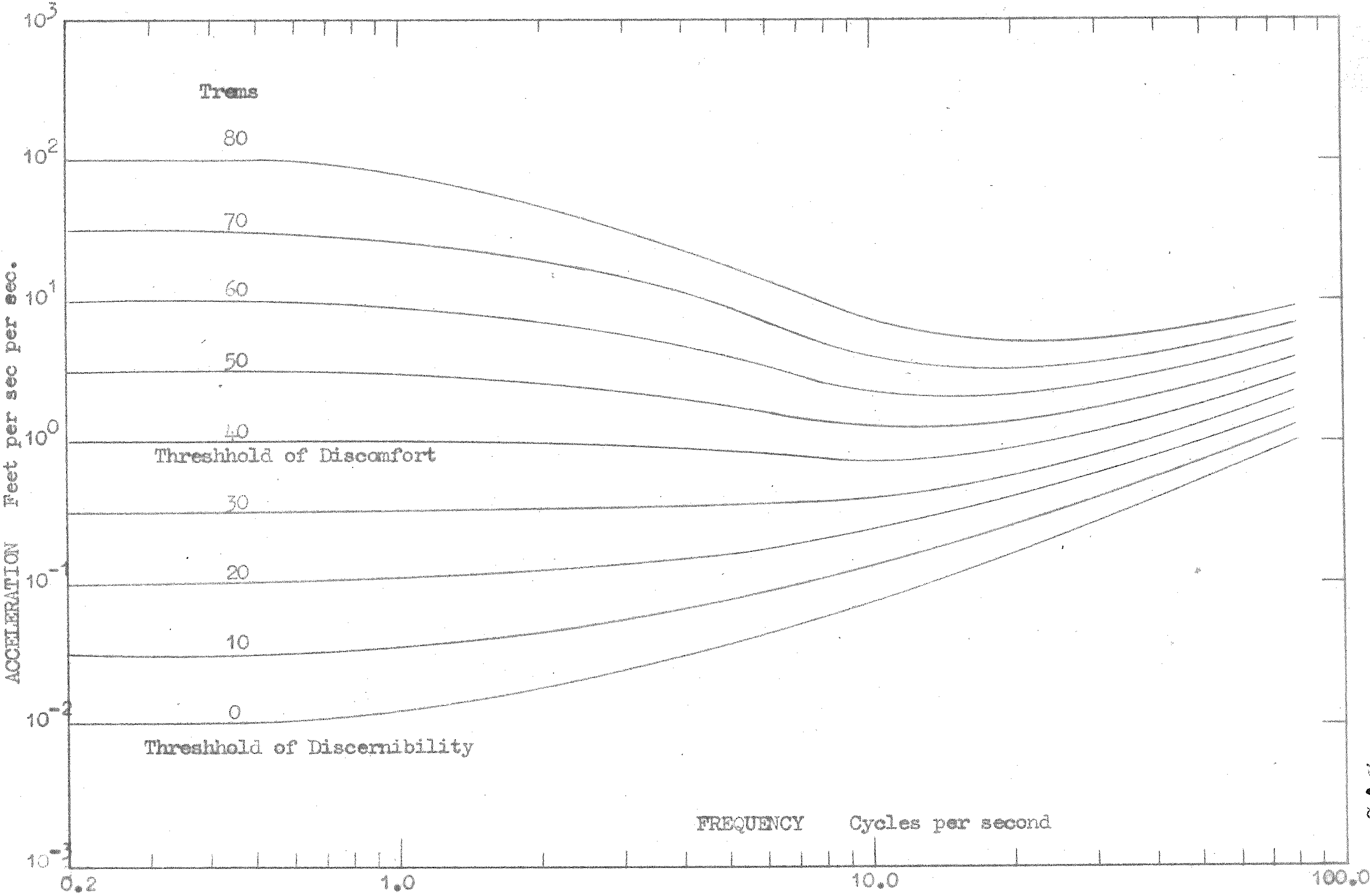



Fig. 2.2. HUMAN SUSCEPTIBILITY TO VIBRATION - POSTLETHWAITE.

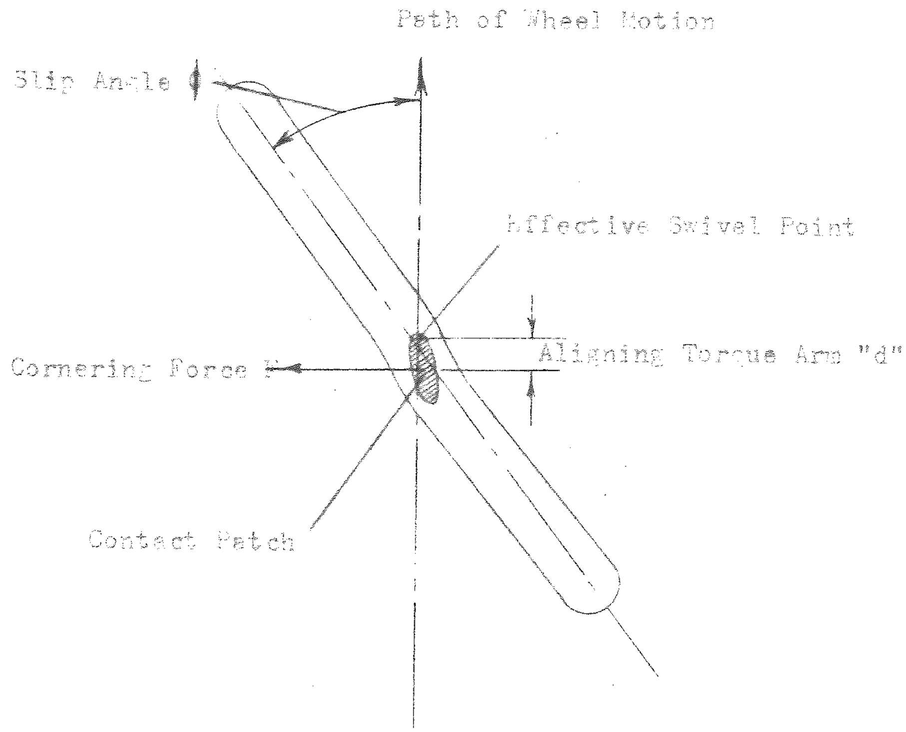



FIG. 2.3. STEERING ACTION OF THE PNEUMATIC TYRE.

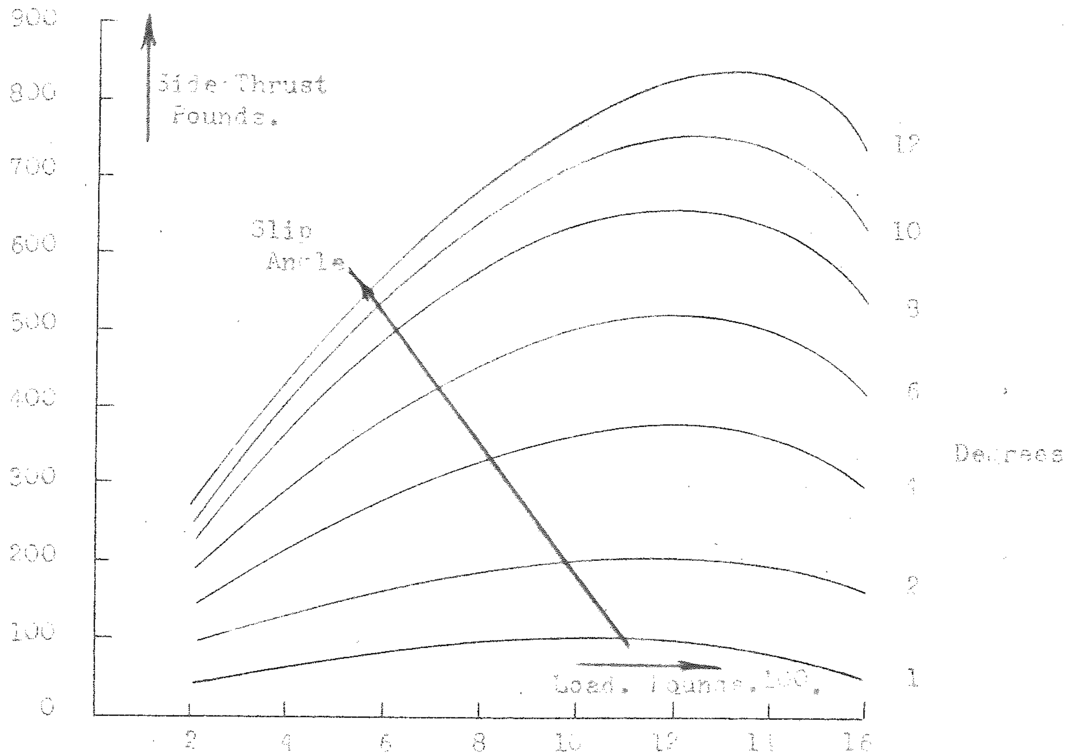
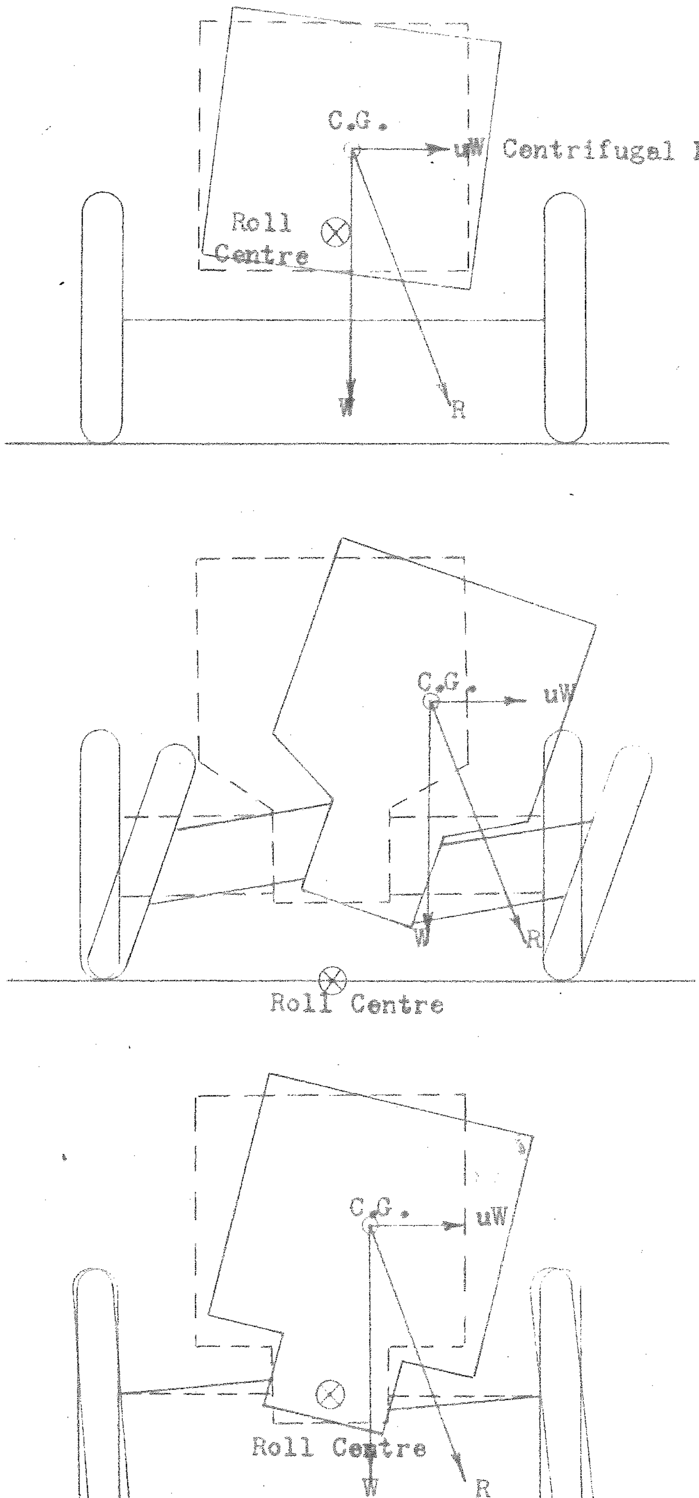



FIG. 2.4 CORNERING POWER OF THE PNEUMATIC TYRE - OLLEY.




(a) Beam Axle Suspension

(b) Parallel Action Independent Suspension.

(c) Swing Axle Independent Suspension

Fig. 2.5. ROLL ACTION OF TYPICAL SUSPENSION SYSTEMS .

# Chapter 3

## OPERATING CONDITIONS.

## 3. OPERATING CONDITIONS.

The conditions under which the vehicle suspension is required to provide satisfactory performance vary widely. The most important variables are those of load, speed and road surface condition, but, the nature of the path, and the state of motion of the vehicle may have significant influence on the dynamic performance.

Road surface irregularities are naturally extremely important, and these vary widely throughout the world, depending largely on the methods of construction and surface preparation. Of approximately 500,000 miles of roads in Australia at the present time, over 50% have natural surface, while the remainder are either bitumen sealed or concrete paved. The smooth finish road surfaces normally are troubled with disturbances of long wavelength only. Such irregularities exceed 20 feet in length, and arise from the natural contours of the ground. However, these may be aggravated by the construction, as with concrete paving which tends to settle after use, and establish a regular succession of joints. Irregularities of short wavelength in such surfaces are usually associated with breakdown of the construction. Thus undermining can lead to the formation of pot-holes in a bitumen surface, or bad cracks with sharp slope discontinuities in paving.

The high temperatures which are often achieved in bitumen surfaces in this country, can lead to the generation of small regular surface ripples similar to the "washboard" surfaces common on unsealed roads. The deterioration of a road surface in this manner is extremely rapid for the irregularities are self-generating. Thus with a surface of low mechanical strength, a single irregularity may cause the axle to vibrate and cause periodic loading on the road surface which is consequently deformed. Subsequent loading from the passage of other vehicles, which all have similar characteristics of axle vibration, will further deform the surface and extend the duration of the irregularities. On unsealed roads, washboard surfaces have been observed to exist continuously for many miles. Fogg (1957) reports that information on such surfaces from Africa, America, Australia, and India was very consistent, the wavelengths being 30" plus and minus 2". The amplitudes naturally showed greater variation, but he concluded that ½" was satisfactory for the concrete surface of a proving track.

Many irregularities of short wavelength exist in all road surfaces. If these are of small amplitude, they are absorbed by the tyres, but they tend to produce tyre noise. Short irregularities of large amplitude, such as potholes, excite the suspension to transient oscillation, and tend to generate a succession of such irregularities, as with the potholed or washboard surfaces.

In general, the long wave irregularities, which tend to excite large body motions, are in the original construction of the road surface, and these are not greatly modified by the passage of large volumes of traffic. On the other hand, sharp irregularities caused by local breakdown of the surface can lead to an extremely rapid decay, especially if the road has low mechanical strength.

The increasing speeds which are available from the power units of the modern road vehicles place even greater importance on the original construction of the road. As the speed at which a vehicle is driven is set by the driver, it is adjusted to some extent to suit the other operating variables. Nevertheless, there is the need to travel at reasonable speeds if journey time is to be comparable with that of other forms of transport. The average speed range of modern vehicles extends to 80 m.p.h.. Such speeds emphasize the need for good control in road surface preparation, as for example a road wave length of 88' corresponds to 1 cycle per sec. excitation at 60 m.p.h. At such a frequency the body would tend to resonate, and the consequent decrease in stability could be very dangerous. Thus it is essential to restrict even long road waves to small amplitude if the higher speeds available are to be used with safety.

In operation the road vehicle is also subject to quite wide variations in load and load distribution. This is particularly true of the commercial and public transport vehicles. A normal passenger car may have a live load of 50% of the vehicle weight, whereas the commercial vehicle must withstand increases of the order of 100% of its unladen weight. In order to maintain consistent braking and steering characteristics, it is desirable to have this live load evenly distributed between the front and rear suspension. Nevertheless variations in load distribution do occur, especially in the commercial vehicle because of the large load range.

A restriction which is imposed on the vehicle suspension by other considerations of height and body styling is the maximum spring deflection. These "ride clearances" simply place a limit to the allowable motion of the axle towards and away from the body for each position of static equilibrium. It is thus necessary to design the suspension such that it is capable of absorbing bump energies for all irregularities by spring deflections less than the available ride clearances. For extreme cases of excitation, this requires the use "bump stops", and consequently great deterioration of the isolating capabilities of the suspension. The problem is further emphasized with simple suspensions where ride clearances change with load, and has led recently to the introduction of compensating suspensions which maintain constant standing height.

# Chapter 4

# DYNAMICS OF THE VEHICLE SUSPENSION

## 4. DYNAMICS OF THE VEHICLE SUSPENSION.

The satisfaction of performance criteria in operation, obviously depends on the dynamic response of the suspension to the prevailing operating variables. Fortunately, forward speed the variable which establishes dynamic conditions of operation, is under the control of the driver. Acceptable performance may thus be achieved on all reasonable surfaces by selection of a satisfactory forward speed. It is nevertheless desirable to employ suspension characteristics which provide the maximum range of acceptable speeds for any given road surface, thus allowing higher speeds, and reducing the danger associated with sudden changes in road conditions.

The basic form of the suspension of self propelled road vehicles has been the same since its inception when the pneumatic tyre, and the isolating body spring both proved to be essential. Improvement in suspension performance enabling higher road speeds, available from the power units, to be utilized, has been along the lines of successive modification as the dynamic features became apparent during use. The variety of geometrical arrangements and mechanical characteristics employed for vehicle suspensions naturally stems from the conflicting requirements of the performance criteria.

The basic suspension system considered as a system of lumped elements is shown diagrammatically in Fig 4.1. This shows the body mass supported by four isolating springs and dampers from the four wheel masses which in turn are supported through tyre springing from the road surface. The vehicle suspension is thus composed of five masses, and therefore has 30 degrees of freedom which may be dynamically excited.

Fortunately not all of these are of major importance, but many of the lesser axle motions may be intercoupled causing undesirable effects under certain operating conditions, so they cannot be entirely neglected.

4.1. PRIMARY MOTIONS.

The six degrees of freedom of the vehicle body are illustrated in Fig. 4.2. They comprise the three translational motions of bounce, side-shift and fore and aft shift, and the rotations of pitch, roll and yaw. Body motions are of primary importance in establishing the comfort conditions in the vehicle, but they can also have a marked effect on stability. As such, only three of the body motions are of major concern, these being bounce, pitch and roll. The remaining three degrees of freedom are only of minor interest as they are subject only to small excitation, and may be rigidly constrained without compromise in performance.

The criterion of passenger comfort has demanded that any large motions to which the passenger is subjected be of low frequency. As the body motion at any time is a composite of surface profile and transient oscillation it follows that the three primary body motions all have low natural frequencies.
There are however practical limitations which exist in the simple conventional suspension where softer springs cause a decrease in roll stability, and wide variations in standing height with load.
For this reason most modern vehicles have the natural frequencies of primary motions in the range 60 to 75 cycles per minute.
Several compensating suspensions have been introduced which present the lowest frequencies by eliminating standing height variations

and maintaining high roll stiffness.

4.1.1 ACTION IN PITCH AND BOUNCE.

Apart from the frequencies of large body motions, it is essential that the body should move in a manner to produce the least possible amount of neck-whipping of the passengers. The greatest tendency to produce neck-whipping arises from the pitching of the vehicle. It is however, impossible with conventional suspension to eliminate body pitching as front and rear wheels must travel almost identical paths. Two features of the suspension design may however be controlled to minimize both the magnitude and the effect of the pitching on all road surfaces.

Thus the "inertia distribution" and the "spring rate distribution", which govern the coupling between pitch and bounce oscillations, can be adjusted to give the most desirable ride action of the body. The effects of these two factors can most conveniently be explained by considering the simple lever analogy for the vehicle body shown in Fig 4.3. Here the centre of gravity of the body is situated distances "a, and b" from the front and rear springs respectively. These springs have stiffnesses $K_f$ and $K_r$, while the body has mass m, and radius of gyration k about the centre of gravity.
This problem is solved in most vibration texts, and has two natural frequencies corresponding to pure rotational oscillatory motions about centres whose distances from the centre of gravity of the body are expressed as ;

$$OC_{1,2} = -\frac{((a^2-k^2)K_f + (b^2-k^2)K_r)}{2(aK_f - bK_r)} \mp \frac{\sqrt{((a^2-k^2)K_f + (b^2-k^2)K_r)^2 + 4k^2(aK_f - bK_r)^2}}{2(aK_f - bK_r)} \quad \ldots\ldots(4.1)$$

These two centres thus posses the property that

$$OC_1 . OC_2 = k^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

so each oscillation centre behaves as a centre of percussion to motions about the other centre. Furthermore the natural frequencies of oscillation about these centres are

$$f_{1,2} = \frac{1}{2\pi}\sqrt{\frac{1}{m}\left( K_f\left( 1 + \frac{a}{OC_{12}} \right) + K_r\left( 1 - \frac{b}{OC_{12}} \right) \right)} \quad \ldots\ldots(4.3)$$

These formulae are unfortunately too complex to enable an optimum selection of suspension parameters to be shown immediately. There are nevertheless two cases which are of special interest.

(1) If front and rear suspensions have the same static deflections under the body load, then the condition arises where $(aK_f - bK_r)$ is zero. Consequently equation 4.1 indicates that the vibration centres are situated at the centre of gravity and at infinity, so that natural motions of pitch and bounce are not coupled. This condition also determines the ratio of the natural vibration frequencies as $\sqrt{k^2/ab}$ .

(2) If $k^2 = ab$, the values of OC become $-a$, and $+b$, indicating that the centres of natural oscillation are situated at the spring centres. Consequently each spring separately controls one of the natural modes.

The ratio $k^2/ab$, which defines the inertia distribution, thus plays an important part in determining the performance available from a particular suspension. The closer this value is made to unity, the less is the interaction between front and rear wheel impulses. As the necessity for almost equal wheeltracks must cause similar impulses to be applied to the front and rear suspensions to be separated only by a finite time interval, it is advantageous to ensure that front impulses can never cause motions of the rear suspension which detract from its ability to absorb the direct bump energy. This is obviously available when $k^2/ab = 1$, as then each suspension is an oscillation centre for deflections of the other. Such optimization is closely approached in all modern road vehicles where $k^2/ab = 0.8$ to $0.9$. Practically it has been proved that higher values are in fact unnecessary as the inertia decoupling effect improves very little, whereas the increase in inertia of yaw, which is closely related to k, causes greater demands on the steering forces developed from the tyres.

The second feature which is of importance is referred to as the spring rate distribution, though this must be considered simultaneously with weight distribution, for the effect really depends on the relative static deflections of the front and rear suspensions. Accepting inertia decoupling, the body action can be adjusted under singular road impulses to give an essentially "flat ride", by the selection of these two variables. As front wheel impulses cause rotary oscillations of the body about the rear suspension and vice-versa, a minimum degree of body pitch can be obtained when the oscillations of the front suspension are of lower natural frequency than those of the rear suspension. This is obtained by providing greater static deflections in the front than in the rear suspension. Naturally the optimum difference in natural frequencies depends on the relationship betwwen wheelbase and the forward speed of the vehicle, as these determine the time delay between the initiation of front and rear oscillations. It is thus general practice to incorporate a static deflection ratio which gives optimum performance at the average speed at which the vehicle is likely to be driven. This is found to give improved ride qualities over a wide range of speeds.

To provide adequate adequate braking and steering traction, the front wheel load is normally greater than that on the rear wheels when unladen. This also enables a greater percentage of the live load to be taken at the rear while conserving almost equal weight distribution. Consequently, as the weight distribution is fixed by considerations other than comfort, the desired static deflection ratio then automatically fixes the optimum spring rate distribution to be employed.

Modern suspensions use an unladen weight distribution ratio, weight on the rear wheels/ weight on the front wheels of 0.8 to 0.92, together with a static deflection ratio static deflection of rear suspension/ static deflection of front suspension of 0.6 to 0.95 .

4.1.2 ACTION IN ROLL

The rolling motion of the vehicle body plays an important part in establishing the position stability of the vehicle, and may have an appreciable influence on the comfort conditions. Roll is of greatest consequence when the motion of the vehicle is along a curved path, the effect of road bumps being small except when supplementary to this cornering roll. Basically both comfort and stability criteria are concerned with the magnitude of the roll angles developed, though comfort is also influenced by roll frequency, and roll jerk to which the passenger is most susceptible. To satisfy these requirements the roll frequency is maintained at values close to the pitch and bounce frequencies, but generally slightly higher than these so that all the motions blend together to give a smooth ride. Roll jerk however, is mainly determined by driver action, and as such is not controllable by design, though selected damper placement may reduce the severity of jerk generated from the road profile.

The magnitude of cornering roll developed in a vehicle body, depends naturally on the centrifugal force associated with the curved path, the position of the roll axis, and the distribution of the roll resistance between the front and rear suspensions. Because of the general assymetry of road irregularities causing roll, and the symmetry of cornering requirements, the spring rate distribution on each side of the vehicle is equal, although this may be modified in special circumstances of known assymetric load. This results in a roll axis which lies in the vertical fore and aft centreline plane of the vehicle. The inclination of the roll axis to the horizontal may however be quite high as it depends on the relative heights of the roll centres of the front and rear suspensions. These roll centres are in turn established from the geometric arrangement of the respective suspensions. Thus, with the beam axle type of suspension, a high roll centre is achieved with body roll taking place about the spring centres, whereas with most of the existing independent wheel suspensions the roll centre is quite close to the ground. With high roll centres, the resultant rolling couple of the centrifugal force about the roll axis is small and consequently the roll angles will be small for reasonable roll resistance rates from the springs. However, with low roll centres, the centrifugal couple developed about the roll axis is far greater and results in greater magnitudes of roll angle for the same roll resistance. Many modern road vehicles have the axle rear suspension and independent front suspension which results in a roll axis which is inclined to the horizontal, passing through the front roll centre at about ground level and the rear roll centre at spring centre height. High roll angles may be developed in such a suspension unless some attempt is made to share the roll couple between the front and rear suspensions. Thus "anti-roll" or "stabilizing" springs are fitted to increase the roll resistance of the front suspension without increasing its resistance to pitching and bouncing.

It has been found that an optimum height for the roll axis does exist if safety and stability are to be maintained. Thus, with a low roll centre , cornering forces may cause exceptionally high roll angles which result in a complete loss of position stability. With low roll centres there is the danger that the driver, having little indication of the cornering forces that he is demanding from the tyres, may cause a loss of adhesion with consequent disruption of the directional stability of the vehicle. Between these extremes the magnitude of roll angle may be used to give the driver an indication of the performance being achieved by the vehicle, and hence to reduce greatly the likelihood of loss of stability of either form.

## 4.2. SECONDARY MOTIONS.

The secondary motions in suspensions are those involving the unsprung masses, and they are dependent mainly on the springing and damping efforts of the tyres. These motions are far more complex than the primary motions, as the various degrees of freedom of the unsprung masses may be coupled both geometrically and gyroscopically. Consequently they have a wide range of exciting road conditions. In some cases they may even become self-excited, drawing power from the forward vehicle motion to sustain the vibrations in the absence of road irregularities.

Being essentially oscillations of the axle masses on the tyres, the secondary motions are of far higher frequency than the primary motions ranging from 450 to 1000 cycles per minute. (Schilling and Fuchs 1941) Consequently these vibrations may take place without causing any appreciable movement of the body, though the forces transmitted to it may be quite high. The comfort conditions, which at these frequencies are determined by acceleration, may be seriously affected, though in general the resilient seat cushions are effective in isolating the passengers.

Large secondary motions do however have disastrous effects on the stability of a vehicle, particularly its directional stability, for they may quite easily result in axle motions of greater magnitude than the static tyre deflections. In such cases the tyres lose contact with the road completely and cannot then supply any steering side force. Violent axle motions while the vehicle is following a curved path may may thus lead to a complete loss of stability.

The tendency of the tyre to lose contact with the road surface is quite marked even on good surfaces, and it is one of the major factors contributing to the deterioration of a road surface. Because of the relatively small static tyre deflections, generally of the order of $\frac{3}{4}$ inch, a road irregularity of about the same size can cause the axle to jump. When the tyre is fed back into contact with the road surface there is a period of high contact pressure which tends to mould the surface into a depression. As most modern vehicles have very similar characteristics for tyres and unsprung masses, a succession of these will tend to cause a progression of depressions in the road surface, all ideally spaced to maintain and magnify the violent oscillations of the axle. If the road surface is unable to withstand this action because of insufficient surface strength, undermining or temperature softening, it will rapidly deteriorate into a corrugated or pot-holed surface, which seriously reduces the maximum safe speeds of vehicles travelling upon it.

A feature of the secondary vibrations is that they have a strong non-linear character, arising once again from the common tendency for the tyres to lose contact with the road. Thus while the tyre and road are in contact the spring determining the motion is almost entirely controlled by the tyre spring rate. However, immediately the contact is lost only the body suspension spring force is available to feed the axle back to its neutral position. Consequently the axle motion exhibits the properties of a softening non-linear system. This effect is shown diagrammatically in Fig.2.4 which plots amplitude of axle motion versus vehicle speed for a particular sinusoidal road profile. This figure illustrates immediately the dangers which can arise in a vehicle which suddenly passes onto a section of road which has degenerated into corrugations in the manner described, at a speed sufficient to cause the wheel to hop. Almost immediately the wheel is thrown clear of the road surface, the interval during which contact is resumed being only a small fraction of the vibration cycle. If the driver attempts to slow the vehicle, the wheel motions become even more violent because of the effect of decreasing frequency on the softened system. Eventually the axle motions would reach the limit stops, and further speed reduction will cause it to vanish so that permenent contact is resumed. This will however take a considerable time, during which the driver has little directional control over the vehicle. On the other hand, if the driver increases speed, he may eliminate wheelhop completely, though not necessarily so, but the increased forward velocity may be dangerous.

Thus it is obvious that whenever the secondary motions build up to the extent that they cause wheel-hop, they are most dangerous, and provision must be made to reduce the possibility of this happening.

The foregoing remarks have applied to a single wheel having only one degree of freedom. In fact, each wheel mass has six degrees of freedom. The geometric arrangement of the suspension is of great importance in determining the coupling between the various degrees of freedom of a particular wheel, and the coupling between motions of different wheels. This latter in particular can cause some very complex problems, with combined intercoupling of many degrees of freedom to form self-excited oscillations in a system free from external disturbances. In particular, the rigid coupling provided in the front suspension by the use of a single axle is capable of maintaining a complex vibration known as "shimmy", which may seriously affect directional stability. The actual mechanism of this phenomenon involves oscillation of the steering angle which is maintained by side-shift and tramp of the axle through gyroscopic and geometric coupling. Over a wide range of speeds these motions may be so phased that they are self- excited, drawing power from the forward vehicle motion. Uneven braking , and out of balance wheels in such a suspension may also cause motions akin to shimmy.

Similar effects are noticed in the rear suspension where rigid axles are used, though in this case they are not so dangerous. Here the vibrations depend largely on the state of motion, assuming a rear wheel drive, and power, or brake-hop may result in violent oscillations and loss of road contact which seriously detracts from either the application of power or braking. Once again the mechanism of the motions is extremely complex, involving interaction of many degrees of freedom and can generally only be reduced to a safe level by practical tests and minor modifications to the suspension geometry.

By eliminating the direct interaction between wheel motions, the independent suspension has greatly reduced the severity of secondary effects of this nature, especially in the front suspension where each wheel can change its camber without seriously affecting the ather. Consequently, with independent suspension secondary vibrations of the unsprung masses are dependent mainly on the road surface irregularities. The reduction in unsprung weight available with this type of suspension has also served to alleviate the problem slightly, for it allows softer suspension spring rates to be used. However the wide range of road surface conditions to which the vehicle is subjected does not allow a mere increase in natural wheel-hop frequency to eliminate the likelihood of wheel-hop occuring. Although it does increase the vehicle velocity at which wheelhop will occur, there is still the problem of maintaining road contact without increasing the body force transmission unduly.

## 4.3. HARSHNESS.

The vibrations of the highest frequency range which are detected in the vehicle body are termed "harshness". These are in fact the lowly damped transient vibrations of the resilient body structure caused by sudden impactive loading.

Increases in tyre flexibility in recent years have decreased the amount of harshness tremendously. Small high frequency variations in road surface are absorbed in the more flexible tyres so that direct transmission is reduced. It has been found that a definite lower limit to the tyre flexibility does exist, quite apart from that indicated by the increase in tyre wear and steering effort. This limit is brought about by the large changes in rolling radius which can occur in a soft tyre as it rolls over a sudden bump. As forward speed is maintained the wheels angular velocity is rapidly accelerated to accomodate the radius change, and this causes a sharp impulse in the fore and aft direction to be delivered to the axle. As most suspensions are quite rigid in this direction, especially the independent types, these impulses are transmitted directly to the body frame causing harshness. The same reasoning can be applied to the fore and aft component of the wheel-bump reaction.

A factor which has tended to increase the extent of harshness is the increased structural rigidity of the body caused by unit construction In this case the degree of damping presented to vibrations of the body structure is much reduced and the harshness persists for longer periods. This may be alleviated to some extent by rubber isolators, but the reduction in effective torsional rigidity of the vehicle which this entails cannot always be tolerated. Nevertheless, it is obvious that the duration of harsh vibrations can be reduced by some suitable damping medium placed in a position to rapidly decay any transient frame or shell oscillations, though the existence of harshness indicates that tyre flexibilities cannot be increased indefinitely.

## 4.4 DAMPING AND "SHOCK ABSORBERS".

Damping is present in the vehicle suspension in many varied forms. It is a factor which has great influence on both ride and road-holding, as it may control amplitudes of resonant vibrations of all forms, and causes a rapid decay of transients thus regaining quickly the condition most suited to the absorption of further disturbances.

Unfortunately it is only possible to apply damping, apart from inherent damping, between the sprung and unsprung masses of the vehicle.
This means that the damper introduces a strong coupling between the primary and secondary vibrations which would not otherwise exist. Consequently the body may be subjected to high damping forces at secondary motion frequencies which naturally detract from the isolation effectiveness of the suspension though they are necessary to maintain satisfactorily small secondary motions.

The magnitudes and types of damping existing in the suspension may considerably modify this interaction and are thus of importance.
Coulomb damping is a mathematical ideal in which a constant force opposes relative motion of the damper elements. It is closely approached by hydraulic dampers in which pressure is kept constant by relief valves
Actual dry friction has a resisting force inversely proportional to velocity, and it is influenced by normal reaction, materials and surface finish of the contact areas. This is the common form encountered when surfaces rub.
Viscous damping is another mathematical ideal in which the resisting forces are proportional to the velocities of motion.
Degenerate viscous damping is very common in practice, its characteristics being derived from variations in viscosity with temperature and pressure. Here the resisting force is proportional to a power of velocity that is less than one.
Hydraulic damping has resistance proportional to the square of velocity and is obtained by pumping fluid through a sharp edged orifice.
Hysteresis damping arises from the area of the stress-strain loop for materials under cyclic loading. Energy dissipation is normally pro-

portional to stress raised to a power greater than one.

All these damping types are known to exist in the road vehicle suspension, though in some cases the magnitude may depend on the mechanical structure and condition of the suspension elements and consequently may vary quite markedly through the life of the vehicle. A particular example is the coulomb and dry friction effects which are present in the plain leaf spring and the early types of friction dampers. (O'Connor 1946)

The realization that damping has a considerable effect on the performance criteria has led to the general use of "frictionless" spring types with a separate damping unit, so that the response can be controlled and optimized.

The hysteresis damping of the pneumatic tyres plays an important part, for it is ideally situated to influence the secondary motions without introducing additional forces between the sprung and unsprung masses. Unfortunately the magnitude of this damping is not sufficiently high to eliminate wheel-hop phenomena entirely. Furthermore its value is simply a by-product of the tyre design, and is not constant through the tyre life. Consequently, although it is of an advantageous form, hysteresis damping must be supplemented by suspension damping to achieve consistent performance.

The characteristics of modern suspension dampers, or "shock-absorbers" as they have been mis-named, vary widely. This stems from the relative importance of comfort and stability in a particular application. As these are necessarily compromised, model characteristics are adjusted to suit the average speeds and road surfaces which are most likely to be encountered. This does of course mean that performance under conditions remote from those of design will not be optimized.

The type of damper which is most commonly used is the hydraulic damper. The resisting force which suwh a damper presents to relative motion is dependent on the velocity of motion. For low velocities, as generated by primary motions, these forces are low and of viscous damping form, being provided by small bleeds in the hydraulic circuit. As the velocities increase, however, the maximum force developed is made less and less dependent on velocity, by utilizing spring loaded pressure relief valves. Consequently the secondary frequencies are resisted by a characteristis approaching coulomb damping. This force-limited characteristic enables enables control of primary motions to be achieved without greatly increasing force transmission at wheelhop frequencies or during transients.
Another feature of these dampers is that they have directionally controlled characteristics. Thus when axle and body are approaching the "compression" forces developed in the damper are usually smaller than the "rebound" forces developed when the axle and body separate at the same velocity. The object of this of this practice is to reduce the transmission of high forces during compression transients, which are the most prevalent, but it is thought to be irrational by some writers (DenHartog), and in fact the difference has decreased to such an extent that it is practically insignificant in many modern designs.

Many other attempts have been made to reduce the compromize of comfort and stability by employing special damper characteristics, but these generally required special road conditions for optimum results.

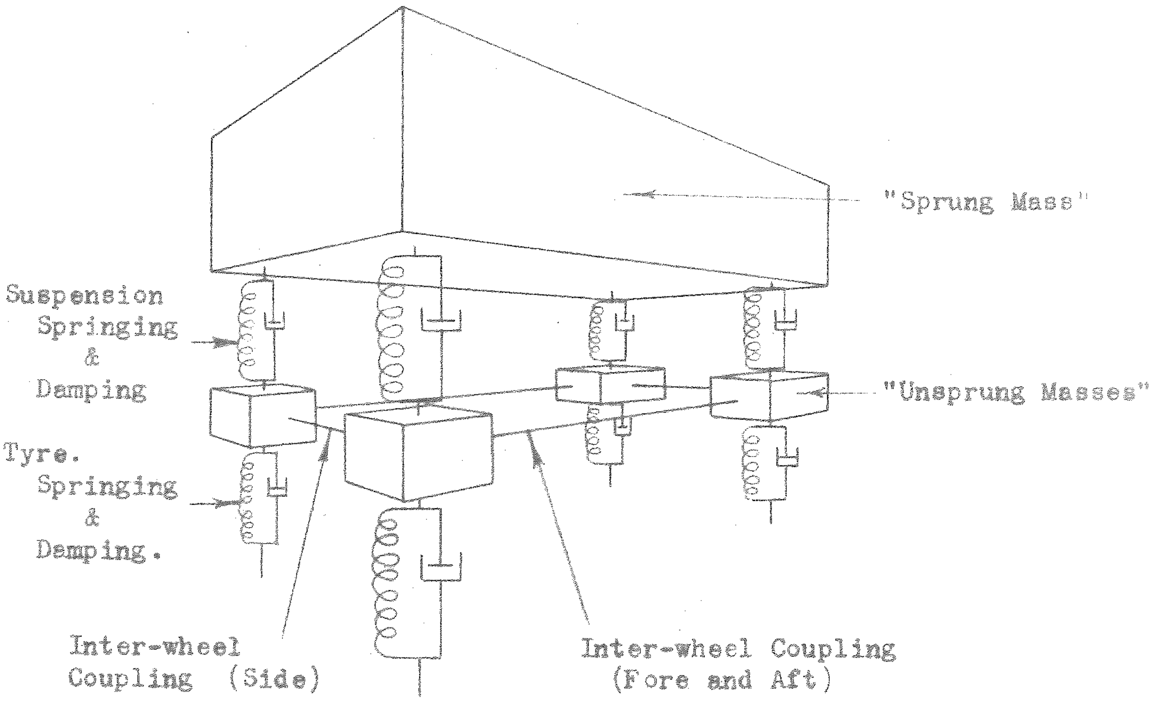



Fig. 4.1. THE BASIC VEHICLE SUSPENSION, AS A SYSTEM OF CONCENTRATED ELEMENTS .

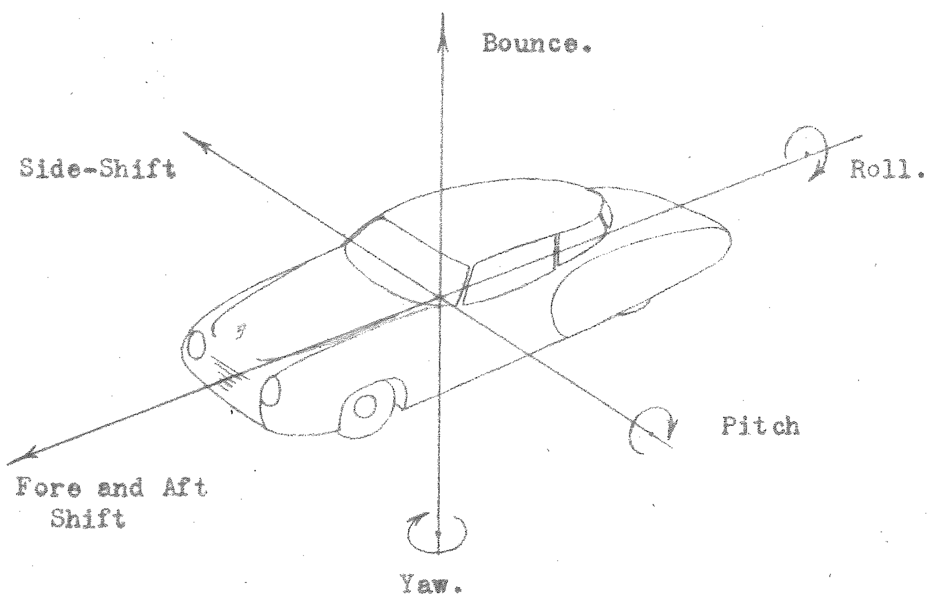



Fig. 4.2. PRIMARY MOTIONS OF THE VEHICLE BODY.

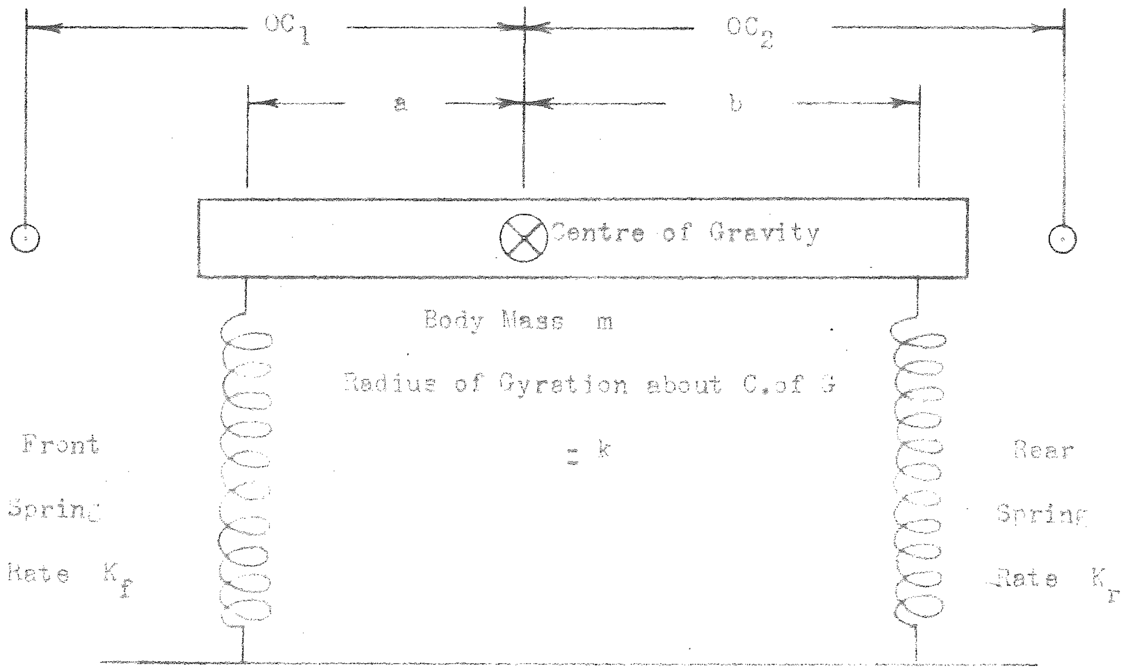



FIG.4.3. LEVER ANALOGY OF THE VEHICLE BODY.

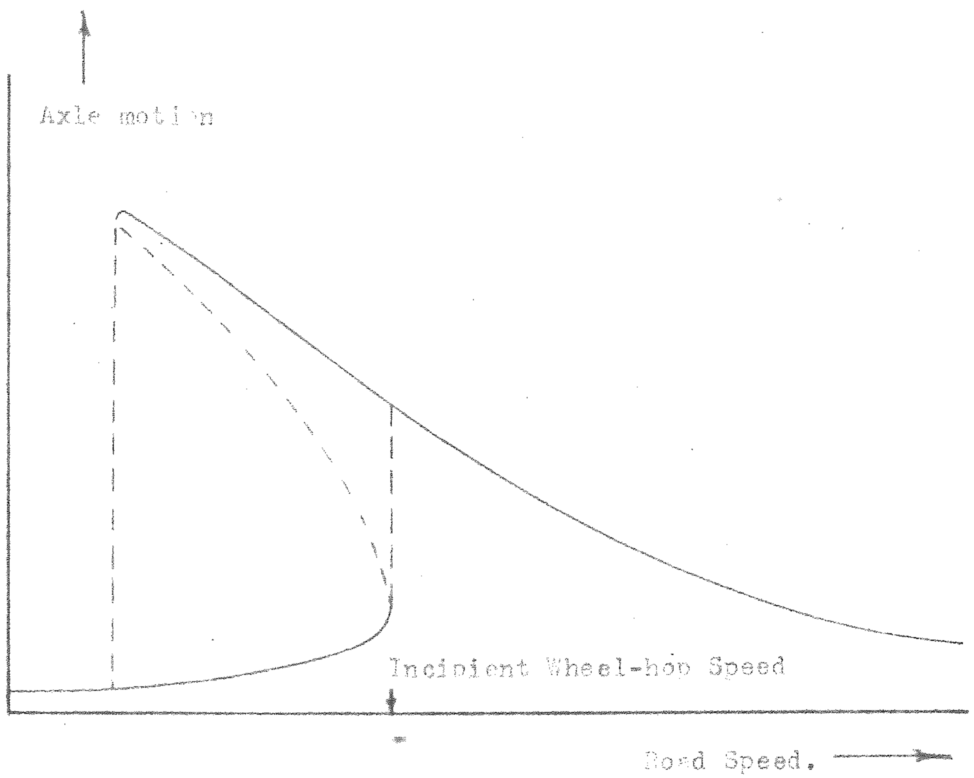



FIG.4.4. SOFTENING NON-LINEAR CHARACTERISTIC OF SECONDARY VIBRATIONS

# Chapter 5

# THE LINEAR SUSPENSION

# 5. THE LINEAR SUSPENSION .

The complete dynamic behaviour of the vehicle suspension has been shown in the previous chapter to be extremely complex . It is possible however to obtain a great deal of information indicating the performance of the system by considering simple arrangements which have the same basic characteristics. This tecnique has been used in the previous chapter to illustrate the method of optimization of pitch and bounce coupling to give an essentially flat ride , by considering the vehicle body as a rigid beam supported by two springs . Further application of the method may be used to investigate the interaction between primary and secondary motions of the suspension . Thus , if the complicating effects of coupling are neglected the suspension may be regarded as a simple two mass system having only two degrees of freedom as illustrated in fig.5.1. In such a case , the body and axle could only move vertically , and the system would possess two natural frequencies corresponding to the primary, or body, and the secondary, or axle resonance . However,even with such simplification , the response of the system can only be determined relatively easily when further restrictions are placed on the characteristics of the suspension elements and the nature of the exciting road profile . Thus, rigorous solution can only be obtained for linear suspension characteristics, that is in the case where spring forces are proportional to their displacements, and damping forces proportional to the relative velocity between elements. Besides restricting the characteristics of the suspension elements to linear form, this also demands that the tyres never lose contact with the road surface, and the ride clearances never be exceeded, conditions which can both be easily disrupted in practice. It is nevertheless convenient to use the linear response to indicate the general trends in the behaviour of such a system, with a view to improving performance by the introduction of non-linear characteristics for the suspension elements .

Again, for the sake of mathematical convenience, it is necessary to restrict the exciting road forms to types amenable to simple solution . As indicated in chapter 3, road irregularities may be either regular or singular, so it is necessary to obtain both the harmonic and transient responses of the simplified system to illustrate the overall performance .

## 5.1. HARMONIC RESPONSE .

A rigorous solution for the harmonic response may be determined from the following analysis, usigg the nomenclature of fig.5.1.

The equations of motion of the system are thus -

$$m_B D^2 x_B = K_B( x_A - x_B ) + C_B D( x_A - x_B ) \quad \text{........(5.1)}$$

$$m_A D^2 x_A - m_B D^2 x_B = K_A( x_0 - x_A ) + C_A D( x_0 - x_A ) \quad \text{.....(5.2)}$$

Equation (5.1) may be re-arranged to give $x_A$ in terms of $x_B$ thus,

$$x_A = \frac{m_B D^2 + C_B D + K_B}{C_B D + K_B} \cdot x_B \quad \text{..............(5.3)}$$

By substitution of equation(5.3) in (5.2) the relationship between body motion and excitation can be determined,

$$\text{thus} \quad \frac{x_B}{x_0} = \frac{(K_A + C_A D)(K_B + C_B D)}{(m_A D^2 + C_A D + K_A)(m_B D^2 + C_B D + K_B) + m_B D^2( C_B D + K_B)}$$

$$\text{.............(5.4)}$$

The harmonic response of the system is most conveniently determined from this equation by introducing the following parameters ,

$$D = jw \quad , \quad D^2 = -w^2 \quad ,$$

$$R = \frac{w}{w_A} \quad , \quad r = \frac{w}{w_B} \quad , \quad P = \frac{w}{w_{AB}}$$

$$B = \frac{C_A}{2m_A w_A} \qquad b = \frac{C_B}{2m_B w_B} \quad \text{...........(5.5)}$$

Where $w_A = \sqrt{\dfrac{K_A}{m_A}}$ , $w_B = \sqrt{\dfrac{K_B}{m_B}}$ , $w_{AB} = \sqrt{\dfrac{K_A}{m_B}}$ ...(5.6)

$$\text{Thus} \quad \frac{x_B}{x_0} = \frac{(1 + 2BRj)(1 + 2brj)}{(1 - R^2 + 2BRj)(1 - r^2 + 2brj) - P^2(1 + 2brj)} \quad \text{........(5.7)}$$

$$\text{And} \quad M_B = \frac{x_B}{x_0} = \sqrt{\frac{(1 - 4BRbr)^2 + 4(BR + br)^2}{((1 - r^2)(1 - R^2) - 4BRbr - P^2)^2 + 4((1-R^2-P^2)br + (1-r^2)BR)^2}}$$

.........(5.8)

Similarly by substituting for $x_B$ in equation (5.2)

$$M_A = \frac{x_A}{x_0} = \sqrt{\frac{(1 - r^2 - 4BRbr)^2 + 4(BR(1 - r^2) + br)^2}{((1 - r^2)(1 - R^2) - 4BRbr - P^2)^2 + 4((1-R^2-P^2)br + (1-r^2)BR)^2}}$$

...........(5.9)

Furthermore the total transmission to the body is determined by

$$F_T = m_B x_B w^2 = K_B x_0 r^2 M_B \quad \text{..............................(5.10)}$$

For a particular excitation frequency, variables r,R,P, are fixed as functions of the masses and spring characteristics of the elements , while B and b represent the degrees of damping present in the tyre and body dampers respectively.

The harmonic response presented by these equations may be most conveniently illustrated in graphical form, but unfortunately as there are three independent variables involved,a series of graphs is necessary. As tyre damping is the least controllable variable, it is convenient to ignore it in determining the optimum characteristics of the suspension elements . The results obtained after such an omission will then always be pessimistic as all tyre damping is advantageous for it opposes secondary motions without transmitting force to the body.

Response of a system having zero tyre damping is obtained from equations (5.8),(5.9) and (5.10) by substituting $B = 0$.

Thus

$$M_A = \sqrt{\frac{(1-r^2)^2 + 4b^2r^2}{((1-r^2)(1-R^2)-P^2)^2 + 4b^2r^2(1-R^2-P^2)^2}} \quad \ldots(5.11)$$

$$M_B = \sqrt{\frac{1 + 4b^2r^2}{((1-r^2)(1-R^2)-P^2)^2 + 4b^2r^2(1-R^2-P^2)^2}} \quad \ldots(5.12)$$

$$\frac{F_T}{K_B\, x_0} = r^2 . M_B . \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.13)$$

Graphical representation of these equations is given in figures 5.2, 5.3, and 5.4 , in which $M_A$ , $M_B$ , and $\frac{F_T}{K_B x_0}$ are plotted versus frequency ratio r, for various degrees of suspension damping b. The particular relationship between r, R, and P used to plot these graphs is selected to represent a typical passenger car suspension of the present time , and is exactly the same as that produced on the mechanical analogue described in chapter 7 .

The effects of linear suspension damping as indicated by these figures is largely dependent on frequency ratio r . Thus at either of the resonances damping prevents infinite motions of body and axle from developing, the resultant magnification being dependent only on the degree of damping present. However, the total force transmission to the body at the resonances tends to reach a minimum limit as the magnitude of damping increases . Furthermore while damping causes reduced axle motions throughout practically the entire frequency range, it does so at the expense of increased body motions at frequencies away from resonance. Together with this, the total force transmission at non-resonant frequencies becomes larger as the degree of damping rises .

It is thus obvious that linear damping cannot satisfy all the performance criteria of the suspension system, for although it limits the magnification of axle and body motions at resonance and causes reduced axle motions throughout the frequency range, it also results in increased body motions and far greater force transmission at frequencies remote from resonance.

## 5.2 TRANSIENT RESPONSE.

The response of any linear system to an input which causes transient oscillation is most easily determined by the Laplace Transform method. In the case of the vehicle suspension it is of interest to determine the response of the simple two mass system of fig.5.1. to a step function disturbance in the road profile. In order to reduce the number of variables, and emphasize the effects of suspension damping $C_B$, the inherent tyre damping is neglected as in the presentation of the harmonic response.

The basic equation for the motion of the body mass $m_B$ is then given by modifying equation (5.4) to -

$$\left( (m_A D^2 + K_A)(m_B D^2 + C_B D + K_B) + m_B D^2 (C_B + K_B) \right) x_B = K_A (C_B D + K_B) x_0 \quad \ldots\ldots\ldots\ldots\ldots (5.14)$$

On dividing by $m_A . m_B$, and making the substitutions of equation (5.6) this gives -

$$\left( D^4 + 2bw_B \left(1 + \frac{w_A^2}{w_{AB}^2}\right) D^3 + \left(w_A^2 + w_B^2 + \frac{w_A^2 w_B^2}{w_{AB}^2}\right) D^2 + 2bw_A^2 w_B D + w_A^2 w_B^2 \right) x_B =$$

$$= (2bw_A^2 w_B D + w_A^2 w_B^2) x_0 \quad \ldots\ldots\ldots\ldots (5.15)$$

Taking the Laplace Transform of this equation with a step function excitation of magnitude $X_0$ with $x_B$ and all its derivatives zero at time $t = 0$ results in -

$$x_B(p) = \frac{X_0 (2bw_A^2 w_B p + w_A^2 w_B^2)}{p\left(p^4 + 2bw_B\left(1 + \frac{w_A^2}{w_{AB}^2}\right) p^3 + \left(w_A^2 + w_B^2 + \frac{w_A^2 w_B^2}{w_{AB}^2}\right) p^2 + 2bw_A^2 w_B p + w_A^2 w_B^2\right)} \quad \ldots (5.16)$$

Similarly -

$$x_A(p) = \frac{X_0(w_A^2\,p^2 + 2bw_A^2 w_B p + w_A^2\,w_B^2)}{p(p^4 + 2bw_B(1 + \frac{w_A^2}{w_{AB}^2})p^3 + (w_A^2 + w_B^2 + \frac{w_A^2 w_B^2}{w_{AB}^2})p^2 + 2bw_A^2 w_B p + w_A^2\,w_B^2)} \quad \ldots\ldots(5.17)$$

And

$$F_T(p) = \frac{K_B X_0 p(2bw_A^2 w_B p + w_A^2 w_B^2)}{w_B^2(p^4 + 2bw_B(1 + \frac{w_A^2}{w_{AB}^2})p^3 + (w_A^2 + w_B^2 + \frac{w_A^2 w_B^2}{w_{AB}^2})p^2 + 2bw_A^2 w_B p + w_A^2 w_B^2)} \quad \ldots\ldots(5.18)$$

The standard method of obtaining the inverse trasnforms for such functions involves the rearrangement of the right hand side of equations (5.16),(5.17) and (5.18) into partial fraction form. Unfortunately, the fourth order equation in p which exists in the denominator of these expressions has no general solution and must be evaluated for particular combinations of suspension parameters. Once again, the values of $w_A$, $w_B$, and $w_{AB}$ are chosen equal to those parameters existing on the mechanical analogue described in chapter 7, which is also representative of a typical independent suspension, while the damping ratio b is given a range of values from 0 to 1 to illustrate the effects of suspension damping.

The solution of the characteristic equation of the system, which coincides with the denominator of the previous equations, is given below for various degrees of damping b.

| Damping Ratio "b" | Factorized Characteristic Equation |
|---|---|
| 0 | $(p^2 + 6248)(p^2 + 38.70)$ |
| 0.1 | $(p^2 + 9.570p + 6237)(p^2 + 1.157p + 38.77)$ |
| 0.3 | $(p^2 + 28.67p + 6147)(p^2 + 3.518p + 39.34)$ |
| 0.5 | $(p^2 + 47.61p + 5959)(p^2 + 6.038p + 40.58)$ |
| 0.8 | $(p^2 + 75.32p + 5450)(p^2 + 10.52p + 44.36)$ |
| 1.0 | $(p^2 + 92.60p + 4841)(p^2 + 14.71p + 49.95)$ |

The actual solution of $x_A$, $x_B$, and $\frac{F_T}{K_B X_0}$ as time functions may then be obtained from the partial fraction rearrangement of the transform equation by reference to tables of standard inverse transforms, giving the following solutions -

Case 1. $b = 0$

$$x_B = X_0 ( 1 + 0.0062\sin(79.04t + 90^\circ) + 1.0062\sin(6.2209t - 90^\circ)$$

$$x_A = X_o ( 1 + 0.9542\sin(79.04t - 90^\circ) + 0.0458\sin(6.2209t - 90^\circ)$$

$$F_T = K_B X_0 ( 0.954\sin(79.04t + 90^\circ) + 0.954\sin(6.2209t - 90^\circ)$$

Case 2. $b = 0.1$

$$x_B = X_0(1 + .01647e^{-4.785t} \sin(78.83t + 152.4^\circ) + 1.015e^{-.5783t} \sin(6.200t - 82^\circ)$$

$$x_A = X_0(1 + .9560e^{-4.785t} \sin(78.83t - 94^\circ) + 0.0480e^{-.5783t} \sin(6.200t - 72.8^\circ)$$

$$F_T = K_B X_0(2.520e^{-4.785t} \sin(78.83t + 158.8^\circ) + .968e^{-.5783t} \sin(6.200t - 70^\circ)$$

Case 3. $b = 0.3$

$$x_B = X_0(1 + .0476e^{-14.33t} \sin(77.08t + 153.6^\circ) + 1.0585e^{-1.759t} \sin(6.021t - 75^\circ)$$

$$x_A = X_0(1 + .9860e^{-14.33t} \sin(77.08t - 98^\circ) + 0.0520e^{-1.759t} \sin(6.021t - 25^\circ)$$

$$F_T = K_B X_0( 7.15e^{-14.33t} \sin(77.08t + 175^\circ) + 1.026e^{-1.759t} \sin(6.021t - 37.5^\circ)$$

Case 4. $b = 0.5$

$$x_B = X_0(1 + .0850e^{-23.81t} \sin(73.43t + 143.6^\circ) + 1.1953e^{-3.0192t} \sin(5.608t - 61.5^\circ)$$

$$x_A = X_0(1 + 1.0390e^{-23.81t} \sin(73.43t - 105^\circ) + 0.0606e^{-3.0192t} \sin(5.608t - 5^\circ)$$

$$F_T = K_B X_0( 12.45e^{-23.81t} \sin(73.43t + 180^\circ) + 1.190e^{-3.019t} \sin(5.608t)$$

CASE 5. $b = 0.8$

$$x_B = X_0(1 + .1849e^{-37.66t} \sin(63.50t + 123^\circ) + 1.8793e^{-5.259t} \sin(4.087t - 38^\circ)$$

$$x_A = X_o(1 + 1.280e^{-37.66t} \sin(63.50t - 122^\circ) + .0900e^{-5.259t} \sin(4.087t + 20^\circ)$$

$$F_T = K_B X_0( 24.8e^{-37.66t} \sin(63.5t + 184.4^\circ) + 2.05e^{-5.259t} \sin(4.087t + 66.3^\circ)$$

Case 6 . $b = 1.0$

$$x_B = X_0(1 + .3310e^{-46.3t} \sin(51.93t + 107^\circ) + 1.711e^{-5.32t} - 3.027e^{-9.39t}$$

$$x_A = X_0(1 + 1.681e^{-46.3t} \sin(51.93t - 130^\circ) - .0706e^{-5.32t} + .3587e^{-9.39t}$$

$$F_T = K_B X_0( 39.6e^{-46.3t} \sin(51.93t + 188^\circ) - 1.19e^{-5.32t} + 6.58e^{-9.39t}$$

Fig 5.5 shows the effect of linear damping on the most important features of the transient response i.e. maximumovershoot and decay time of the axle and body, and the maximum transmission to the body . This indicates that the overshoot of axle and body can be limited by linear damping, but only at the expense of increased transmission . Furthermore, as the damping ratio increases, the maximum transmission occurs closer to the point of initiation of the transient and hence becomes more impulsive in character . The graph also indicates a minimum in the decay times for both axle and body at damping ratio b approximately 0.6 . Thus, as for the harmonic response,linear damping cannot satisfy all the performance criteria of the vehicle suspension so that in operation it can only represent a compromize .

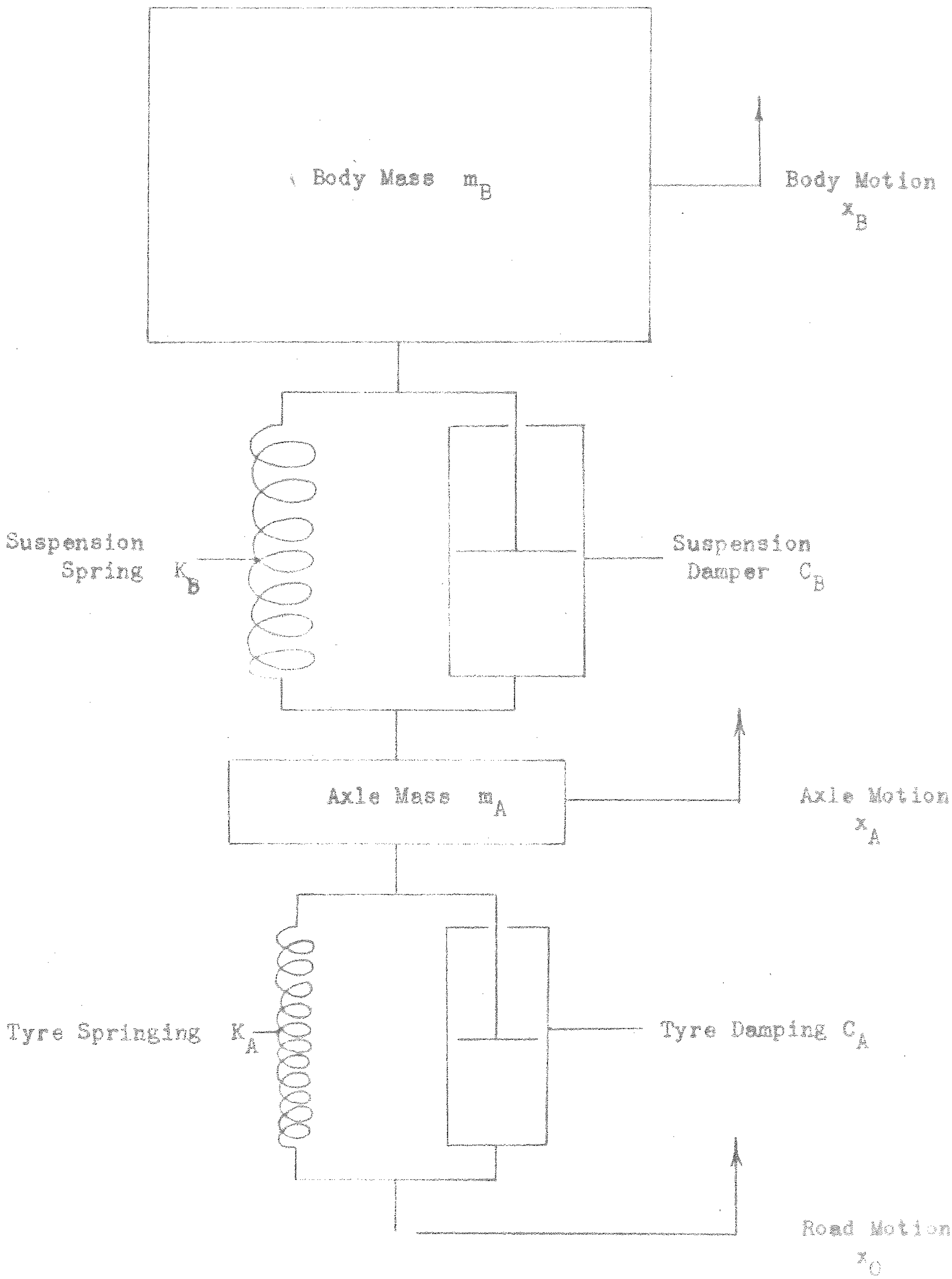



Fig. 5.1. SIMPLIFIED TWO MASS SYSTEM .

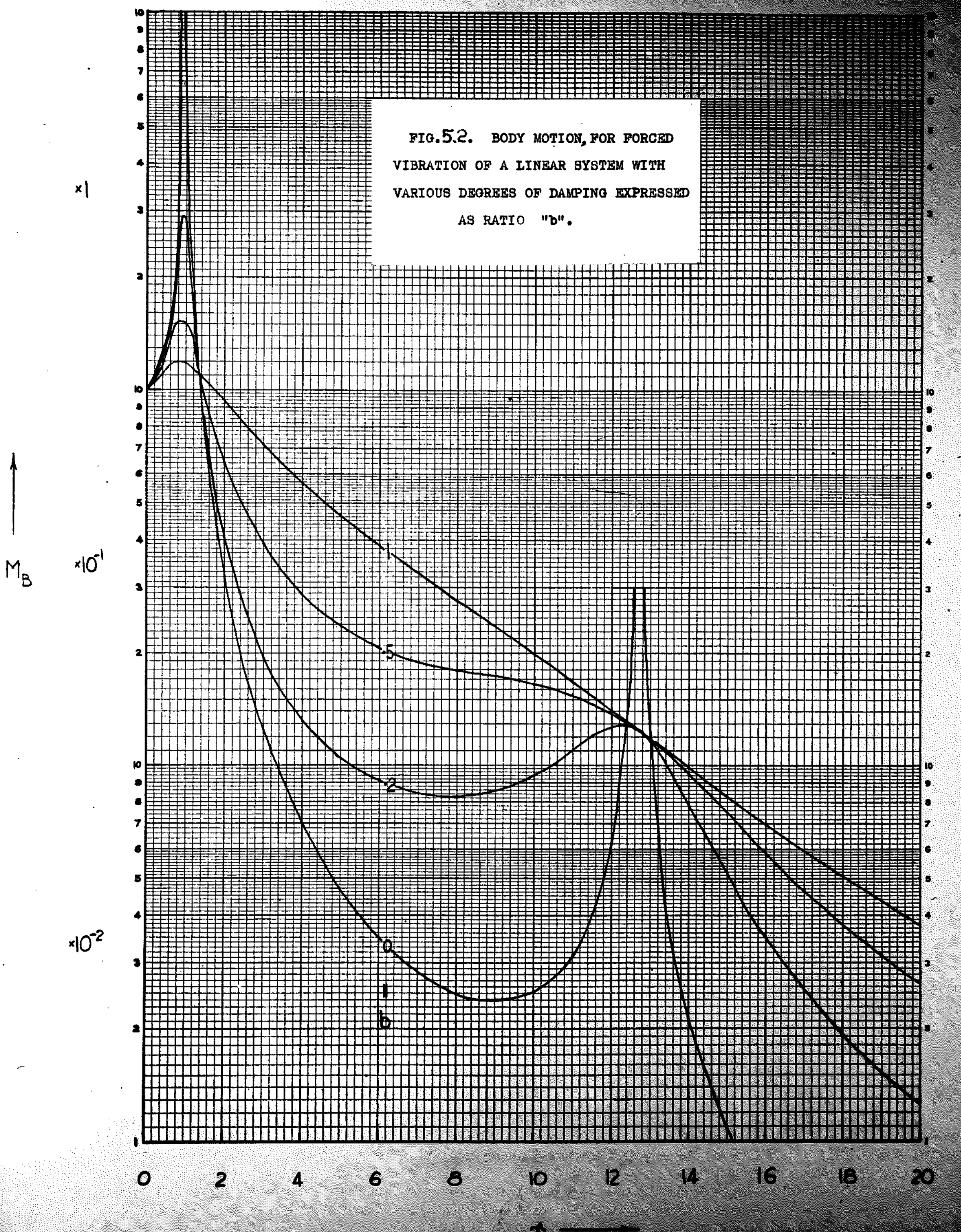



FIG. 5.2. BODY MOTION, FOR FORCED VIBRATION OF A LINEAR SYSTEM WITH VARIOUS DEGREES OF DAMPING EXPRESSED AS RATIO "b".

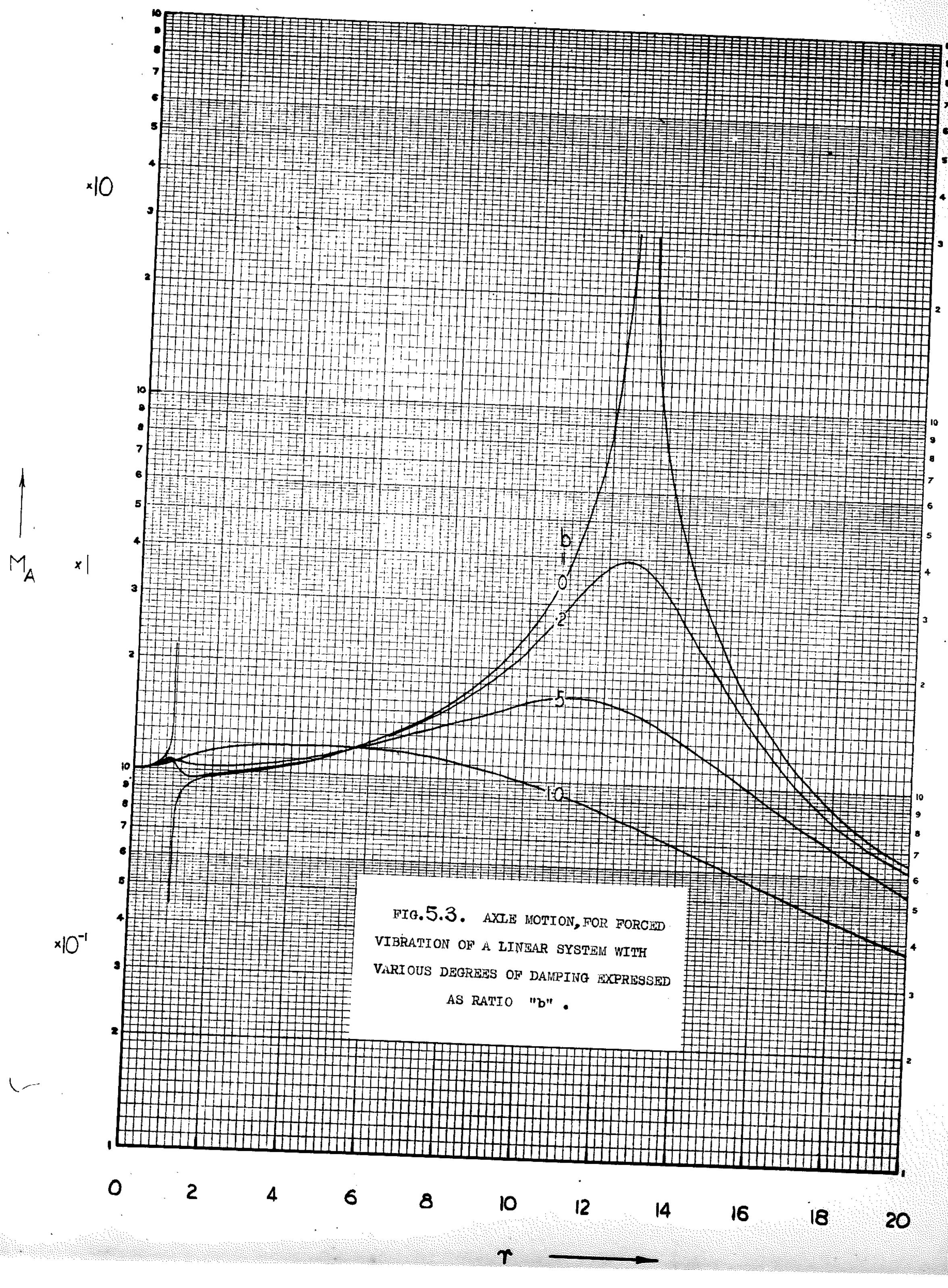



FIG. 5.3. AXLE MOTION, FOR FORCED VIBRATION OF A LINEAR SYSTEM WITH VARIOUS DEGREES OF DAMPING EXPRESSED AS RATIO "b".

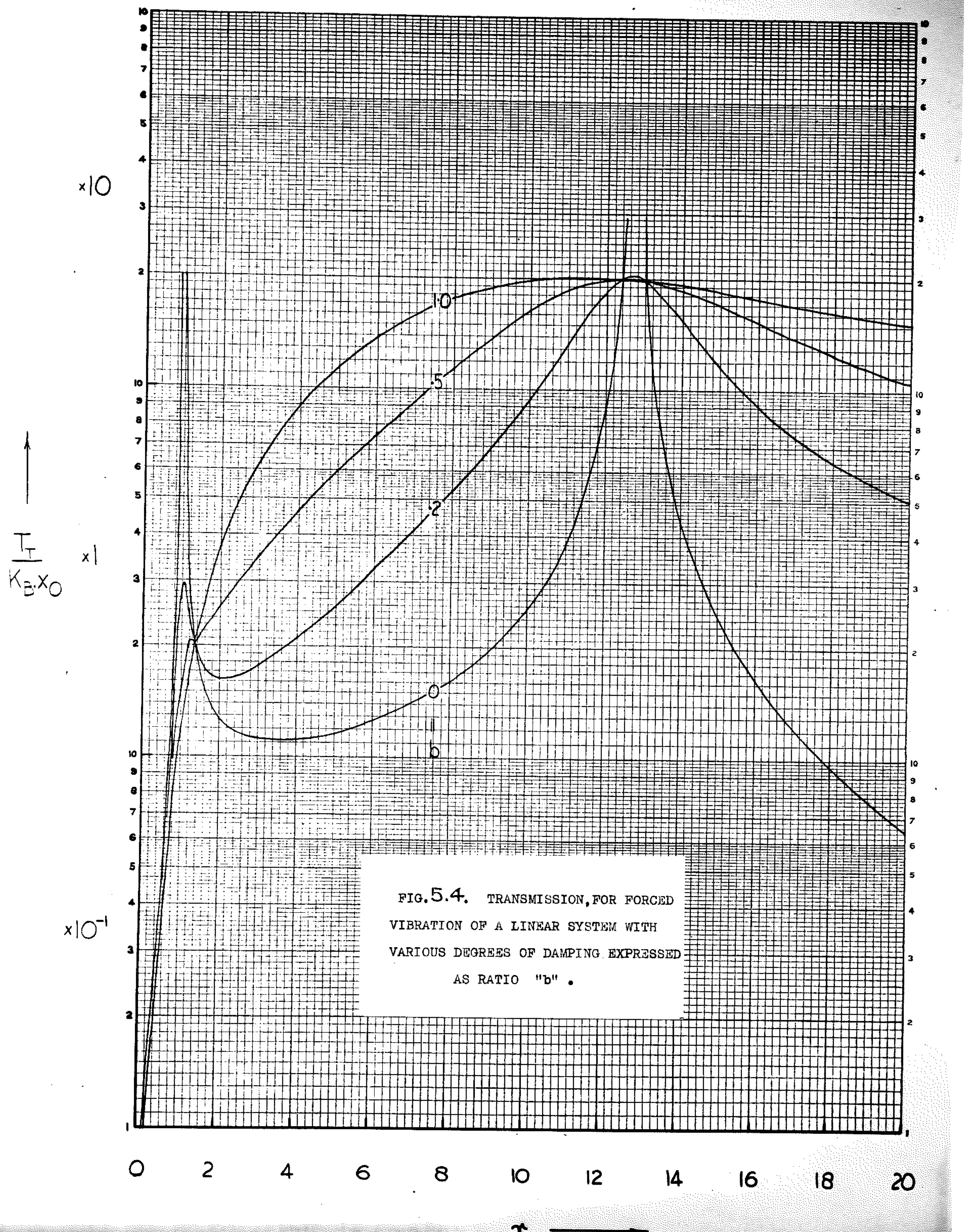



FIG. 5.4. TRANSMISSION, FOR FORCED VIBRATION OF A LINEAR SYSTEM WITH VARIOUS DEGREES OF DAMPING EXPRESSED AS RATIO "b".

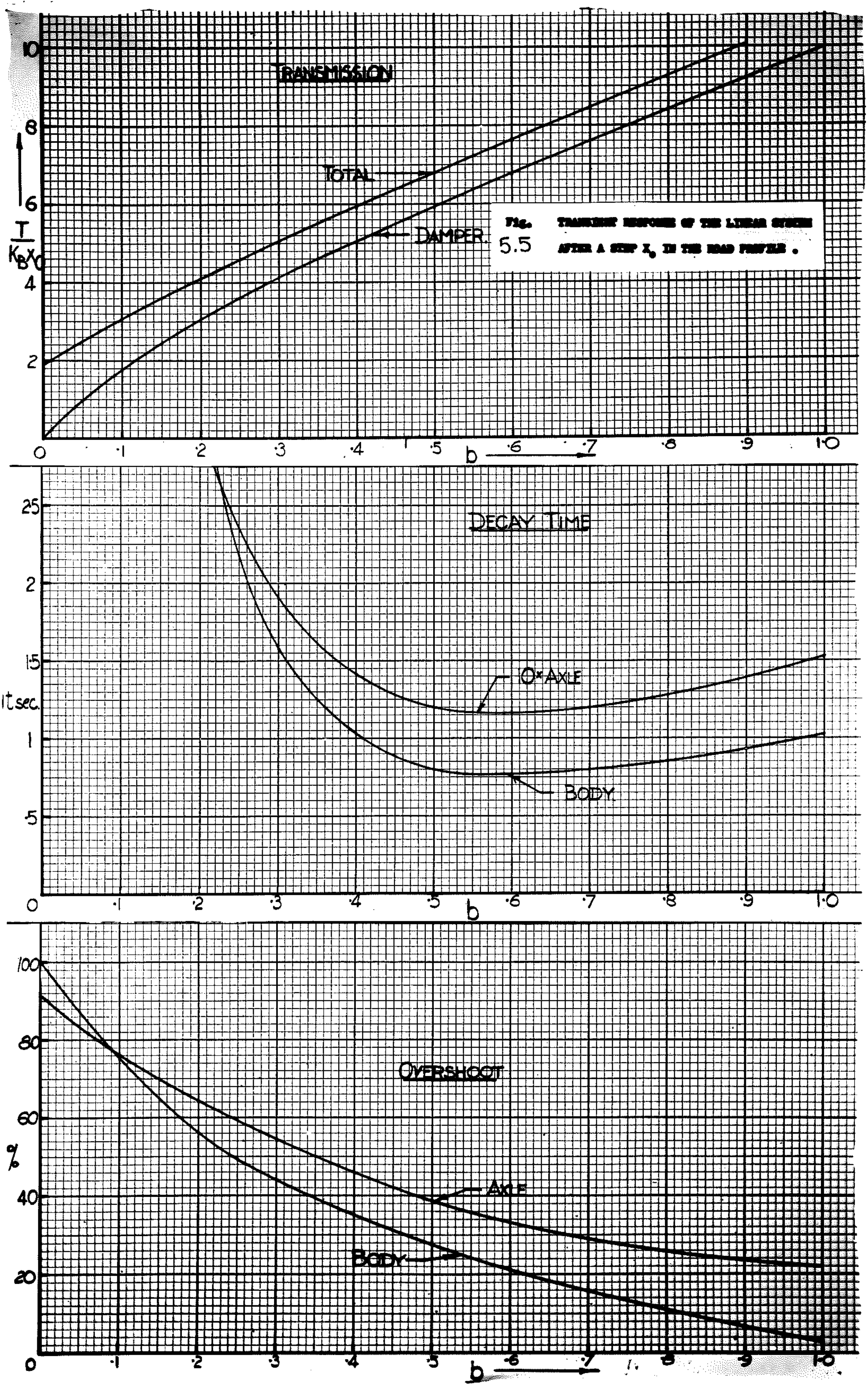



Fig. 5.5 TRANSIENT RESPONSE OF THE LINEAR SYSTEM AFTER A STEP $X_0$ IN THE ROAD PROFILE.

# Chapter 6

# NON-LINEAR VIBRATIONS

## 6. NON-LINEAR VIBRATIONS.

The rigorous solutions obtained in the previous chapter are only applicable to systems which have linear characteristics. This implies that spring forces are always proportional to their deformations from the static equilibrium position, and damping forces are always proportional to the relative motion between the damper elements.
A third condition which must be satisfied by linear systems is that the masses themselves be unaffected by any vibration, however practically all mechanical systems comply with this requirement, and in particular it is not applicable to vehicle suspension behaviour.

On the other hand, non-linearities of both springing and damping are most marked in the suspension, and these may seriously modify the performance estimated from linear theory.

### 6.1. DEFINITION OF NON-LINEAR CHARACTERISTICS.

#### 6.1.1. NON-LINEAR SPRING CHARACTERISTICS.

A non-linear spring characteristic is obviously one in which the restoring force brought about by deformation of the spring is not directly proportional to the magnitude of this deformation. Consequently there are two basic types of non-linearity, one in which the restoring force increases at a greater rate than the deformation, and one in which the restoring force increases at a lesser rate than the deformation. These are termed "hardening" and "softening" non-linearities respectively, for they exhibit an increase or decrease in the effective spring rate in the deformed position.

These basic forms may be further combined to give composite non-linear spring characteristics in which the relationships between restoring force and deflection are not identical for deformations on either side of the static equilibrium position.

Typical examples of non-linear spring characteristics are given in Fig. 6.1 (a),(b),(c).

Non-linear spring characteristics may in some cases be specified mathematically by introducing cubic or exponential relationships in the dependence between load and deflection. Generally however it is more convenient to specify the non-linearity graphically for practical springs, in "spring characteristic" curves.

## 6.1.2. NON-LINEAR DAMPING CHARACTERISTICS.

The condition of linear damping in which the resisting force is proportional to the relative velocity across the damper is rarely obtained in practical suspensions. The actual damping forms which do exist differ widely, depending largely on their sources as indicated in Chapter 4. Thus damping force may be proportional to powers of velocity both greater and less than unity, and may even become independent of velocity, merely resisting motion with a constant force. In a similar way to composite non-linear springing, damping may offer resistance of different magnitude or form to motions in one direction than it does to motions in the reverse direction.

Consequently, although the resisting effort provided by a damper may in many cases be specified by a constant of proportionallity which relates it to a function of velocity, the performance of most practical dampers must be presented graphically. This is done by plotting the damper effort versus the relative position of the damper elements as this varies with simple harmonic motion. Such a figure is termed the damper "work diagram" as its area indicates the total energy dissipated by the damper under the particular operating conditions. Obviuosly this indicates the damper behaviour through a complete vibration cycle and covers a wide range of velocities of motion, but it is often necessary to determine "work diagrams" at various frequencies to fully specify the speed dependence.

Typical work diagrams are illustrated in Fig.6.2.

## 6.2. NON-LINEARITIES OF VEHICLE SUSPENSION COMPONENTS.

Non-linearities which do exist in the suspension of road vehicles arise in the main from two causes,

(a) The basic physical arrangement of the system which is force-closed by gravity to the source of excitation. The system may thus modify the effective disturbances by losing contact with the ground. Thus for axle motions greater than a definite amplitude fixed by the static tyre deflection, the effective tyre spring vanishes during any period of lost road contact.

Also the necessity of presenting finite limits to the magnitudes of axle-body relative motion or "ride-clearances" requires distinct hardening non-linearities whenever these limits tend to be exceeded. Thus, the effective suspension spring rate is greatly increased by bringing rubber snubbers into play.

(b) The need to compromize ride comfort and road holding stability, which has led to the general use of non-linear dampers which have a predetermined limit to the maximum forces that they can develop. Such dampers may also be assymetric in that their characteristics for closing damper motions differ from those when opening.

In addition to these major sources of non-linearity in the springing components of the vehicle suspension, the tyre itself provides a non-linear spring which becomes progressively stiffer as deflected, however the effect is far less pronounced, and tyre springing may generally be assumed linear.

In some commercial vehicles where large variations are to be encountered, the suspension spring is purposely made non-linear with the spring rate everywhere proportional to the load acting. By this means the primary natural frequency is made independent of load, and the general behaviour much less load dependent. Actual road tests on passenger buses with such suspension springs have been carried out as described in Appendix 6.

Non-linearities of damping are in fact very wide spread in the suspension as indicated in Chapter 4. Many of the sources of the sources of damping are associated with friction of the suspension parts and this produces damping forces which are independent of or inversely proportional the velocities of motion. Also the inherent hysteresis damping in the tyres and rubber suspension components is strictly non-linear the energy dissipation depending solely on the stressing of the material.

## 6.3. DYNAMIC CHARACTERISTICS OF NON-LINEAR SYSTEMS.

### 6.2.1. NON-LINEARLY SPRUNG SYSTEMS

The effect of the non-linearities of springing on the dynamic behaviour of simple systems has been known for some time, being first reported by Duffing in 1918. A one mass system with non-linear spring is considered for simplicity. The basic difference of such a system is that the natural frequency depends on the amplitude of oscillation. If the spring has hardening characteristics, the natural frequency will increase with increasing amplitudes of motion, whereas the reverse is true of a softening non-linearity. Thus the frequency of transient oscillation of a non-linear system will depend largely on the severity of the impulse initiating the transient. Also, if damping is present in the system the decaying oscillation will change in frequency as the effective spring rate alters with decreasing amplitude.

As a consequence of this, the steady state harmonic behaviour of the non-linear system has several characteristics which are not associated with linear systems. These stem from the existence of a frequency range in which response is not unique, the actual vibration amplitudes depending largely on the history of the excitation. Typical harmonic response curves for hardening and softening non-linearly sprung masses with a small amount of damping are given in Fig.6.3. The effect of the non-linearity is seen to be a displacement of the point of maximum dynamic magnification away from the base natural frequency. With a hardening spring this displacement is towards the higher frequency range while the opposite is true for a softening spring. A major feature is that the motion cannot build up to infinity at any particular frequency even in the absence of damping. Thus in the hardening spring case, infinite motions of the mass are obtainable only at infinite frequency in an undamped system, whereas infinite motions are never obtainable in a softening system as such would theoretically correspond with zero excitation. With damping in the system, both the amplitude and frequency of the maximum possible motion are dependent on the magnitude of damping present.

Extremely large motions cannot be developed as such become unstable and the system decays to the low energy level mode of oscillation. Damping thus introduces finite discontinuities in the frequency curves known as "jumps". The effects are illustrated in Fig,6.3. for a hardening non-linearly sprung system. With increasing frequency of excitation of a given amplitude, the resultant motions build up along the curve AB. On increasing the frequency above that at B the motion continues to build up until point C is reached. Here the damping present prevents further build up of the motions with frequency as it tends to dissipate more energy than the excitation provides.(Rauscher 1938) . Thus if the excitation frequency is increased, the system adopts the only stable motion possible which is at the lower energy level, and the magnitude of oscillation rapidly decays to the point D. Further increase in frequency causes a slight reduction in amplitude as indicated by the path DE. If however the excitation frequency is decreased from that of state D it is found that stable oscillations of low energy level exist for all frequencies along the path DF. At F, the low energy level oscillation becomes unstable, and amplitudes "jump-up" to those of point B. Thus for all frequencies in the range BC possible amplitudes of motion are not unique, oscillations of both high and low energy level being stable, the resultant motions depending solely on the history of the excitation. If this is one of increasing frequency from a low value, a high level oscillation is most likely but this will "jump-down" to a low level oscillation at a frequency determined only by the degree of damping. With decreasing frequency however, the low level oscillation itself eventually becomes unstable and motions "jump-up" to a much higher energy level. In this case the "jump-up" frequency depends primarily on the characteristic of the non-linear spring, and is only slightly influenced by the magnitude of damping. Thus although a non-linearly sprung sprung system cannot develop infinite amplitudes of motion at any finite frequency, there is still a considerable frequency range in which the minimum achievable motions represent considerable dynamic magnification.

Exactly analagous results apply in the case of non-linearly sprung systems having softening spring characteristics.

6.2.2. NON-LINEARLY DAMPED SYSTEMS.

Non-linearities of damping have an extremely wide range of possible characteristics but basically they may be divided into two classes

(1) Non-linearity which provides damping effort dependent on some function of the resulting vibration other than the first power of velocity.

(2) Non-linearity which provides damping effort independent of the resultant vibrations.

As shown in the results of chapter 5 for a linear system, damping has little effect on the amplitudes except near resonance, but it has a marked influence on transmissibility, i.e. the transmission of force to the isolated body, at all frequencies throughout the range.

In general the same effects are apparent for non-linear damping of all types falling into the category (1) above. If the dependence is on a high power of velocity, the effects are more pronounced than for linear damping, while the opposite is true if lower powers of velocity are involved. The most interesting effect of damping non-linearity occurs in a linearly sprung system with non-linear damping of class (2) above. In this case, it is possible for the system to develop infinite amplitudes at resonance even when some damping exists. This was first indicated by Den Hartog (1930) for the case of Coulomb damping, and is shown to be true where damp-effort is limited in section 6.5. The apparent anomaly is explained by the divergence of excitation energy and dissipated energy possible where both are proportional to the amplitudes of oscillation as in such systems. It is nevertheless possible to overcome this effect by supplying a sufficient magnitude of damping effort, the critical value depending on the incident excitation.

6.2.3 COMBINED NON-LINEARITIES OF SPRINGING AND DAMPING.

There is as yet no information of the behaviour of systems having combined non-linearities of both springing and damping. Basically of course, the characteristics of such systems will be determined by the spring non-linearity, but the damping will definitely affect transmissibility and the "jump-down" frequency, and to a lesser extent the "jump-up" frequency. The modification likely to be introduced in the transmissibility should be analogous to that for a linearly sprung system, but the effect of non-linear damping on the jump frequencies is not entirely predictable. It is thought, however, that any magnitude of damper effort, whether related to the resultant vibrations or not, will always ensure that a finite "jump-down" frequency does exist i.e. that the high energy level oscillations will not continue to be stable throughout the entire frequency range of non-unique solution. This is based on the fact that the existence of high level oscillations of a non-linearly sprung system is a problem of stability rather than energy balance as in the linear system. As the margin of stability of the high level oscillations decreases as they increase in magnitude with change of frequency, it follows that any slight reduction caused by damping will eventually make the oscillations unstable, and this will constitute a finite limit to the "jump-down" frequency.

These effects are verified by the experimental work of Chapter 8.

## 6.4. SOLUTION OF NON-LINEAR VIBRATION PROBLEMS.

Some rigorous mathematical analyses of non-linear vibration problems are known, though generally it is necessary to employ approximate graphical or numerical solutions. Problems of non-linear springing and damping have been investigated separately, but even so the solutions are practically all restricted to systems having only one degree of freedom.

### 6.4.1. SPRING NON-LINEARITY.

(a) Transient response.

The most important feature of systems with non-linearities of springing is the dependence of the frequency of undamped free vibration on the amplitude of oscillation. A second characteristic is the change in the waveform of the vibrations from true simple harmonic motion. These relationships may be rigorously determined if the non-linearity of the spring can be expressed as a simple exponential, polynomial, or transciendental relationship between restoring force and deflection. By equating the maximum kinetic energy of the mass to the maximum stored potential energy of the spring, the oscillation period for a definite amplitude may be determined by integration, although this normally requires the use of elliptic integrals. Typical examples of this procedure are given in most vibration texts.

If the spring characteristic is linear in segments, the natural period may be determined by considerations of simple harmonic motion in each linear region with equality of velocities at all changes of spring rate. (DenHartog and Heiles 1936).

For more complex non-linearities it is necessary to use approximate graphical or numerical solutions. Graphical solutions are based on the method devised by Lord Kelvin in which the mass motions are plotted in increments by using the relationship

$$R = \sqrt{\frac{(1 + (\text{Velocity})^2)^3}{\text{Acceleration}}}$$

Where R is the radius of curvature of the motion.

A modification of this method which is particularly useful employs the "phase-plane" construction in which mass motion is first plotted against mass velocity, for convenience, and later transferred to a time dependent plot. (Jacobsen 1952).

Although these methods are only approximate, it is possible by averaging and refinement of interval to obtain an acceptable accuracy. Graphical procedures are of course essential where practically determined spring characteristics cannot be expressed as a simple mathematical relationship.

Numerical solutions of high accuracy may be obtained by the method

devised by Blaess (1937). In this the motion is expressed in terms of its derivatives by the Maclauren series. A progressive numerical integration of the first three terms of this series is performed for five small time intervals. At this stage, corrections calculated from the incremental values are applied to eliminate the cumulative errors. The method may then be continued in steps of five increments to complete a full cycle.
Other iterative numerical methods of solution are given by Rauscher (1938) and Brock (1951). These facilitate integration of the basic equation relating mass acceleration to spring force, and give success-approximations of the wave form tilla cycle is complete.

Another method of obtaining the transient response of the one mass non-linear spring system is that developed by Linsted and described by Timoshenko, called the method of successive approximations. This can only be used if the spring characteristic can be expressed mathematically. In this the circular frequency and amplitude of the resultant motion are assumed to be an infinite series of the non-linearizing spring constant. The successive approximations are then achieved by assuming a solution containing only one term of these series. This gives an answer which is fed into the two term series and modified to satisfy the boundary conditions. This second answer is then fed into the three term series, modified, and so on till acceptable accuracy is achieved. Provided the degree of non-linearity is small, the series converge rapidly, giving an accurate answer in Fourier series form.

(b) HARMONIC RESPONSE.

Determination of the steady state harmonic motion response of a non-linearly sprung system is a far more complex problem than is the transient response. In general, approximate means of solution must be employed as the only systems having known rigorous solutions are special cases of those with spring characteristics linear in segments. (DenHartog and Heiles, 1936) and (DenHartog and Mikina, 1932)

The simplest approximation which can be used is that the resulting motions are of sine form. This assumption is naturally in error although the wave shape is close to sine form for quite marked non-linearities provided they are symmetric. Such a solution can only satisfy the equations of motion at the instants that the system is at the extreme or centre positions. It is thus possible to establish a formula relating the approximate amplitudes of motion to the exciting frequency which can be solved either iteratively or graphically.

A more accurate approximation is given by the Ritz averaging method (Duffing 1918) in which a series form is assumed for the steady state motion, and the constants of the series selected to make the average value per cycle of the virtual work done by each term equal to zero. This leads to the formation of a relationship between frequency and amplitude which is directly soluble.

An iterative solution suggested by Rauscher(1938) is particularly useful as it may be used for systems having non-symmetrical spring characteristics, and may also include the effect of linear damping. In this the oscillations are first assumed to have the same form as the free undamped vibrations of the system of a selected amplitude. This enables a motivating force throughout the cycle to be determined, and gives an approximate solution for the operating frequency. The latter is then used to modify the form of the motivating force, and facilitates an iterative solution. If the system is damped, the dissipated energy for an assumed amplitude of vibration determines the phase angle and the consequently the form of the motivating force which may then be used to determine the approximate frequency and start the iterative procedure.

A modification of this method recently suggested by Mahalingham (1957), provides a direct graphical solution by plotting the "frequency function", depending on the relationship between free natural frequency and oscillation amplitude, and the "forcing function", dependent on excitation and inertial forces, versus the vibration amplitudes. The method may also be used for damped systems by introducing a phase angle to the excitation, but this does however, require an iterative solution as the phase angle must first be approximated from the motions determined for an undamped system. An example of this method is given in Appendix 3, where it is used to obtain an indication of the severity of wheelhop phenomena when the tyre loses contact with the ground excitation.

### 6.4.2. NON-LINEAR DAMPING.

(a) TRANSIENT RESPONSE.

The main effect of non-linear damping on a linearly sprung system is to modify the rate of decay from the exponential form of linear damping The most convenient method of determining this decay is by the ratio of successive swings of the free oscillation. This may be evaluated from the energy dissipation in the damper during the cycle. Most vibration texts give the example of Coulomb damping where decay is in arithmetic progression with no change in the periods of oscillation.

The special case of damping proportional to the square of velocity was investigated by Milne (1923 and 1929) and he showed that not only does the decay proceed in a complex progression, but that the periods of oscillation are also affected by such damping.

An approximate solution for damping which may be expressed in a simple mathematical form is available by the method of successive approximations in exactly analogous fashion to the solution for non-linear spring characteristics mentioned in the previous section.

The most generally accepted method of solution is an approximate one suggested by Jacobsen(1930) in which a non-linear damper is assumed to have the same performance as an "equivalent linear damper" having the same energy dissipation per cycle when operating under the same conditions. This method does not recognize the possibility of non-linear damping affecting oscillation periods, but in general this effect is so small as to be insignificant. Experimentally the equivalent linear damping concept has proved to give results of high accuracy.

(b) HARMONIC RESPONSE.

There is only one known rigorous solution of the harmonic response of a system with non-linear damping. This is for the case of Coulomb damping in a linearly sprung one mass system, and even this solution is only valid in certain regions.(DenHartog 1931).

The approximate method of allowing for non-linearities of damping by an extension of the equivalent linear damping concept of Jacobsen (1930) provides good agreement with both the rigorous solution and experimental results. The method is suited to non-linearities which may be represented as functions of velocity, and includes the extreme of coulomb damping. In this method the actual damping is replaced by an equivalent viscous damping having the same energy dissipation per cycle at the same amplitude and frequency. Consequently the equivalent viscous damping can only be expressed as a function of the unknown amplitude. With coulomb or velocity square non-linearity the response equation may be arranged to give a simple solution, but this is not true for damping dependent on high or fractional powers of velocity, the solutions of which are quite involved. Furthermore the method may be adopted to composite damping which is dependent on various powers of velocity though the complexity increases tremendously.

This approximate solution may also be extended for use with systems of more than one degree of freedom but once again the method becomes extremely complicated.

## 6.5. HARMONIC RESPONSE WITH CONTROLLED DAMPING.

Although the equivalent viscous damping concept is useful for non-linear damping which is a function of velocity, it is extremely difficult to modify the method to situations where the maximum damper effort is dependent on some other measurable variable of the response. If controlled damping is used however, such situations will arise. The concept of controlled damping, i.e. establishing a damper force which is dependent on a characteristic of the resultant vibration other than the velocity, must always be modified by the basic behaviour of the damper which can only develop resistance to motion. Consequently the effect of controlled damping is to establish a shape and magnitude of the "work-diagram" of the damper under particular conditions of oscillation.

It is possible by re-arrangement of the formulae of linear response to determine the harmonic response of systems in which the magnitude of the work diagram is made dependent on any of the vibration characteristics, but the shape of the work diagram by this method is restricted to that of a linear damper. Thus the distribution of damping force throughout the vibration cycle is directly proportional to the velocity of motion, but the actual magnitude of the damping forces developed may be fixed by any of the vibration characteristics.

This method thus provides a very satisfactory basis of comparison for the experimental results from the mechanical analogue given in Chapter 8, in which the effects of work diagram shape are also investigated.

### 6.5.1. FORCE LIMITED DAMPERS.

The harmonic response of the simplified two mass vehicle suspension previously considered in Chapter 5, may be obtained for a damping characteristic which is linearly distributed through the vibration cycle, but whose maximum value is constant. The suspension is represented by Fig.5.1. and assumes linear spring constants for both the tyre and suspension, with zero tyre damping. The equations of motion of this system are given in Chapter 5, equations 5.1 and 5.2 From these,

The force developed in the damper $F_D = C_B D(x_A - x_B)$ ........(6.1)

Substituting $x_A = \dfrac{(m_B D^2 + C_B D + K_B).\, x_B}{C_B D + K_B}$ ........(6.2)

Gives $F_D = \dfrac{m_B C_B D^3 .\, x_B}{C_B D + K_B}$ ........(6.3)

This may be further rearranged to give;

$$F_D / K_B x_O = m_B/K_B \,.\, C_B D^2/(C_B D + K_B) \,.\, x_B/x_O \quad \text{.......(6.4)}$$

Under steady state conditions of oscillation further substitutions $D = jw$, and $D^2 = -w^2$ can be made together with the previously introduced parameters of the system.

Thus;

$$\frac{F_D}{K_B \cdot x_0} = (-)\frac{2br^3 j}{(1-r^2)(1-R^2) - P^2 + (1-R^2-P^2).\,2brj} \quad \text{.......(6.5)}$$

From this the magnitude of damper force developed;

$$\left|\frac{F_D}{K_B \cdot x_0}\right| = \frac{2br^3}{\sqrt{((1-r^2)(1-R^2) - P^2)^2 + 4b^2r^2(1-R^2-P^2)^2}} \quad \text{.....(6.6)}$$

$F_D/K_B x_0$ may then be used as the damping parameter to characterize the behaviour of a damper which develops a constant maximum force independent of the magnitude of the resulting vibrations. The distribution of damping force throughout the vibration cycle is linear.

Equation 6.6 may then be re-arranged to give;

$$b = \sqrt{\frac{(F_D/K_B x_0)^2((1-r^2)(1-R^2) - P^2)^2}{4r^6 - 4(F_D/K_B x_0)^2 r^2 . (1 - R^2 - P^2)^2}} \quad \text{..(6.7)}$$

This expression for b may now be substituted into the original response equations of the system (5.11) and (5.12), giving;

$$M_B = \frac{\sqrt{1 + (F_D/K_B x_0)^2((1-r^2)(1-R^2) - P^2)^2 - (1-R^2-P^2)^2)}}{((1-r^2)(1-R^2) - P^2)} \quad \text{..(6.8)}$$

$$M_A = \frac{\sqrt{(1-r^2)^2 + (F_D/K_B x_0)^2(((1-r^2)(1-R^2) - P^2)^2 - (1-R^2-P^2)^2)}}{((1-r^2)(1-R^2) - P^2)} \quad \text{..(6.9)}$$

and $F_T / K_B x_0 = r^2 M_B$ ....................................(6.10)

The substitution of equation 6.7. for "b" introduces limited ranges of validity for the damping parameter $F_D/K_Bx_O$.

Thus $4r^6 \nless 4(F_D/K_Bx_O)^2 . r^2 . (1-R^2-P^2)^2$ ..............(6.11)

i.e. $\left|\frac{F_D}{K_Bx_O}\right| \ngtr \left|\frac{r^2}{1-R^2-P^2}\right|$ ..............(6.12)

The graphical representation of these equations is given in Figs.6.4 6.5 , and 6.6 . These figures are markedly different from those which present the response of a linearly damped system ,i.e. Figs 5.2, 5.3, and 5.4. They illustrate that certain limits exist for the damping parameter below which the resonant motions of both masses will become infinite. Furthermore, the limiting values to eliminate resonance of the two masses are very different in magnitude. Thus, the body resonance may be completely eliminated by a damping parameter $F_D/K_Bx_O > (1 - R^2)^{-1} = 1.005$, while axle resonance vanishes for all values of the damping parameter $F_D/K_Bx_O > (1 - R^2)^{-1} = 20$.

It is interesting to note that the limiting values of $F_D/K_Bx_O$ necessary to eliminate body and axle resonances coincide exactly with the limits of validity of the equations presented by Equation 6.12. This is explained by the fact that equation 6.12 also indicates the maximum forces that can be generated in a linear damper by substituting $b = \infty$. At the resonant frequencies, any linear damper will develop exactly the same force independent of the value "b" which it possesses, though of course the lower the value of b, the greater is the resultant motion required to achieve this force. Consequently, if the magnitude of damping force is governed by some external source to be a constant value as implied by the parameter $F_D/K_Bx_O$, no control of the resonance is achieved for values of $F_D/K_B x_0$ less than that which must be developed by a linear damper in limiting resonant amplitudes. Furthermore, if such a damper force is available immediately, i.e. not requiring amplitude build-up to generate it as in a linear system, there is no possibility of dynamic magnification at the resonant frequency, and also no possibility of the damper developing higher forces at these frequencies. Thus, a force limited damper which can supply the critical damping force required will behave as an infinitely stiff linear damper at the resonance i.e. $b = \infty$, and will completely eliminate any signs of the spring resonance.

These facts are borne out in the Figs. 6.4-5-6. on which the dynamic response of the one mass system composed of the locked axle and body oscillating on the tyre spring has been plotted. This illustrates that the limit of validity represented by eqüation 6.12. coincides with infinite suspension damping, under which conditions the system behaves as a single mass system with axle and body locked together.

As the two critical values of force limited damping parameters are so very different, a selected damping value can control body motions for excitation of 20 times the magnitude for which it can control axle resonance. In the case of the vehicle suspension it is necessary to control both resonances, so the body would have to be seriously overdamped. Maximum values of body motion would then occur at a frequency higher than the basic suspension frequency, corresponding to body and axle oscillations on the tyre spring. The comfort conditions in the body would thus be seriously impaired.

Force-limited damping thus has undesirable characteristics arising from the fact that axle and body control cannot be achieved by the same damping parameter. Also with such a characteristic, the system is overdamped for all excitations less than the design value, and consequently will give less favourable performance under all such conditions.

6.5.2. MAXIMUM DAMPER EFFORT PROPORTIONAL TO MAXIMUM SPRING FORCE.

As shown in the previous section, the chief disadvantage of force-limited damping is that the damping force is in no way influenced by the excitation conditions, and thus must present overdamping to any excitation less than the design value. One method of relating the damping effort to the excitation is to use a damping parameter which provides a force proportional to the spring forces developed in the suspension spring.i.e. the relative motion between axle and body is used as an indication of the amplitude of excitation.

Once again the harmonic response of the basic two mass suspension with damping which produces a maximum force which is proportional to the maximum spring force developed in the cycle may be derived from the linear equations of Chapter 5.

Thus, suspension spring force

$$F_S = K_B.(x_A - x_B) \qquad \text{.....................(6.13)}$$

$$= -\frac{r^2 x_0}{(1-r^2)(1-R^2) - P^2 + 2brj(1-R^2-P^2)} \qquad \text{.......(6.14)}$$

Combining equation 6.14 with 6.5.gives

$$\left|\frac{F_D}{F_S}\right| = 2br \qquad \text{..............................(6.15)}$$

Substituting this damping parameter into the equations of harmonic response 5.11 and 5.12 gives ;

$$M_B = \sqrt{\frac{1 + (F_D/F_S)^2}{((1-r^2)(1-R^2) - P^2)^2 + (F_D/F_S)^2 . (1-R^2-P^2)^2}} \quad \ldots\ldots\ldots(6.16)$$

$$M_A = \sqrt{\frac{(1-r^2)^2 + (F_D/F_S)^2}{((1-r^2)(1-R^2) - P^2)^2 + (F_D/F_S)^2 . (1-R^2-P^2)^2}} \quad \ldots\ldots\ldots(6.17)$$

$$F_T/K_B x_O = r^2 M_B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.18)$$

The graphical presentation of these three equations of harmonic response is given in Figs. 6.7-8-9.

These figures indicate that the harmonic response differs considerably from that of either strictly linear damping, or force limited damping as presented earlier.

Firstly, only a low value of the parameter is necessary to control body resonance, but as the damping is increased to values in excess of unity, the magnitude and frequency of the peak body motion both tend to increase. Furthermore, body motions are increased above the minimum obtainable value throughout the entire frequency range except except in the immediate vicinity of the resonances.

A second feature of this damping control is that it does always present a finite limit to resonant axle motions, however a relatively high value of the parameter is required to reduce the greatest axle magnifier to a desirable level, and this does also introduce high axle motions at far lower frequencies, by tending to lock the axle and body together under these conditions. Nevertheless, such damping is advantageous at high frequencies as it reduces axle motions in this range.

Finally, although this damping control reduces transmission in the immediate vicinity of the resonances, there is a limit to the maximum reduction which can be achieved, and at the same time, the transmission at all frequencies remote from resonance is greatly increased so that the selection of a damping parameter of this type would remain essentially a compromize of the same factors involved with linear damping.

## 6.5.3. MAXIMUM DAMPER EFFORT PROPORTIONAL TO MAXIMUM BODY ACCELERATION.

A second alternative method of relating the damper effort to the excitation is by means of the amplitude of the resultant body acceleration. The harmonic response of the system with maximum damper effort proportional to the maximum body acceleration may be determined from the linear equations of Chapter 5 in a method similar to that used in the previous sections. Once again this method implies that the distribution of damping effort throughout the oscillation cycle is linear.

Thus from Chapter 5 ;

$$\frac{F_D}{A_B m_B} = \frac{2brj}{1 + 2brj} \qquad \text{.........................(6.19)}$$

Where $A_B$ is the magnitude of the body acceleration.

This gives immediately

$$4b^2r^2 = \frac{(F_D/A_B m_B)^2}{1 - (F_D/A_B m_B)^2} \qquad \text{.........................(6.20)}$$

Thus the range of damping parameter for which the solution is valid is ;

$$\left|\frac{F_D}{A_B m_B}\right| \not> 1 \qquad \text{.........................(6.21)}$$

Substituting equation 6.20 into the harmonic response equations 5.11-12 ,

$$M_B = \sqrt{\frac{1}{((1-r^2)(1-R^2) - P^2)^2 + (F_D/A_B m_B)^2((1-R^2-P^2)^2 - ((1-r^2)(1-R^2) - P^2)^2)}}$$

$$\text{.........................(6.22)}$$

$$M_A = \sqrt{\frac{1 + (F_D/A_B m_B)^2(1 - (1-r^2)^2)}{((1-r^2)(1-R^2) - P^2)^2 + (F_D/A_B m_B)^2((1-R^2-P^2)^2 - ((1-r^2)(1-R^2) - P^2)^2)}}$$

$$\text{.........................(6.23)}$$

and as before,

$$\frac{F_T}{K_B x_O} = r^2 . M_B \quad \text{.............................}(6.24)$$

The graphs of these functions throughout the frequency range for various values of the damping parameter $F_D/A_B m_B$ are given in Figs. 6.10, 6.11,and 6.12.

These indicate that high values of the damping parameter are necessary to control body resonance, a value $F_D/A_B m_B = 0.9$ being required to limit the maximum dynamic magnifier of the body to 1.2. However, even this value is not sufficient to limit the resonance of the axle motions, a value very close to the limit being required to produce satisfactory axle control. Consequently, if the resonant motions of the axle are to be kept to a minimum with such damping, the body is seriously overdamped, and this causes the maximum body magnifier to occur at a frequency considerably above the true body frequency, and thus impairs the comfort conditions. Furthermore, at frequencies remote from the resonances, a high value of the damping parameter greatly increases the magnitude of the force transmission the the body so that as an overall means of control this form of damping is unsatisfactory.

6.5.4. <u>MAXIMUM DAMPER EFFORT PROPORTIONAL TO MAXIMUM AXLE ACCELERATION.</u>

The third variable which may be used as an indication of the excitation amplitude is the resultant axle acceleration. The harmonic response of the system in which the maximum damper effort is proportional to the amplitude of the resultant axle acceleration may also be obtained in a similar manner to that used in the previous sections. Again this assumes that the distribution of damper effort through the vibration cycle is linear.

Thus from the response equations of Chapter 5

$$\frac{F_D}{A_A\, m_B} = \frac{2brj}{(1-r^2) + 2brj} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.25)$$

Thus

$$4b^2r^2 = \frac{(1-r^2)^2 \cdot (F_D/A_A m_B)^2}{1 - (F_D/A_A m_B)^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.26)$$

With the limit of validity

$$\left|\frac{F_D}{A_A\, m_B}\right| \not> 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.27)$$

By substituting Equation 6.26 into the response equations 5.11-12 these give ;

$$M_B = \sqrt{\frac{1 + (F_D/A_A m_B)^2\, r^2 (r^2-2)}{((1-r^2)(1-R^2) - P^2)^2 + (F_D/A_A m_B)^2\, r^2P^2(2((1-r^2)(1-R^2)-P^2) + r^2P^2)}}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.28)$$

$$M_A = \sqrt{\frac{(1-r^2)^2 + (F\ /A\ m\ )^2 r^2 (r^2-2)}{((1-r^2)(1-R^2) - P^2)^2 + (F_D/A_A m_B)^2 r^2P^2(2((1-r^2)(1-R^2)-P^2) + r^2P^2)}}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.29)$$

$$\frac{F_T}{K_B x_0} = r^2\, M_B \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.30)$$

The graphical representation of these equations is given in Figs. 6.13-14-15 for various values of the damping parameter $F_D/A_A m_B$.

These indicate,

(1) That very little control of body resonance is achieved with this form of damping unless extremely high parameters are employed. This is associated with the existence of zero axle motion, and hence zero damping effort at r = 1, i.e. very close to the body resonance.

(2) Axle resonance is greatly attenuated by even small values of the damping parameter. As the damping increases, the frequency of greatest axle magnification decreases to values far below the true axle frequency, until finally in the limit the axle and body become locked together and oscillate on the undamped tyre spring at damping parameter = 1.

(3) Force Transmission rapidly increases at high frequencies with increasing values of the damper parameter, the frequency of peak transmission gradually decreasing to the locked frequency as the damping increases.

Thus, this form of damping control is only successful in limiting axle motions, and requires only a small parameter to do this. However, even small values of $F_D/A_A m_B$ cause considerable increase in force transmission above the undamped case at frequencies remote from resonance so that it is still desirable to eliminate damping at all times when it is not essential for reducing axle motions. In this respect axle acceleration control of damping may be used to advantage as this variable gives an indication of the tendency of the wheel to leave the road, so that damping could be applied only when axle acceleration became excessive.

### 6.5.5. CONCLUSIONS ON CONTROLLED DAMPER PARAMETERS.

The previous sections indicate that there is no single damping parameter, based on the fundamental vibration characteristics of the two mass system, which can be used to control the magnitudes of damping effort in such a manner that resonant motions of both body and axle are acceptable, while force transmission at all frequencies is the minimum obtainable value. The results do however indicate two separate parameters which could be applied conjointly to achieve this end. These are $F_D/F_S$ , the parameter which implies a maximum damping effort proportional to the maximum spring force developed in the cycle, and $F_D/A_A m_B$, the parameter which implies a maximum damper effort proportional to the maximum axle acceleration recorded in the vibration cycle. The first of these is seen to provide adequate control of body resonance for a low value of the parameter which has the smallest effect on force transmission at higher frequencies of any of the forms investigated. However, such a magnitude of damping effort has insignificant effect on the resonant motions of the axle. In this respect, it is well to recognize that the ultimate object of axle motion control is to prevent the tyre from losing contact with the road surface for some period of the vibration cycle and not necessarily to produce the minimum axle motions. Consequently, if some characteristic of the resultant motions of the axle and body could be used to indicate that high damping effort was required to prevent wheelhop, the overall performance of the system could be greatly improved, as high damping effort would be introduced when, and only when it was essential to prevent excessive axle motions.

From the previous sections, the parameter which shows the greatest promise of providing such an indication of the damping effort required, is the axle acceleration.

It is possible by the method used in the previous sections to determine the relationship between the recorded axle accelerations and the damper effort required to just prevent wheel-hop motion during harmonic excitation.

Thus,

Relative motion between axle and ground

$$x_{OA} = x_O - x_A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6.31)$$

Then making the substitutions for system parameters given by Equations 5.5 and 5.6, this may be expressed as ;

$$\frac{x_O}{x_{OA}} = (+)\frac{(1-r^2)(1-R^2)-P^2 + 2brj(1-R^2-P^2)}{(1-r^2)R^2 + P^2 + 2brj(R^2+P^2)} \quad \ldots\ldots(6.32)$$

Furthermore, by rearranging to give axle acceleration $A_A$ in terms of the relative motions between the axle and ground $x_{OA}$ :

$$\frac{A_A}{w_B^2\, x_{OA}} = \frac{r^2 \cdot x_A}{x_{OA}} = r^2\left\{\frac{x_O}{x_{OA}} - 1\right\} \quad \ldots\ldots\ldots\ldots(6.33)$$

$$= (-).\frac{r^2.((1-r^2) + 2brj\ )}{(1-r^2)R^2 + P^2 + 2brj(R^2 + P^2)} \quad \ldots\ldots\ldots(6.34)$$

The damper effort may also be referred to the relative motion between axle and ground to give ;

$$\frac{F_D}{K_B x_{OA}} = (-)\frac{2br^3j}{(1-r^2)R^2 + P^2 + 2brj(R^2 + P^2)} \quad \ldots\ldots\ldots(6.35)$$

By combining equations 6.34 and 6.35 to eliminate "b" :

$$\left|\frac{A_A}{w_B^2 X_{OA}}\right| =$$

$$\frac{r^2}{((1-r^2)R^2 + P^2)} \sqrt{(1-r^2)^2 + \left(\frac{F_D}{K_B X_{OA}}\right)^2 \frac{(((1-r^2)R^2 + P^2)^2 - (1-r^2)(R^2 + P^2)^2)}{r^4}} \qquad \text{..........................(6.36)}$$

The criterion of wheelhop is obviously the point where $X_{OA} = x_{Ast.}$ = the static tyre deflection, and the relationship (6.36) is plotted in Fig.6.16. for this condition. This figure shows that only a small magnitude of damper effort is required in the low frequency regions to prevent wheelhop, and in the suggested application, this would be provided by the basic relative motion control of damper effort. Furthermore, as the frequency of operation approaches the natural period of the locked axle and body on the tyre spring, very high values of axle acceleration can be tolerated before wheelhop occurs, so that it is unlikely that damping would be necessary. However, as the excitation frequency increases, the axle acceleration at wheelhop tends to become independent of the actual frequency of operation, but decreases as the magnitude of damping effort increases. Consequently if the axle acceleration control of damping effort were employed, it would be necessary to constantly compare the recorded accelerations with the critical values for the particular damper effort existing to ensure that wheelhop was not occuring.

Fig.6.16 also indicates that the damper cannot provide greater effort than $F_D = 18 \cdot K_B x_{Ast}$, for at this value the axle and body become locked together by damping. Thus it is obvious that there is a limit to the magnitude of road undulation which can be traversed by a vehicle in which wheel-hop is prevented by damping effort.

In order to clarify the conclusions relating to the axle acceleration control of damping effort, and to indicate the performance of such a system, the relationship between road undulation amplitude and the damping effort required to just prevent wheelhop at various frequencies throughout the range was also determined.

Thus by combining equations 6.32, and 6.35 ;

$$\left|\frac{X_0}{X_{0A}}\right| = \frac{1}{(1-r^2)R^2 + P^2}\sqrt{((1-r^2)(1-R^2)-P^2)^2 + \left(\frac{F_D}{K_B X_{0A}}\right)^2 \frac{(1-R^2-P^2)^2(R^2(1-r^2)-P^2)^2}{r^4} - \left(\frac{F_D}{K_B X_{0A}}\right)^2 \frac{((1-r^2)(1-R^2)-P^2)^2(R^2+P^2)^2}{r^4}}$$

.....................(6.37)

This equation is plotted graphically in Fig.6.17, and this indicates that below a frequency ratio of $r = 9.5$, any magnitude of damping effort reduces the allowable size of road undulation which can be traversed by a vehicle without causing wheel-hop to occur. However, above $r = 9.5$, damping greatly increases the magnitude of sinusoidal bumps which can be safely absorbed, but, even so, wheel-hop cannot be eliminated in this range by damping effort if the road undulations have an amplitude equal to the static tyre deflection.

When compared with Fig.6.16, it is seen that for frequencies in excess of $r = 9.5$, where high damping is beneficial, the axle accelerations at incipient wheelhop are dependent only on the magnitude of damping effort that exists, so that axle acceleration control of high damping effort is definitely feasible.

Finally, although it is desirable to eliminate wheelhop entirely, from the aspects of vehicle stability and safety, it is obvious that such cannot be achieved if the vehicle is driven over a road having undulations of greater amplitude than the static tyre deflection, at an excessive speed. Nevertheless, axle acceleration control of high damping effort would be advantageous under such conditions, as the axle acceleration, being greater than the critical value, would ensure the maximum available magnitude of damping effort so long as the exciting conditions prevailed, and thus although wheelhop would not be eliminated, the suspension would be in its most satisfactory state to prevent excessive axle motions.

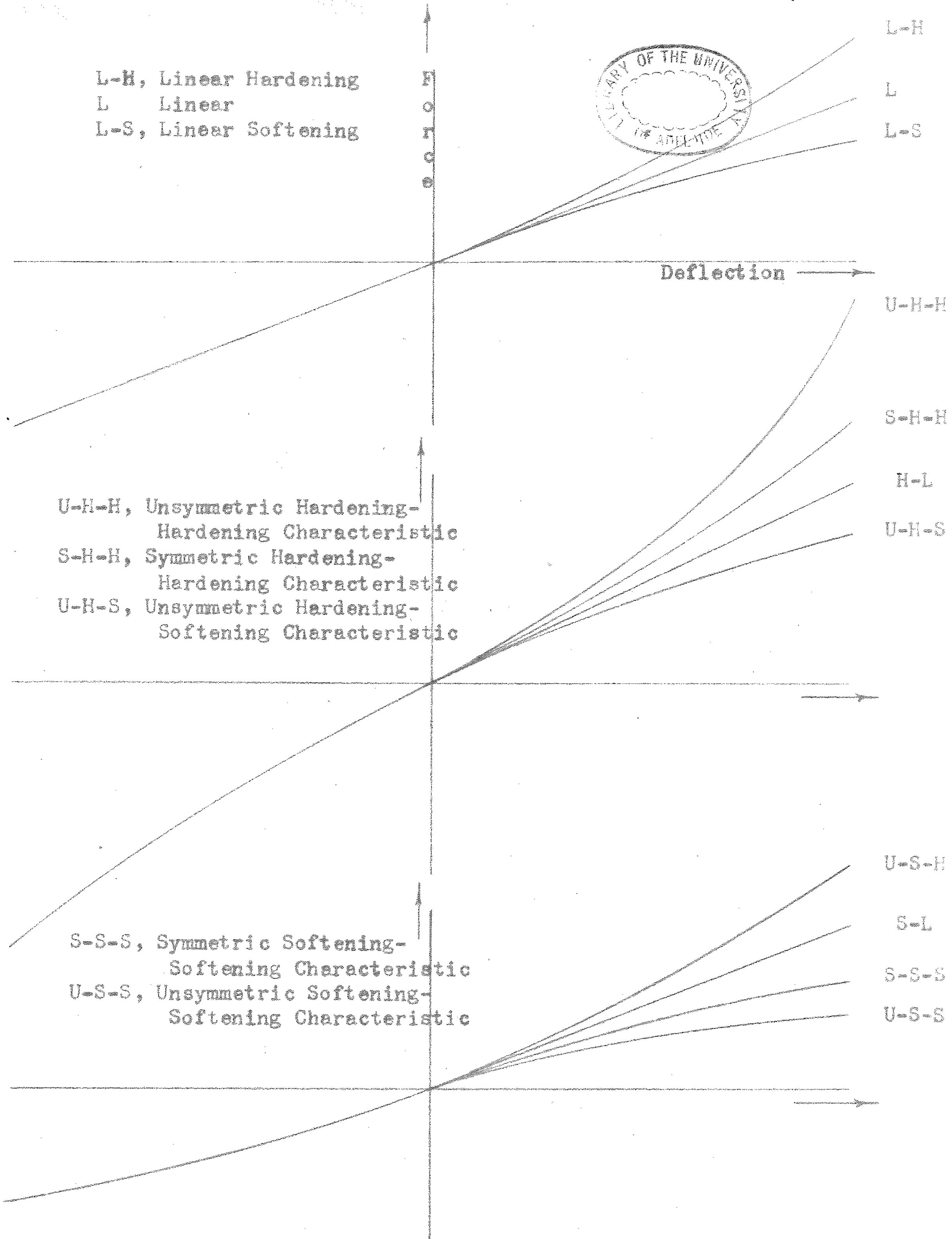



Fig. 6.1. NON-LINEAR SPRING CHARACTERISTICS.

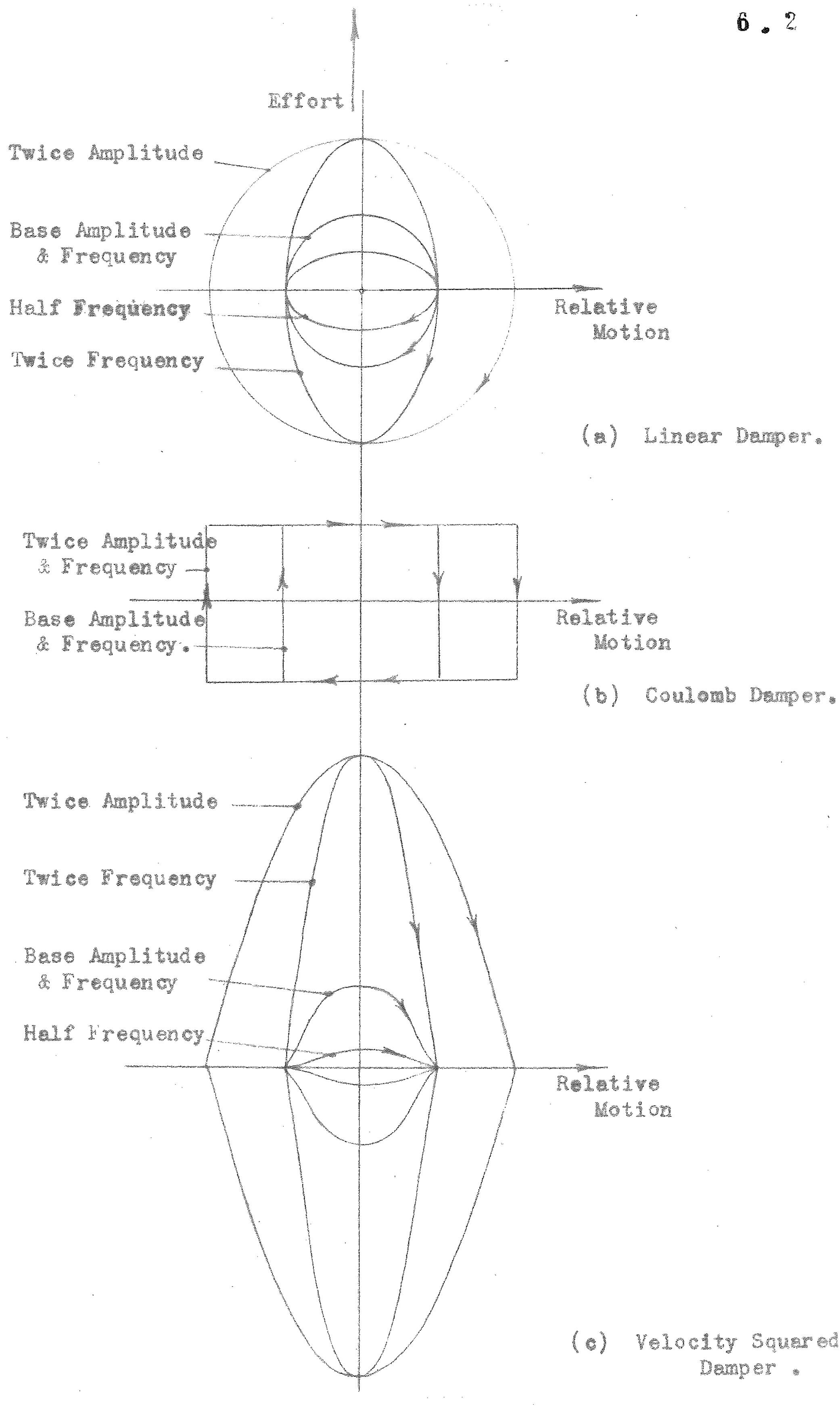
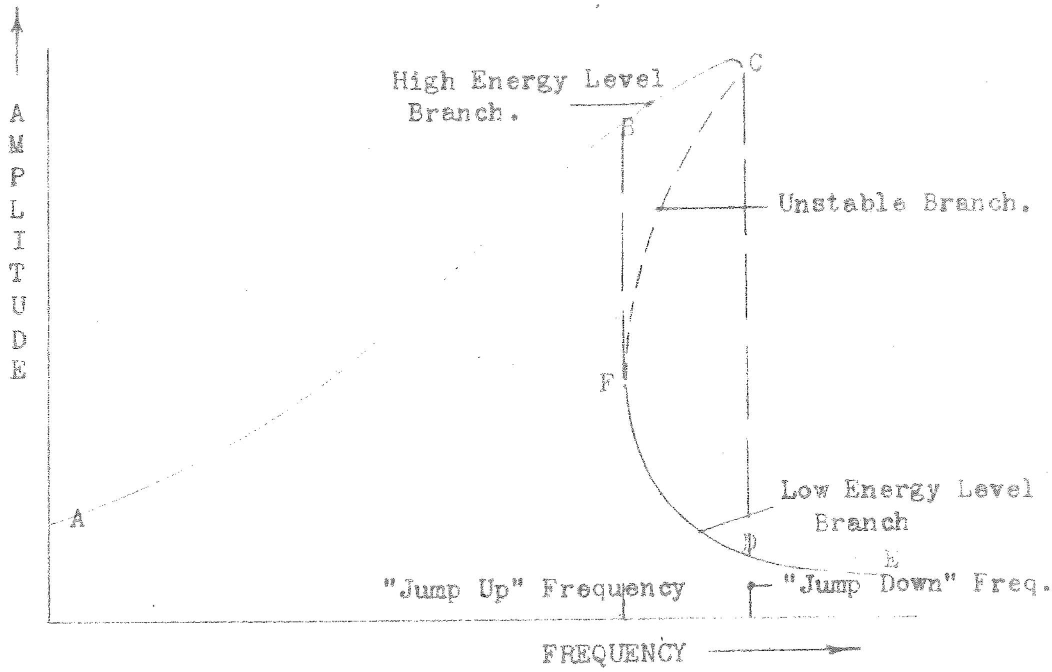
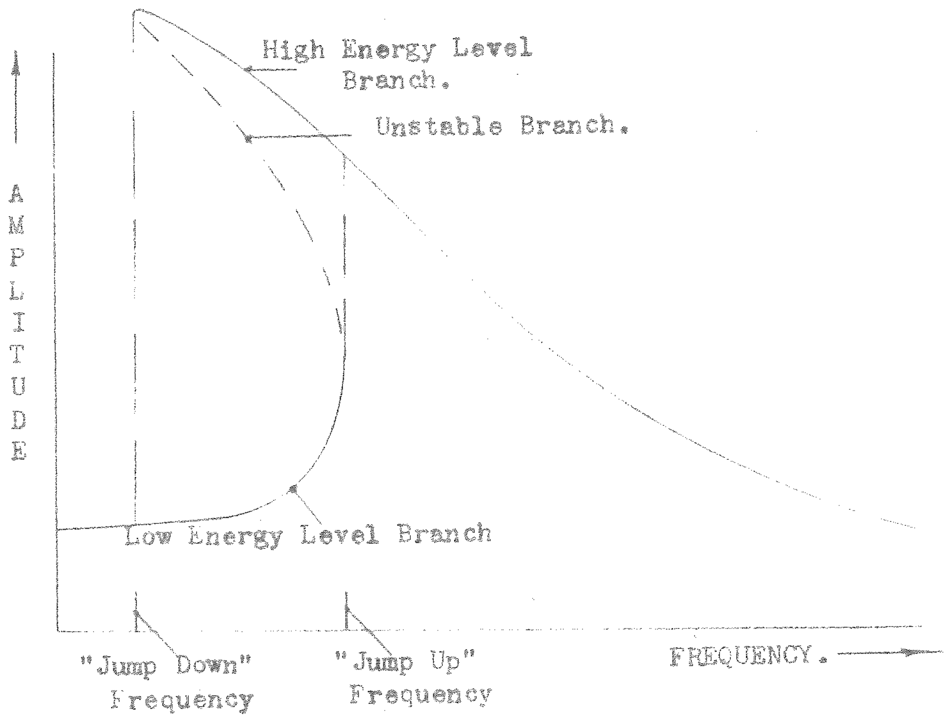



Fig. 6.2. Typical Damper "Work Diagrams"




(a) Stiffening Characteristic.




(b) Softening Characteristic.

Fig. 6.3. HARMONIC RESPONSE OF NON-LINEARLY SPRUNG SYSTEMS.

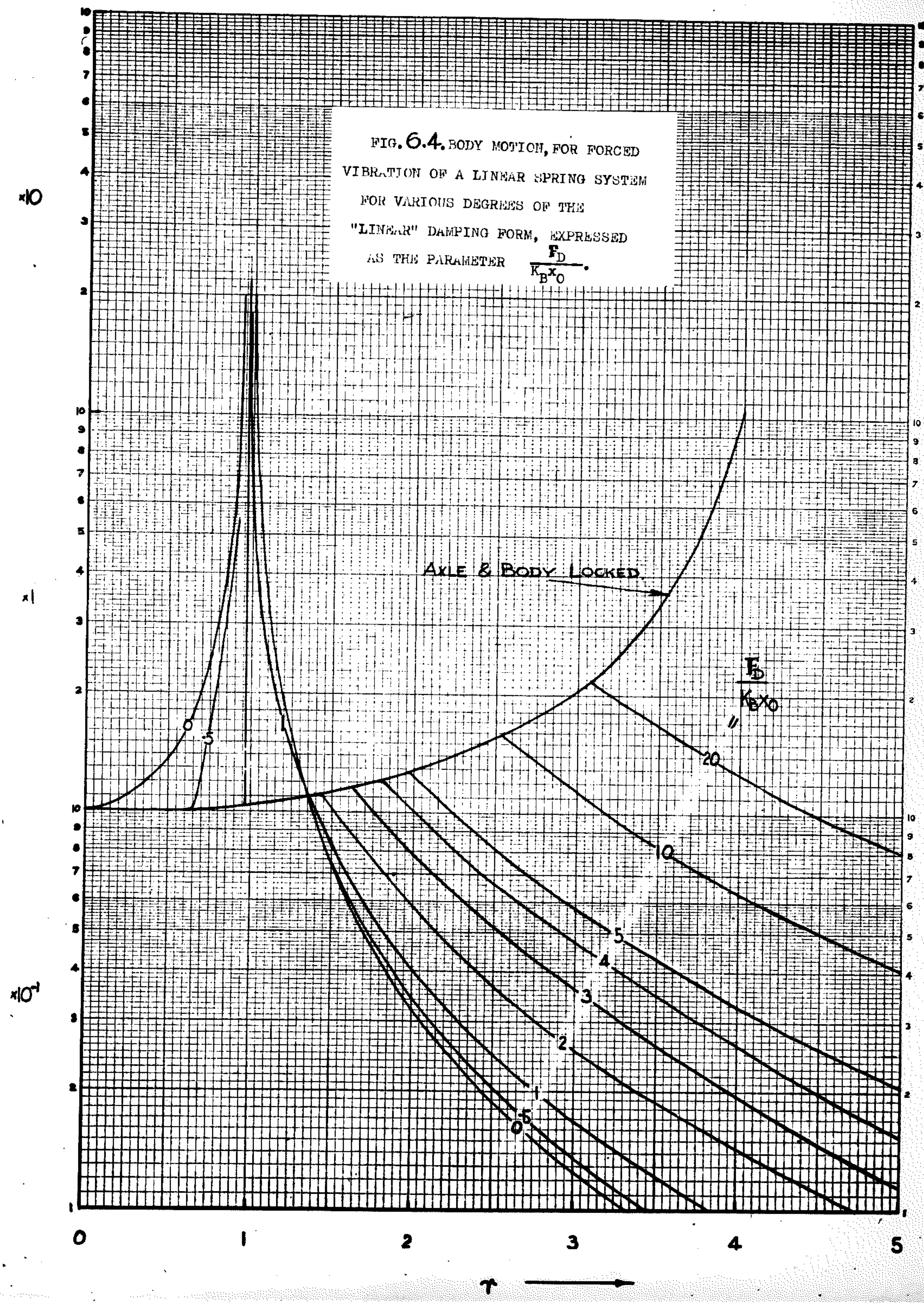


FIG. 6.4. BODY MOTION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "LINEAR" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{F_D}{K_B x_0}$.
AXLE & BODY LOCKED
$\frac{F_D}{K_B x_0}$ =
20
10
5
4
3
2
1
·5
0
×10
×1
×10⁻¹
$M_B$
0
1
2
3
4
5
τ

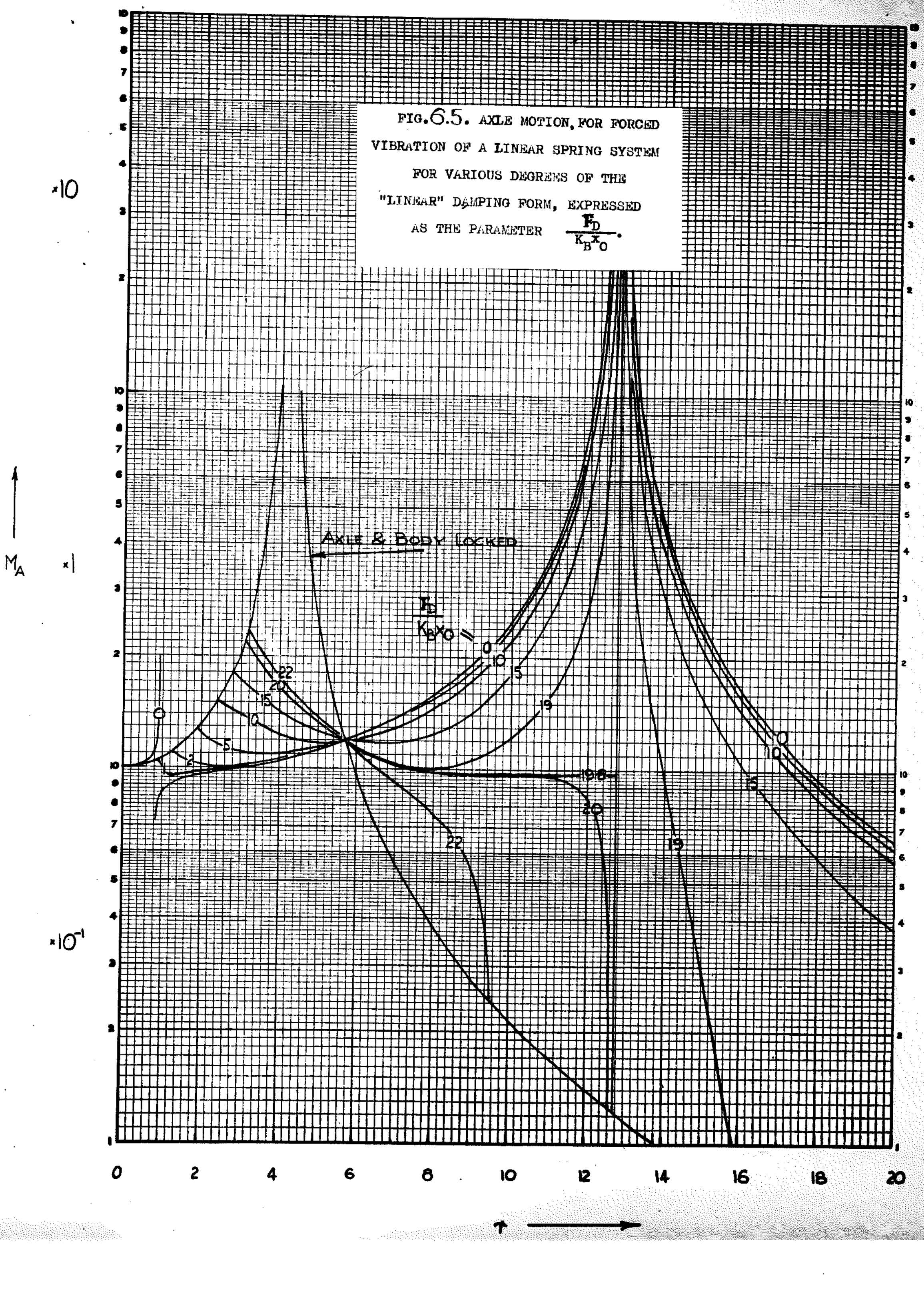


FIG. 6.5. AXLE MOTION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "LINEAR" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{F_D}{K_B x_O}$.
AXLE & BODY LOCKED
$\frac{F_D}{K_B x_O}$ =
0
10
5
19
19.8
20
22
15
2
$M_A$
×10
×1
×10⁻¹
0
2
4
6
8
10
12
14
16
18
20
r

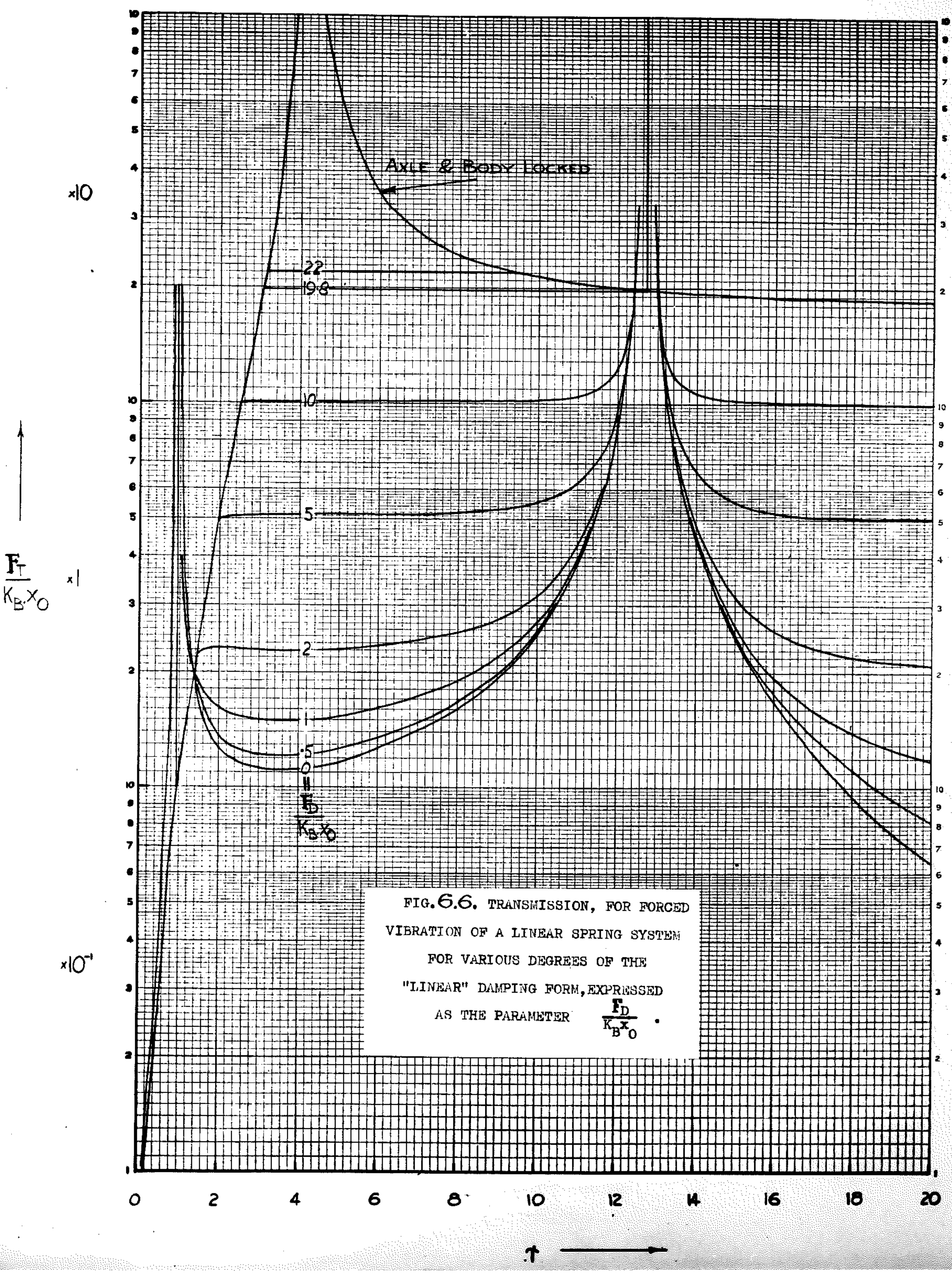



FIG. 6.6. TRANSMISSION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "LINEAR" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{F_D}{K_B x_0}$ .

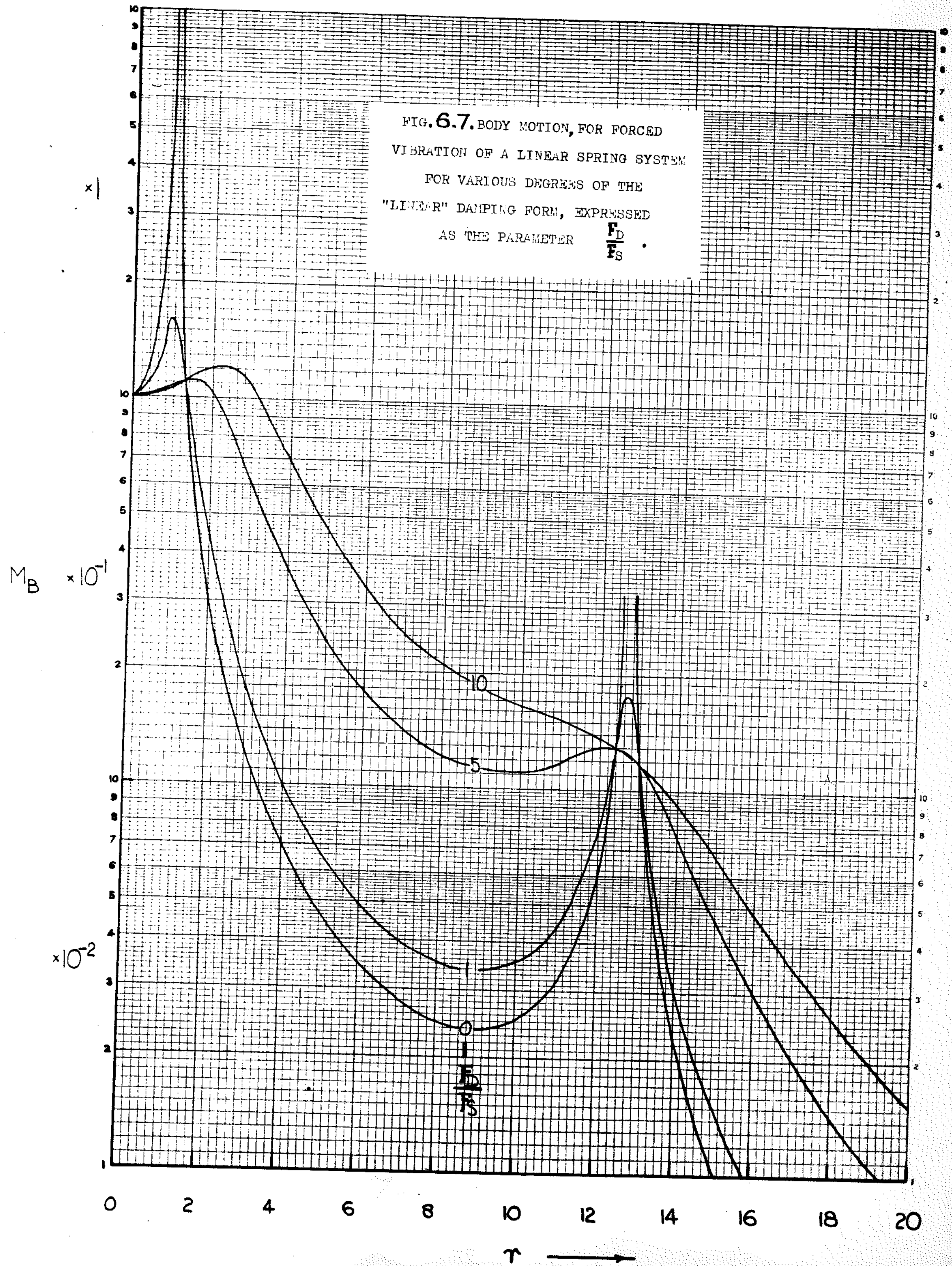



FIG. 6.7. BODY MOTION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "LINEAR" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{F_D}{F_S}$.

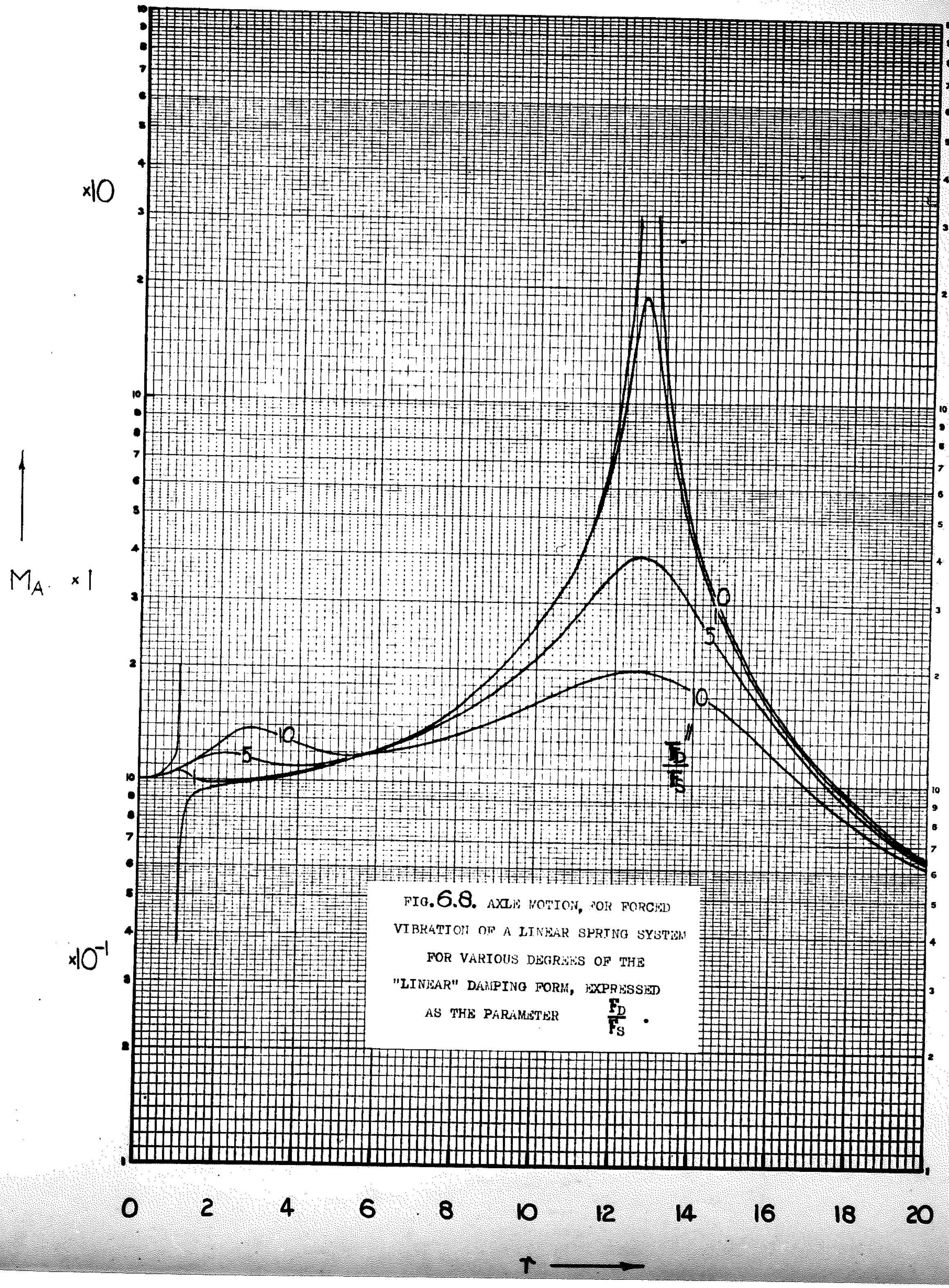



FIG. 6.8. AXLE MOTION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "LINEAR" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{F_D}{F_S}$.

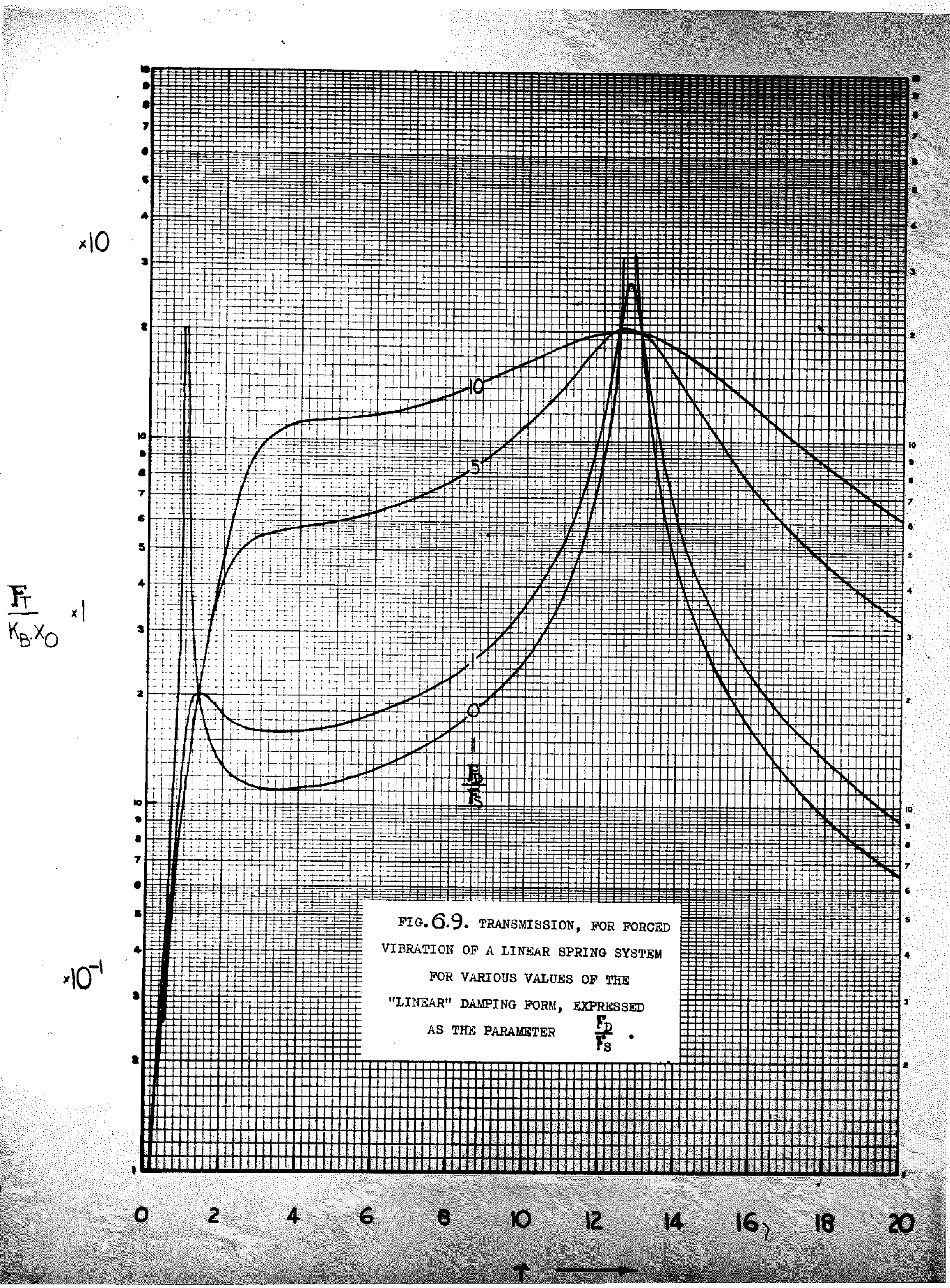



FIG. 6.9. TRANSMISSION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS VALUES OF THE "LINEAR" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{F_D}{F_S}$.

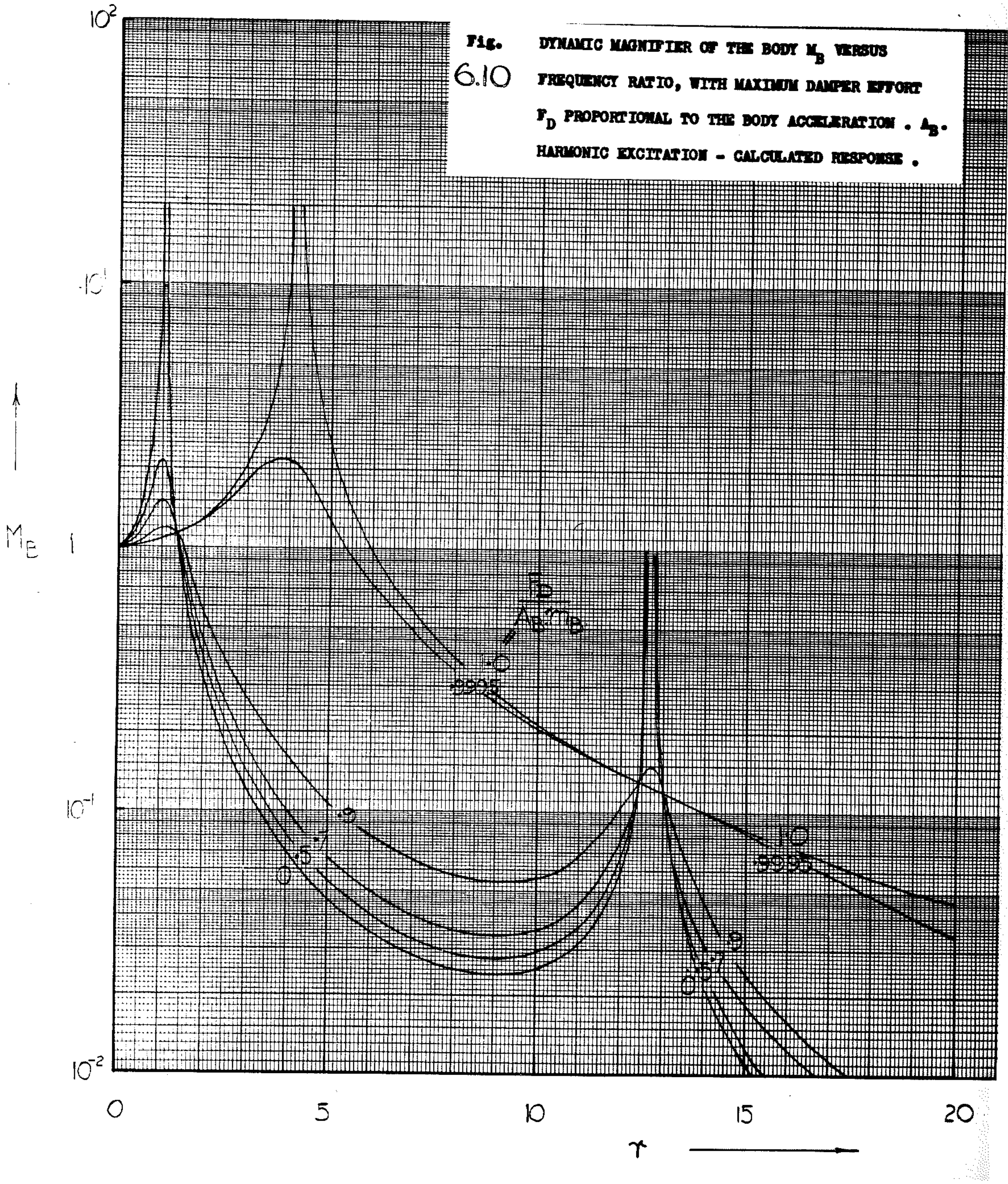
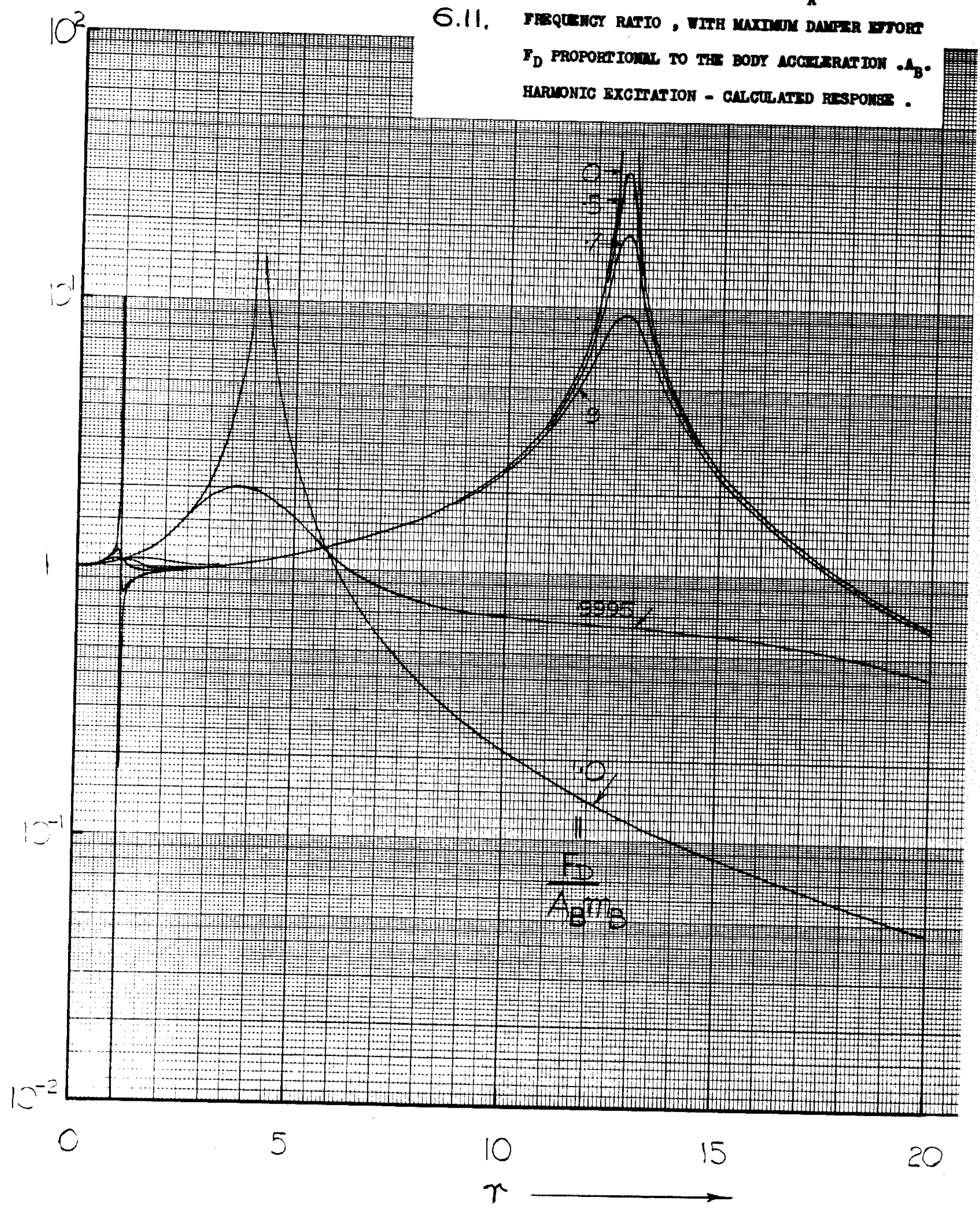



Fig. 6.10 DYNAMIC MAGNIFIER OF THE BODY $M_B$ VERSUS FREQUENCY RATIO, WITH MAXIMUM DAMPER EFFORT $F_D$ PROPORTIONAL TO THE BODY ACCELERATION . $A_B$. HARMONIC EXCITATION - CALCULATED RESPONSE .

Fig. 6.11. DYNAMIC MAGNIFIER OF THE AXLE $M_A$ VERSUS FREQUENCY RATIO, WITH MAXIMUM DAMPER EFFORT $F_D$ PROPORTIONAL TO THE BODY ACCELERATION $A_B$. HARMONIC EXCITATION - CALCULATED RESPONSE.



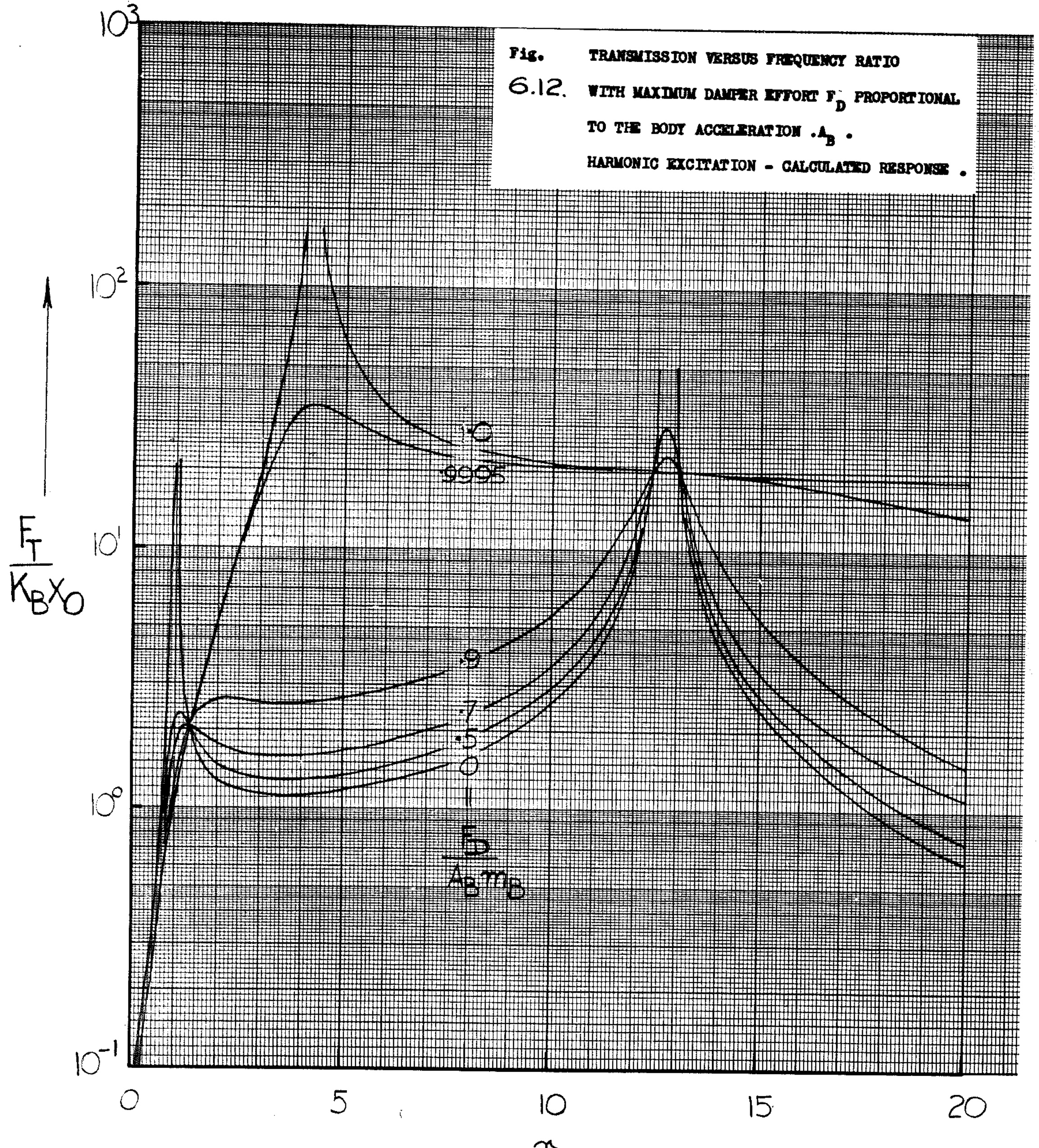



Fig. 6.12. TRANSMISSION VERSUS FREQUENCY RATIO WITH MAXIMUM DAMPER EFFORT $F_D$ PROPORTIONAL TO THE BODY ACCELERATION $A_B$. HARMONIC EXCITATION - CALCULATED RESPONSE.

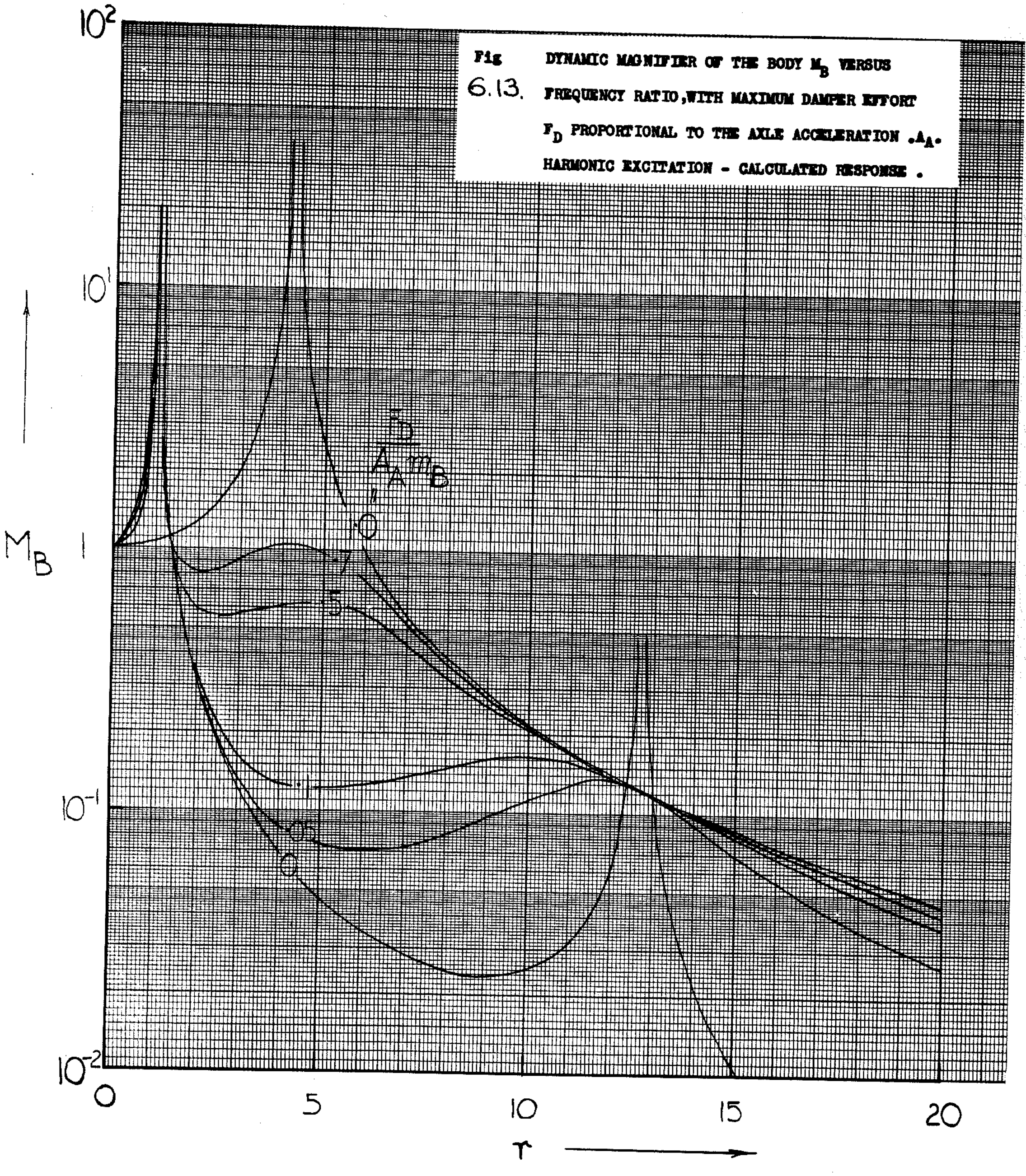



Fig 6.13. DYNAMIC MAGNIFIER OF THE BODY $M_B$ VERSUS FREQUENCY RATIO, WITH MAXIMUM DAMPER EFFORT $F_D$ PROPORTIONAL TO THE AXLE ACCELERATION $A_A$. HARMONIC EXCITATION - CALCULATED RESPONSE.

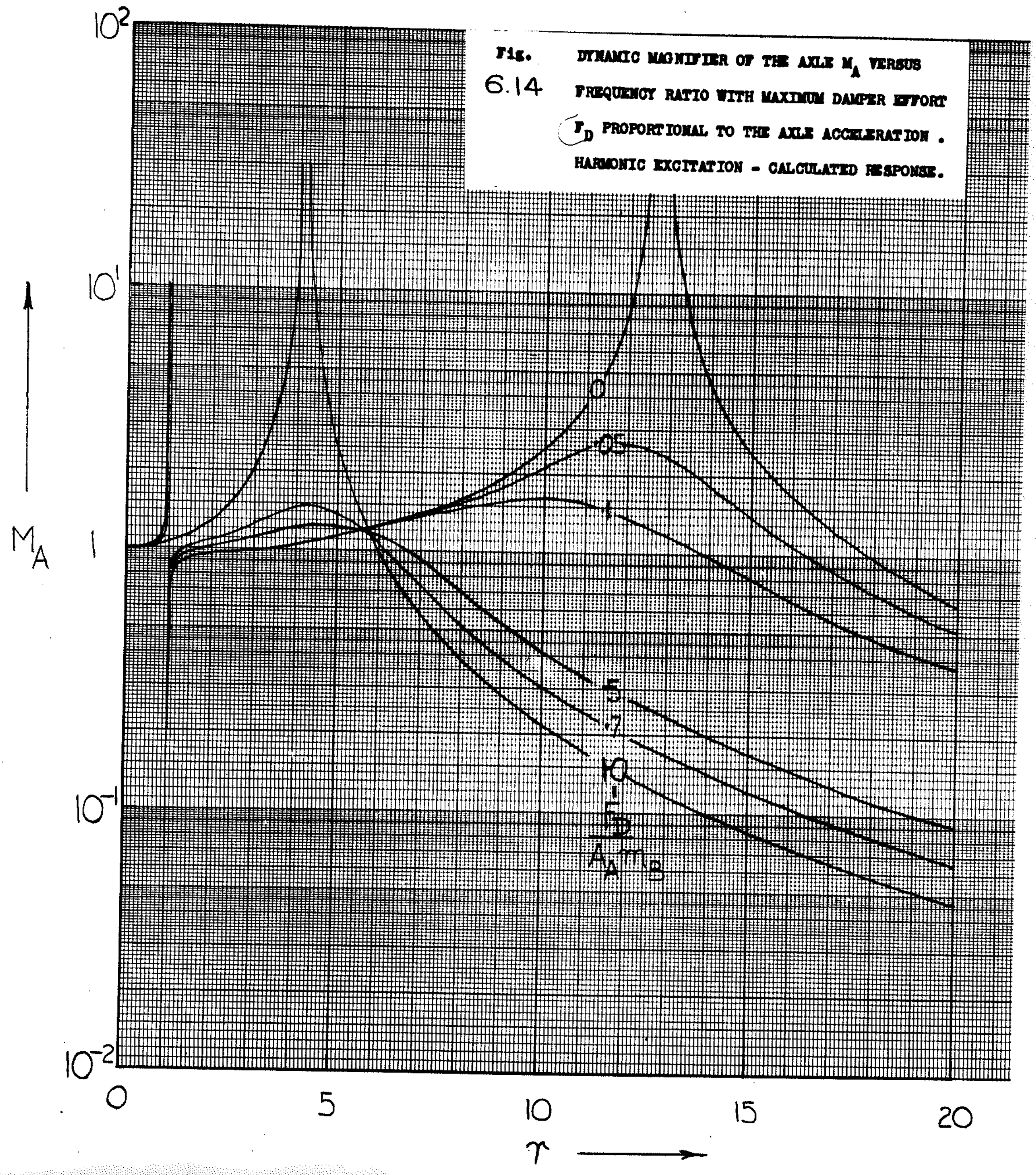



Fig. 6.14 DYNAMIC MAGNIFIER OF THE AXLE $M_A$ VERSUS FREQUENCY RATIO WITH MAXIMUM DAMPER EFFORT $F_D$ PROPORTIONAL TO THE AXLE ACCELERATION . HARMONIC EXCITATION - CALCULATED RESPONSE.

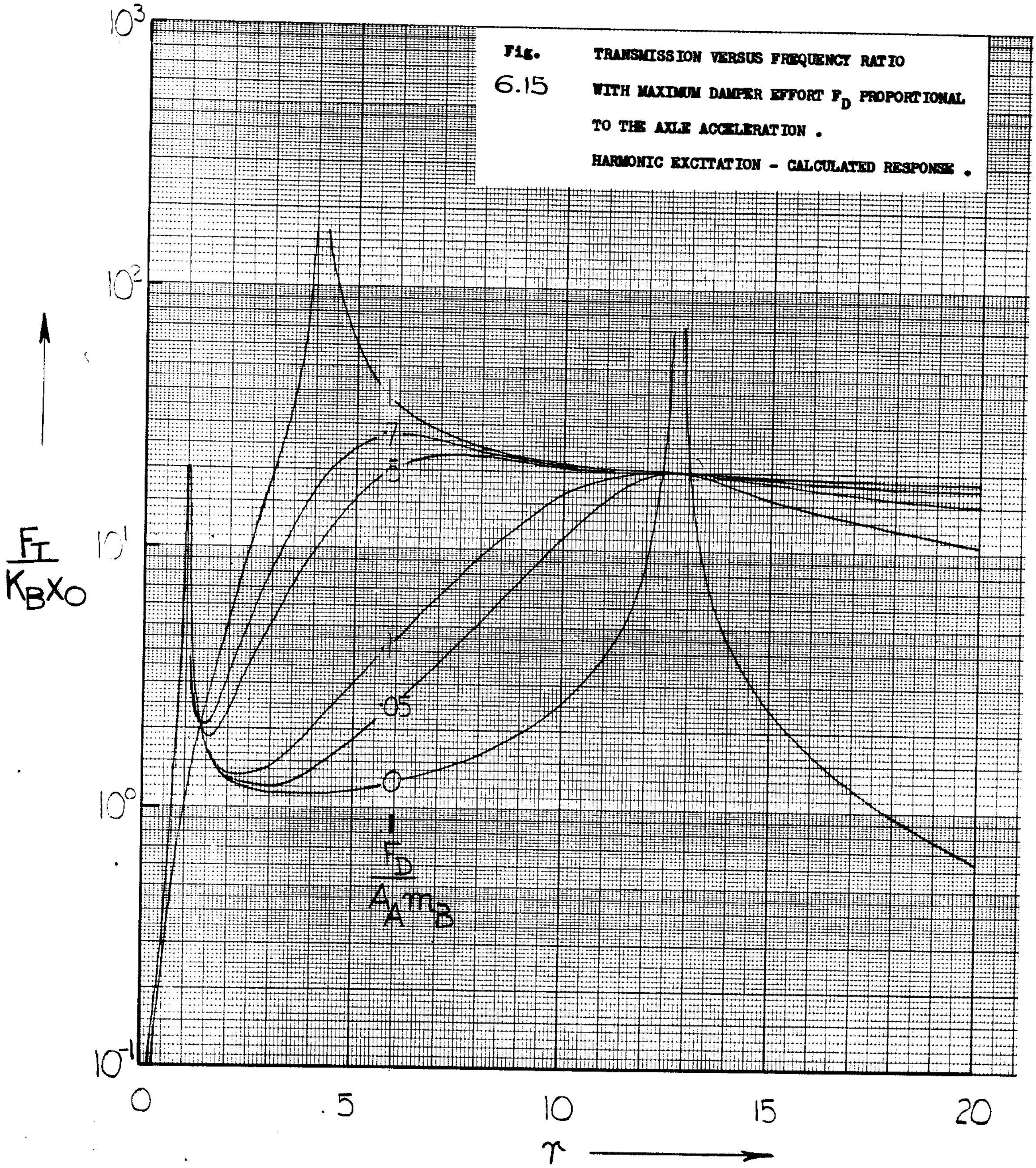



Fig. 6.15 TRANSMISSION VERSUS FREQUENCY RATIO WITH MAXIMUM DAMPER EFFORT $F_D$ PROPORTIONAL TO THE AXLE ACCELERATION. HARMONIC EXCITATION - CALCULATED RESPONSE.

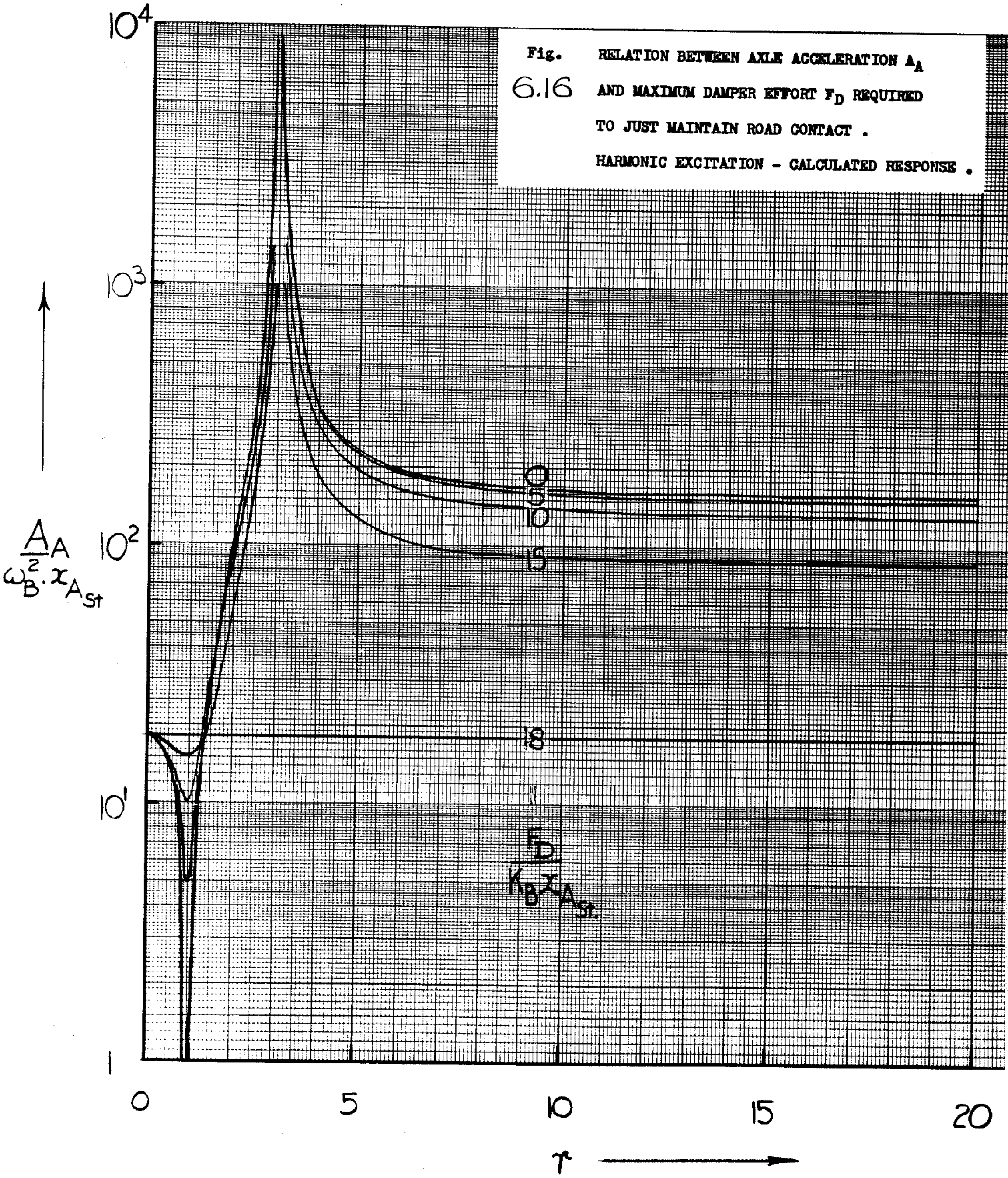



Fig. 6.16 RELATION BETWEEN AXLE ACCELERATION $A_A$ AND MAXIMUM DAMPER EFFORT $F_D$ REQUIRED TO JUST MAINTAIN ROAD CONTACT. HARMONIC EXCITATION - CALCULATED RESPONSE.

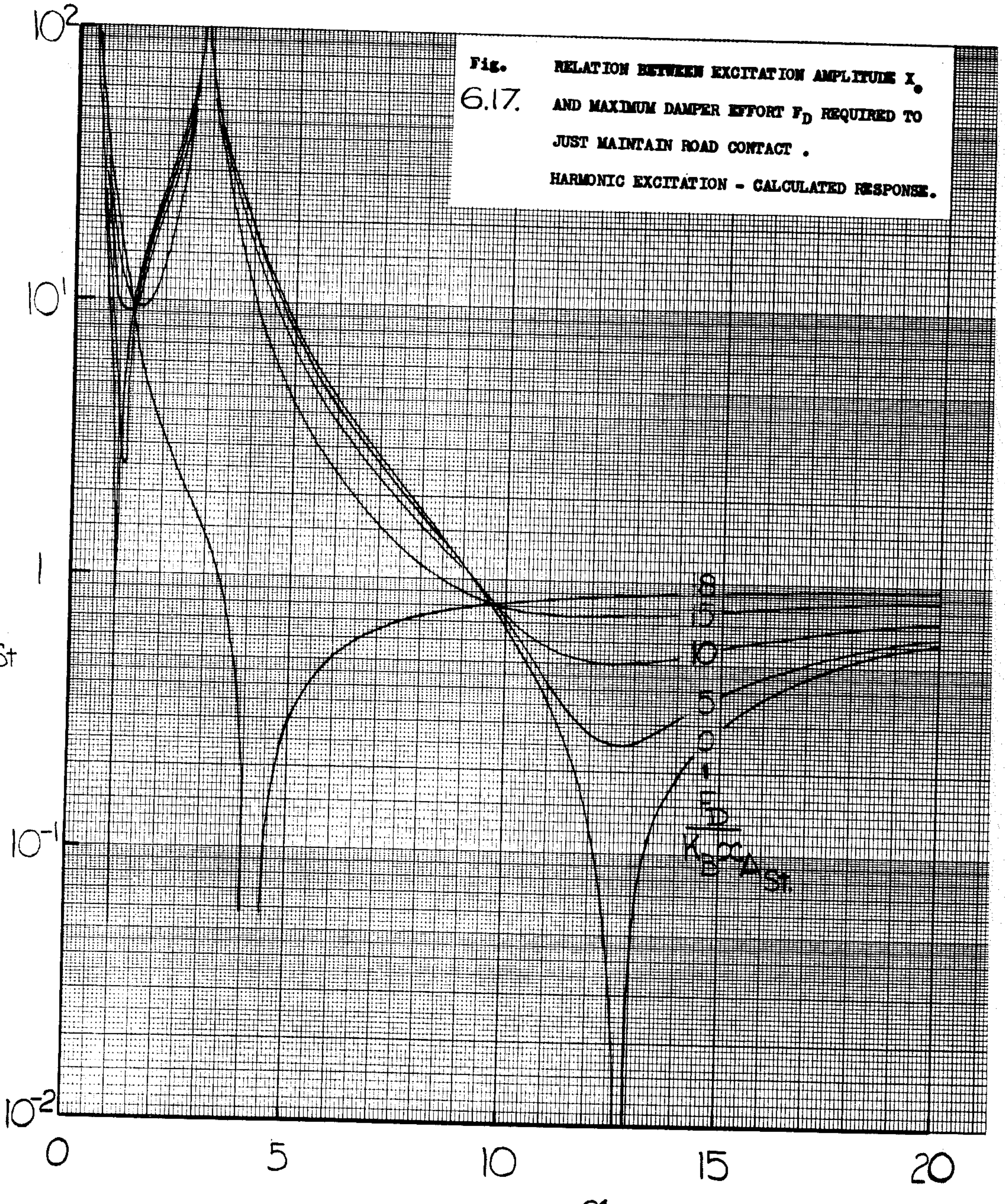



Fig. 6.17. RELATION BETWEEN EXCITATION AMPLITUDE $X_o$ AND MAXIMUM DAMPER EFFORT $F_D$ REQUIRED TO JUST MAINTAIN ROAD CONTACT. HARMONIC EXCITATION - CALCULATED RESPONSE.

# Chapter 7

# MECHANICAL ANALOGUE

## 7. MECHANICAL ANALOGUE.

It has been shown in the previous chapters that the vehicle suspension presents an extremely complex system whose dynamic characteristics must be determined in order to optimize the suspension parameters. Furthermore, although the harmonic and transient responses may be obtained, though with considerable complexity, for the system if it is assumed to have linear coefficients for the spring and damping characteristics, the responses defy rigorous mathematical solution where non-linear characteristics exist. Investigation of the responses of such a system is best achieved experimentally with either a prototype or an analogous system. Even so, if the number of parameters is to be kept within reasonable practical limits in an effort to assess their individual effects without intercoupling, it is desirable to subdivide the complete unit into simpler systems with fewer degrees of freedom which can be relied on to give equivalent performance. In the case of the vehicle suspension, the greatest simplification which can be tolerated is to two basic two mass systems as illustrated in Fig.4.3 and Fig.5.1. The beam simplification of Fig.4.3. has already been used in Chapter 4 to illustrate the intercoupling of primary motions, while the independent suspension simplification of Fig.5.1. has been introduced in Chapters 5and 6 to facilitate investigations of coupling between primary and secondary motions. The experimental work of this thesis is restricted to that concerning an analogue of the latter system, for it is realized that this enables optimization of the compromize which must exist between comfort and stability in the vehicle suspension.

A mechanical analogue was selected because it eliminated problems of simulation in that the components used in the construction of the analogue were made from the same material and of similar form to the actual units which would be employed in the vehicle itself. This is particularly true of the damper which uses a magnetic liquid whose damping effort can be separately controlled in the manner proposed for the prototype damper. Fig.7.1. shows the torsional system which is a mechanical analogue of the simplified independent suspension with associated body mass. As such, the system has only two degrees of freedom, and can represent only vertical motions of the sprung and unsprung masses. The inertias A and B represent the unsprung (axle) and Sprung (body) masses respectively. The effective road surface given by the input pulley is connected to the axle inertia A by a linear spring equivalent to the pneumatic tyre, while the body inertia B is connected to the axle by a parallel spring-damper combination equivalent to the suspension elements. Throughout the experiments the tyre spring is maintained as a linear characteristic, and the tyre damping is assumed zero, while non-linear components are substituted for the spring and damper combination of the suspension elements. It is in the positive contact between the guide pulley (road surface) and the tyre spring, that is maintained with the analogue, that the system differs most from the true vehicle suspension. The responses from the analogue are thus strictly limited to cases where the tyre and road surface do not lose contact. In fact, this does not constitute a limitation of the analogue for the aim of the investigation is not to determine the response when the tyre loses contact with the road, but to optimize the suspension parameters so that axle hop is minimized.

## 7.1. ANALOGUE EXCITATION.

Both harmonic and transient responses of the system are available on the mechanical analogue of Fig.7.1. These responses give the behaviour of the suspension when the vehicle travels over either regular undulations or a sharp step in the road profile. Simple harmonic excitation is achieved by oscillation of the input pulley of the analogue by means of the Scotch Crank and balance mechanism of Fig7.2. The amplitude of oscillation can be adjusted simply by varying the throw of the Scotch Crank between its limits of 0to 1 inch. This corresponds to 0.67 Radians as the maximum available amplitude of rotational excitation. The counterbalance of the Scotch Crank was considered necessary to reduce the structural vibrations of the welded frame on which the analogue was mounted and in fact proved most satisfactory.

The crank is driven by a variable speed D.C. motor having both armature control from a Ward-Leonard set and a supplementary fine field control. This provides steady motor speeds throughout the range 150 to 2000 R.P.M. It is necessary to greatly extend this speed range so that full coverage of the spectrum is provided for harmonic responses. This is achieved by two supplementary epi-cyclic gear boxes in series with the main drive Fig 7.3. Each of these gearboxes has a reduction of 4:1 and facility is provided so that either may have the outer annulus locked to the drive shaft or to ground, the whole thus constituting a three speed gear-box having reduction ratios 16 : 1 , 4 : 1 and 1 : 1 . In conjunction with the motor speed control, steady oscillation frequencies can be achieved anywhere in the range 10 to 2000 cycles per minute. This corresponds to a frequency spectrum of 0.167 to 33.0 cycles per sec. and completely covers the important ranges of the analogue.

Transient response of the analogue to an effective finite step in the road surface profile may be obtained by the method shown in Fig.7.4. Here, the input pulley is locked to ground so that it cannot oscillate at all, and the axle inertia is deflected from its true equilibrium position by applying a torque to it with a strong light cord. As the body inertia has no external torque applied to it, the relative deflection across the suspension spring is thus zero, and the body assumes the same position relative to its true equilibrium position in space as does the axle. With negligible tyre damping in the analogue, the transient oscillations of the system which result when the external deflecting torque is "instantaneously" removed by cutting the cord, are the same as the response of the system to a finite step in the road surface profile.

## 7.2. SPRING CHARACTERISTICS.

Throughout the entire series of tests, the characteristic of the tyre spring of the analogue is kept linear. This greatly reduces the complexity of presentation of the effects of non-linearities in the suspension elements, and is consistent with the requirement of the optimum suspension to maintain road contact.

On the other hand, various types of spring characteristic are used for the analogue suspension spring. In the first series of experiments, a linear spring is used while investigating the effects of damping non-linearities. In later series, symmetric hardening springs of various degrees of non-linearity, and also non-symmetric hardening-softening springs are used. In both the latter cases, non-linearity is achieved geometrically on the analogue by a simple tension spring connected between the axle and body inertias as shown in Fig.7.5. If the axis of this tension spring is adjusted to be co-planar with the oscillation axis of the analogue, the effective torsional stiffness of the spring is symmetric and hardening with increasing deflections on either side of the equilibrium position. Furthermore, if the initial tension of the hardening spring is adjusted to zero in the set-up position, its effective spring rate is zero for small deflections about the equilibrium position. Thus when used in conjunction with the basic linear suspension spring of the analogue, the non-linearizing tension spring can be adjusted to give hardening non-linearity which is symmetrically distributed about the basic linear rate at the equilibrium position. The effect of non-linearity can thus be investigated with reference to a linear spring having the same basic rate .

Unsymmetric hardening-softening non-linear spring characteristics may also be introduced with the simple tension spring, but in this case it is necessary to place the axis of the tension spring skew to the oscillation axis of the analogue, and also to pretension the spring . With such an arrangement, deflections of the spring which tend to increase the angle of skew introduce hardening spring characteristics, whereas deflections which tend to decrease the angle of skew cause softening characteristics compared to the equilibrium spring rate. Consequently there is a limit to the minimum rate which can be achieved with this arrangement and this is equal to the basic rate of the helical torsion spring. Thus it is necessary to limit the relative motions between axle and body during tests on the analogue, to values less than the relative static deflection introduced by the initial pretension of the non-linearizing spring, otherwize the overall spring characteristic would exhibit a second hardening region on the originally soft side for deflections greater than this value.

Furthermore, the equilibrium rate of such a spring combination depends greatly on the initial pre-tension of the non-linearizing spring, which in turn establishes the degree of hardening and softening which can be achieved. Thus the equilibrium rate can only be adjusted to equal the original linear spring rate if the basic helical spring is changed to a softer one. Although possible on the analogue, this requires considerable disassembly and was not regarded as essential, so that all experimental work carried out on unsymmetric hardening-softening springs is associated with different basic spring rates from the linear and symmetric hardening spring tests.

## 7.3. DAMPING CHARACTERISTICS.

Damping of the analogue is achieved by means of magnetic liquid dampers which have been specially devised for this purpose. A diagrammatic section of the damper is given in Fig.7.6. The cup of the damper houses an electromagnet winding, and is shaped in such a way that all the magnetic flux passes through the magnetic liquid except for a small percentage of leakage through a brass insulating ring. This cup is rigidly fixed to the body inertia of the analogue and contributes to this, whereas the torque disc which is immersed in the magnetic liquid with small clearance from the cup, is rigidly connected to the axle inertia by a shaft passing through the centre of the helical torsion spring. As the analogue is mounted with the axis vertical it is not necessary to seal the torque disc shaft against the cup to prevent fluid leakage, so that all the damping torque developed on the damper shaft originates in the magnetic liquid, and is thus controllable. The characteristics of the magnetic liquid as a damping medium were investigated prior to analogue tests and are recorded in appendix. Basically this behaves in the manner anticipated in that the effort of the damper depends primarily on the magnetic flux estalished by current in the electromagnet windings. A second feature is that with constant magnetic flux, the damping torque developed is practically independent of velocity. Consequently it is obvious that if magnetic flux is controlled in some pre-arranged manner, the torque developed in the damper will depend only on this flux and not necessarily on the relative motion between the damper elements. Furthermore, the only way that the damper effort can be made to depend on the relative motions of the two elements is by using these to control the dampers magnetic flux.

These features are made use of in the experimental investigation of :

(a) The effects of variations in shape of the damper "work diagram"

(b) Control of the maximum damper effort during the vibration cycle by a characteristic of the motion other than relative velocity.

Harmonic response tests of the analogue for these damping conditions were achieved by means of the geared mechanism illustrated in Fig.7.3. This provides a drive, by means of the two speed gear train from the input shaft to the scotch crank mechanism, to the wiper of a voltage forming resistor. The wave form of the voltage picked up by this wiper is thus determined solely by the design of the resistor, while the frequency of voltage fluctuation is either exactly the same or twice that of the excitation of the analogue, and may thus be used to supply a varying magnetic flux to the damper. Also the phase angle between the voltage former and the input pulley may be adjusted slowly through a complete cycle while operating, by the hand worm adjustment for rotating the axis of the voltage forming resistor. In fact this adjustment is used in conjunction with the visual observation of the existing work diagram presented on a C.R.O. as explained in section 7.4.2. With a particular voltage former resistor in place, a desired work diagram is obtainable provided the voltage former is in the correct phase with the relative motion across the damper. This latter naturally depends greatly on the frequency of operation, and also on the motions of the analogue elements resulting from the damper diagram configuration which exists at any time. Consequently as either the frequency of operation, or the magnitude of damping is varied, the phase of the voltage former must be adjusted until the required damper diagram shape persists under steady state conditions of oscillation. In actual fact, the transient oscillations arising from adjustment of the voltage former phase are extremely short lived and it is easy to maintain a desired diagram shape. Thus the harmonic response of the system with damper characteristics of chosen shape and magnitude can be easily determined on the analogue by exciting the damper with an external voltage source which is not directly dependent on the motions of the analogue elements. Then by recording the resultant motions of the system, it is possible to relate damping effort to any selected characteristic of the vibration, and thus determine the dynamic response for conditions where damping torque is an automatically controlled function of this characteristic.

The transient response of the analogue under conditions of selected damper diagram shape is far more difficult to achieve. The direct approach in which the voltage determining the damper effort is "formed" by the actual motions of the analogue elements proved to be the most reliable. However such a method cannot be generalized so that it would have been necessary to devise numerous mechanisms to transduce the recorded oscillation parameters to a controlling damper voltage. This was not considered necessary, however, as earlier harmonic response tests indicated that some of the damper diagram shapes investigated were impractical. Furthermore it was realized that the decay time of the transient is directly related to the control of the dynamic magnifier of the harmonic response so that a qualitative indication of the effects of the less important of the damper diagram shapes could be obtained from the performance during the first cycle of the transient only . The only transient responses which were considered worthy of detailed investigation were those with "constant" and "return" damping forms. A detailed description of these forms is given in Chapter 8 . Basically the first requires simply a constant voltage of excitation, whereas the latter needs a voltage proportional to the deflection of the suspension spring during its rebound to the static equilibrium position and zero at all other times. This was achieved on the analogue by the friction switch and rheostat illustrated diagrammatically in Fig.7.7. The contacts of the friction switch were rigidly connected to the body inertia while the insulating cylinder rolled and slipped on the axle inertia thereby operating the switch. The rheostat was fixed to the body and the wiper to the axle, and adjusted to the centre position when the system was in static equilibrium. Thus during transient oscillation, whenever the relative motion tended to return to the static zero position, the appropriate contact was made by the minute frictional drag on the insulated cylinder of the switch and a voltage proportional to the relative deflection of the suspesion spring was then fed to an amplifier and thence controlled the magnetic flux in the damper, whereas at all other times the voltage was zero. In practice this simple device proved most satisfactory and gave consistent results for the transient response under "return" damping conditions.

## 7.4. INSTRUMENTATION.

The instrumentation required for the analogue has two purposes firstly to record the various responses, and secondly to provide an accurate knowledge of the operating conditions. The variables which constitute the response of the analogue are the motions of the input, axle and body, and the total force transmission to the body, while the operating conditions include the amplitude and frequency of excitation and the performance of the magnetic liquid damper.

### 7.4.1. MOTION RECORDING.

The analogue is particularly suited to direct recording as the resultant motions are relatively large, and this is achieved on the built-in pen recorder shown in Fig.7.1. Any angular motion of the input, axle or body inertias, causes the light pens attached to them to be pulled across the surface of the recording paper by a thin brass shim which is guided by rolling bearings. Each pen slides freely along a chrome plated guide bar, the whole providing a positive pen drive which presents negligible frictional drag to the analogue masses..

Records are obtained on teledeltos paper which is pulled past the pens at a fixed rate by the recorder drive. To ensure that the paper passes the pen marking planes at right angles and does wander, all paper guide and tensioning rollers are mounted on a separate frame which is aligned and bolted to the major frame. Marking of the teledeltos is achieved by making the platinum pen points 450 volts positive with respect to the earthed guide rollers and limiting the current to 6 milliamperes. The quality of the line is critically dependent on the reaction of the pen points on the teledeltos. Provided this is maintained by the special pen springs at ½ gram, an extremely sharp well defined line is obtainable. This allows motions to be estimated to the nearest 0.1 millimeter on the recording paper, which represents 0.004 Radians movement of the analogue masses. As the maximum allowable motions of the analogue elements are 1.44 Radians, the accuracy of motion recording is better than ½% of the maximum allowable deflection.

The paper drive, a converted Mark IV Selsyn which operates as a synchronous motor driven from the mains, provides a constant paper speed, and also enables recording at any excitation frequency to be of reasonable wavelength by incorporating a three speed gear-box.

This provides paper speed of approximately $\frac{1}{2}$, 2, and 8 inches per second, thus giving wavelengths of about 1 inch in the important frequency ranges. In order to accurately determine the paper speed during each test, a series of timing marks is also made on the teledeltos. A multivibrator is used as a mains frequency counter to control the firing of an impulse generator which energizes the timing pen. Distinct timing dots are thus made every 0.1 sec.

As the dynamic magnifiers of the axle and body depend on the input motion, it is necessary to know this value accurately. The recording of input motion on the teledeltos is not entirely satisfactory as its value is small in comparison to the resultant axle and body motions, and thus has relatively greater error as the limitation of measurement still applies. To overcome this, the input motion is accurately measured by a calibrated telescope directed at the input pulley Fig.7.1. This enables input motion to be measured to 0.01 millimeters corresponding to less than $\frac{1}{2}$% of the minimum excitation employed.

7.4.2. DAMPER PERFORMANCE.

The performance of the damper is assessed by the instantaneous recording of its work diagram. This diagram simply plots the torque developed in the damper versus the relative motion of the damper elements. It is thus necessary to record two variables the instantaneous damper torque and the instantaneous position and direction of motion between the axle and body inertias.

7.4.2.(a) DAMPER TORQUEMETER.

Damper torque is measured by the electrical resistance strain gauge torquemeter of Fig.7.9. which is housed in the shaft connecting the damper torque disc to the axle inertia through the centre of the helical torsion spring . A diagrammatic representation of the torquemeter is given in Fig7.10.

The torque disc is rigidly connected to the lower portion of the shaft, which, in turn, is supported radially and axially in the top shaft which is connected to the axle,by miniature ball-bearing races. Rotational alignment is achieved by the reaction of the semi-circular cantilevers which are built in to the top shaft and connected to the boss on the bottom shaft by a thin brass shim . The cantilevers are pre-stressed against each other in the equilibrium position, and transmit any torque developed on the damper disc by reduction of preload in one and increase of preload in the other. Phillips PR9212, 600 ohm strain gauges are glued to the inner and outer surfaces of both these cantilevers, the electrical connections from these being arranged in a bridge circuit to give the maximum signal with complete temperature compensation . By using this system, a sufficiently large electrical signal is available without undue sacrifice in the torsional stiffness of the damper shaft.

In the original construction of the torquemeter, tests indicated that the effective stiffness of the unit was approximately 200 pound inches/Radian, which is over 50 times as stiff as the suspension spring, and this was considered satisfactory. Also the natural frequency of the damper disc on the torquemeter was measured to be 440 cycles per second, that is about 15 times greater than the maximum available frequency of excitation.

Thus, throughout the tests, the torque indicated by the torquemeter signal is a true value of damper torque and suffers no dynamic magnification due to torquemeter resonance.

The torquemeter has proven experimentally to have exactly similar linear characteristics for positive and negative torques up to the maximum pre-load torque which is far in excess of the test values used. Also tha torquemeter was extremely consistent, having negligible "stiction" a feature which was eliminated during development by the flexible brass shims applying the preload.

7.4.2.(b) RELATIVE MOTION.

An electrical signal which is proportional to the relative motion between the axle and body inertias, necessary for plotting the work diagram of the damper, is provided by the linear rotary potentiometer shown in Fig.7.9.

The resistance winding is fixed to the body inertia and contributes to it, while the earthed wiper is fixed to the damper shaft. An external centre position is established electrically by potentiometer so that the signal can be accurately balanced about the wiper position at static equilibrium. The resultant signal between the external centre tap and earth is then directly proportional to the relative position of the analogue elements.

The position potentiometer is wound of extremely fine wire, and relative motion of 0.004 Radians, equivalent to the minimum detectable motion of the analogue elements on the built-in pen recorder, is easily obtainable.

Calibration of the potentiometer is obtained by static relative deflections measured on the teledeltos pen-recorder.

7.4.2.(c) DAMPER "WORK DIAGRAM".

The electrical signal from the torque-meter is first amplified by a direct coupled pre-amplifier, and then is fed to the Y-plates of two monitored Cathode Ray Oscillograph tubes. At the same time the electrical signal from the relative position potentiometer is fed to the X-plates of these C.R.O. Providing the phasing of electrical connections to the C.R.O. is correct, the resultant locus plotted by the beam spot is the existing work diagram of the damper.

The first C.R.O. is equipped with a long persistence screen and facilitates visual observation of the complete work diagram, thus enabling the operator to maintain any required diagram shape and magnitude. The second C.R.O. tube is used for photographic recording of the work diagram during test, and consequently has a short persistence screen and camera attachment.

During transient tests, the work diagram does not persist as a closed figure as the amplitudes of oscillation are decaying. Nevertheless the work diagram tecnique for presenting damper performance throughout the transient may be used for convenience, though the figures may become extremely involved. It is more satisfactory to record transient performance as a time function, and this may be done photographically on the C.R.O. for the first few oscillations by having the time base triggered at the initiation of the transient, or by using a high speed pen-recorder in place of the C.R.O. where the complete transient response is required. All three tecniques have been used in the transient tests for the presentation of damper performance.

### 7.4.3. EXCITATION FREQUENCY.

Satisfactory determination of the harmonic response of the analogue naturally depends on an accurate knowledge of the excitation frequency. This is particularly true for excitation close to the natural resonances of the system. There are three separate systems for indication of the excitation frequency of the analogue, and each of these supplements the others to give an accurate and simple speed indication throughout the frequency spectrum.

Firstly, the drive motor has an accurate D.C. tachometer generator permanently mounted on its shaft. The voltage from this generator is directly proportional to the motor speed. For initial setting of the excitation frequency, the generator output is measured on a calibrated voltmeter giving the motor speed in revolutions per minute to the nearest 10.R.P.M. This setting is not sufficiently accurate, however, near the resonances as amplitudes of motion are then critically dependent on speed. Furthermore, in these ranges, there is a tendency for the motor speed to drift from the desired value by absorbing some of the oscillation energy. This was anticipated in the original design, and an effort made to minimize it by including a relatively large flywheel at the scotch crank to make the rotational energy far greater than the oscillation energy. Nevertheless, the necessity of ensuring a particular oscillation frequency near the resonances was apparent in the preliminary testing, To this end, the output voltage from the tachometer generator was also accurately measured by a calibrated potentiometer using a standard cell as reference. The balance of voltages from the potentiometer and the generator was made visual by using an external galvanometer. Thus, once a particular frequency of operation had been selected, a movement of the galvanometer pointer immediately indicated any tendency to drift from this value. By this means, speed variations of only 1 R.P.M. could be detected, but it was not possible to control the motor speed over a reasonable time interval to greater accuracy than 2.R.P.M.

The stroboscopic pattern method of speed setting provides the means of calibration of the tachometer and the potentiometer, and was also found to be the most convenient method of ensuring a constant excitation frequency. However , this has the restriction that it can only be used for frequencies in finite steps, and consequently must be used in conjunction with the other methods of speed measurement. Stroboscopic patterns of 15 points were fixed to the motor shaft and to the outer annulus of the first epi-cyclic gearbox Fig.7.3. When illuminated by light flashes 50 times per second these produced stationery figures for shaft speeds an integral multiple of 100.R.P.M. An accurate 50 c.p.sec light flash was provided by a gas-discharge tube triggered from a Tinsley tuning fork. In operation, the excitation speed was set a approximately by reference to the voltmeter tachometer, and then accurately set by stroboscope if 100R.P.M. intervals were satisfactory, or by calibrated potentiometer if a finer subdivision of the frequency spectrum was demanded .

## 7.5. CALIBRATION OF ANALOGUE PARAMETERS.

It is obviously essential that the characteristics of the torsional analogue be accarately determinable. Thus it is necessary to calibrate the axle and body inertias, the tyre and suspension spring characteristics and the suspension damper characteristic. In fact the last mentioned characteristic is capable of such wide variation by the use of magnetic liquid dampers, that it is necessary to treat it as an operating variable and effectively re-calibrate at every condition of operation throughout the tests as indicated in section7.4.2.(c) .

Spring characteristics however remain consistent once the physical form of the spring has been established. It is thus only necessary to calibrate the spring characteristic when the arrangement of the system has been disturbed or modified to introduce some change. Throughout the entire series of tests the tyre spring was maintained of the same linear characteristic whereas the suspension spring was altered from the basic linear form to various types and degrees of non-linearity. Calibration of the spring characteristics was achieved by the method illustrated in Fig.7.11. Here a static torque is applied to the spring being calibrated and the resultant deflection measured by means of the telescope and scale fixed to the analogue masses. To calibrate the body spring, the axle was locked to the input by a rigid collar, and thence to earth by a locking screw, while positive and negative calibrating torques were applied to the body inertia whose deflection was measured. Calibration of the tyre spring was most satisfactorily achieved by locking both input and body inertia to earth and applying the calibrating torques to the axle inertia while measuring its deflection.

This calibration is then of the tyre and suspension spring in parallel i.e. the added rates of the two springs, from which the tyre spring rate can be deduced.

The accuracy of the calibration is satisfactory as the calibrating torques have negligible error while the deflections can be measured to 0.001 Radians which represents approximately 0.1 % of the maximum allowable spring deflection.

Calibration of the axle and body inertias was achieved when the system had linear spring characteristics. The body inertia was determined from the natural period of oscillation of the body on the calibrated suspension spring with the axle locked to earth . The greatest source of error in this calibration is the time measurement as this must be squared in determining the inertia. The 0.1 sec timing dots provided on the teledeltos recording greatly improve this accuracy which after statistical evaluation from a number of cycles is estimated to be better than one percent.

Similarly the axle inertia is determined from its natural period of oscillation on the tyre and suspension springs in parallel with the body and input locked to earth.

The slight increase in inertias brought about by the introduction of non-linearising springs is negligible as it is only their inertia which contributes, the anchors being included in the original calibration.

Facility is made for calibration of the damper torquemeter throughout the tests. This procedure is shown in Fig.7.9. Calibration torques are applied by dead-weights to the lower section of the damper shaft by a fine strong wax thread, while the damper is de-energized and the excitation zero. The effective torque-arm of the applied load was measured with the thread strained against the damper shaft and is accurate to within 1%. It will be noticed from Fig.7.12. that the calibration torque is not pure, but provides an associated radial load on the damper shaft. This nevertheless proved to be satisfactory experimentally, as the radial load was taken by the miniature ball bearings and had no discernible influence on the torque signal provided by the torquemeter. The damper torquemeter was calibrated by this means before and after each series of tests to ensure that amplifier gain did not drift throughout the series.

## 7.6. ANALOGUE EQUIVALENCE.

The torsional system illustrated in Fig.7.1. is a direct mechanical analogue of the simplified translational system of independently sprung axle and body assembly of the modern road vehicle introduced in Chapter 5. Fig.5.1. The components of the analogue were designed to have characteristics that agreed as closely as possible with an average modern suspension.

Analogue equivalence is proven by comparison of the equations of motion of the linear torsional and translational systems.
Thus for the translational system, the equation of motion of the body mass is given by:

$$m_B D^2 x_B = K_B(x_A - x_B) + C_B D(x_A - x_B) \quad \ldots\ldots\ldots\ldots\ldots(7.1.)$$

Also the equation of motion of the axle mass is :

$$m_A D^2 x_A = K_A(x_O - x_A) - K_B(x_A - x_B) - C_B D(x_A - x_B) \quad \ldots(7.2.)$$

While for the torsional system of the analogue, the equations of motion are ;

$$I_B D^2\theta_B = K_{\theta B}(\theta_A - \theta_B) + C_{\theta B}D(\theta_A - \theta_B) \quad \text{.................(7.3.)}$$

$$I_A D^2\theta_A = K_{\theta A}(\theta_0 - \theta_A) - K_{\theta B}(\theta_A - \theta_B) - C_{\theta B}D(\theta_A - \theta_B) \quad \text{....(7.4)}$$

Equivalence of the analogue and the translational suspension components is thus ;-

| ANALOGUE | | UNITS | SUSPENSION | | UNITS |
|---|---|---|---|---|---|
| Inertia | $I$ | $lb.in.sec^2$. | Mass | $M$ | $lb.sec^2/in.$ |
| Spring Rate | $K_\theta$ | lb.in/RAD. | Spring Rate | $K$ | lb/in. |
| Rotation | $\theta$ | RADIANS. | Motion | $x$ | inches. |
| Time | $t_\theta$ | seconds. | Time | $t$ | seconds. |
| Torque | $T$ | lb.in. | Force | $F$ | lb. |

If the equivalence relations are selected as ;-

$$\theta = a.x \quad \text{..............................(7.5.)}$$

$$T = b.F \quad \text{..............................(7.6.)}$$

$$t_\theta = c.t \quad \text{..............................(7.7.)}$$

where a,b,and c, are constants.

Then,

$a = \theta/x$ has dimensions Radians/inch. .....(7.8)

$b = T/F$ has dimensions inches. .......(7.9)

$c = t_\theta/t$ is dimensionless ...............(7.10)

It then follows from the equivalence relations that

$$\text{Inertia } I = \frac{b.c^2}{a}. \text{ Mass}(M) \quad \text{................(7.11)}$$

$$\text{Spring Rate } K_\theta = \frac{b}{a} . K \quad \text{................(7.12)}$$

Provided a unity time scale is selected, i.e. $c = 1$, the relationships reduce to

$$\text{Inertia } I = b/a \,.\text{Mass}(M) \quad \text{and} \quad \text{Rate } K_\theta = b/a \,.\, K \quad \text{................(7.13)}$$

It is thus obvious that further selection of scale factors is not necessary before the torsional system can be completely defined.

The values of the analogue constants fit very closely to average suspension characteristics for scale factors ;-

$$b/a = 0.0644 \text{ square inches}.$$

$$c = 1.0$$

The relationships are then ;-

| TORSIONAL ANALOGUE | TRANSLATIONAL SUSPENSION |
|---|---|
| Axle Inertia $I_A = 0.0128$ lb.in.sec$^2$ | $M_A = 0.199$ lb.sec$^2$/in. = 77 lb |
| Body Inertia $I_B = 0.0949$ lb.in.sec$^2$ | $M_B = 1.475$ lb.sec$^2$/in. = 560 lb |
| Tyre spring $K_{A\theta} = 76.1$ lb.in/RAD. | $K_A = 1180$ lb/in. |
| Suspension $K_{B\theta} = 3.86$ lb.in/RAD. | $K_B = 60$ lb/in. |

These are the parameters for which the calculated responses of chapters 5 and 6 apply.

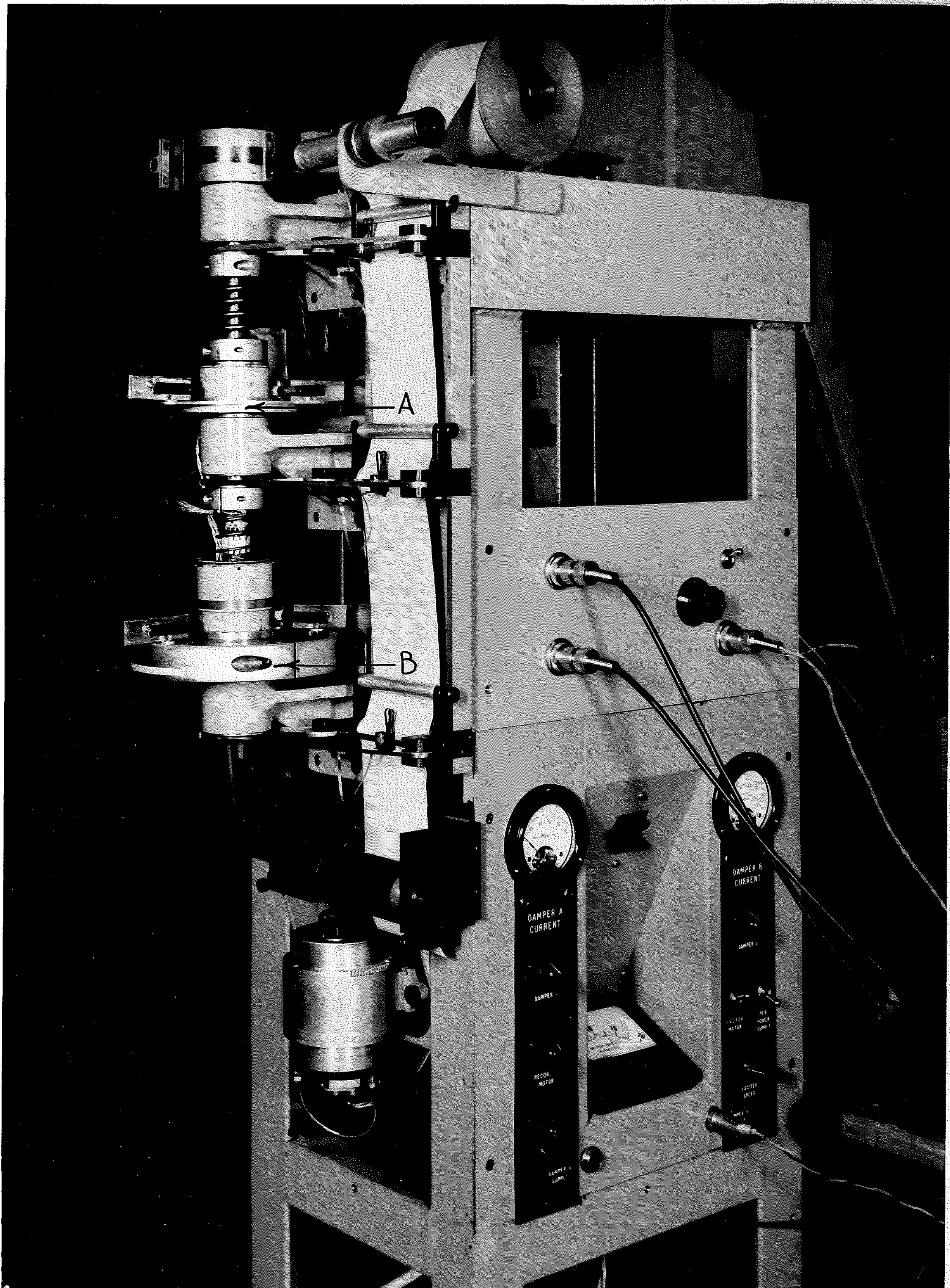



FIG.7.1. TORSIONAL MECHANICAL ANALOGUE.

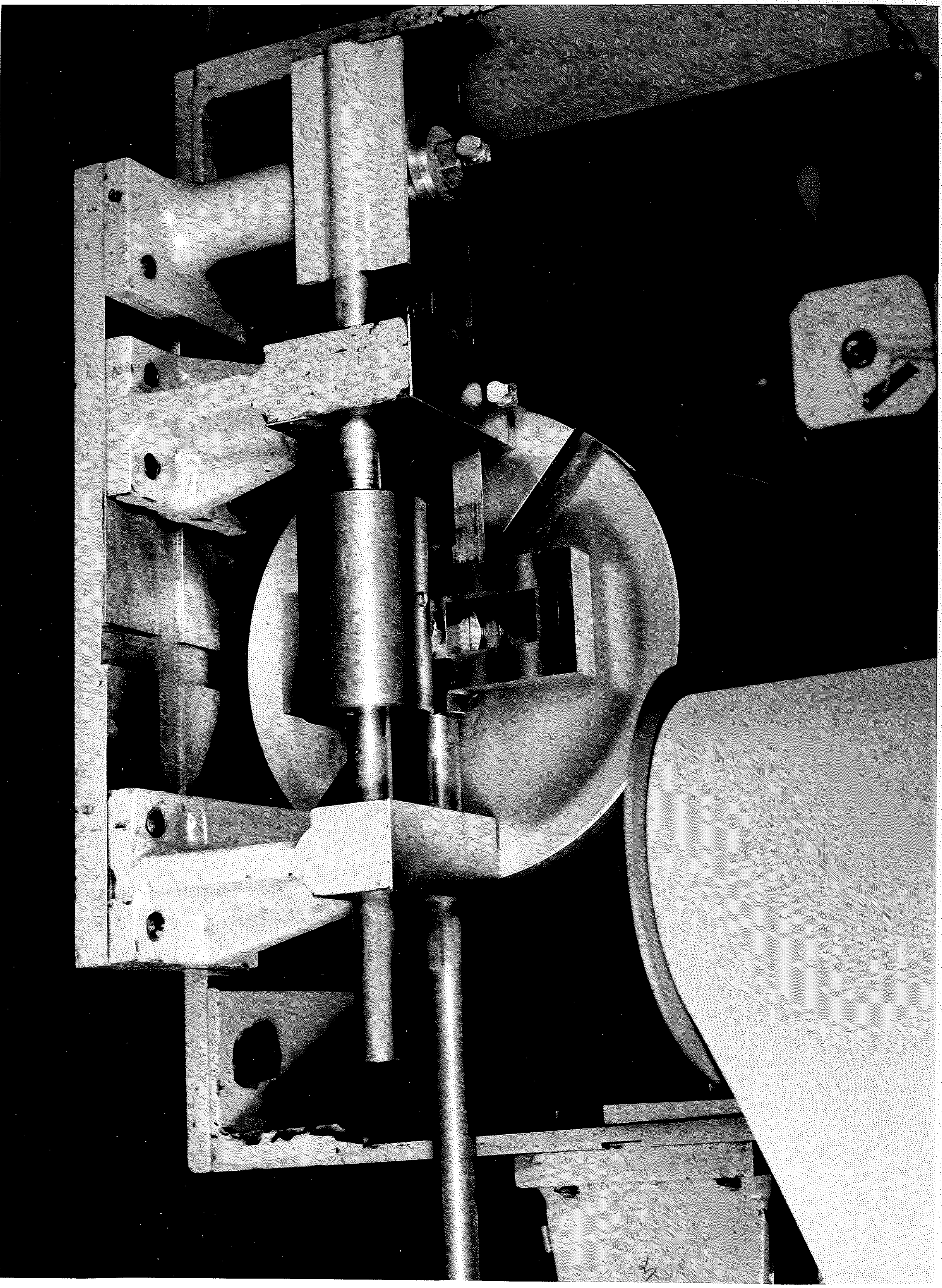


FIG.7.2. SCOTCH CRANK AND BALANCE MECHANISM.

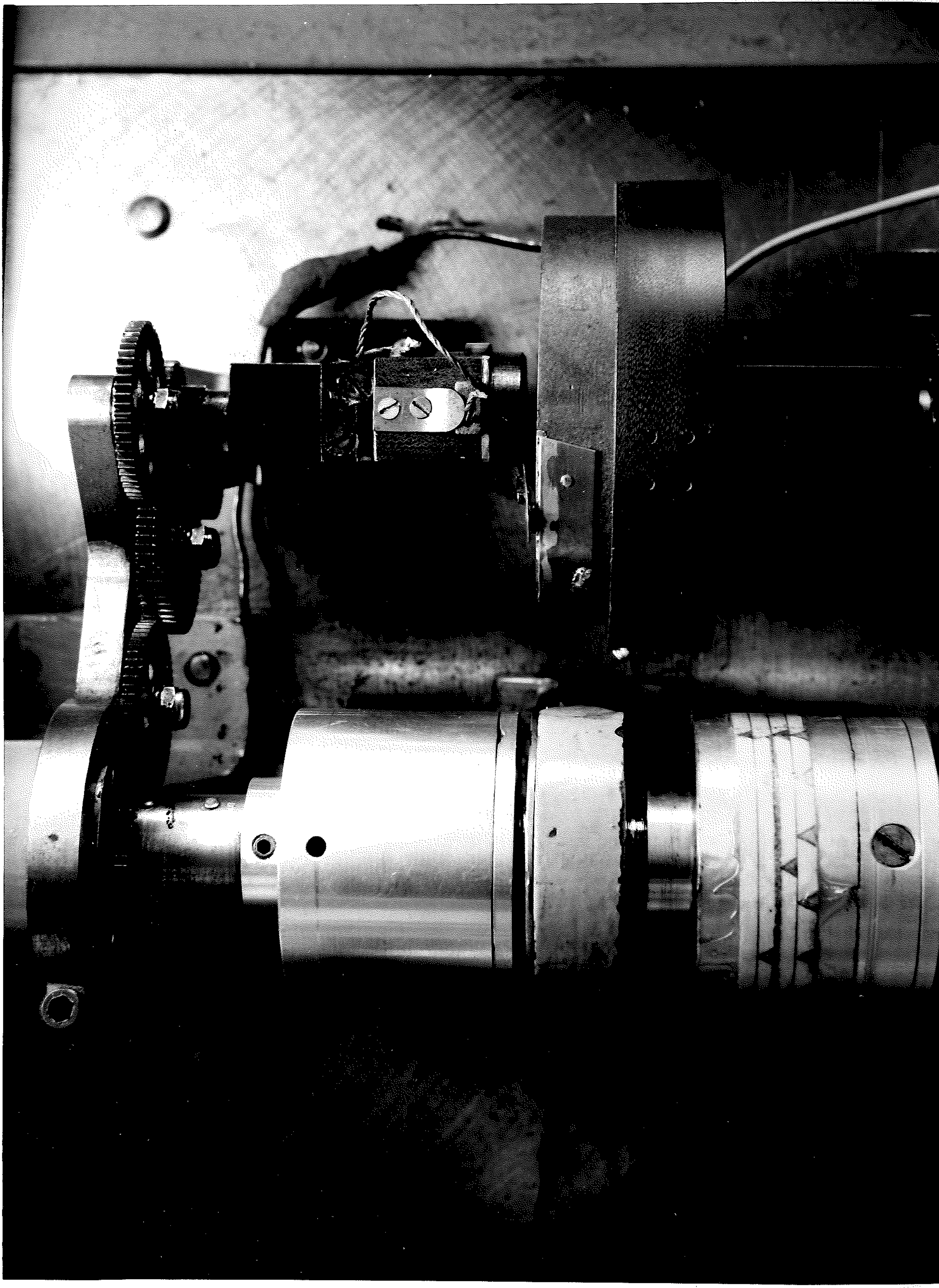


FIG.7.3. MAIN DRIVE GEAR-BOXES AND VOLTAGE FORMER DRIVE.

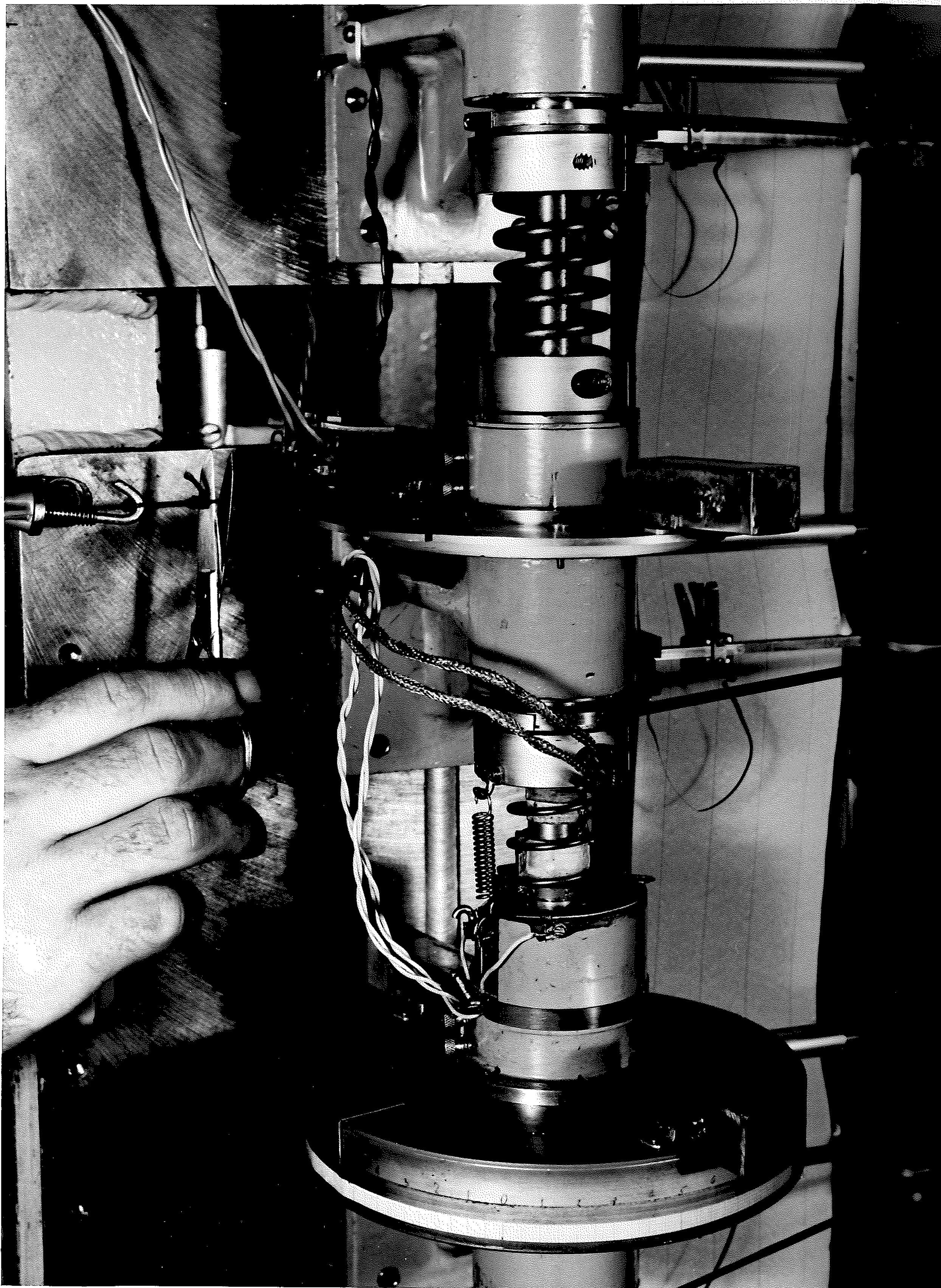


FIG.7.4. TRANSIENT EXCITATION OF THE ANALOGUE.

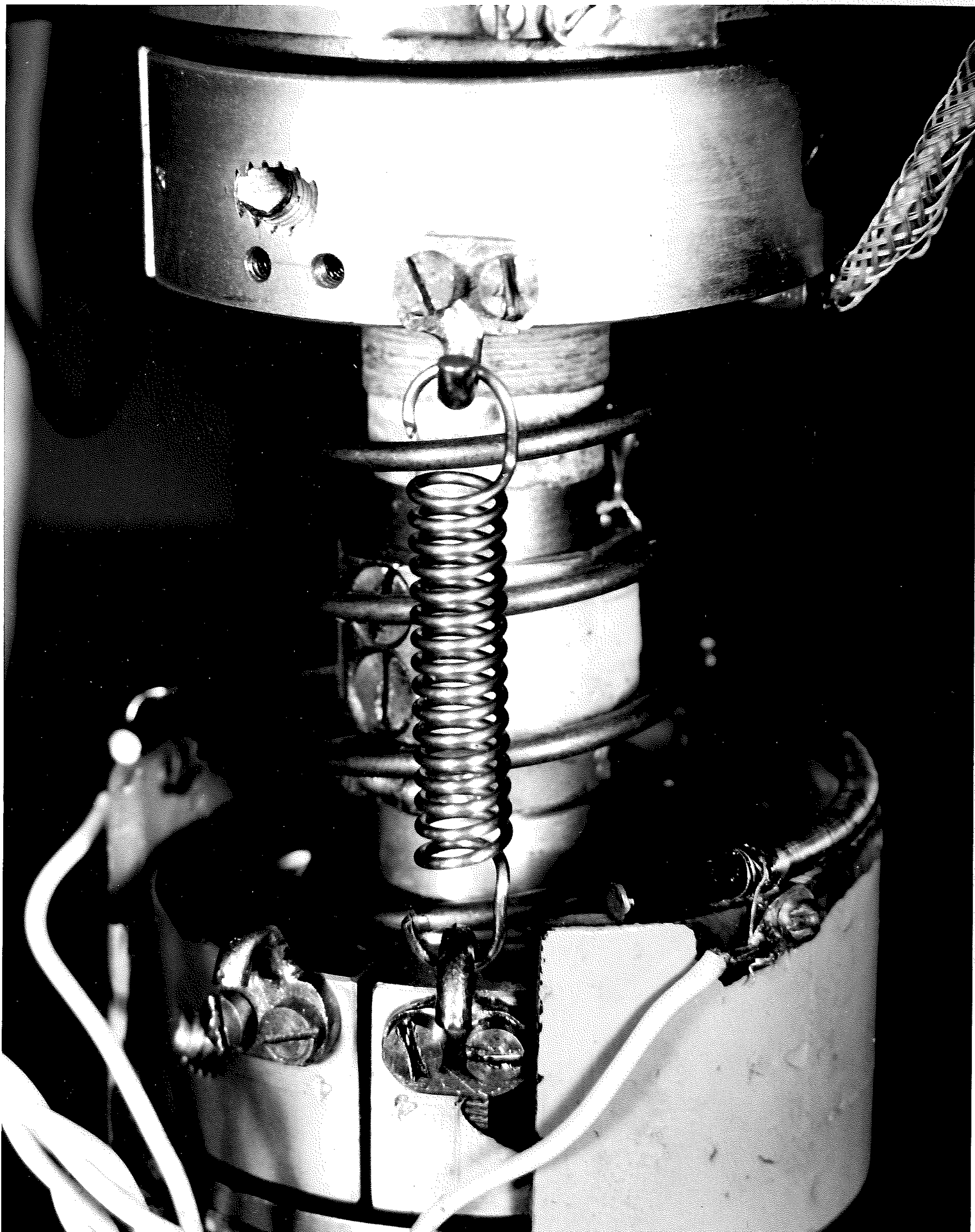


FIG.7.5. ARRANGEMENT FOR NON-LINEAR SPRING CHARACTERISTIC.

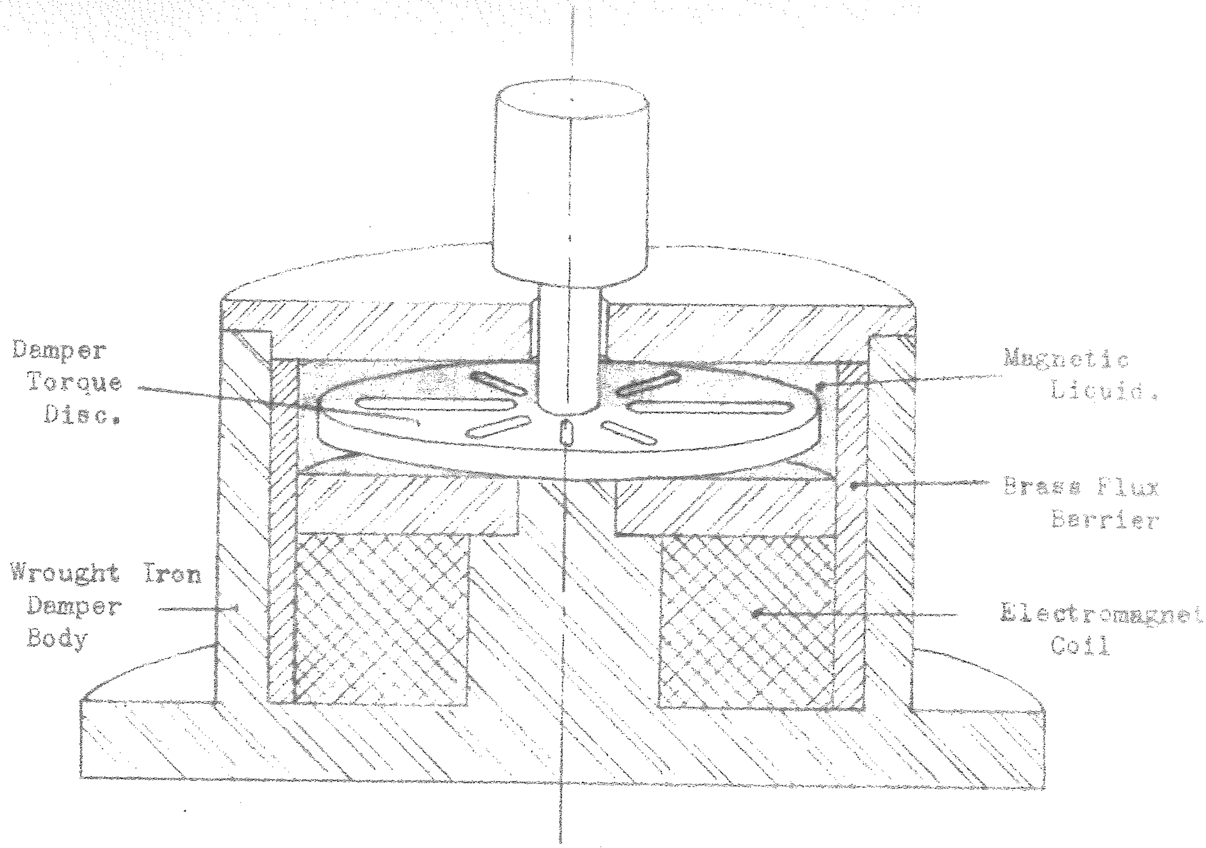



Fig.7.6. DIAGRAMMATIC SECTION OF THE ANALOGUE MAGNETIC LIQUID DAMPER.

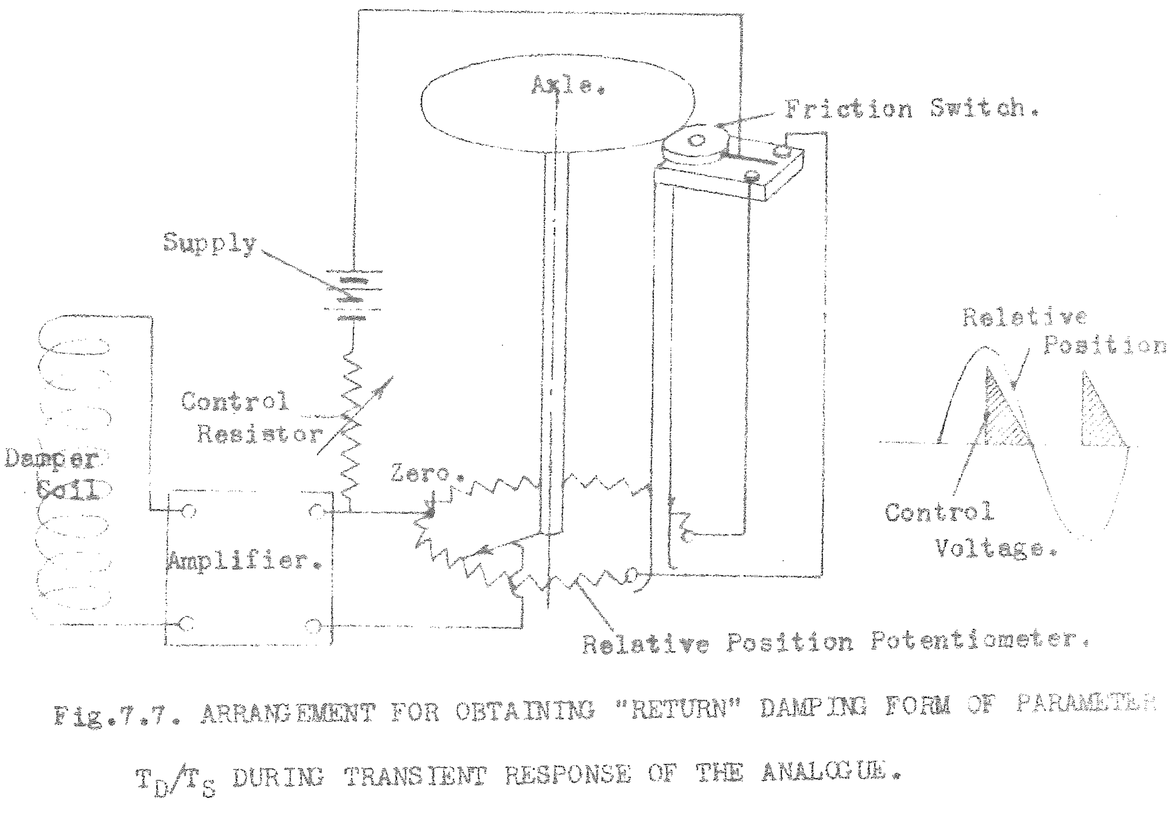



Fig.7.7. ARRANGEMENT FOR OBTAINING "RETURN" DAMPING FORM OF PARAMETER $T_D/T_S$ DURING TRANSIENT RESPONSE OF THE ANALOGUE.

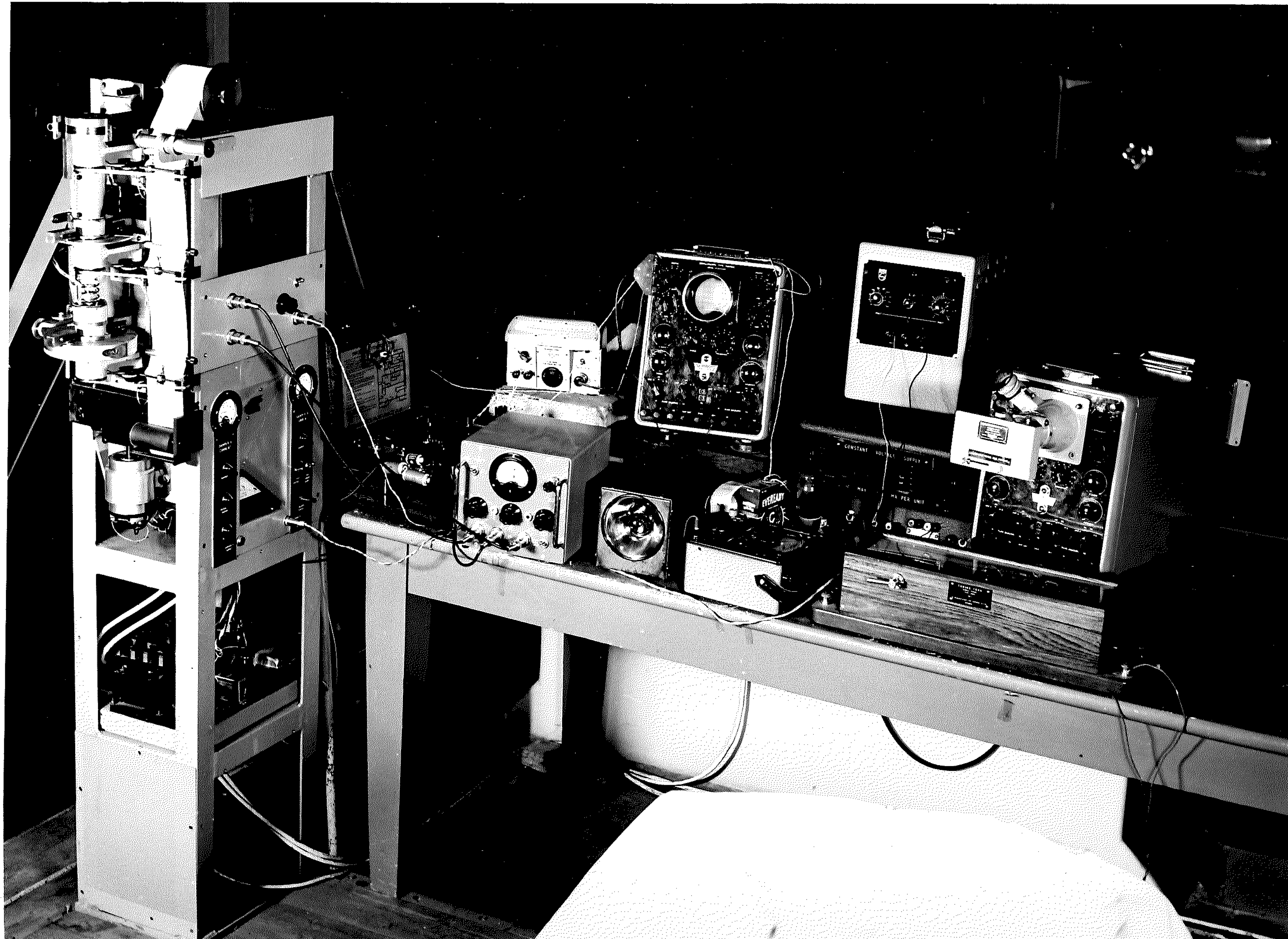



FIG.7.8. GENERAL ARRANGEMENT OF THE ANALOGUE INSTRUMENTATION.

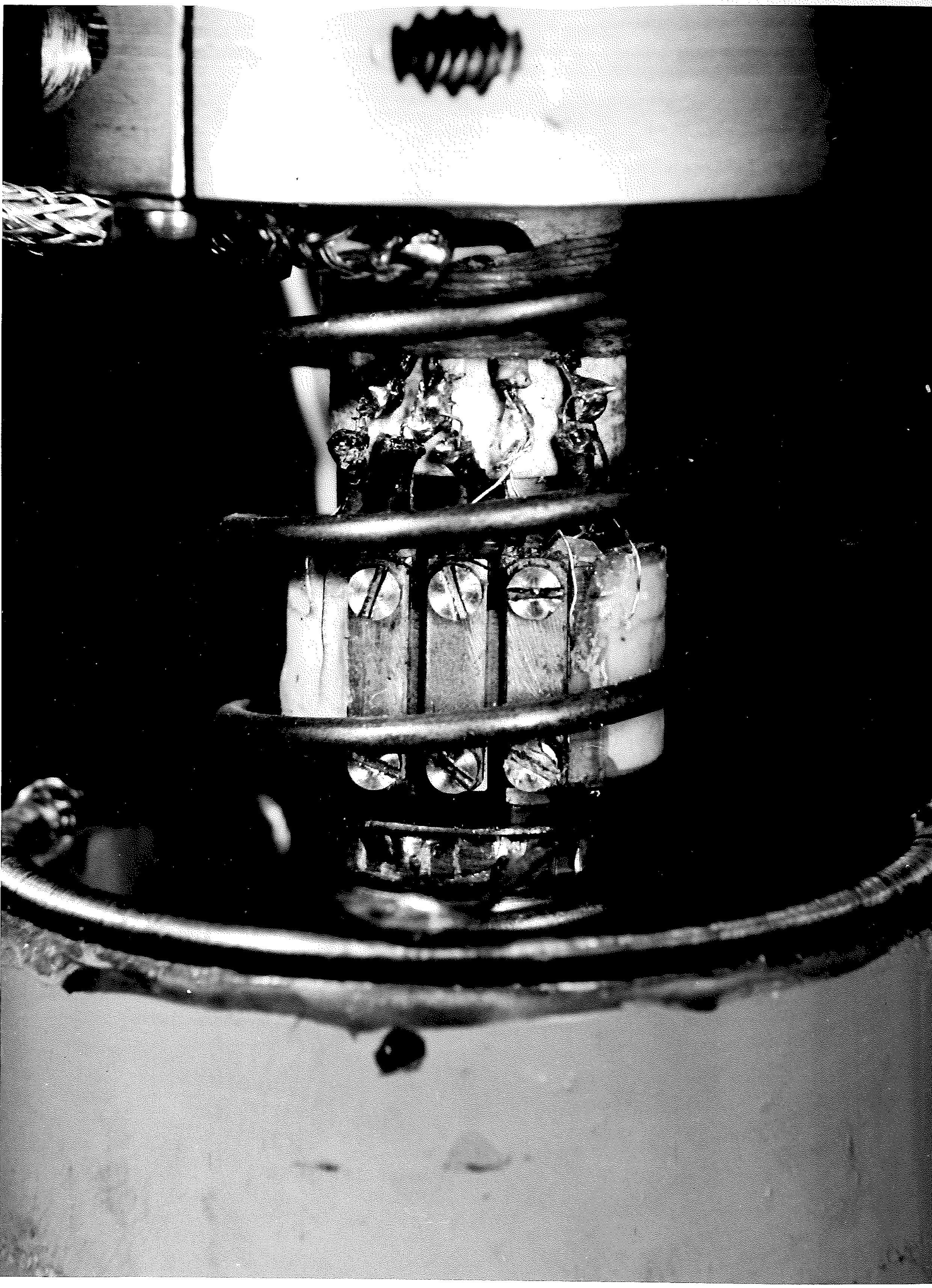


FIG.7.9. THE ANALOGUE DAMPER TORQUE-METER.

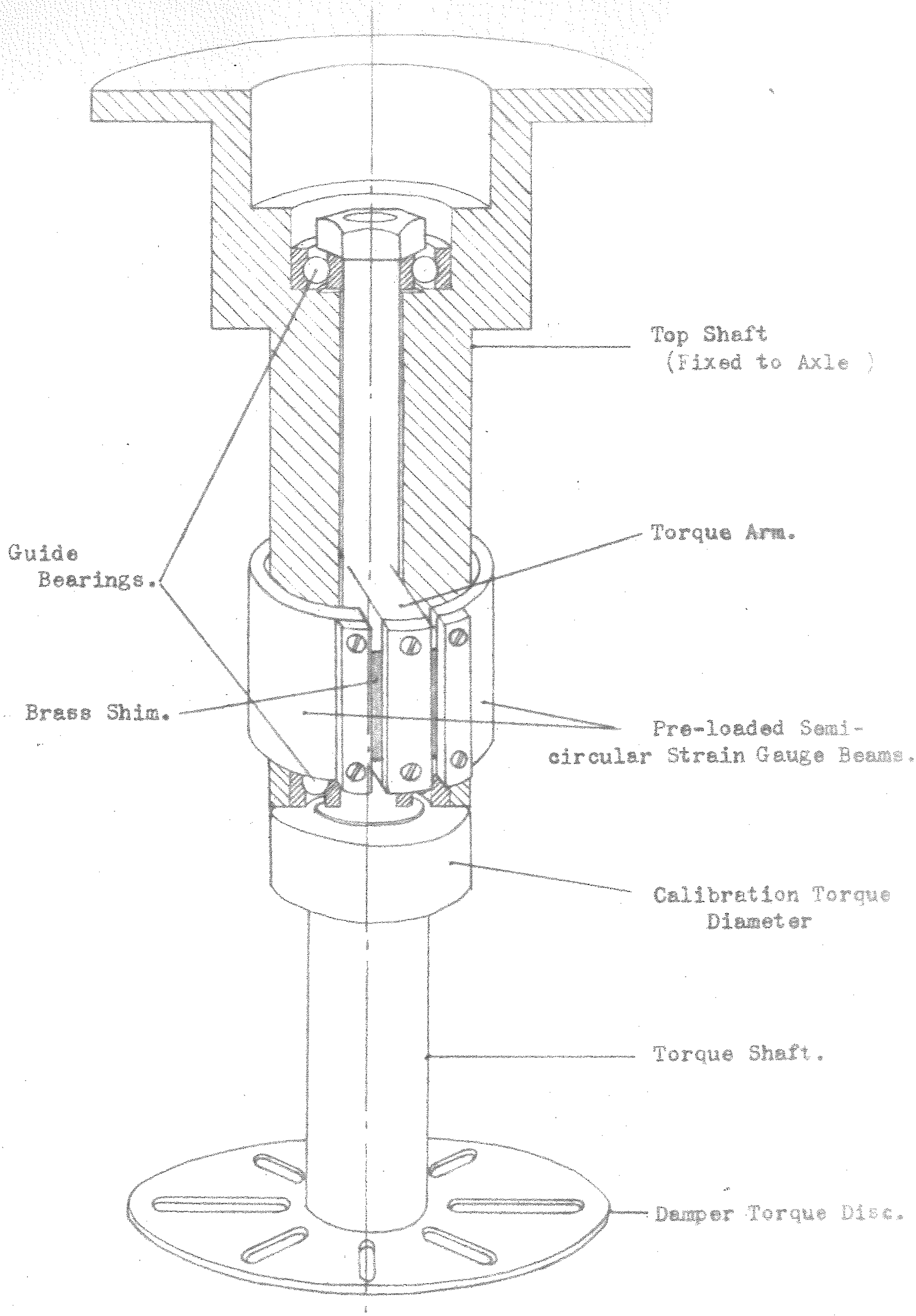



Fig. 7.10. DIAGRAMMATIC ARRANGEMENT OF DAMPER TORQUEMETER

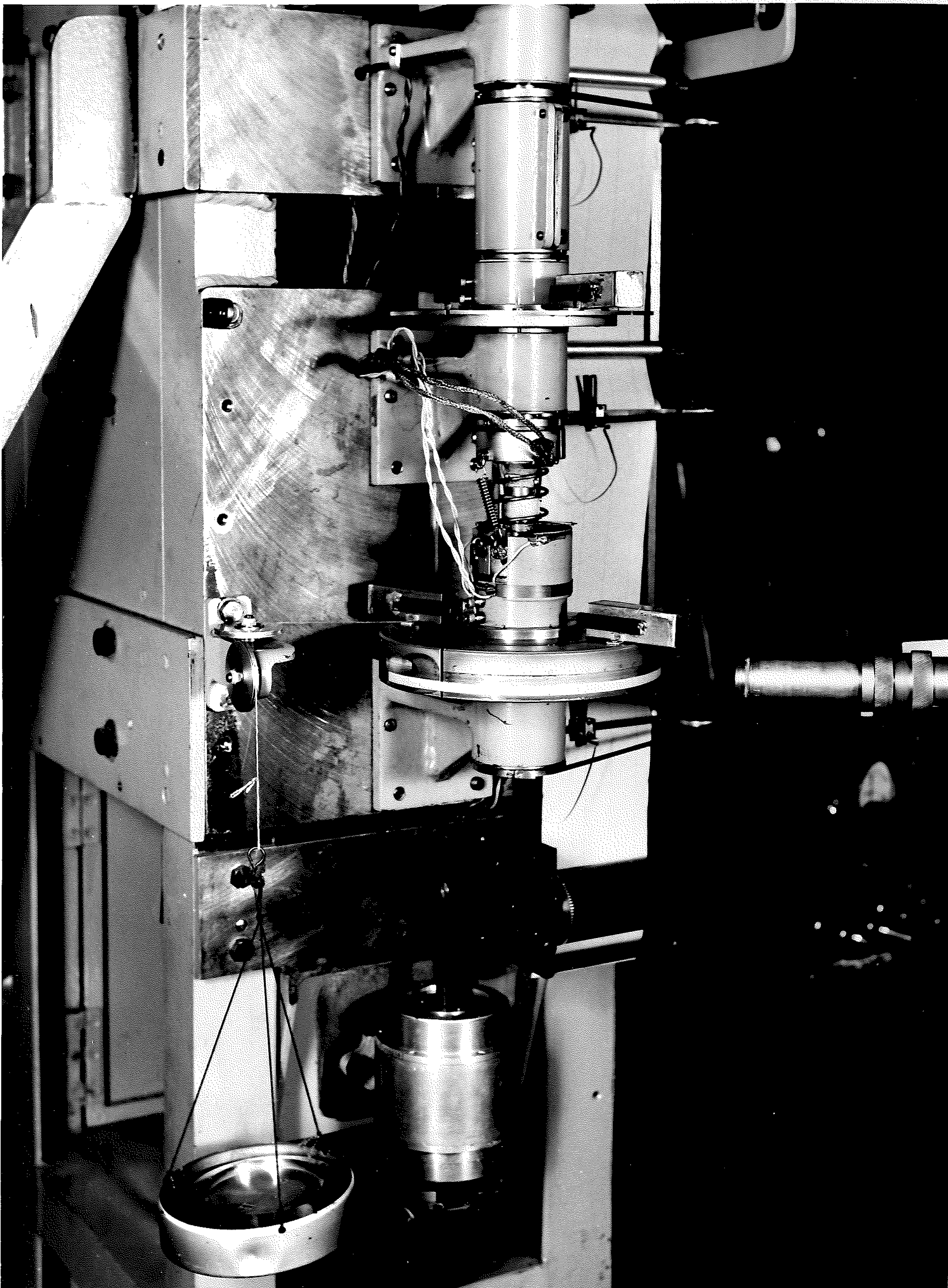


FIG.7.11. CALIBRATION OF SPRING CHARACTERISTICS.

FIG.7.12. CALIBRATION OF DAMPER TORQUE-METER .

# Chapter 8

# NON-LINEAR PERFORMANCE OF THE MECHANICAL ANALOGUE

## 8. NON-LINEAR PERFORMANCE OF THE MECHANICAL ANALOGUE.

The harmonic and transient responses of the analogue, when fitted with non-linear components for suspension damping and springing, were investigated progressively, so that the individual features could be easily separated, and the interacting effects thus made more amenable to analysis. Thus, the first series of experiments were concerned with non-linear damping in the system which was fitted with linear springs, and the tests with non-linear springs were conducted with the magnetic liquid dampers de-magnetised so the system was practically undamped. Then, finally, the tests were broadened to include the effects of damping in the non-linearly sprung system.

### 8.1. NON-LINEAR DAMPING IN A LINEARLY SPRUNG SYSTEM.

Investigation of the performance of a suspension system having linear spring characteristics, but non-linear damping, is facilitated by the magnetic liquid damper incorporated in the mechanical analogue. This damper is described in section 7.4.2. and its basic characteristics are presented in Appendix.2.

The analogue unit was fitted with a slotted wrought iron torque-disc throughout the tests, and consequently the damping effort was independent of the speed of relative motion of the elements and was determined solely by the magnetic flux passing through the liquid, so its performance was essentially non-linear.

The damper work diagram, available from the signals of torquemeter and relative position potentiometer, was used to indicate the performance of the damper under all conditions of operation, so that it was necessary to specify both the shape and the magnitude of this diagram in order to ensure similar performance at all speeds throughout the frequency spectrum.

### 8.1.1. EFFECT OF DAMPER WORK DIAGRAM SHAPE ON HARMONIC RESPONSE

The response of the analogue was obtained at various frequencies throughout the range under the action of several different damping forms. The object of these experiments was to determine whether any particular distribution of damping effort in the vibration cycle was more satisfactory than that provided by the linear damper.

The shapes of damper work diagrams investigated were:

(1) "Constant" damping C, in which the damper energising current was kept constant. The torque developed in the damper was then independent of the relative velocity of the elements, and the characteristic shape closely approached coulomb damping, provided the amplitudes of motion were reasonable as described in Appendix.2.

(2) "Away" damping A, in which the damper effort was concentrated in the region where the elements move away from the relative zero position, the return motion towards relative zero not being resisted.

(3) "Return" damping R, in which damping torque was concentrated only to resist the return of the elements to the relative zero position, any motion away from this point having greatly reduced opposition.

(4) "End" damping E, where damping torque was concentrated in the outer semi-amplitudes of the motion, movement through the relative zero position having only a small opposing torque.

In these experiments, Away, Return, and End damping were achieved simply by switching the damper current on and off twice per cycle for a period of $\frac{1}{4}$ cycle using the rotrary switch coupled to the drive motor. By controlling the phase of this switch with the worm gear adjustment described in section 7.3., each of these damping forms were available, thus Away damping was achieved by phasing the switch to go on just as the relative motion across the damper passed through the zero position, and to go off just as the motion reached a maximum, while similar arrangements were possible to give both Return and End forms.

Although the damper effort was independent of velocity when operating under constant energizing currents, it was found that the magnitude of the Away, Return, And End damping forms did depend on the frequency of excitation. This was a secondary effect arising from the necessary inductance of the electromagnet coil, which introduced a time constant in the electrical circuit supplying the damper. Consequently, as the excitation frequency increased, the magnitude to which the magnetic flux built up during the "on" time of the switch decreased, and it was necessary to increase the supply voltage progressively under these conditions to maintain similar damper diagrams.

Typical damper work diagram shapes recorded during these tests are illustrated in Fig.8.1, and these show the effect of exponential build-up and decay of magnetic flux in the switched energization forms.

As the effects of damping depend largely on the relationship between the exciting frequency and the natural frequencies of the system, it was necessary to operate at several frequencies above, below and near the natural resonances, to obtain an overall indication of the effects of damper torque distribution. This also required the selection of exciting amplitudes at the various frequencies so that the resultant motions were within the allowable limits, and controllable by the magnitudes of damping torque available.

The results of these tests are given graphically in Figs. 8.2 and 8.3. For comparison, the response of the system to linear damping is also plotted on these graphs. This was obtained by calculation as indicated in section 5, using the calibrated analogue parameters.

Test Results.

In the immediate vicinity of body resonance, any form of damping is desirable, as it tends to decrease dynamic magnification of the body, and thus also the magnitude of the total torque transmitted. However, for a given value of damper torque developed, the "Constant" distribution gives the minimum dynamic magnifier. This is attributed to the property of maximum energy dissipation which is possessed by this damping form. Nevertheless, reasonable body motions can be achieved with all damping types developing forces which are small in comparison with the undamped spring forces, the total transmission being only dependent on the resultant dynamic magnifier, as shown in Fig.8.3.

Slightly above body resonance, "return" damping is the only form which does not increase both body magnifier and torque transmission above the undamped case. However, the effect is conditional, and above a definite value of "return" damper torque, body magnifier and torque transmission increase. This limiting value of "return" damper torque proved to be approximately equal to the maximum spring torque developed.
This effect was quite pronounced throughout the range between the two natural frequencies. A second effect which was noticed in this region was the tendency for "return" damping to increase axle motions, but this was extremely small.

Close to axle resonance, the "constant" damping form was most satisfactory in controlling axle motion, and thereby reducing transmission. Other forms of damping did reduce the axle magnifier, but required a greater magnitude of damping effort to do so, and consequently caused far greater torque transmission to the body. This was particularly obvious in the case of "return" damping in which increasing damper torque indicated a minimum in the torque transmission. This is understandable, as the tyre spring governs the axle resonance, and being far stiffer than the body spring, requires far greater magnitudes of damper effort to reduce the motions. Thus with increasing damper torque, the total transmission to the body at first decreased due to the reduction in axle motion, but eventually the reduction in spring force caused by reduced axle motions was outweighed by the increased damper forces necessary to cause this decrease in motion.

Above axle resonance, conditions were similar to those between the resonances, but all forms of damping caused a slight decrease in axle motions. Here again, the "return" damping form provided the minimum torque transmission, especially where the damper torque was less than the developed spring torque.

The test results thus indicate :-

(a) In the close vicinity of the resonances, the "constant" damping form is the most satisfactory. This is especially true at the axle resonance where the damping torques required are high, whereas at body resonance there is small variation even though greater damping torques are necessary with other forms to give the same motions.

(b) At intermediate frequencies betwwen the resonances "return" damping provides smaller body motions, and results in the minimum torque transmission. This latter is most obvious if the damping effort is less than the spring torque developed. Although "return" damping does increase axle motions slightly in this range, the increase is negligible for damping values giving the minimum transmission.

(c) Above axle resonance "return" damping provides the minimum transmission, but decreases axle motions slightly less than the "constant" form.

(d) Throughout the entire spectrum, "away" and "end" damping forms produce results which are inferior to one or other of the "return" and "constant" forms.

8.1.2. RESPONSE OF THE SYSTEM WITH "CONSTANT" DAMPING FORM.

As the preliminary experiments proved that the "constant" damping form gave the most satisfactory performance in the region of the resonances, the complete harmonic and transient responses of the system using linear springs and "constant" damping form, were obtained on the analogue.

The harmonic response of the system was determined from records taken at a fine subdivision of the frequency range which was further concentrated in the region of the resonances. At any particular frequency, the excitation amplitude was adjusted to give resultant motions within the limits of the analogue. The dynamic response for various values of the maximum damper torque was then obtained by simultaneously recording the analogue motions and the damper work diagram. The dynamic magnifiers of the axle and body, together with the maximum damper torque developed and the maximum transmission to the body, were then evaluated from these results.

As it was necessary to excite at different amplitudes throughout the frequency range, it was desirable to select parameters to represent the degree of damping and the magnitude of torque transmission, so that the response throughout the frequency range could be generalized. In this respect, it was convenient to use the parameters developed in section 6, for dampers of linear form but controlled magnitude, and consequently, results for damper parameters of $T_D/K_B x_0$ and $T_D/T_S$ with "constant" form are included. It was decided to restrict the number of parameters to these two, so that the results would not be unnecessarily complicated.

The transient response of the analogue to a step in road profile with "constant" damping form was restricted to the case where the magnitude of damper effort remained constant throughout the duration of the transient, and thus the parameter $T_D/K_B x_0$ was employed. This was considered satisfactory for the transient response, as the overall conclusions could be drawn from this case, and thus the mechanical complexity involved in a variation in damping parameter throughout the transient was not justified.

8.1.2-1 HARMONIC RESPONSE WITH FIXED DAMPER EFFORT.

The harmonic response of the mechanical analogue with "constant" damping form of fixed effort represented by the parameter $T_D/K_B x_0$ is presented graphically in Figs.8.4-5-6. These figures compare closely with those calculated for a force limited damper of linear distribution as presented in chapter 6, Figs. 6.4-5-6.

From Fig.8.4. it is seen that the body motion may become very large at resonance for all values of damping parameter less than 1, so there is no control of the resonant body amplitude by damping unless the damper develops a torque equal to that necessary to statically deflect the suspension spring through the excitation amplitude. The greatest difference between the calculated and experimental results for this damping magnitude occurs at the resonance where a linear distribution may theoretically allow any

vibration amplitude, whereas with the "constant" form the magnifier does not exceed 1.7. This difference is caused by the slightly greater energy dissipation for the same maximum torque developed which is possessed by the "constant" damping form.
For parameters in excess of unity, the body shows no tendency to resonate at its true natural frequency, the maximum dynamic magnifiers occuring at higher frequencies as the damping increases. This is caused by the increasing tendency for the body and axle to become locked togetherby damping and vibrate as a unit on the undamped tyre spring, which presents a higher resonant frequency. The experimental results differ quite widely from those calculated in the regions of locked axle and body because of the change in characteristic of the magnetic liquid damper having small relative motion between its elements as described in Appendix 2, and consequently the build up of body magnifier with high values of damping is less pronounced in the experimental than in the calculated responses.

Similar effects are noticeable in the motions of the axle mass illustrated in Fig.8.5. Variations in the axle magnifier at body resonance are small provided damping is sufficient to limit the body motions to finite values, but it is necessary to employ far higher damping values at the axle resonance in order to limit the axle magnifier to a finite value. The experimental results indicate that finite axle motions can only be achieved if the damping parameter $T_D/K_B x_O$ exceeds 20. This agrees quite well with the calculated results of chapter 6, however, the experimental results indicate higher values of the axle magnifier for the damping values exceeding 20

This may again be attributed to the change in work diagram shape of the experimental damper when the relative motion between the damper elements becomes small. This reduces the tendency for the axle and body to become locked together, even though high damping forces are developed, and consequently the experimental results do not indicate a tendency for the system to behave as a one mass system near the axle resonance, while this is a feature of the calculated response for high values of the damping parameter.

The torque transmission curve of Fig.8.6. agrees very closely with the calculated Fig.6.6., indicating infinite transmission at the body and axle resonances if the damping parameters are less than 1 or 20 respectively. Between the resonances the "constant" experimental form gives values in excess of the calculated values for a linear distribution. This results from the existence of maximum damping effort at the extremes of the relative motions in the "constant" form. The maximum transmission is thus the arithmetic sum of the maximum spring and damper torques. However, with a linear distribution of damping effort in the vibration cycle, the maximum spring and damper forces are out of phase by a quarter cycle, and the vector sum is little affected by spring effort under conditions of high damping parameter.

8.1.2-2. <u>TRANSIENT RESPONSE WITH FIXED DAMPER EFFORT.</u>

The transient response to a step in road profile was obtained on the mechanical analogue with constant current excitation of the suspension damper. The results of these tests are given graphically in Fig.8.7., and this shows that although the response is very, is very similar to that of a linearly damped system for low values of the damping parameter, there is a limit to the magnitude of damping effort which can be used if the body is to return to a true zero. Thus as $T_D/K_Bx_0$ increases above 1, the decay time of the body and axle motions tend to increase above the minimum obtainable value. This is caused by the rapid decay of axle motions of high frequency with greater damping values, which eliminates the axle frequency before the body first reaches its true zero position. Consequently the damping forces which are high in comparison with the suspension spring forces which exist at this time, cause the body and axle to become locked together, and the two masses oscillate as a unit on the undamped tyre spring with energy equal to that of the body at the time of locking. Thus the decay time of the axle increases, and that of the body becomes infinite as it never regains its true zero position relative to the axle. As a result of this limitation, the axle overshoot cannot be reduced to less than 84% with "constant" damping which persists throughout the duration of the transient.

On the other hand axle and body overshoots continue to decrease for far greater values of the damping parameter, so that if this form of damping is used, there must be a compromise between the overshoot and the static position error which high damping will introduce.

Furthermore, the transmission is seen to be a minimum for zero damping, as was the case with linear damping, so once again, any reduction in the overshoot can only be achieved at the expense of increased transmission to the body.

When compared with the response of a linear damper given in Fig. 5.5., it is seen that for a given decay time of the body motion, the transmission is far less, but the initial overshoots are rather more with the "constant" form of fixed effort, and this is recognised to be a feature of the arithmetic decay available with the fixed damper effort.

Although these transient tests indicate that the "constant" damper form of fixed effort is not entirely suitable for use during the whole period of the transient, it is still feasible that it might be used to advantage in the early stages of the transient to rapidly damp out axle motions, and hence reduce the likelihood of wheelhop. In this respect is possible to employ a recorded value of axle acceleration as an indication of the need for high damping effort in the initial stages of the transient as suggested in the calculated response of section 6.5.5. for the control of resonant axle motions.

With this in view, the transient response of the analogue was obtained with high magnitudes of damping effort during the first axle overshoot, and these results are plotted in Figs.8.8 and 8.9. in a manner to illustrate the relationship between the initial recorded acceleration of the axle, and the magnitude of damper effort of "constant" form required to just prevent an axle overshoot equal to the equivalent static tyre deflection $x_{Ast}$. Also plotted on these graphs is the performance of a linear damper achieving the same purpose, as determined from the results of Chapter 5 using the calibrated analogue parameters.

These graphs show that the axle acceleration gives a satisfactory indication of the damping effort required to just prevent excessive

axle overshoot in response to a step in the road profile, and furthermore they indicate that the constant damping form does actually produce slightly smaller total transmission to the body in achieving this than does the linear damper, provided the transient road step does not exceed 4 times the static tyre deflection.
However, as the magnitude of the road step increases above this value the "constant" damping form loses the ability to keep the axle overshoot below the required value, and at the same time causes greater transmission than in the linear case. These effects may be easily explained, as for low damping efforts the body remains almost motionless during the first axle overshoot due to the great difference in the natural frequencies of the two masses. Thus, as the "constant" damping form provides the greatest energy dissipation for a given maximum developed effort and amplitude of motion, it is slightly more effective in attenuating axle overshoot than is the linear damper. However, as the damping magnitude increases, the axle and body tend more and more to be locked together with the "constant" form, and oscillate as a unit on the undamped tyre spring instead of dissipating the bump energy.

Thus there is a limit to the magnitude of road step, below which the "constant" form of damper effort can be used to eliminate axle overshoot by developing a resistance proportional to the initial recorded axle acceleration, but it is felt that this magnitude is more than adequate for a practical suspension, bearing in mind that both tyre curvature and damping are ignored in the analogue results.

## 8.1.2-3. HARMONIC RESPONSE WITH MAXIMUM DAMPER EFFORT PROPORTIONAL TO MAXIMUM SPRING TORQUE DEVELOPED.

The harmonic response of the mechanical analogue with "constant" damping form of magnitude proportional to the maximum spring torque developed in the cycle is presented graphically in Figs. 8.10 8.11 and 8.12. These figures are very similar to those given in section 6, for a damper of linear distribution with parameter $F_D/F_S$. From fig.8.10, it is seen that only a small value of the parameter is required to greatly reduce the resonant motions of the body mass, but as the damping increases, there is a marked tendency for the body motions to reach a peak amplitude at a higher frequency than the true body resonance, so that overdamping is definitely undesirable. Furthermore, the damping parameter required to limit axle resonance to a reasonable value greatly exceeds that necessary to obtain minimum resonant body motions, so that control of axle motion can only be achieved at the expense of overdamping the body and thus introducing the intermediate quasi-resonance with the axle and body oscillating as a unit on the tyre spring.

Thetotal transmission to the body at frequencies remote from the resonances is also increased for all values of the damping parameter employed, and in all cases the transmission exceeds that of the damper with linear form, as it represents an arithmetic summation of the spring and damper contributions, instead of a vector sum as obtained with the linear distribution.

8.1.3. RESPONSE OF THE SYSTEM WITH "RETURN" DAMPING FORM.

To complete the results on the effect of damper diagram shape on the performance, the response of the analogue was obtained under conditions of "return" damping form. Although the "return" damping used in section 8.1.1. was achieved simply by switching damper current on and off for quarter cycle periods in the appropriate phase relationships as indicated in that section, it was noticed that the improvements in performance which were obtained by using "return" damping depended largely on the magnitude of the maximum damper torque developed. It was realized that if the maximum damper torque were available at the instant of motion reversal, instead of at the mean position as with the simple switched form, then approximately twice the magnitude of damper torque could be employed without increase in the transmission, as such torques would be concentrated in the region where spring and damper torques oppose. In addition to this, the damper diagram could then be further tailored to give a greater energy dissipation per cycle without increasing transmission, by reducing damper torque to zero at the other extreme of motion along a line parallel to the spring characteristic. This ideal "return" damper diagram is illustrated in Fig.8.13(a).

In attempting to produce this diagram shape, the simple on-off rotary switch was replaced by a specially formed resistor, which was wired to control the grid bias of a power output valve that had the damper coil in its plate circuit. By driving the rheostat wiper at twice the excitation speed, the damper was then switched on and turned off gradually throughout the following half cycle. Then by adjusting the phase of the swept resistor relative to the input motion by the worm adjustment, the position of switch-on could be made to coincide with the point at which the motion started to return to the relative zero and thus "return" damping form was achieved. Once again the visual observation of the existing damper work diagram provided on the monitored C.R.O. facilitated accurate adjustment of the phase control.

It was found under test that the maximum damper torque could not be developed at the instant of reversal of motion because of the time lags in the electrical and magnetic circuits, and also because the damper required some definite motion to achieve a peak value as explained in Appendix.2. Consequently the ideal "return" form was not achieved, there being some rounding of the characteristic after change in the direction of motion, which caused the maximum effort to be developed at approximately quarter stroke as illustrated in Fig.8.13(b), but this was nevertheless accepted as the standard form for this series of experiments.

8.1.3-1 HARMONIC RESPONSE WITH FIXED DAMPER EFFORT.

The harmonic response of the analogue with fixed damper effort of "return" form is presented in Figs.8.14-15-16. Once again these are very similar to both the calculated response for linear distribution, and the analogue results for "constant" distribution presented earlier.

In this case however, it is noticeable that no control of body motion is achieved unless the parameter $T_D/K_B x_O$ approaches 1.5, whereas axle resonance requires a parameter of at least 25 to produce finite motions. Also, if the damping parameter exceeds 2, the body and axle exhibit the quasi-resonance at frequencies above the true body frequency caused by the tendency for axle and body to lock together and oscillate on the tyre spring. Furthermore with high values of damping, the transmission greatly exceeds that of the undamped case, but for the same value of parameter, it is less than that for either the linear or the constant form, as is to be expected from the opposition of spring and damper efforts with the "return" form.

The most important feature of these responses is the behaviour of the body for values of the damper parameter less than 2. Thus, in Fig.8.14, it is seen that the body motion is reduced below that of the undamped case for a considerable speed range above the true body resonance for damping parameters in this range, and Fig.8.16. shows that that the transmission for $T_D/K_B x_0 = 1$ is less than that for zero damping throughout the entire frequency spectrum.

Thus it is obvious that the "return" form of damping is very advantageous for controlling the body resonance, as it can completely eliminate any dynamic magnification of the body, and at the same time presents the minimum of transmission at frequencies above the resonance.

8.1.3-2 HARMONIC RESPONSE WITH MAXIMUM DAMPER EFFORT PROPORTIONAL TO THE MAXIMUM SPRING TORQUE DEVELOPED.

The harmonic response of the analogue with "return" damping form of maximum effort proportional to the maximum spring torque developed in the cycle is given in Figs.8.17-18-19.
These figures are again similar to those for linear and "constant" distribution presented earlier, showing that any magnitude of the parameter $T_D/T_S$ is satisfactory in limiting resonant axle and body

motions to finite values, but in general slightly greater damping effort is required with return form as it does possess smaller energy dissipation for the same parameter. Furthermore only low values of the parameter are required to completely eliminate body resonance, but far higher values are necessary to reduce axle motions to a satisfactory level, and these tend to cause increased body and axle motions at frequencies between the two resonances. However, this latter tendency is greatly reduced from that of the linear and "constant" forms, as the body and axle do not lock together to the same extent.

As for the previous section, the most important feature of these figures is the behaviour of the body at low values of the damping parameter. Thus it is seen that a value of $T_D/T_S = 1$ gives adequate control of the body resonance, and at the same time reduces body motions for frequencies above the resonance below those of the undamped case. Furthermore, the transmission with this damping magnitude is less than that for zero damping throughout the entire frequency spectrum.

Thus, this form and magnitude control of damping effort would seem to be the optimum for body control, as it provides adequate reduction of resonant body motions without causing increased transmission at any frequency in the range. Consequently, it would be the ideal form of basic damping on which the high damping effort necessary to control axle resonance could be superimposed when, and only when, it was essential as suggested in section 6.5.5.

## 8.1.3-3. TRANSIENT RESPONSE WITH MAXIMUM DAMPER EFFORT PROPORTIONAL TO THE MAXIMUM SPRING TORQUE DEVELOPED.

The transient response of the analogue to a step in the road profile was obtained with a "return" damping form of maximum effort proportional to spring torque by the method described in section 7.3. As the mechanism providing the damper current was driven by the resultant motions of the analogue masses, the shape of the work diagram was rather different from that used in the harmonic response tests, but it was considered that the further complexity involved in reproducing similar diagrams for the transient tests was not justified. The equivalent work diagram of the "return" damping form used in these transient tests is given in Fig.8.13(c).

Also, as the harmonic response tests indicated that the "return" damping form was particularly suited to the control of the body resonance, two series of transient tests were performed, the first being for the complete system, and the second for a step in the axle position which thus constituted a body-only transient. The purpose of this was to separate the axle and body motions, in order to assess the performance of a system having axle acceleration control of high damping effort as suggested earlier to reduce wheelhop in the early stages of the transient.

(a) TOTAL SYSTEM TRANSIENT.

The transient response of the analogue to a step in the road profile, with "return damping form of magnitude proportional to the maximum spring torque developed, is presented in

Fig.8.20.

The most important feature of these results is that the initial axle overshoot is in no way attenuated by the "return" damping form. This must of course occur as the axle reaches its first overshoot before the body motion becomes perceptible due to the great difference in the natural frequencies of the two masses. Consequently, as this entire motion is in the realm where the damper elements are separating from their static equilibrium position, the "return" damping form provides the minimum resistance, and the axle is virtually undamped. As a result of this, the transmission to the body is a minimum being a function only of the suspension spring deflection, however, this is only true for values of the damping parameter less than 1.25 which is the extent of the experimental values. If higher parameters were employed, the peak transmission could occur during the rebound of the axle from its first overshoot, when the high damping forces introduced could exceed this spring force. The use of such high damping parameters of "return" damping would however be pointless, as they would still allow the maximum first axle overshoot, and would later seriously overdamp the body.

Another feature of the response is that high values of the parameter can cause the body to be permanently deflected from its true zero position at the completion of the transient, but if the optimum value suggested by the harmonic response is employed. the minimum decay time of the body is achieved.

(b) BODY TRANSIENT.

The transient response of the body to a step in the axle position with "return" damping form of magnitude proportional to the maximum spring torque developed, is given graphically in Fig.8.21. This shows that the motion of the body can be dead-beat by a damping parameter $T_D/T_S = 0.75$ and for higher values of damping the body suffers a steady-state position error. Furthermore, this damping parameter gives the minimum decay time that can be achieved.

The value 0.75 arises from the fact that the "return" damping form actually achieved on the analogue developed its maximum effort at approximately $\frac{1}{4}$ stroke, by which time the spring effort had decreased to 75% of its maximum value being a linear spring. Consequently the damper and spring efforts were almost identical over the last $\frac{3}{4}$ stroke, and the damper dissipated just sufficient energy to overcome the kinetic energy that the mass developed in the first $\frac{1}{4}$ stroke, and hence the motion became dead-beat.

Fig.8.21 also shows that the maximum transmission to the body is unaffected by the degree of damping, being simply the initial set-up force in the suspension spring.

Thus it is obvious that the "return" damping form is very satisfactory for damping body transients, but if the parameter $T_D/T_S = 1$ , as suggested by the harmonic response is employed, it is likely that the body will be slightly overdamped, though any axle motions which would accompany such a transient would tend to eliminate position errors.

## 8.2. NON-LINEAR SPRINGING IN AN UNDAMPED SYSTEM.

Non-linear spring characteristics could be introduced on the analogue by employing tension springs in combination with the basic helical torsion spring as described in section 7.2. By this means, hardening-hardening characteristics which were symmetrical about the same base rate as the linear suspension could be employed, but unsymmetric hardening-softening spring characteristics caused an increase in the basic rate, and were thus not strictly comparable. Nevertheless, the performance of the analogue when fitted with such spring characteristics can be directly related to that of a suspension having the same form, and consequently, the results serve as a qualitative indication of dynamic performance with unsymmetric non-linear springs.

In order to simplify the experimental procedure during these tests, the suspension damper was maintained in a de-magnetized condition, so that the results apply essentially to undamped arrangements.

### 8.2.1. HARMONIC RESPONSE WITH SYMMETRIC HARDENING - HARDENING SPRING CHARACTERISTICS.

The spring characteristics $S_1$, $S_2$, and $S_3$ used in this series of tests are given graphically in Fig.8.22. This shows that each of the springs had the same basic rate as presented by the helical torsion spring, but the degree of non-linearity was made progressively more pronounced for each characteristic.

The harmonic responses of the analogue, to a particular amplitude of excitation, were obtained with each of these springs as a suspension characteristic, and these were compared with the linear response of the unit for the same conditions of the magnetic liquid damper. The results of this series of tests is given graphically in Fig.8.23., and this shows that as the degree of non-linearity of the suspension spring increases, the amplitude and frequency of the maximum obtainable body motion increases rapidly, so that the overall performance of the isolating body spring becomes progressively worse. It is noticeable from Fig.8.23, that as the degree of non-linearity increases, the "jump-down" frequency increases very much more rapidly than the "jump-up" frequency , so the band of frequencies in which two separate levels of oscillation energy are stable rapidly broadens. It is known that the jump-up frequency is primarily dependent on the spring characteristic and the excitation amplitude, where as the jump-down frequency depends solely on the degree of damping present. Consequently, as the non-linearity of the suspension increases, the system becomes less dependent on the damping which exists.

Although Fig.8.23 only concerns the effect of non-linear springs on body motions, tests were conducted throughout the entire frequency spectrum, and it was found that the degrees of non-linearity available with $S_1$, $S_2$, and $S_3$ had negligible influence on the harmonic response of the axle. This is of course only to be expected, since the body spring rate is only about 5% as stiff as the tyre rate on the analogue, so throughout the remainder of tests on non-linear springs the harmonic responses were only recorded in the region of body resonance.

### 8.2.2. TRANSIENT RESPONSE WITH SYMMETRIC HARDENING-HARDENING SPRING CHARACTERISTICS.

The transient response of the analogue to a range of values for steps in the road profile is given graphically in Fig.8.24. for each of the spring characteristics $S_1$, $S_2$ and $S_3$ of Fig.8.22. This shows that the body overshoot is not affected by the non-linearity, but remains equal to the magnitude of the exciting step, whereas the percentage axle overshoot decreases with increasing non-linearity and increasing magnitude of step. Nevertheless, the actual reduction in overshoot of the axle by spring non-linearity in the range tested is small, and is only achieved at the expense of increased transmission to the body above that for the linear system. Furthermore when compared with the effect of damping on the axle overshoot, and the resultant increase in transmission, it is obvious that spring non-linearity causes far greater increase in transmission for the same reduction in overshoot, and hence cannot be regarded as a satisfactory method of preventing axle motions from developing to a state of wheel-hop.

### 8.2.3. HARMONIC RESPONSE WITH UNSYMMETRIC HARDENING-SOFTENING SPRING CHARACTERISTICS.

The spring characteristics employed on the analogue for investigation of the harmonic response of unsymmetric springs are given graphically in Figs.8.25 and 8.26. Unfortunately these characteristics can only be achieved on the analogue by pre-loading the non-linearizing springs against the basic helical torsion spring as explained in section 7.2. Consequently, variation of the degree of non-linearity automatically changes the basic spring rate at the equilibrium position, and this can only be adjusted to the original linear rate by changing the basic helical spring. This was not considered to be warranted in these investigations as the degree of non-linearity available was not sufficient to influence the axle motions, so tests were restricted to the region of body resonance where the system behaved effectively as a one mass system, and a change of basic rate represented only a change in the basic frequency. Figs.8.25 and 8.26, show that the characteristics employed had different degrees of non-linearity, and also different basic rates. Characteristic $U_1$ had a spring rate variation proportional to the deflection from static zero about a base rate of 10 lb.in per RAD., while characteristic $U_2$ had a spring rate variation proportional to the applied torque about a basic rate of 12.5 lb.in per RAD. The characteristic $U_2$ was thus similar to the "constant" frequency springs employed in some commercial vehicles and described in Appendix 6, for maintaining a constant ride frequency independent of load.

The harmonic responses of the analogue with these spring characteristics and demagnetized dampers are given in Figs 8.32, and 8.33 for a particular amplitude of excitation.

These results indicate that the system possesses a softening non-linear dynamic characteristic for both the spring forms used. Thus as the frequency of excitation is increased from a low value, the body motion suddenly increases to a high value as the "jump-up" frequency is exceeded, while very much higher motions are available at lower frequencies if the system is gradually slowed. It is thus obvious that with unsymmetric spring characteristics, the softening region has the greatest influence in determining the harmonic response of the system.

### 8.2.4. TRANSIENT RESPONSE WITH UNSYMMETRIC HARDENING-SOFTENING SPRING CHARACTERISTICS.

The transient response of the analogue to steps of various magnitudes in the road profile is given in Fig.8.27. The spring employed for the suspension spring was $U_2$ of Fig.8.26, this being a constant natural frequency spring type.

As the characteristic was not symmetrical for deflections on either side of the static equilibrium position, it was necessary to perform the transient tests for both positive and negative steps in the road profile to determine the overall response.

Fig.8.27 shows that for positive steps, the percentage overshoot of the axle decreases with increasing step size, while the body overshoot increases to values far in excess of the step initiating the transient, while for negative steps the reverse is true with axle overshoot increasing, and body overshoot decreasing for increasing magnitude of step. The reason for this apparent anomaly is that when traversing a positive step, the initial axle overshoot, which occurs before the body motion becomes appreciable, causes a deflection of the suspension spring in the hardening region, and hence the overshoot is smaller than that developed for a linear spring. At the same time, the body transient which is in fact initiated by the first axle overshoot commences with the spring deflected in the hardening region, and consequently in its overshoot the body must deflect the suspension spring further in the softening region to absorb the same energy. Thus the overshoot of the body exceeds the magnitude of the step causing the transient.

Conversely, when traversing a negative bump, the opposite conditions apply, and the body overshoot is reduced, while the initial axle motion causes increased deformation of the suspension spring in the softening region.

Fig.8.27 also shows that the transmission to the body increases with the magnitude of positive bumps, but decreases with the magnitude of negative bumps, so although the actual deflection of the suspension spring is less at the first axle overshoot in response to a positive step than in response to a similar negative step, this is more than outweighed by the change in spring rate involved in the two cases. Thus, as the positive step is the transient excitation of greatest importance in the actual suspension, it follows that the constant natural frequency spring does give rather worse performance than a linear spring of the same basic rate. Consequently, if such a spring characteristic is used to reduce load dependence of the suspension, it is essential that some form of damping be introduced to restrict body motions.

8.3. NON-LINEAR DAMPING IN A NON-LINEARLY SPRUNG SYSTEM.

Originally it was planned to investigate the effects of "constant and "return" damping forms on the response of the system fitted with both symmetric hardening-hardening and unsymmetric hardening-softening spring characteristics. It was proposed to achieve these responses in a similar manner to that used with the linearly sprung system, as described in section 8.1. Unfortunately however, this procedure proved to be unsatisfactory for the cases with non-linear springs as it was found that when operating in the frequency range between the "jump-up" and "jump-down" frequencies, the magnitude of the damper effort was critical in determining the stability of the high energy level oscillation. Consequently, as frequency or damping effort was varied with operation in this range, it was not possible to maintain the desired damper diagram shape, and whenever a change from high to low energy level oscillation did occur, it was not known whether this was a true "jump" value associated with the desired diagram shape, or an effect caused by the transient relaxation of the diagram shape. Also the presence of the non-linearising spring made it impossible to use the friction switch mechanism shown in Fig.7.7. which was originally devised for transient tests with "return" damper effort generated by the analogue mass motions in the linearly sprung system. Thus the tests were restricted to the "constant" damping form which did not require to be manually maintained.

Nevertheless, in view of these results, and the comprehensive series of tests on non-linear damping in linearly sprung systems, it is felt that a direct extrapolation is satisfactory to indicate qualitatively the performance of a non-linear spring system with "return" damping form.

8.3.1. HARMONIC RESPONSE OF THE SYMMETRIC HARDENING-HARDENING SPRING SYSTEM WITH FIXED DAMPER EFFORT OF "CONSTANT" FORM.

In this series of tests, the response of the analogue was detailed only in the region of body resonance, as the undamped spring tests indicated that the magnitudes of spring non-linearity employed had no significant effect on the axle resonance.

The spring characteristic chosen for the tests was $S_2$ of Fig. 8.22., and the harmonic response of the system was obtained for four different values of the excitation amplitude, so that the overall performance could be evaluated.

The results of these tests are given graphically in Figs.8.28-29-30-31. All of these figures are very similar, and they indicate that as the magnitude of damping effort increases, so both the "jump-up" and "jump-down" frequencies decrease, but the latter changes at a greater rate than the former. Consequently a value of damping effort is finally reached where the two frequencies become coincident, and the system loses the characteristic band where two oscillation energy levels are stable. The critical magnitude of damping effort required to achieve this is given by the parameter $T_D/K_B x_0 = 1$, and this is seen to be exactly the same as the damping required to produce finite body motions in a linear suspension of the same basic rate as investigated in section 8.1.2. Furthermore, although the experiments indicated that the "jump" frequencies increased with the excitation amplitude for a particular magnitude of damping parameter $T_D/K_S x_0$, the system always exhibited the characteristic that for the parameter = 1 the two frequencies coincided, and the oscillation amplitudes were unique throughout the entire frequency spectrum.

However, even with the critical magnitudes of damper effort, the magnitudes of body motions far exceeded those obtained with a linear suspension spring. Thus, greater damping effort was required in the non-linear system to control the maximum body amplitudes to a reasonable level, and consequently at frequencies above those of test, the transmission to the body would be greater than that for a linear spring system.

8.3.2. HARMONIC RESPONSE OF THE UNSYMMETRIC HARDENING-SOFTENING SPRING SYSTEM WITH FIXED DAMPER EFFORT OF "CONSTANT" FORM.

The harmonic response of the analogue in the region of body resonance, when fitted with the unsymmetric hardening-softening springs $U_1$ and $U_2$ of Figs.8.25-26 as a suspension, and subject to damping of "constant" form and fixed effort is presented graphically in Figs.8.32-33.

These figures are quite similar in form, but they are displaced relative to each other along the excitation frequency axis due to the difference in basic rates at the equilibrium position of the characteristics $U_1$ and $U_2$ as mentioned earlier.
In this case, the effect of damping is to reduce the body motions and at the same time to increase the "jump-down" frequency at a far greater rate than the "jump-up" frequency. Thus as the damping increases, the frequency band over which both high and low

energy level oscillations can exist becomes progressively smaller, till eventually the two critical frequencies coincide, and the body motions become unique throughout the range.

As in the previous section, the critical value of damping parameter required to just eliminate this band of high level oscillation energies, is $T_D/K_B x_0 = 1$, which is also the damping parameter required in a linear spring system of the same basic rate to produce finite body motions at resonance. This is seen to apply in both cases of spring $U_1$ and $U_2$, even though they are concerned with different values of the basic spring rate $K_B$, so the parameter is universally applicable for springs of this form.

When compared with the responses for linear springs, Figs8.32-33 also show that the dynamic magnifiers of the body achieved with the critical damping parameter are far greater, and occur at a higher frequency relative to the same base, if the spring form is non-linear. Thus in an actual suspension fitted with this spring characteristic it would be necessary to employ higher damping effort to restrict body motions to a satisfactory level, than is required with a linear spring, so that transmission at higher frequencies would undoubtedly increase above the minimum obtainable value.

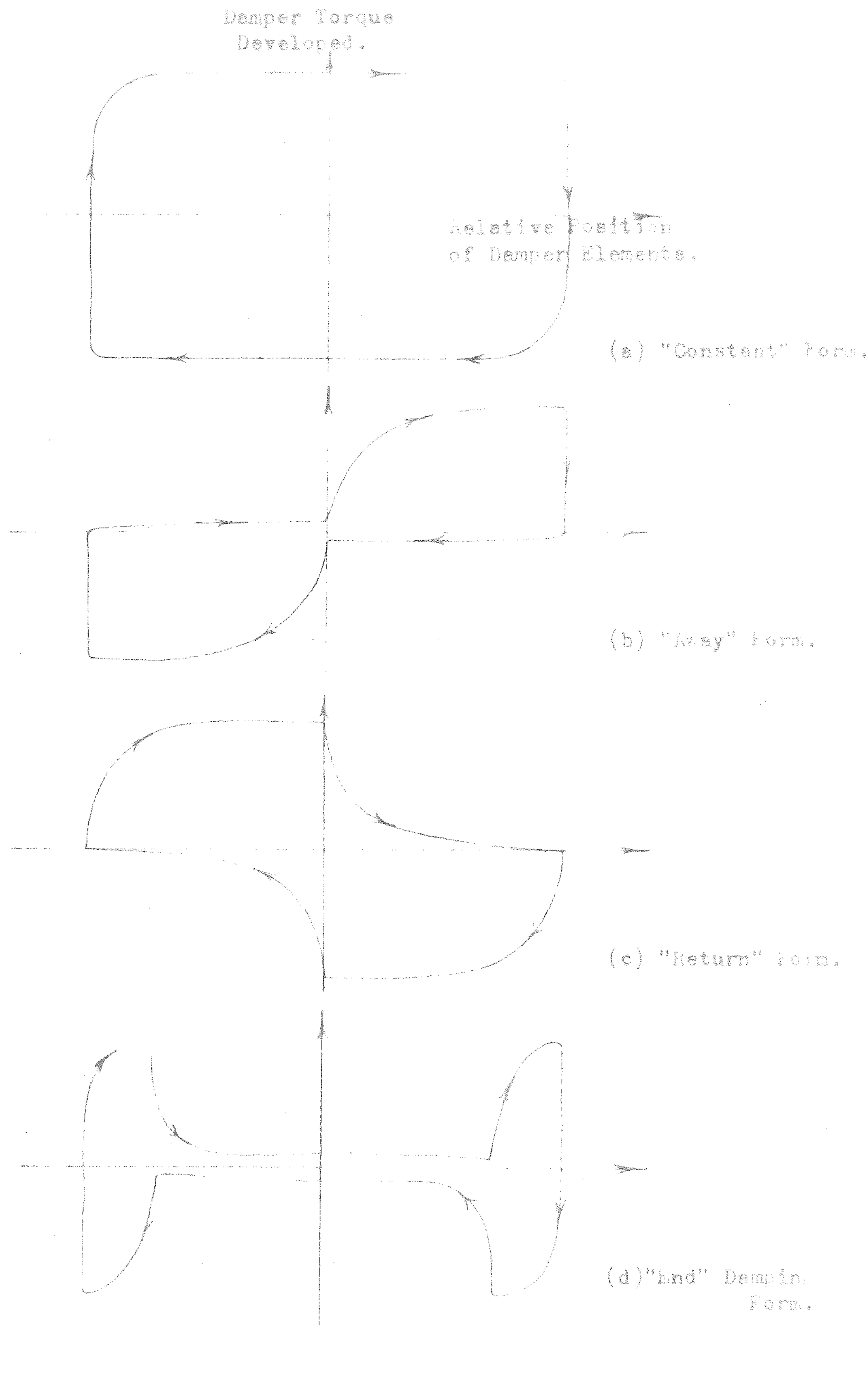



Fig.8.1. DAMPER DIAGRAM SHAPES OBTAINED DURING ANALOGUE TESTS
OF SECTION 8.1.1

Fig.8.2. EFFECT OF DAMPER DIAGRAM SHAPE ON THE HARMONIC RESPONSE OF THE SYSTEM AT FREQUENCIES REMOTE FROM RESONANCE.

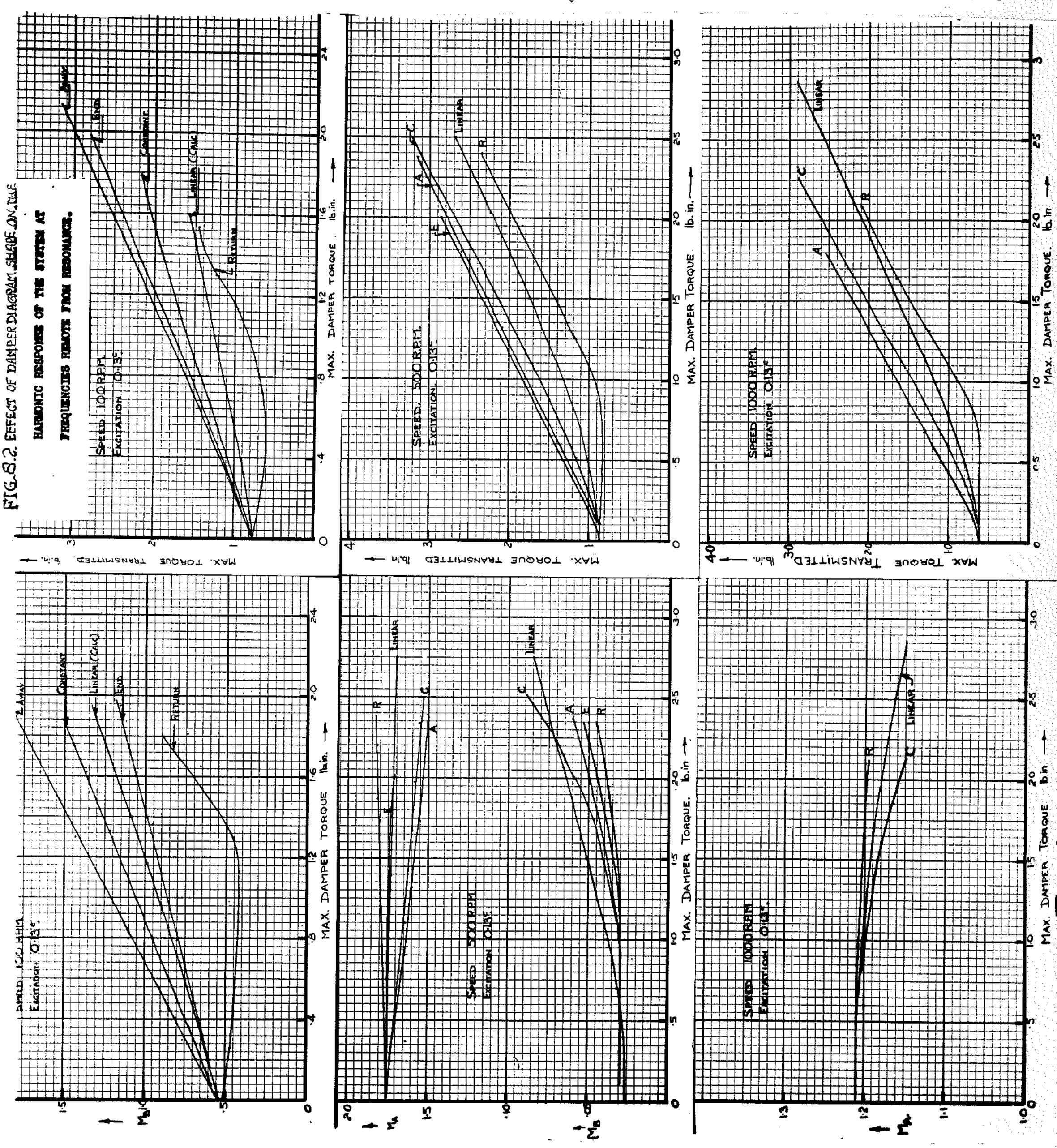
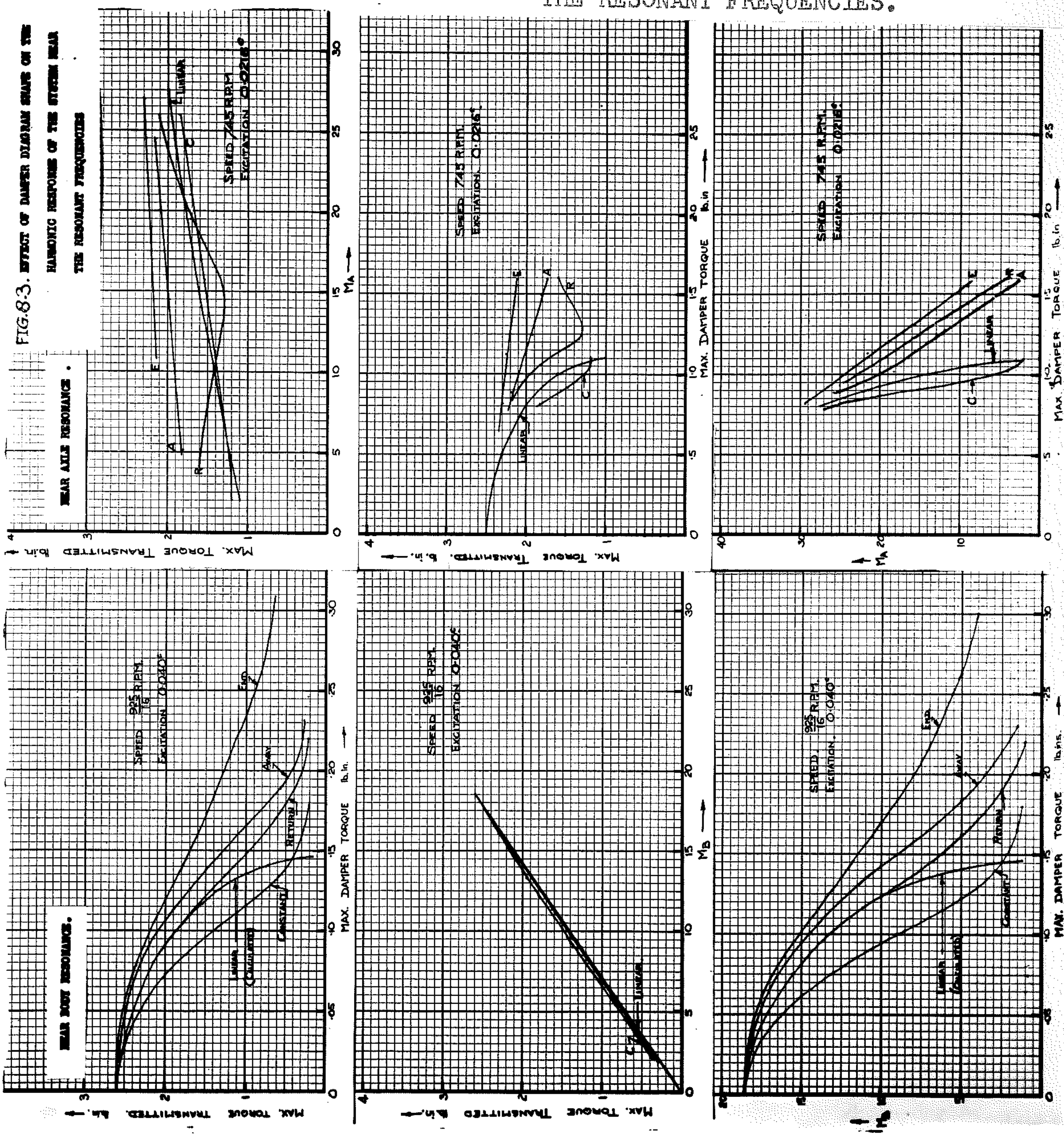


Fig.8.3. EFFECT OF DAMPER DIAGRAM SHAPE ON THE HARMONIC RESPONSE OF THE SYSTEM NEAR THE RESONANT FREQUENCIES.



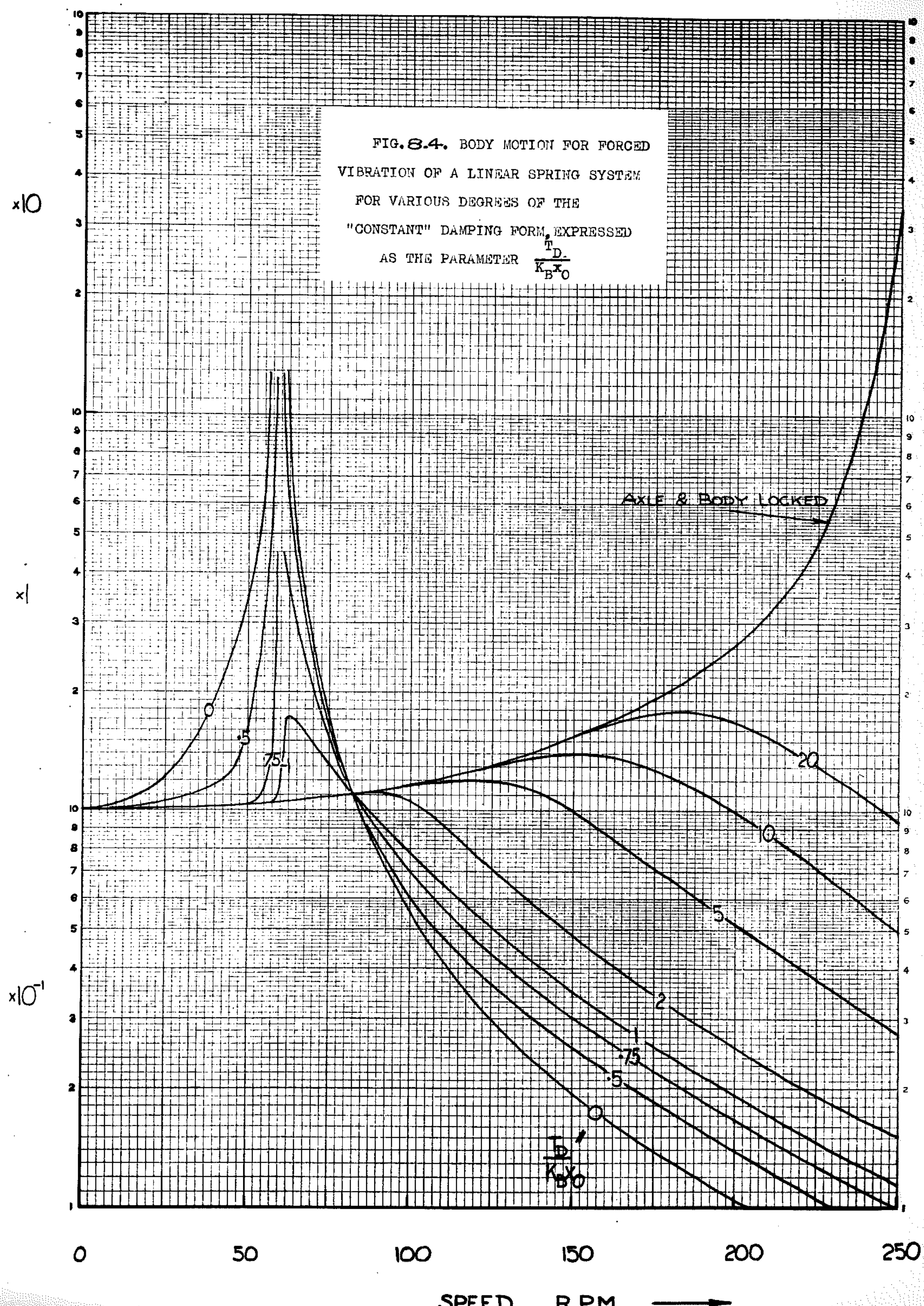


FIG. 8.4. BODY MOTION FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "CONSTANT" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{K_B x_0}$
$M_B$
x10
x1
x$10^{-1}$
AXLE & BODY LOCKED
0
·5
·75
1
2
5
10
20
$\frac{T_D}{K_B X_0}$
0
50
100
150
200
250
SPEED . R.P.M.

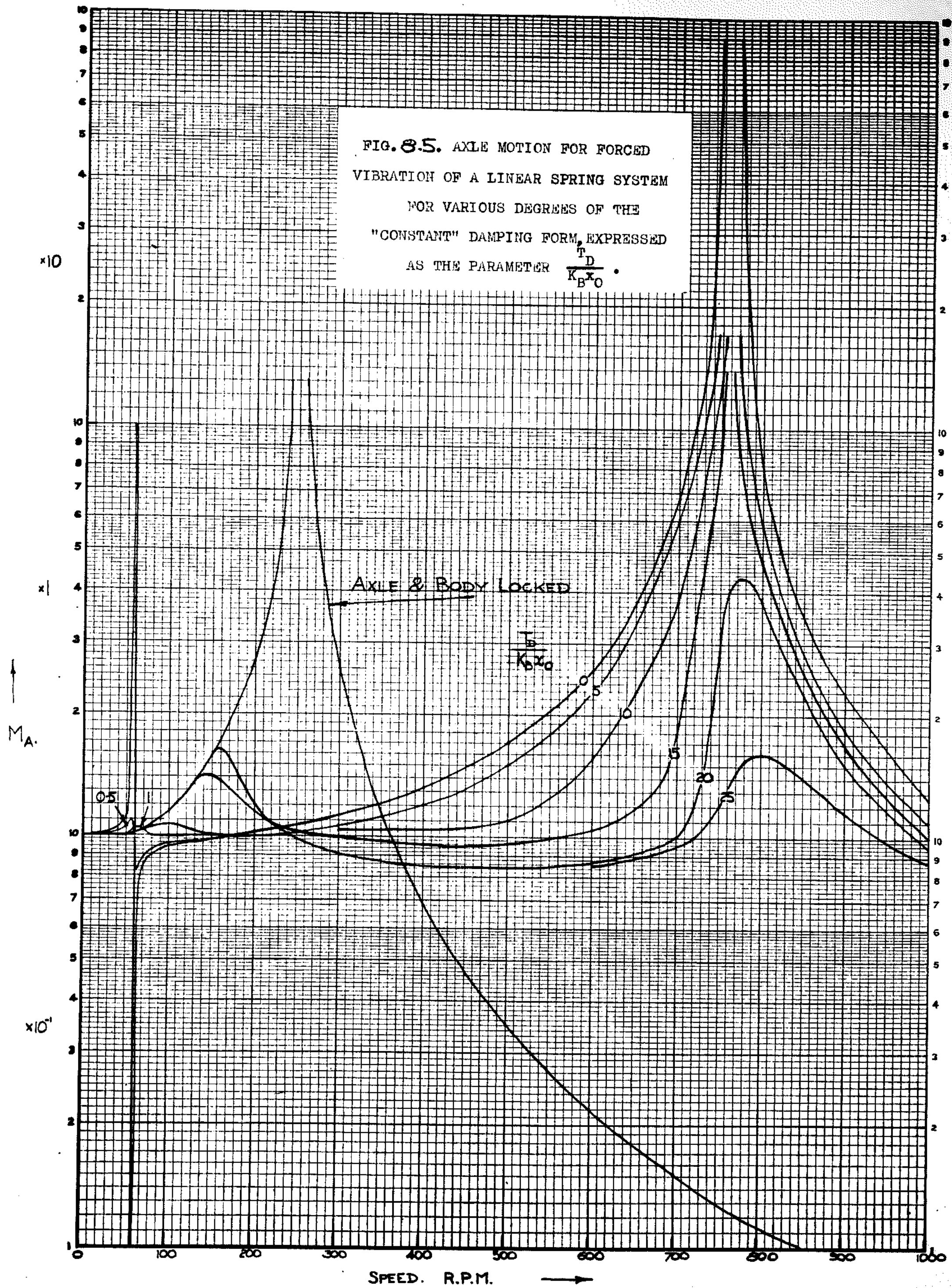



FIG. 8.5. AXLE MOTION FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "CONSTANT" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{K_B x_0}$.

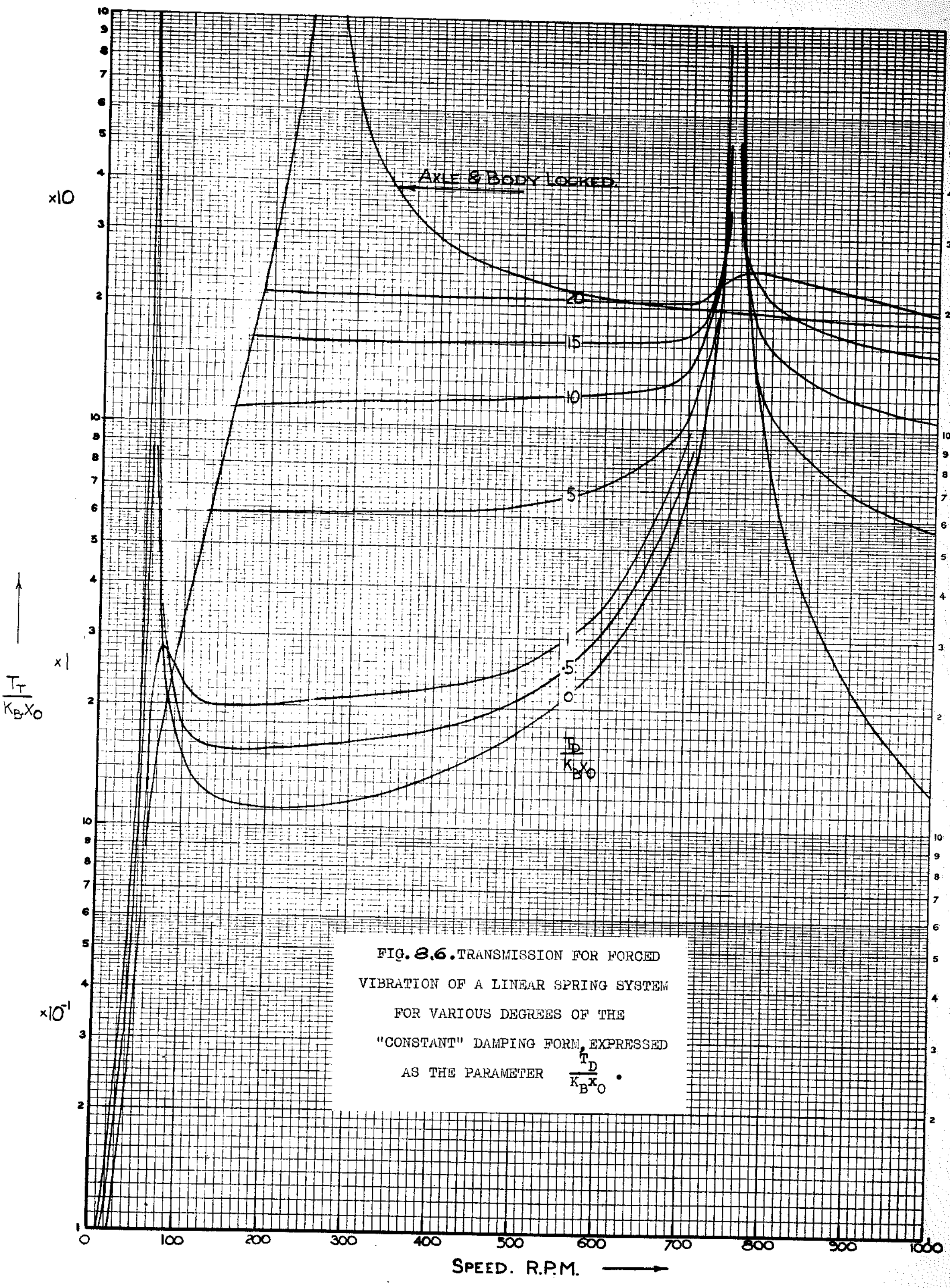
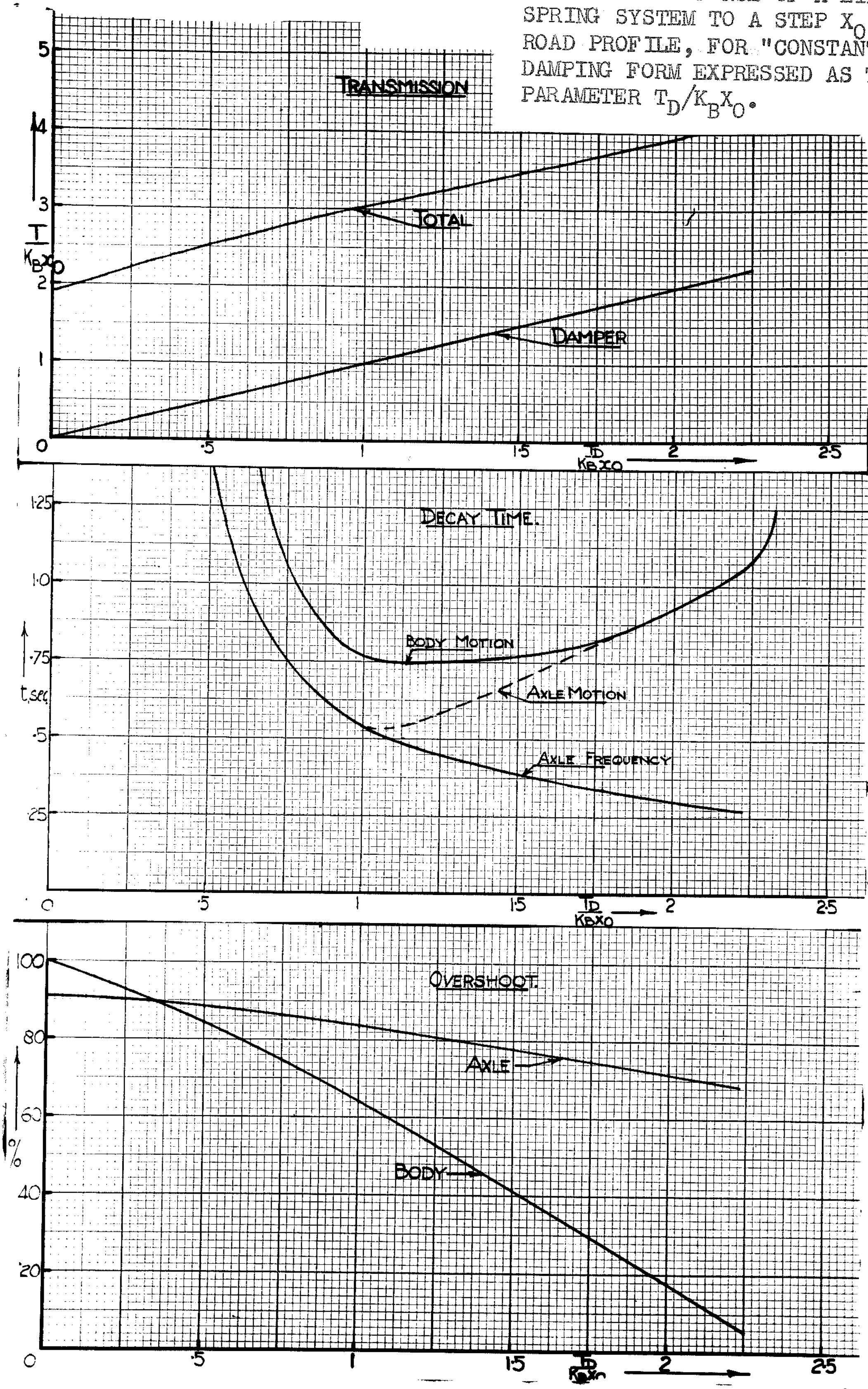



FIG. 8.6. TRANSMISSION FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "CONSTANT" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{K_B x_0}$.

Fig.8.7. TRANSIENT RESPONSE OF A LINEAR SPRING SYSTEM TO A STEP $X_0$ IN ROAD PROFILE, FOR "CONSTANT" DAMPING FORM EXPRESSED AS THE PARAMETER $T_D/K_BX_0$.



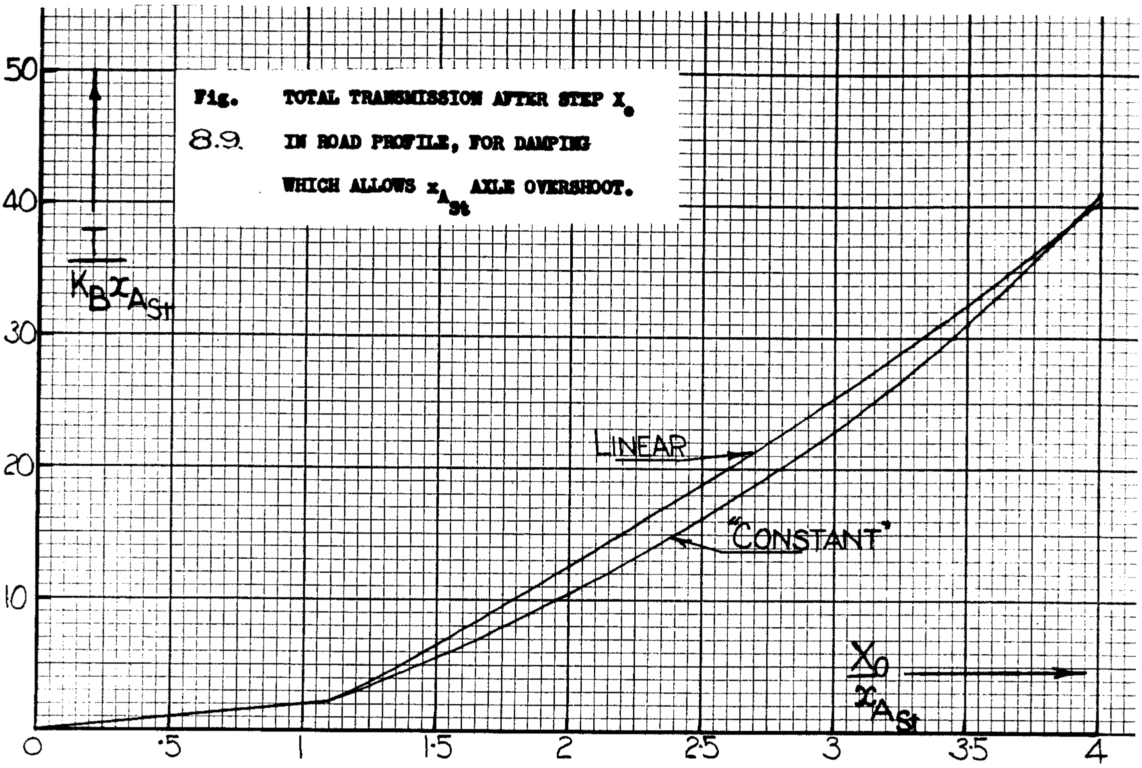



Fig. 8.9. TOTAL TRANSMISSION AFTER STEP $X_0$ IN ROAD PROFILE, FOR DAMPING WHICH ALLOWS $x_{A_{St}}$ AXLE OVERSHOOT.

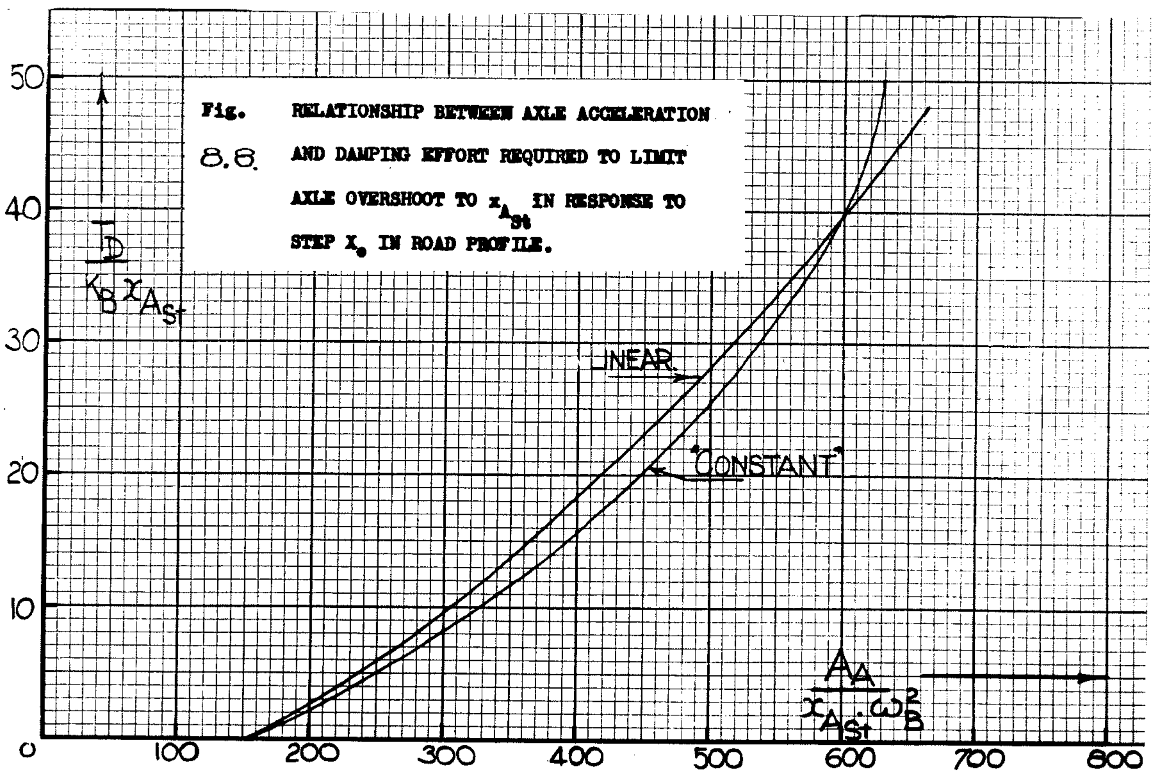



Fig. 8.8. RELATIONSHIP BETWEEN AXLE ACCELERATION AND DAMPING EFFORT REQUIRED TO LIMIT AXLE OVERSHOOT TO $x_{A_{St}}$ IN RESPONSE TO STEP $X_0$ IN ROAD PROFILE.

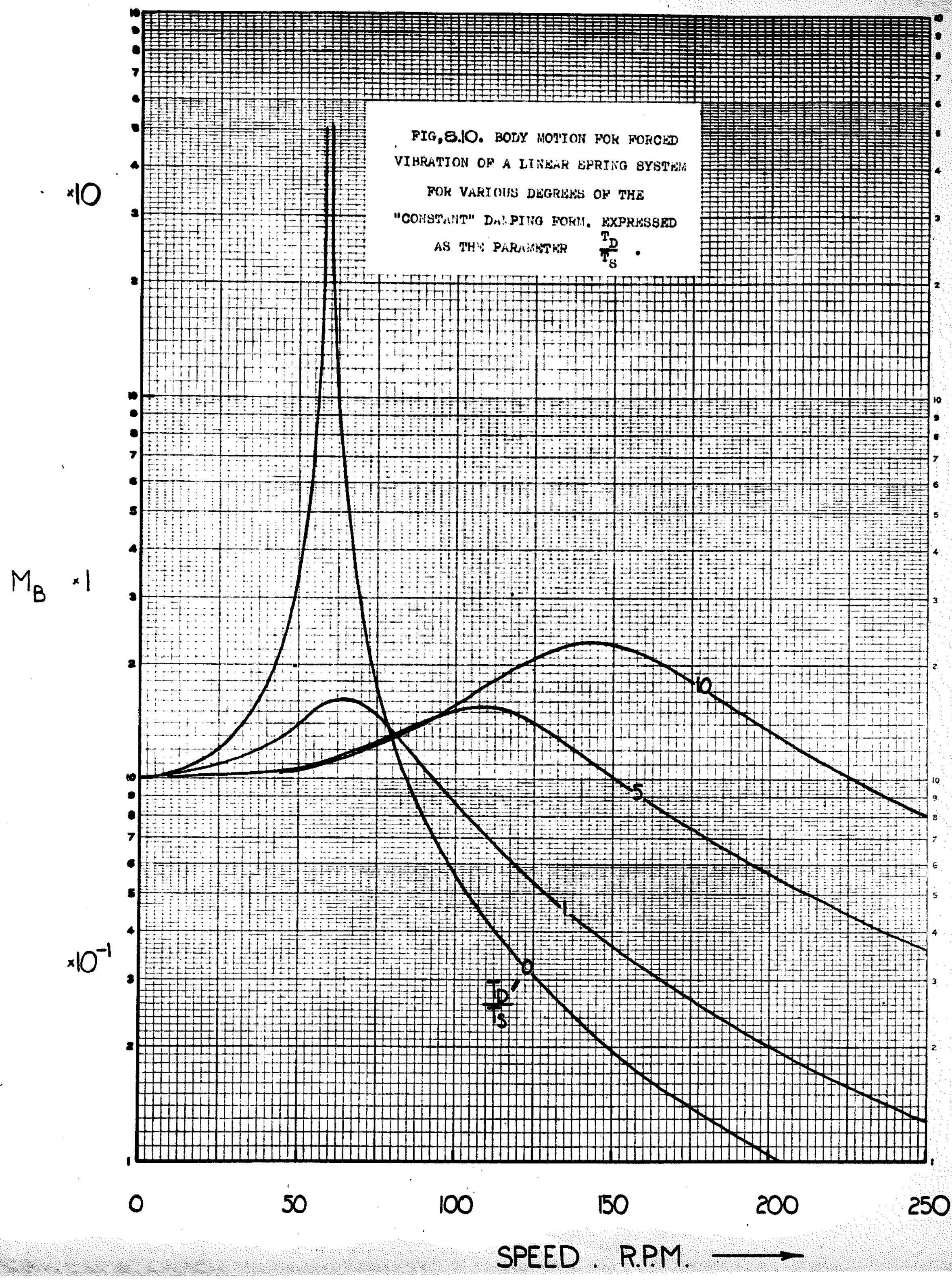



FIG. 8.10. BODY MOTION FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "CONSTANT" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{T_S}$.

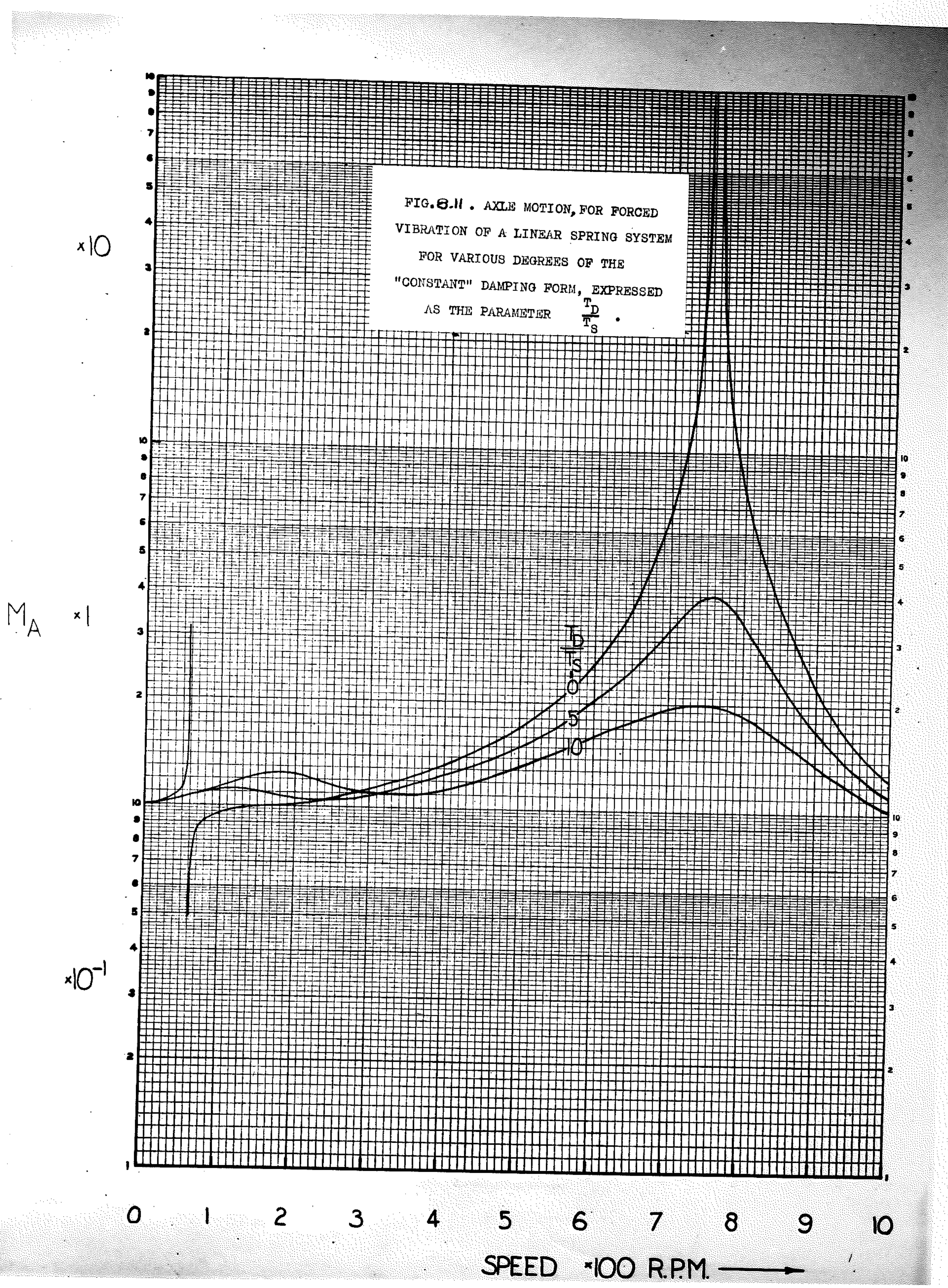


FIG. 8.11. AXLE MOTION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "CONSTANT" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{T_S}$.
$M_A$
×10
×1
$\times 10^{-1}$
$\frac{T_D}{T_S}$
0
5
10
0 1 2 3 4 5 6 7 8 9 10
SPEED ×100 R.P.M.

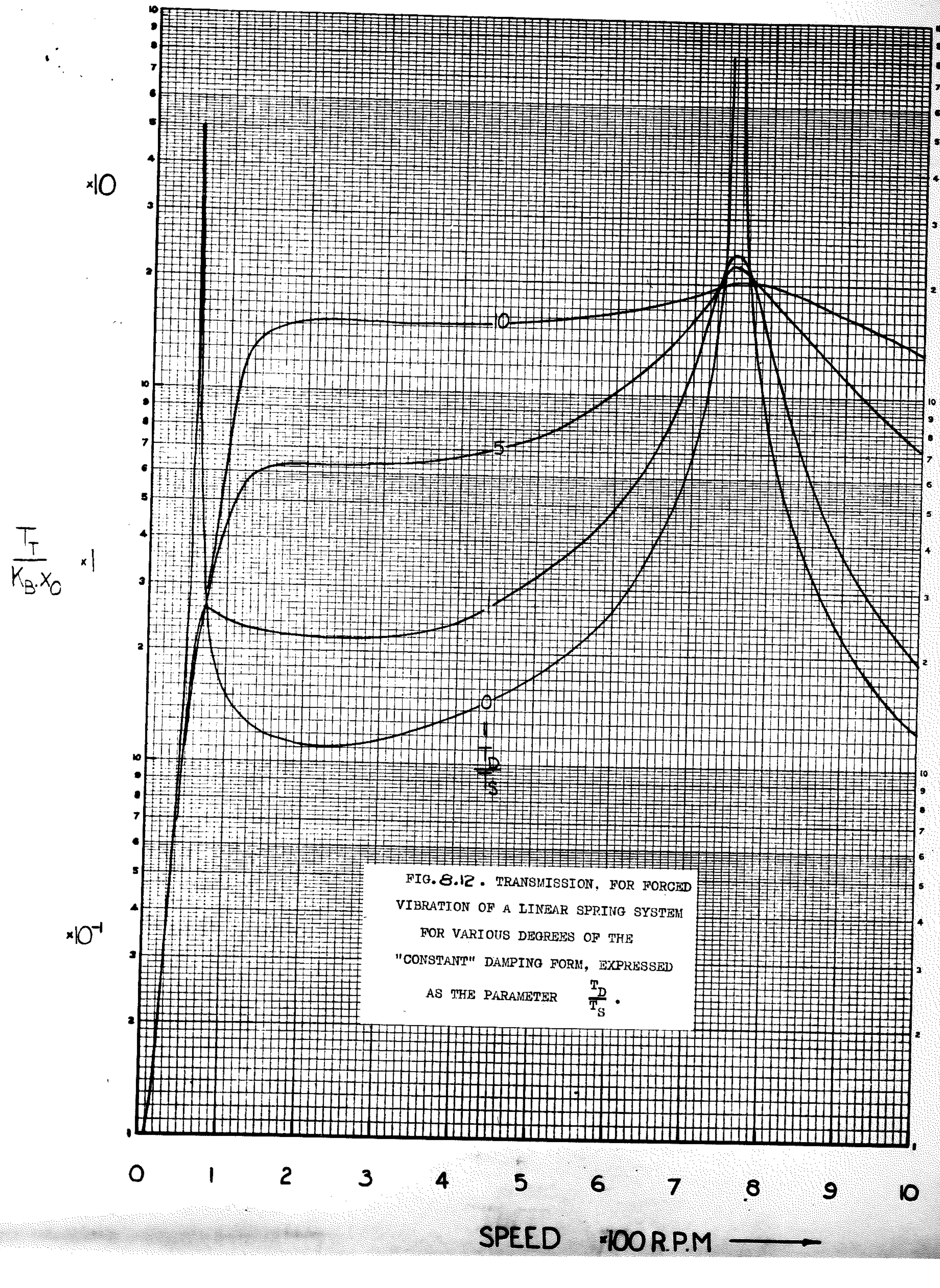
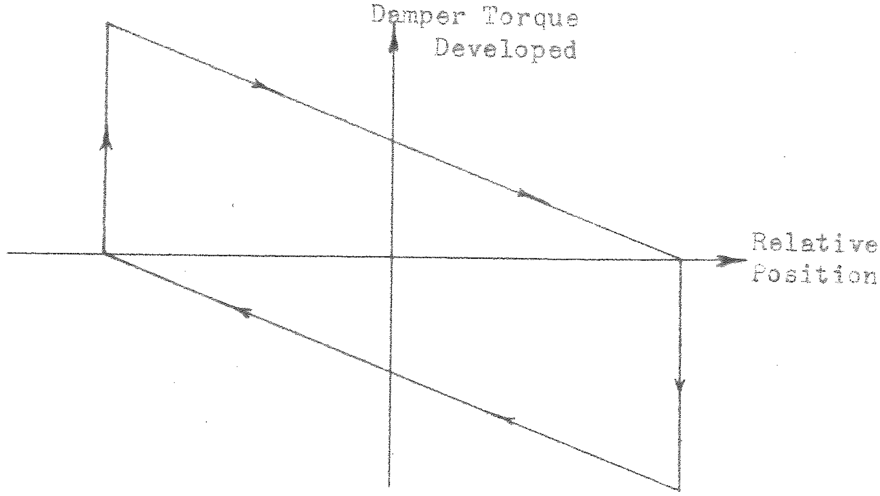
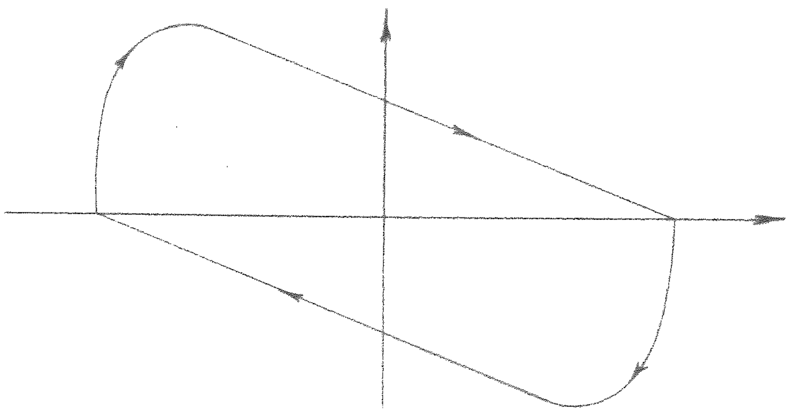


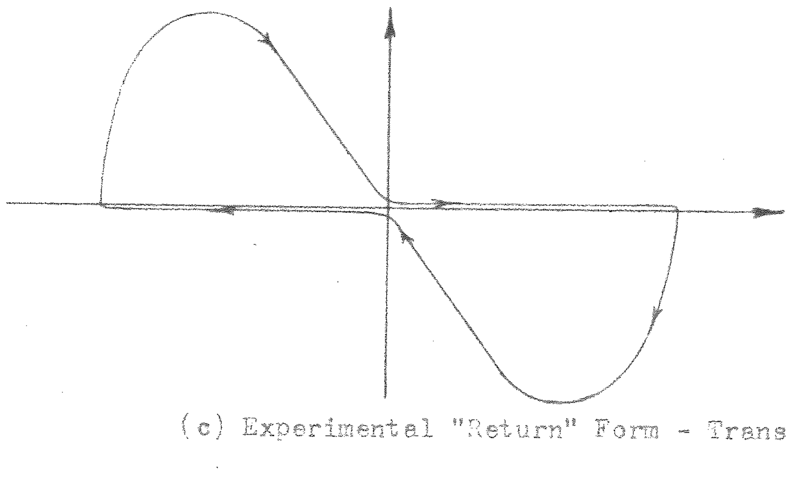

FIG. 8.12. TRANSMISSION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "CONSTANT" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{T_S}$.




(a) Ideal "Return" Damping Form.



(b) Experimental "Return" Form - Harmonic Response



(c) Experimental "Return" Form - Transient Response

FIG. 8.13 IDEAL AND EXPERIMENTAL "RETURN" DAMPING DIAGRAMS OBTAINED DURING ANALOGUE TESTS OF SECTION 8.1.2.

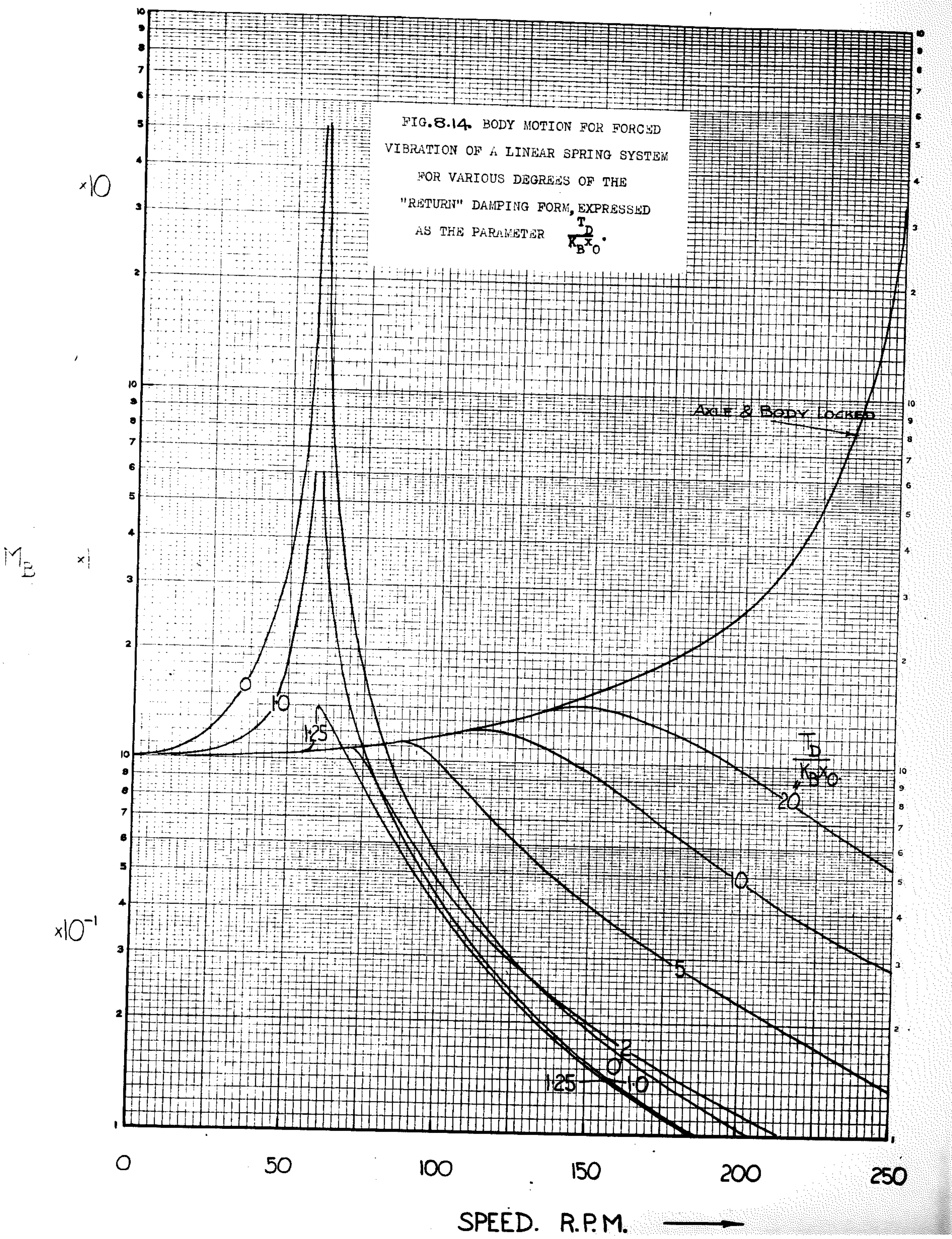



FIG.8.14. BODY MOTION FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "RETURN" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{K_B x_0}$.

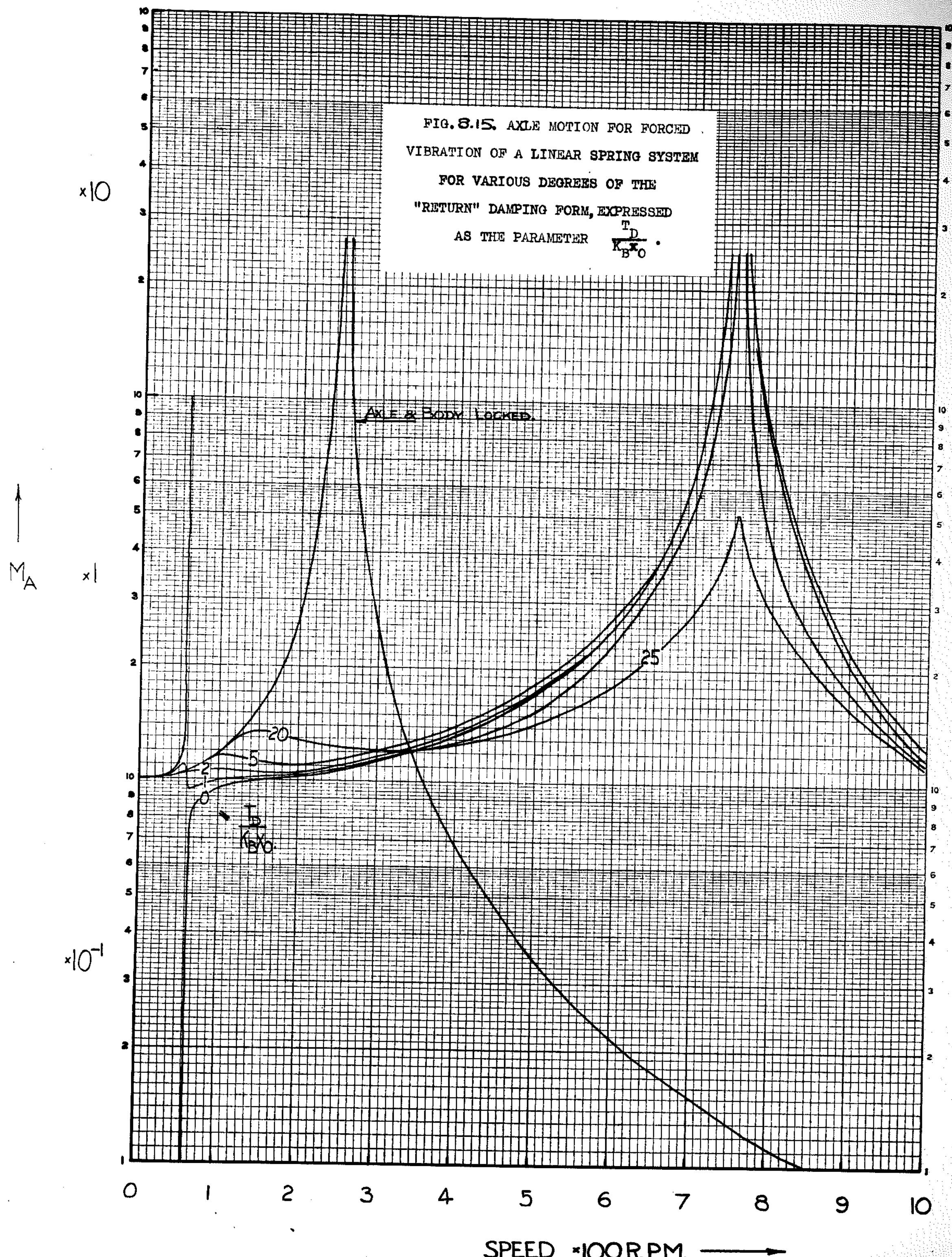



FIG. 8.15. AXLE MOTION FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "RETURN" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{K_B x_0}$.

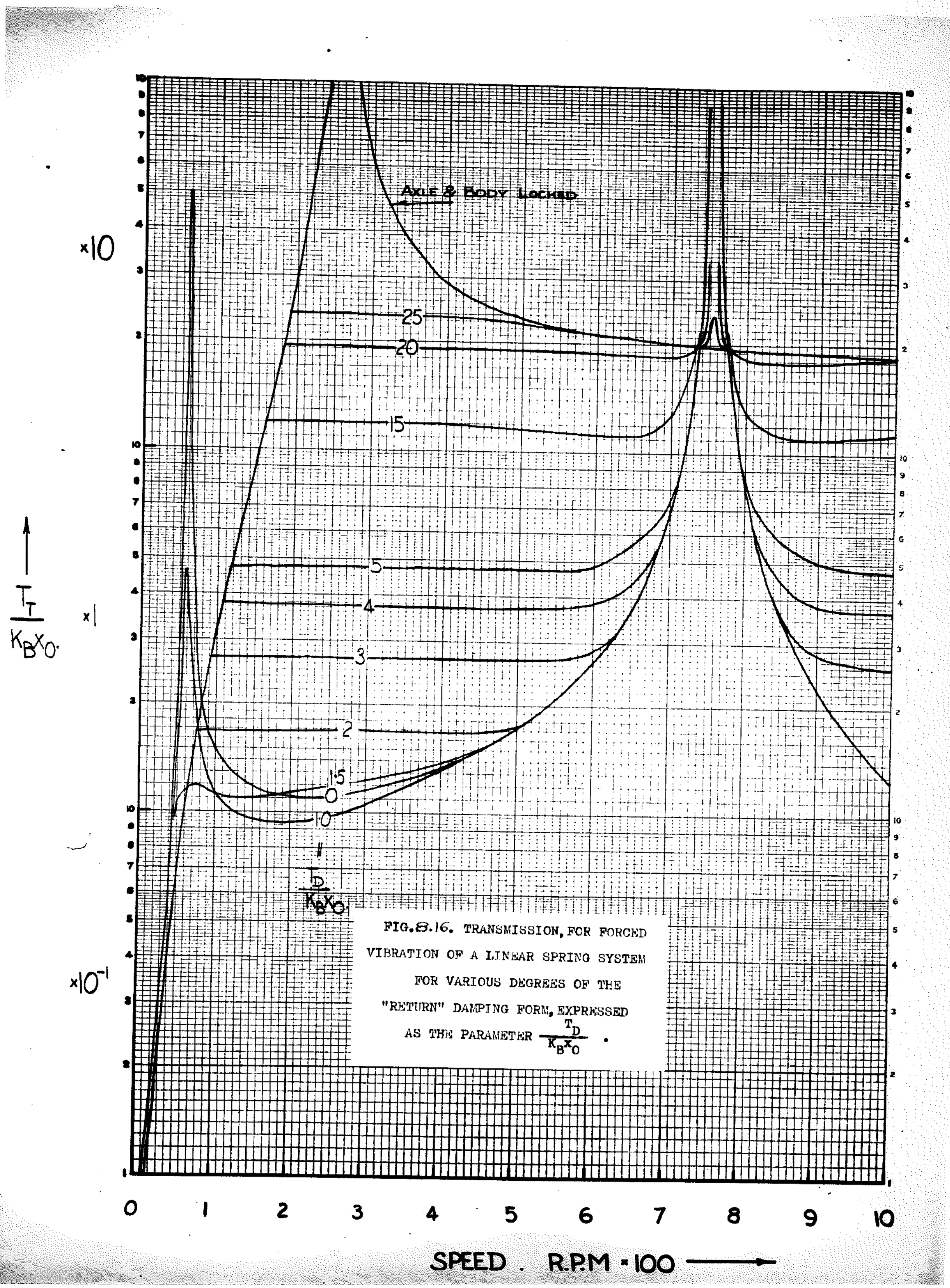



FIG. 8.16. TRANSMISSION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "RETURN" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{K_B x_O}$.

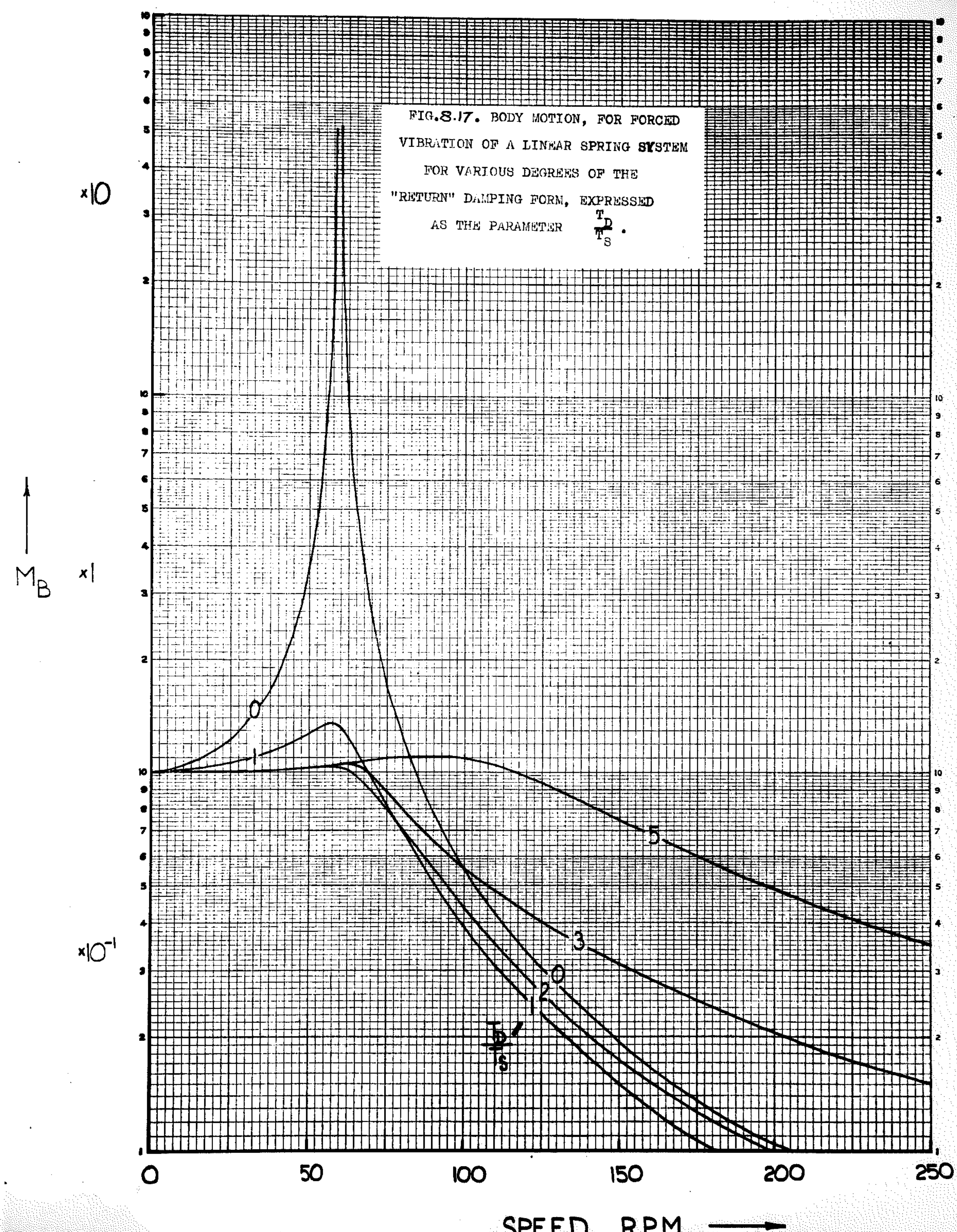



FIG. 8.17. BODY MOTION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "RETURN" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{T_S}$.

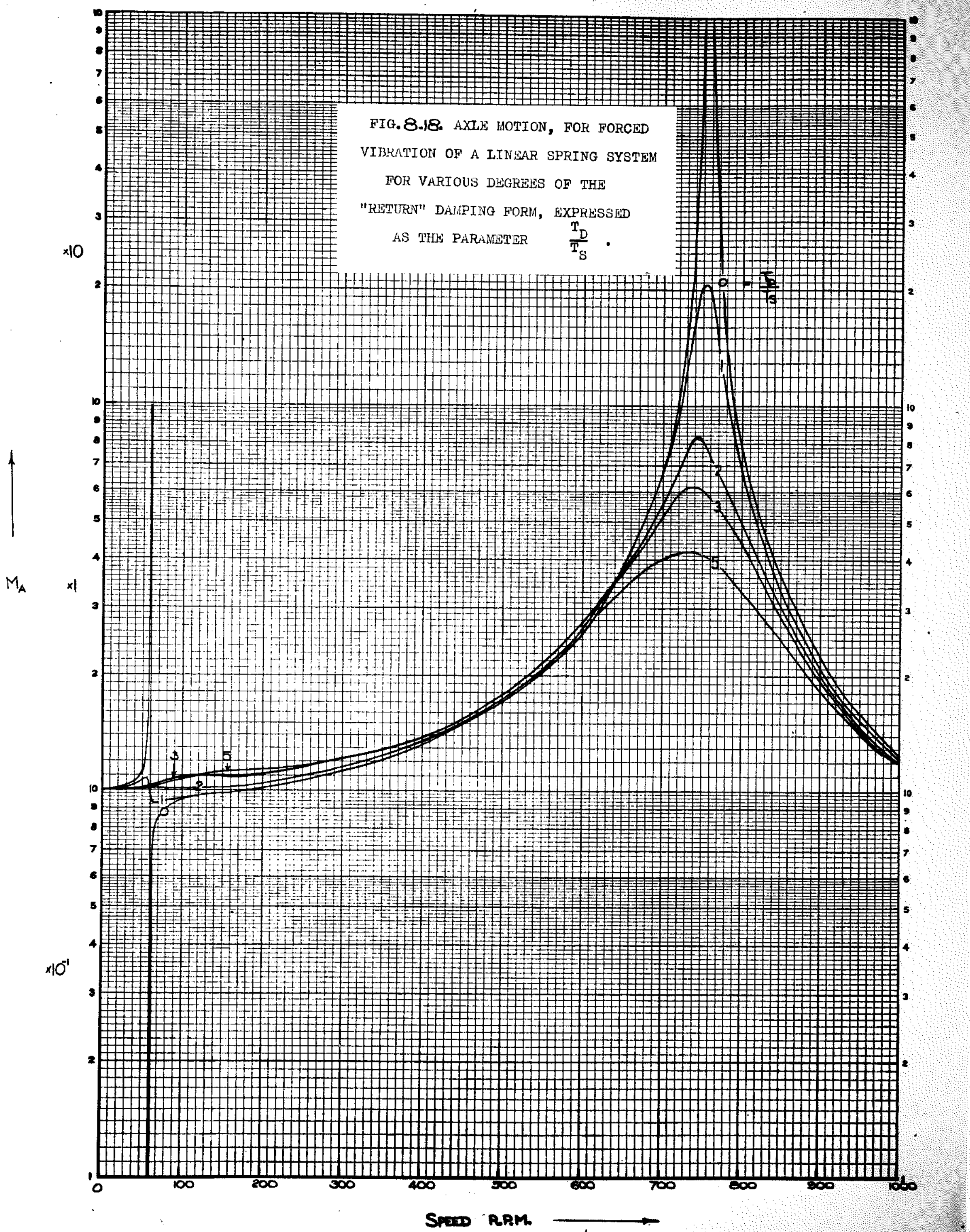



FIG. 8.18. AXLE MOTION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "RETURN" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{T_S}$.

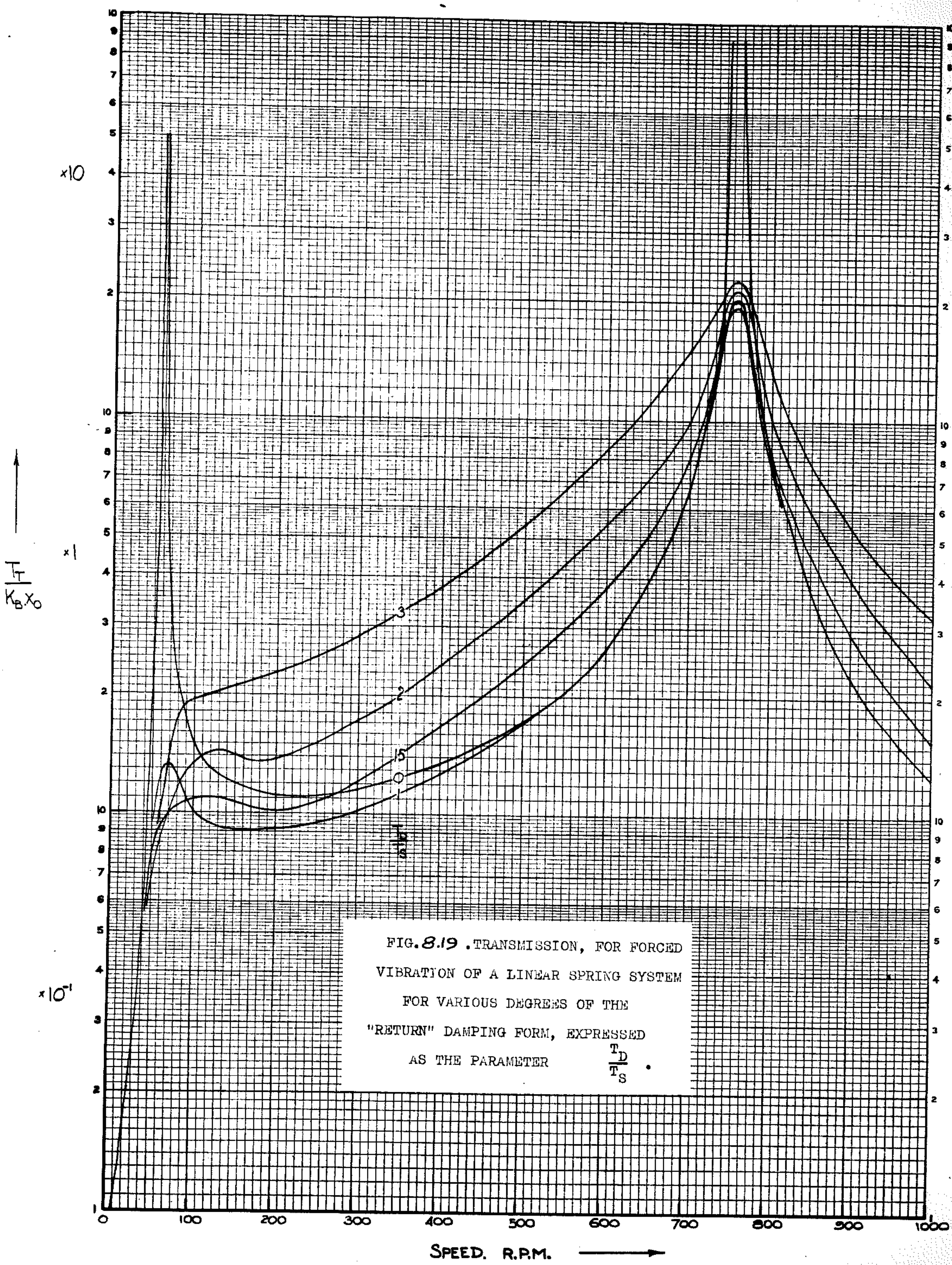



FIG. 8.19 . TRANSMISSION, FOR FORCED VIBRATION OF A LINEAR SPRING SYSTEM FOR VARIOUS DEGREES OF THE "RETURN" DAMPING FORM, EXPRESSED AS THE PARAMETER $\frac{T_D}{T_S}$ .

Fig.8.20. TRANSIENT RESPONSE OF A LINEAR SPRING SYSTEM TO A STEP $X_0$ IN ROAD PROFILE, FOR "RETURN" DAMPING FORM EXPRESSED AS THE PARAMETER $T_D/T_S$.

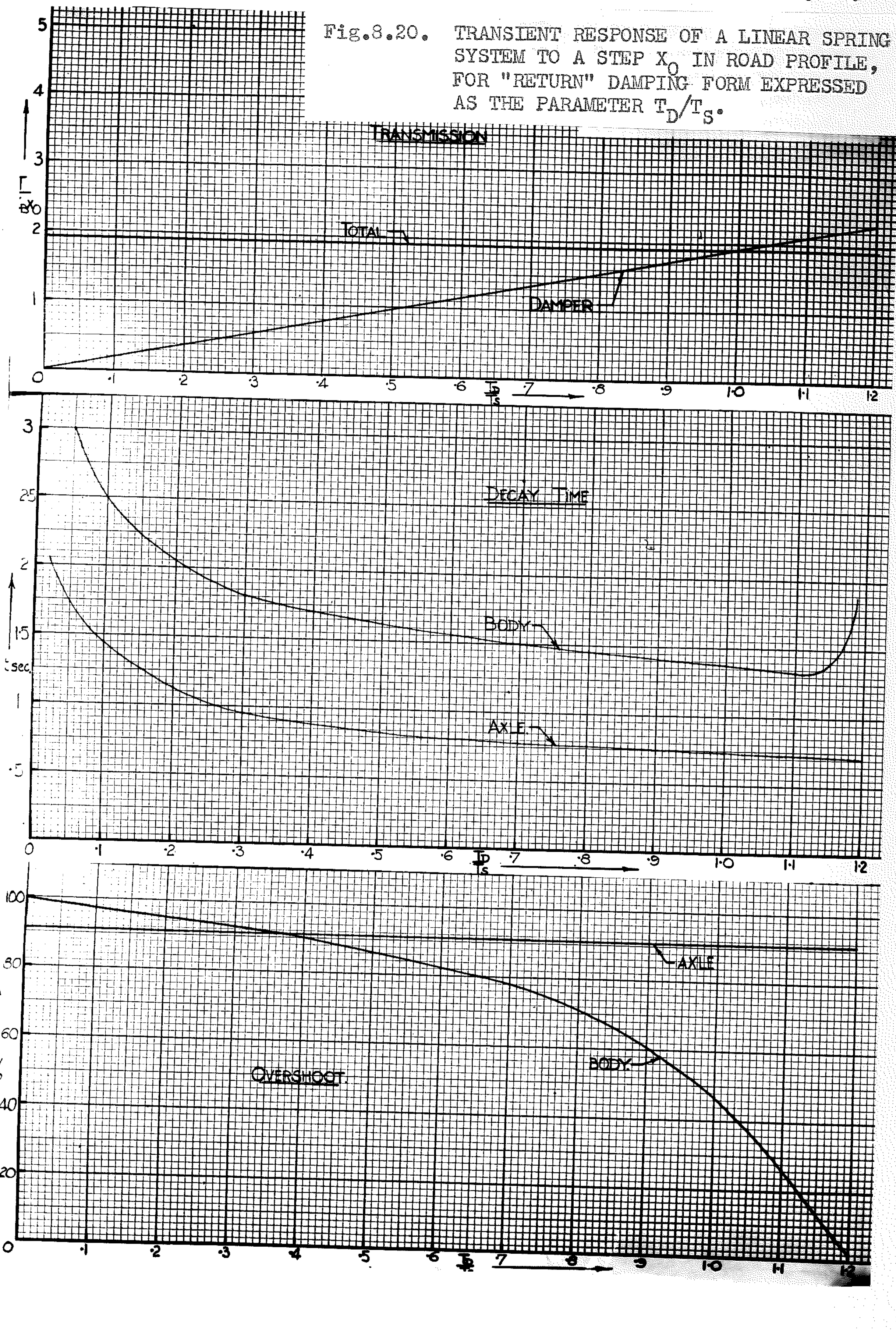
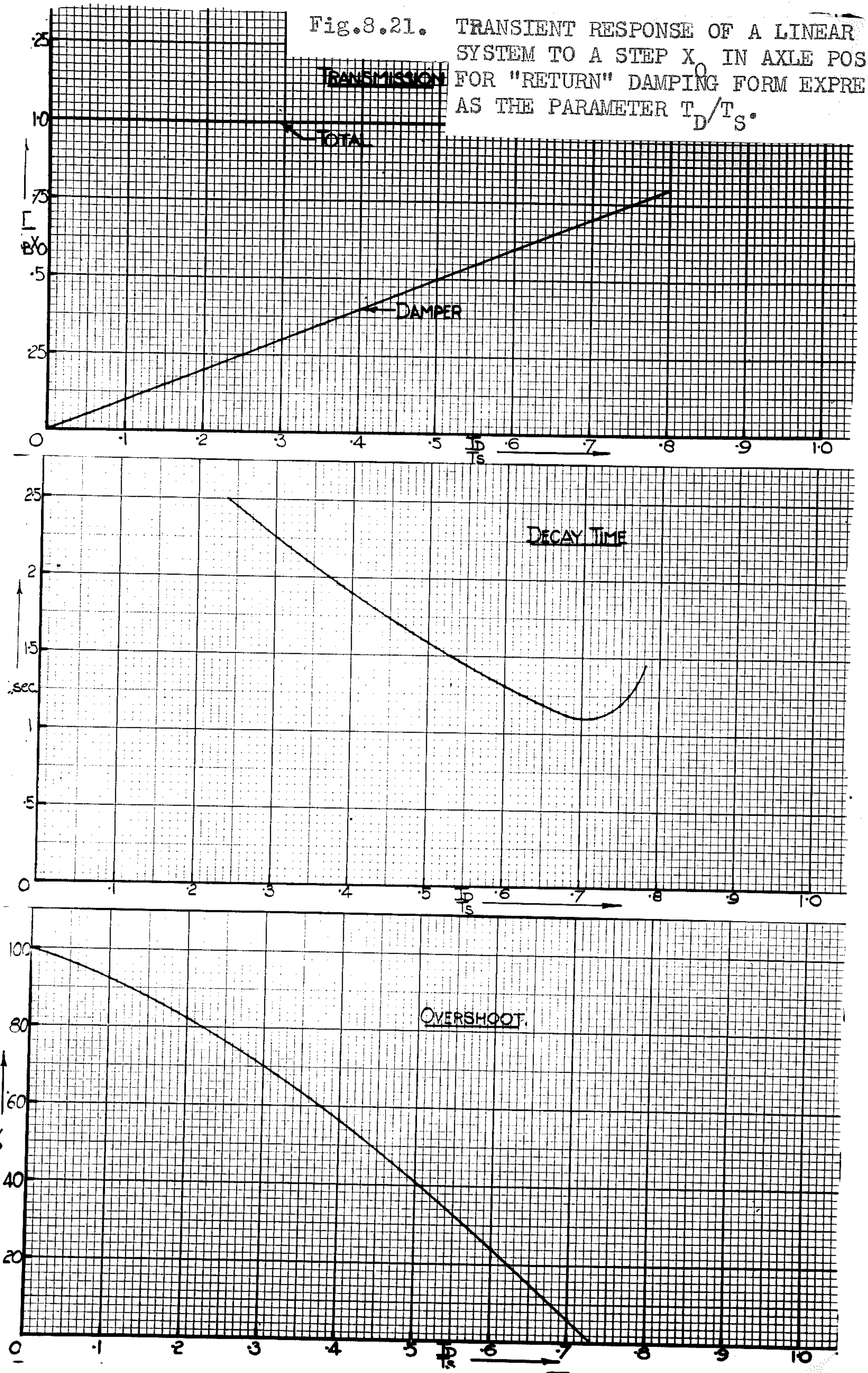


Fig.8.21. TRANSIENT RESPONSE OF A LINEAR SPRING SYSTEM TO A STEP $X_0$ IN AXLE POSITION, FOR "RETURN" DAMPING FORM EXPRESSED AS THE PARAMETER $T_D/T_S$.



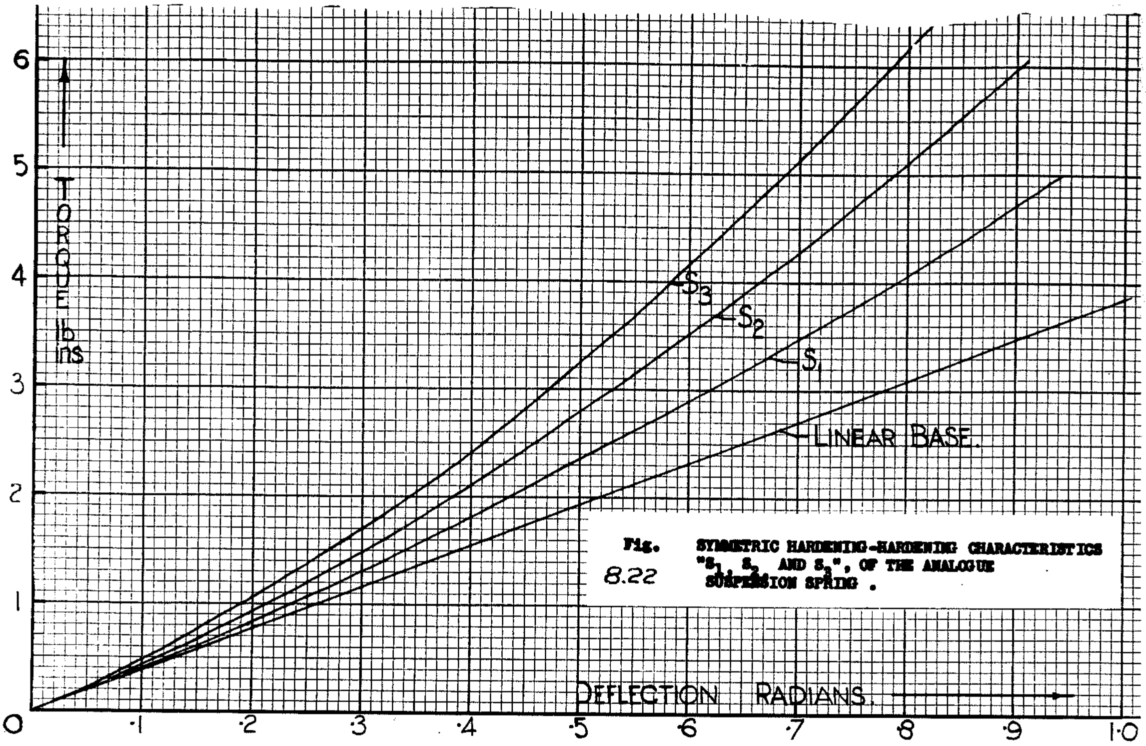



Fig. 8.22 SYMMETRIC HARDENING-HARDENING CHARACTERISTICS "$S_1$, $S_2$ AND $S_3$", OF THE ANALOGUE SUSPENSION SPRING.

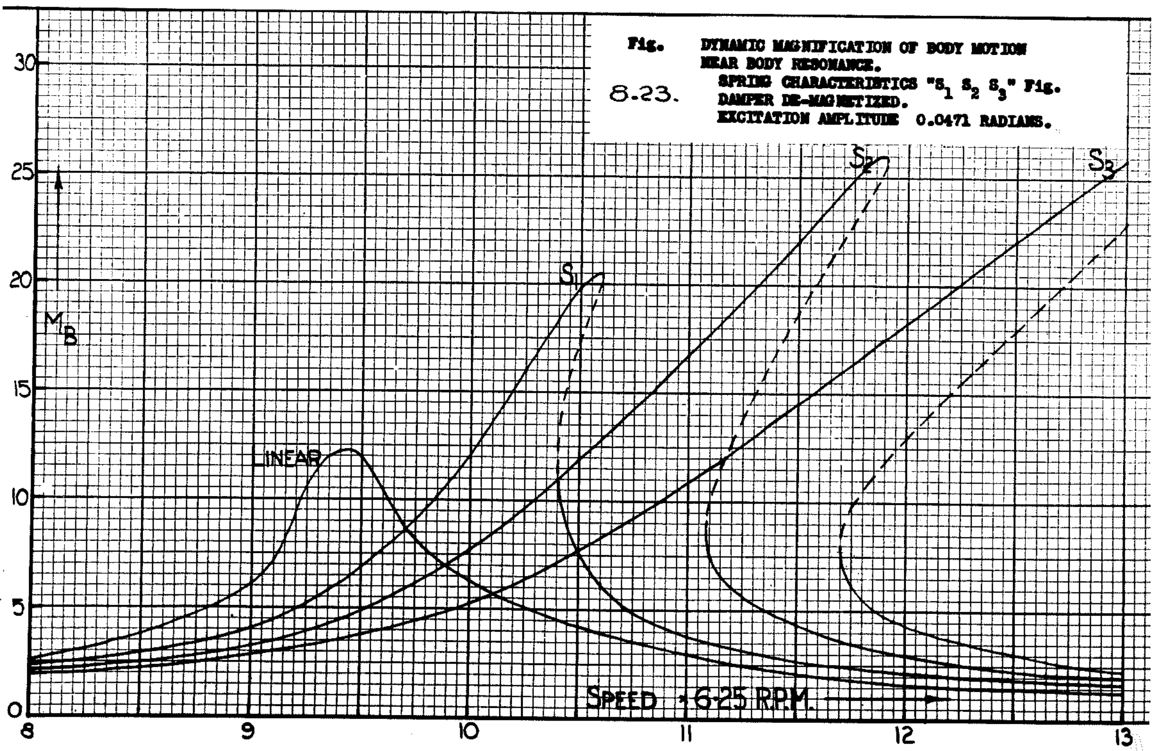
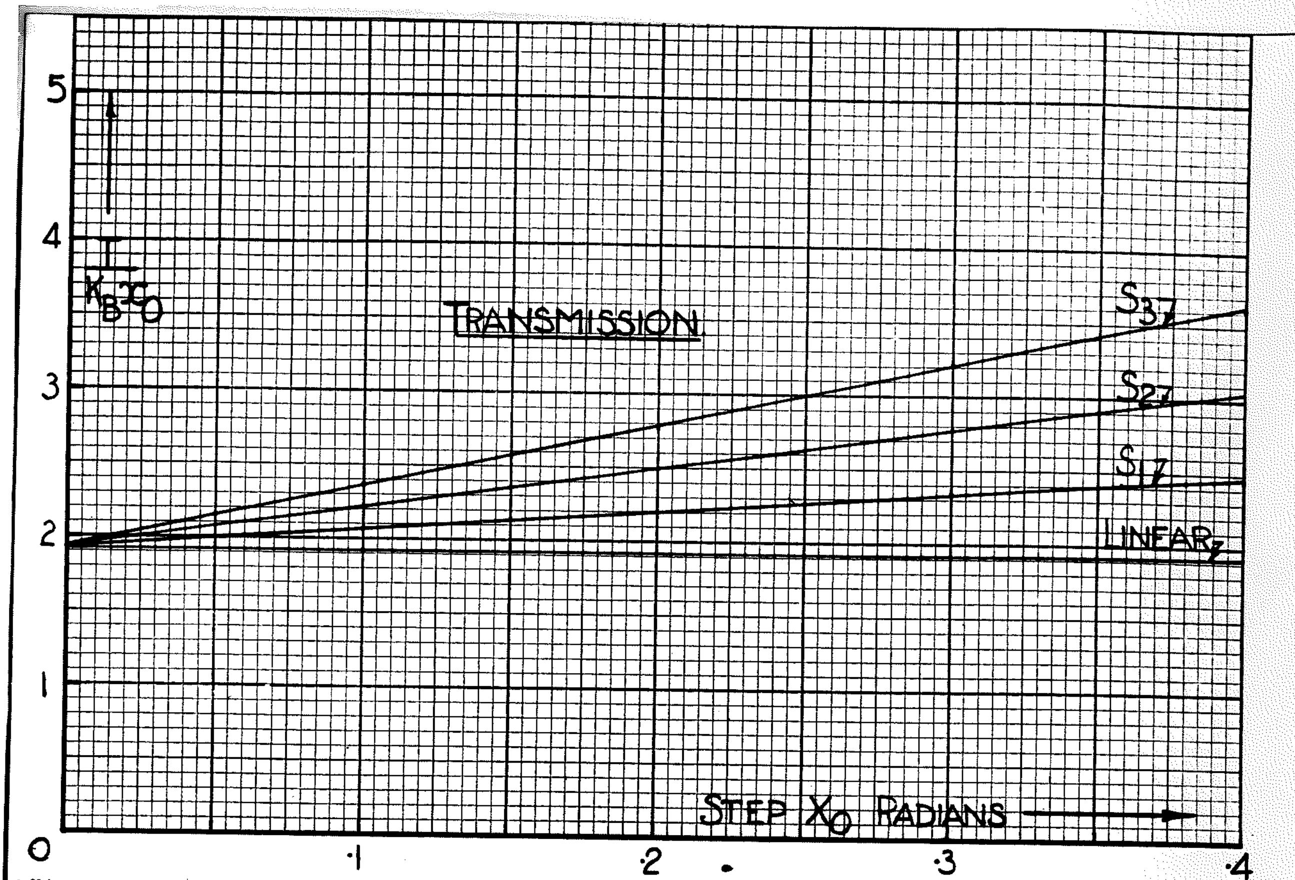



Fig. 8.23. DYNAMIC MAGNIFICATION OF BODY MOTION NEAR BODY RESONANCE. SPRING CHARACTERISTICS "$S_1$ $S_2$ $S_3$" Fig. DAMPER DE-MAGNETIZED. EXCITATION AMPLITUDE 0.0471 RADIANS.



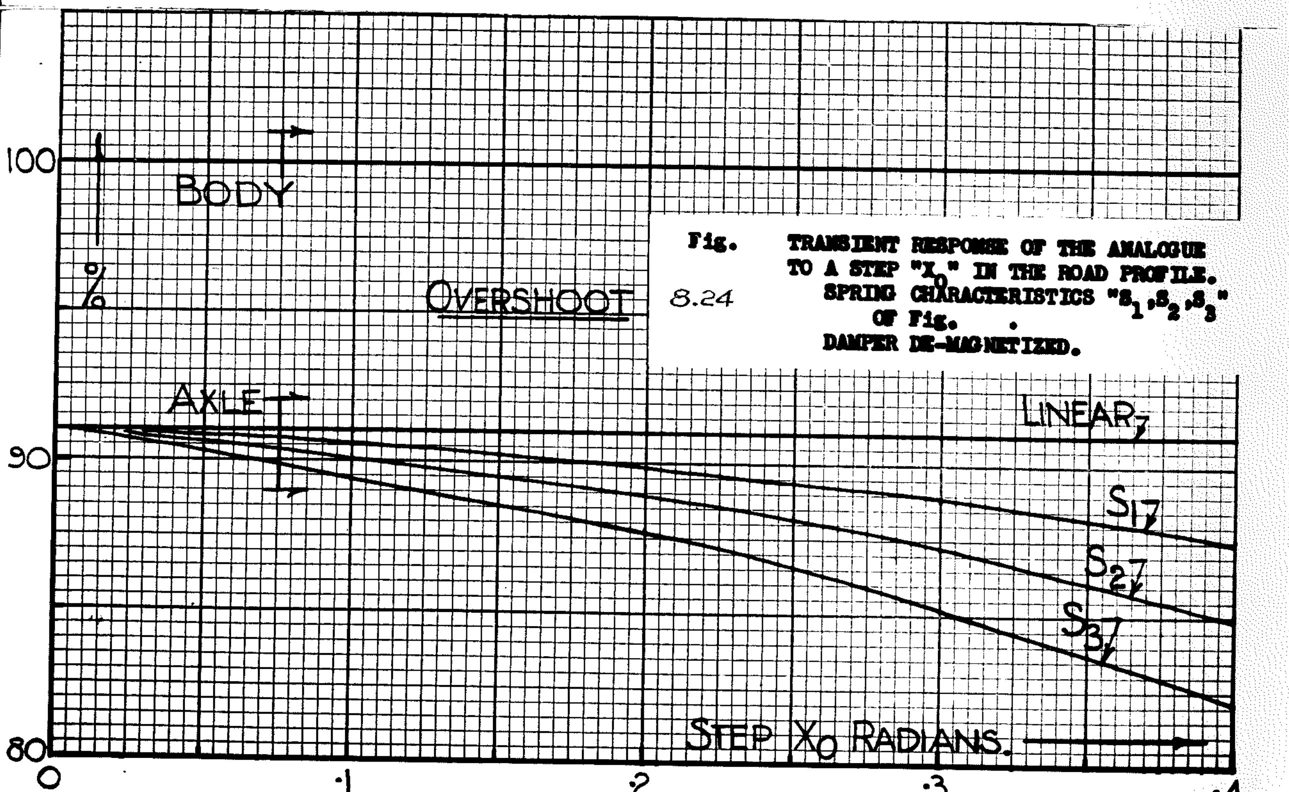




**Fig. 8.24** TRANSIENT RESPONSE OF THE ANALOGUE TO A STEP "$X_0$" IN THE ROAD PROFILE. SPRING CHARACTERISTICS "$S_1, S_2, S_3$" OF Fig. . DAMPER DE-MAGNETIZED.

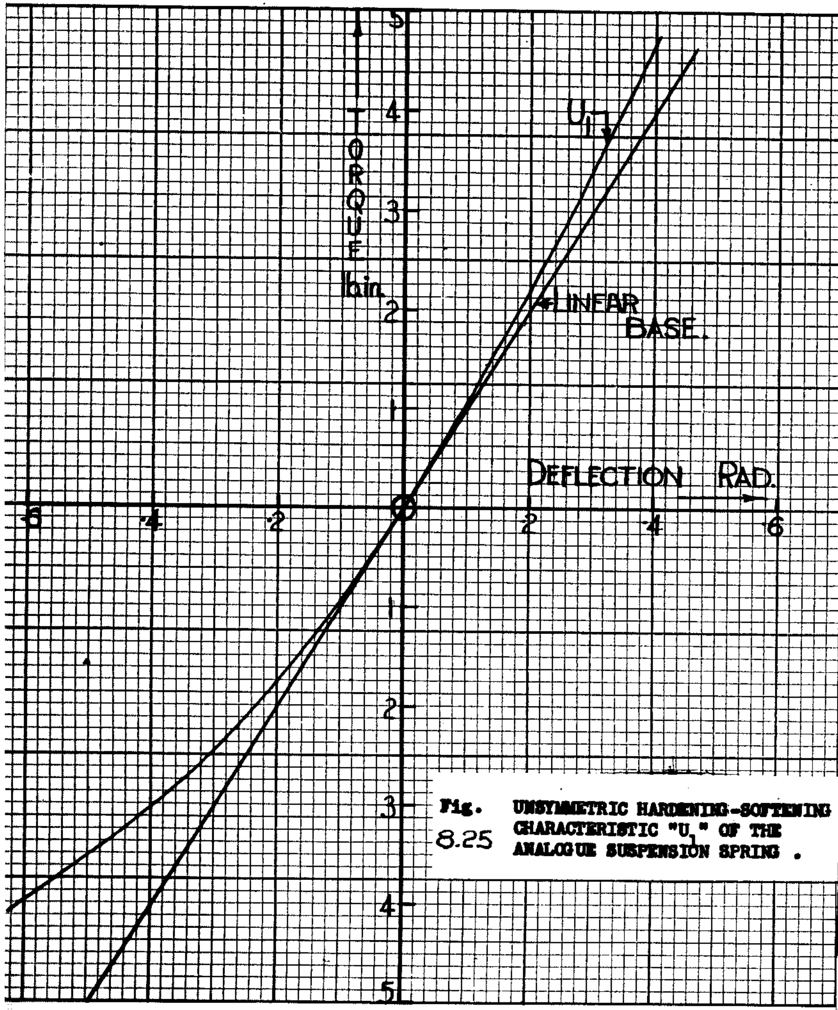



Fig. 8.25 UNSYMMETRIC HARDENING-SOFTENING CHARACTERISTIC "$U_1$" OF THE ANALOGUE SUSPENSION SPRING.

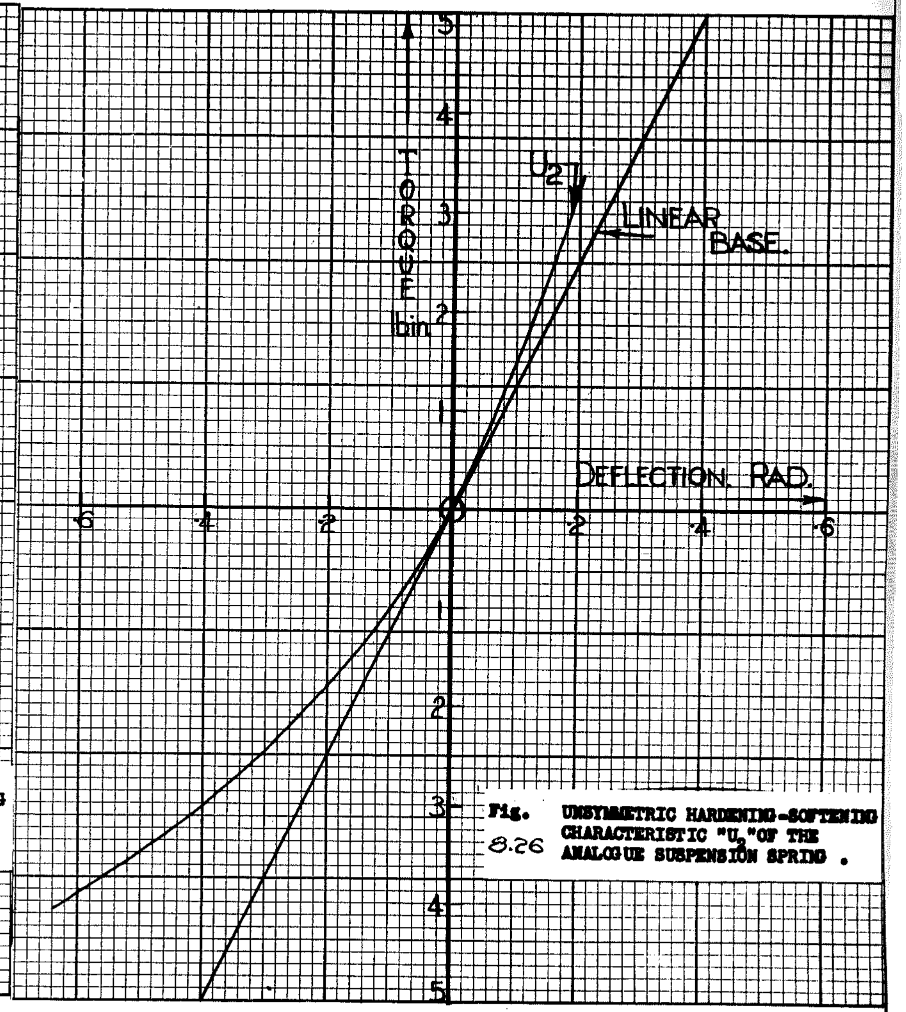



Fig. 8.26 UNSYMMETRIC HARDENING-SOFTENING CHARACTERISTIC "$U_2$" OF THE ANALOGUE SUSPENSION SPRING.

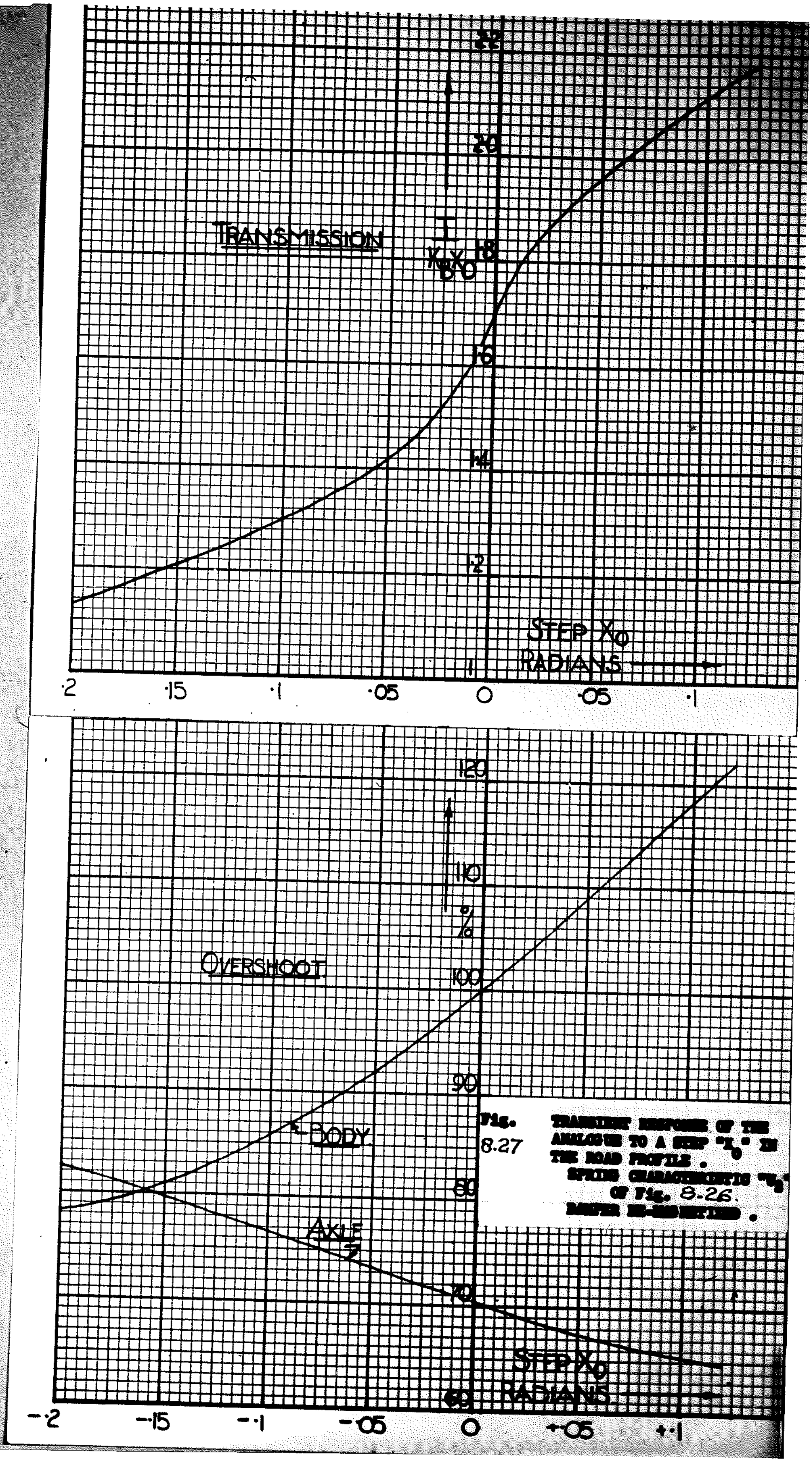



Fig. 8.27 TRANSIENT RESPONSE OF THE ANALOGUE TO A STEP "$X_0$" IN THE ROAD PROFILE. SPRING CHARACTERISTIC "K" OF Fig. 8.26. DAMPER RE-ADJUSTED.

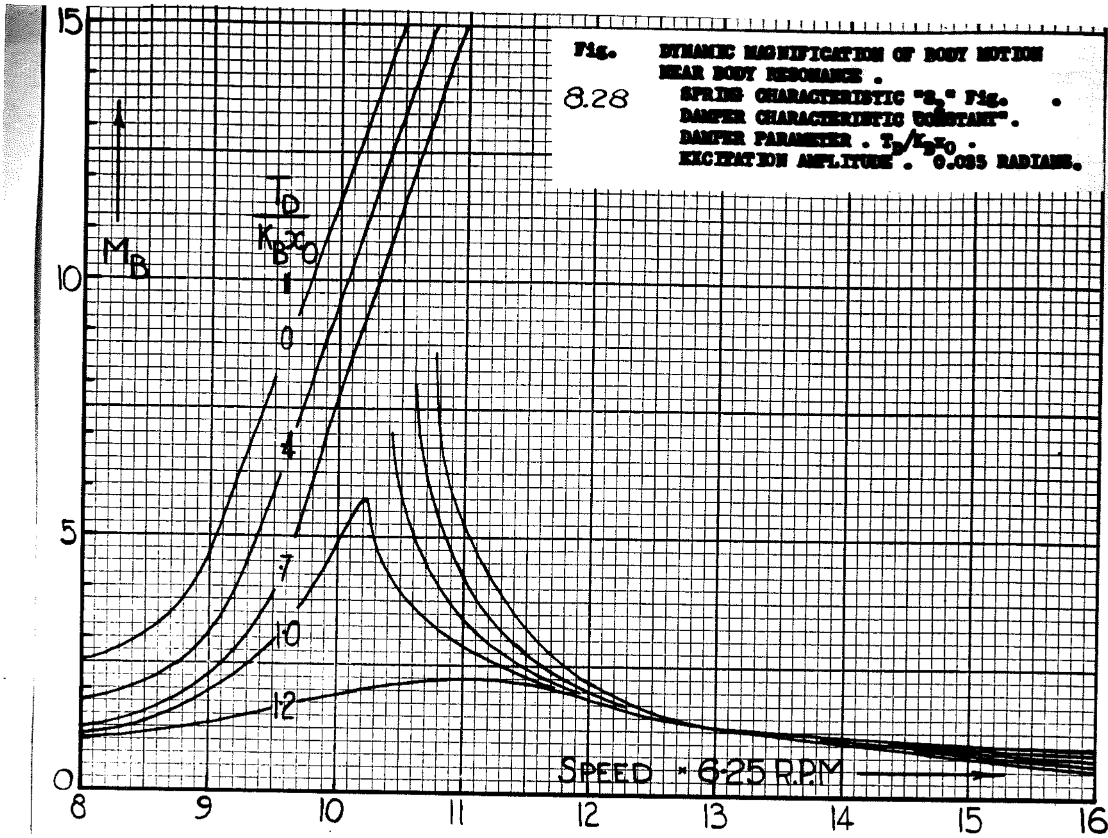



Fig. 8.28 DYNAMIC MAGNIFICATION OF BODY MOTION NEAR BODY RESONANCE. SPRING CHARACTERISTIC "$S_2$" Fig. . DAMPER CHARACTERISTIC "CONSTANT". DAMPER PARAMETER . $T_D/K_B x_0$ . EXCITATION AMPLITUDE . 0.035 RADIANS.

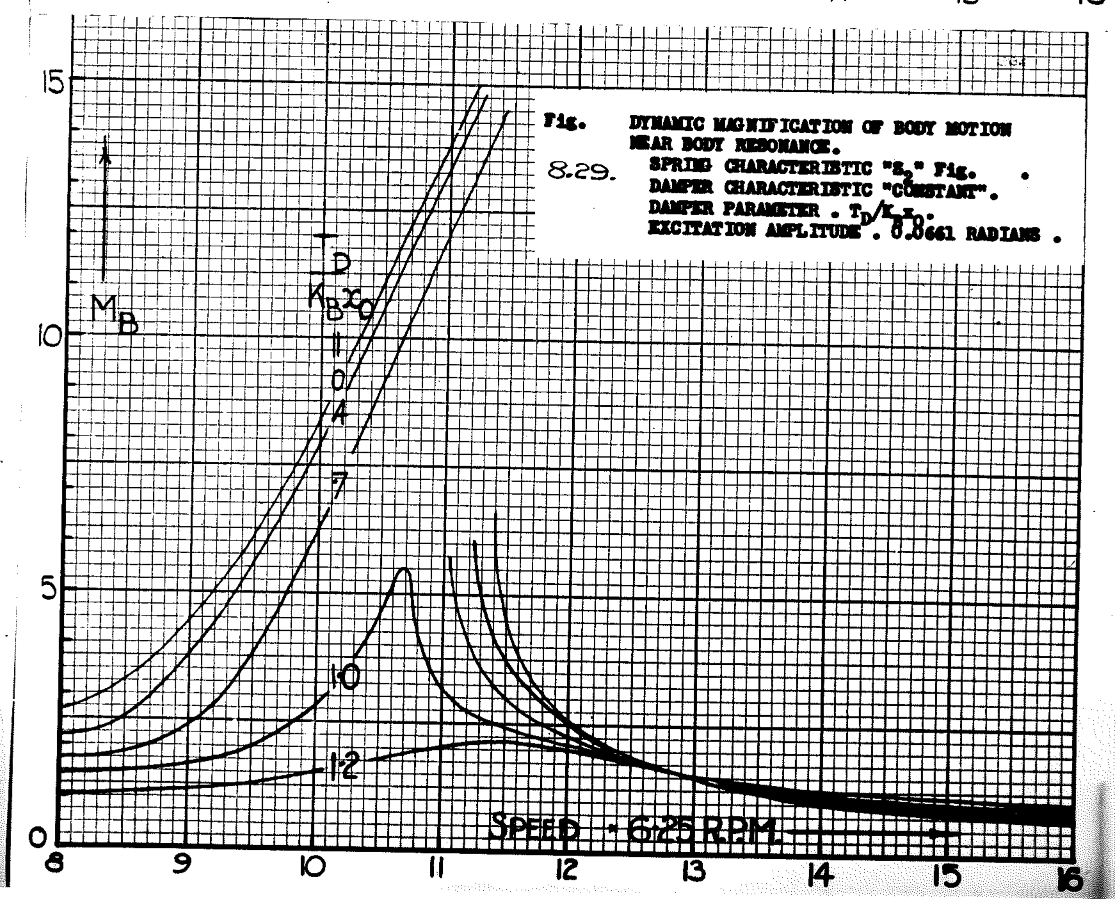



Fig. 8.29. DYNAMIC MAGNIFICATION OF BODY MOTION NEAR BODY RESONANCE. SPRING CHARACTERISTIC "$S_2$" Fig. . DAMPER CHARACTERISTIC "CONSTANT". DAMPER PARAMETER . $T_D/K_B x_0$. EXCITATION AMPLITUDE . 0.0661 RADIANS .

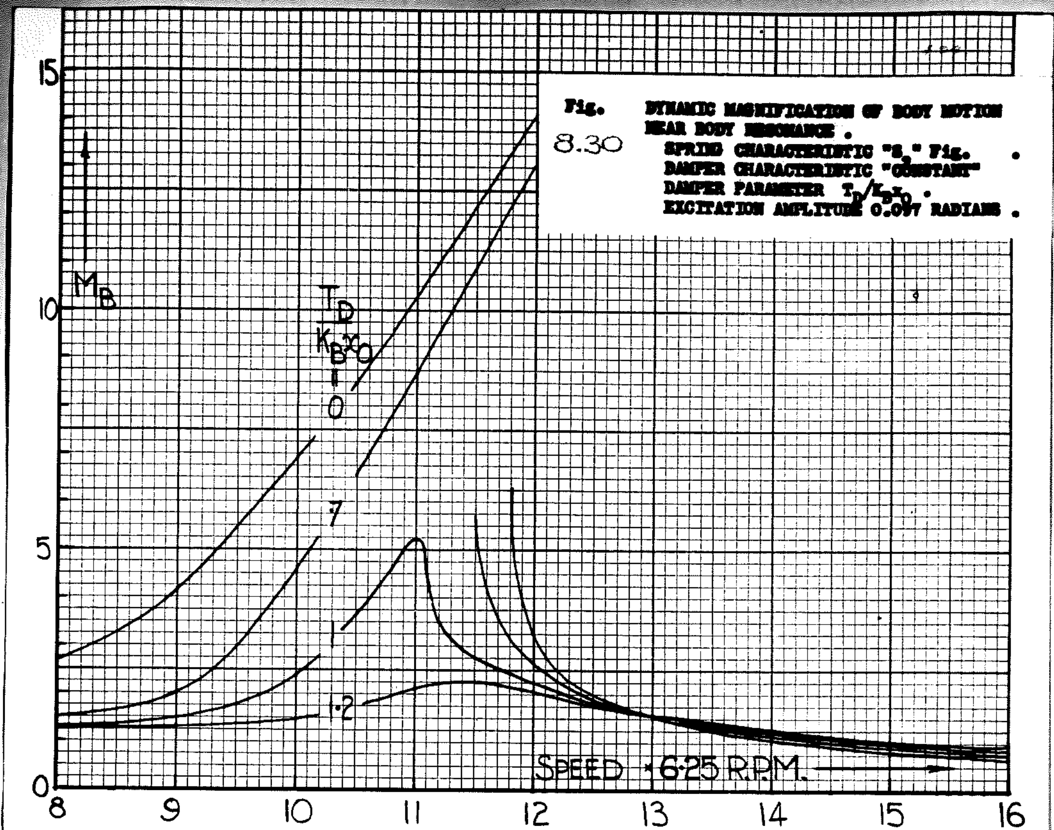



Fig. 8.30 DYNAMIC MAGNIFICATION OF BODY MOTION NEAR BODY RESONANCE.
SPRING CHARACTERISTIC "$S_2$" Fig. .
DAMPER CHARACTERISTIC "CONSTANT"
DAMPER PARAMETER $T_D/K_B x_O$.
EXCITATION AMPLITUDE 0.097 RADIANS.

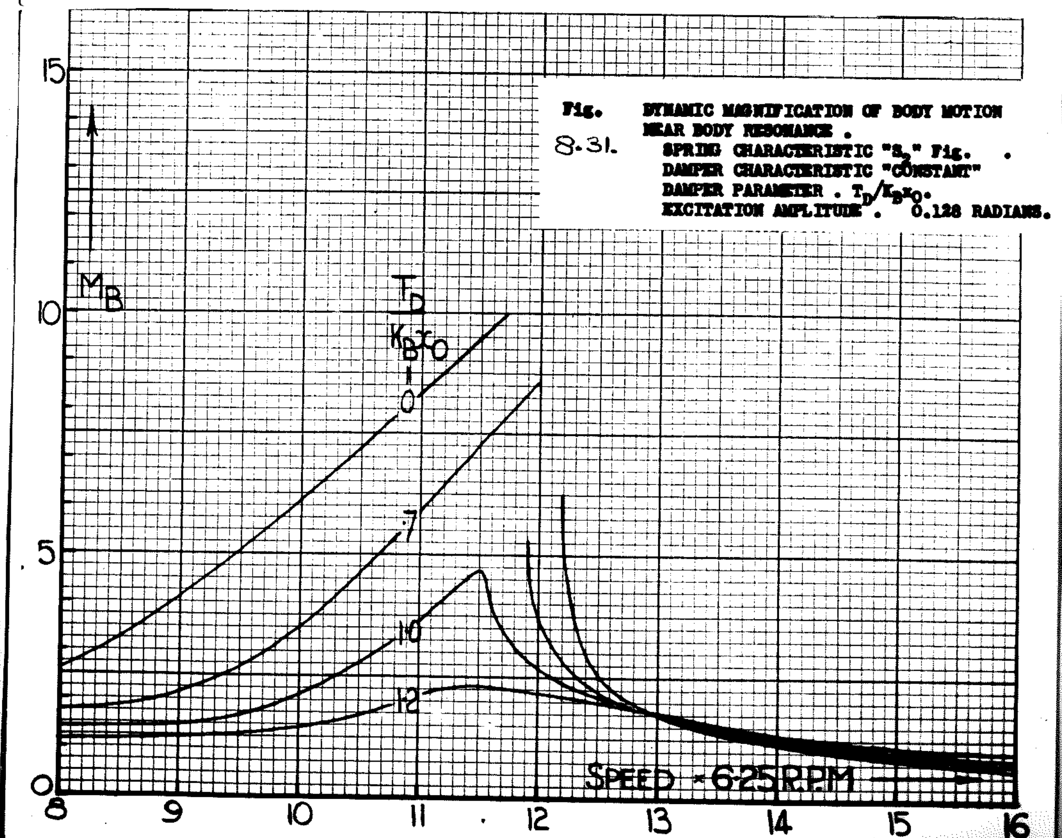



Fig. 8.31. DYNAMIC MAGNIFICATION OF BODY MOTION NEAR BODY RESONANCE.
SPRING CHARACTERISTIC "$S_2$" Fig. .
DAMPER CHARACTERISTIC "CONSTANT"
DAMPER PARAMETER . $T_D/K_B x_O$.
EXCITATION AMPLITUDE . 0.128 RADIANS.

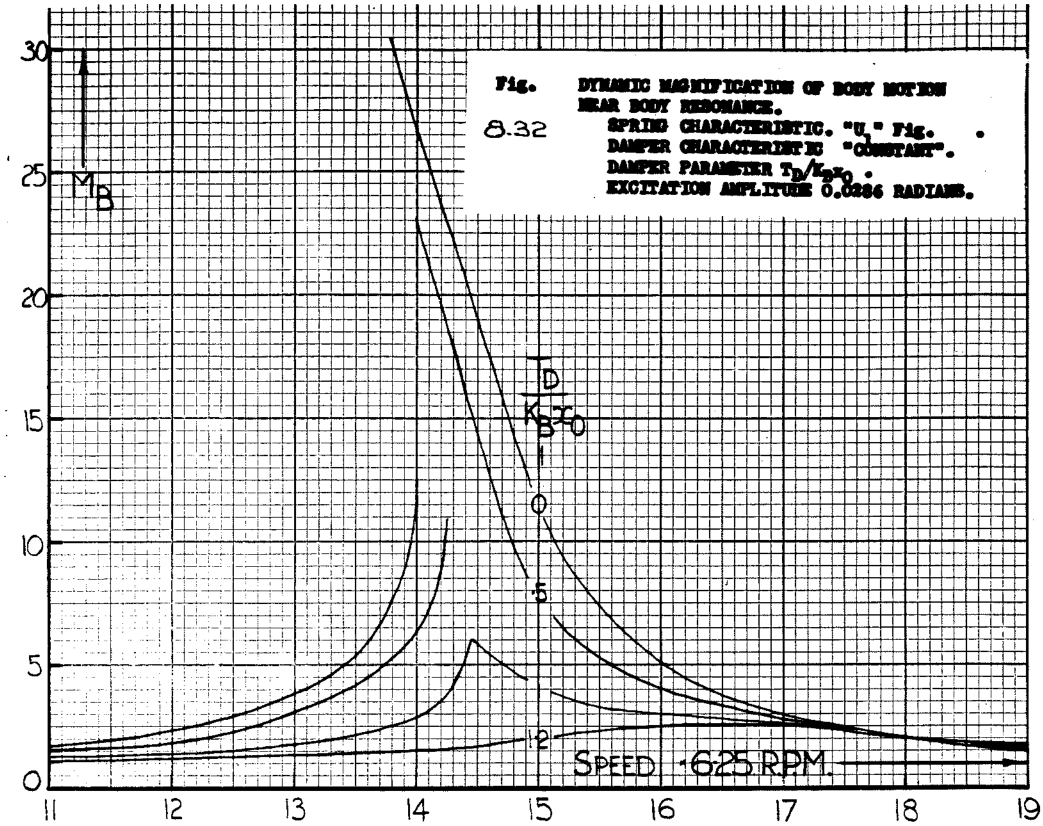



Fig. 8.32 DYNAMIC MAGNIFICATION OF BODY MOTION NEAR BODY RESONANCE.
SPRING CHARACTERISTIC. "$U_1$" Fig. .
DAMPER CHARACTERISTIC "CONSTANT".
DAMPER PARAMETER $T_D/K_B x_0$ .
EXCITATION AMPLITUDE 0.0286 RADIANS.

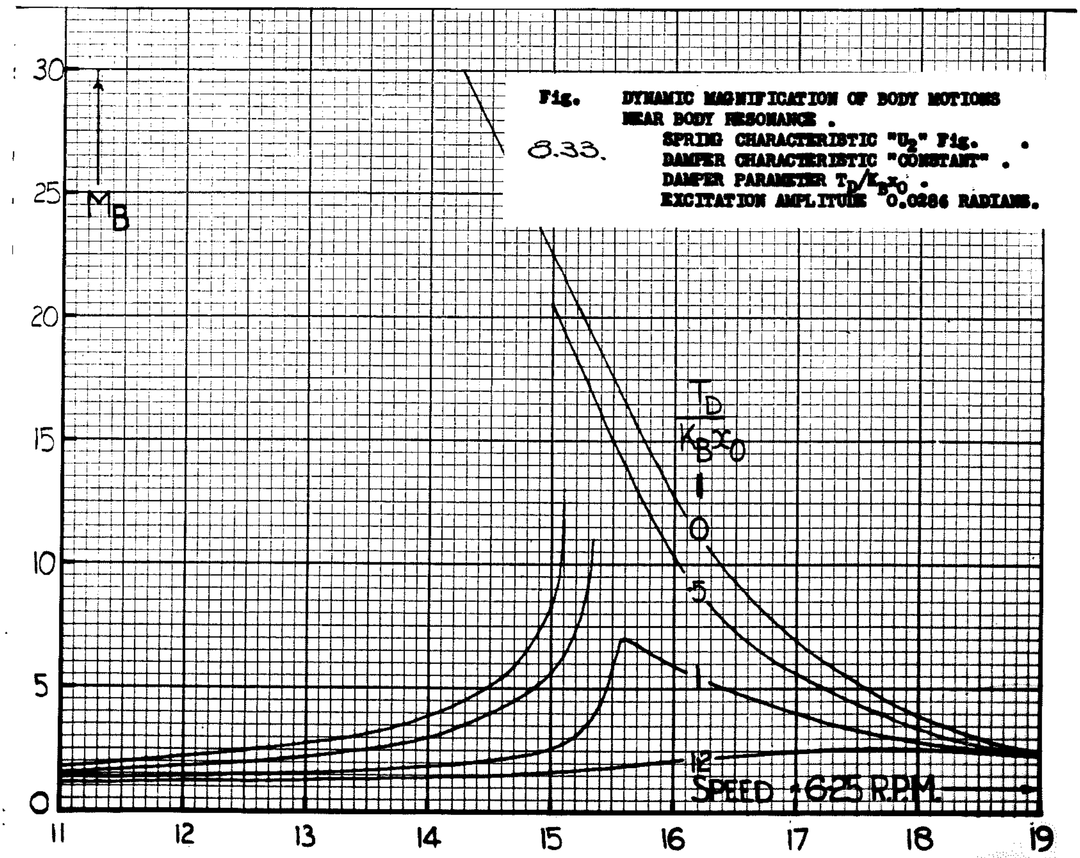



Fig. 8.33. DYNAMIC MAGNIFICATION OF BODY MOTIONS NEAR BODY RESONANCE .
SPRING CHARACTERISTIC "$U_2$" Fig. .
DAMPER CHARACTERISTIC "CONSTANT" .
DAMPER PARAMETER $T_D/K_B x_0$ .
EXCITATION AMPLITUDE 0.0286 RADIANS.

# Chapter 9

# CONCLUSIONS AND RECOMMENDATIONS

## 9. CONCLUSIONS AND RECOMMENDATIONS.

Although comments and conclusions regarding the performance of the analogue and the mathematical analyses of the system are included in earlier sections under the appropriate headings, this chapter is included as an overall summary of the conclusions reached after experimental and analytic investigation of the effects of controlled non-linear damping, and non-linear springing in the suspension of modern road vehicles.

(1) The most general conclusion which can be drawn from both the experimental and analytic work of this thesis, is in relation to the behaviour of a damper in a linear vibratory system. Till the present time, it has been suggested that the basic object of a damper is to dissipate the energy fed into any system of springs and masses by an exciting force whose frequency of variation coincided with a resonance of the system. The supposition was thus that motion built up till the energy balance was reached. Although this is physically exactly what happens, the results of this thesis indicate that the energy dissipation concept of damping is misleading, as the real effect of a damper is to provide a force to balance the excitation force in the condition where spring and inertia forces become self-balancing, and phased at 90 degrees to the excitation.

This conclusion is based on the results for force limited dampers, which do not prevent resonant motions from becoming infinite unless they contribute a definite magnitude of damper effort which is related only to the excitation amplitude. This critical damping effort is also seen to be the same as that developed by a linear dmaper at the resonance, quite independent of the damping ratio which it possesses.

Furthermore, the experimental results indicate that the effect of damper work diagram shape is quite small in comparison with the effect of magnitude of damper effort, having significant influence only on the transmissibility.

Thus the method developed in section 6.5. is suggested to be a sound analytical approach to determine the harmonic response of a system incorporating any non-linear damping type, though of course the accuracy will be greatest if the damper work diagram is similar in shape to that of a linear unit.

(2) With respect to the particular application of the vehicle suspension, it is seen that no single characteristic of the motions of either axle or body is satisfactory to control damping effort in the ideal manner to produce acceptable resonant motions and minimum transmission.

The combined experimental and analytical results do however show that this may be achieved by a composite control arrangement. Thus, if the system is supplied with a basic magnitude of damping effort that is directly proportional to the force developed in the suspension spring, and arranged into a "return" damping form, adequate control of both transient and harmonic body motions can be achieved, while, providing high frequency excitation does not cause axle resonance, the transmission is everywhere a minimum. The results show that if such damping control were selected to give the optimum response to harmonic excitation, the transient response would be overdamped. However, the only effect overdamping of the transient would have, is a small position error, and this would undoubtedly be eliminated in practice by axle motions accompanying such a transient.

Control of axle motions can then be achieved by an overriding axle acceleration control of high damping effort of "constant" form, which would only be introduced when the recorded axle acceleration exceeded a predetermined limit which was known to precede wheelhop in the lowly damped suspension. In this respect the transient analogue results show that high damping effort is only required to eliminate wheelhop if the initial axle acceleration $A_A$ exceeds $150.w_B^2 x_{Ast}$, and this value corresponds with that determined analytically for the harmonic response in the high frequency region.
The transient response also shows that there is a unique relationship between the optimum damping effort and the initial recorded axle acceleration, so this could be used to initially set the level of high damping force.

However, the harmonic response indicates that a different relationship exists between the optimum damper effort and the recorded axle accelerations, so, having first been set by the transient criterion, the magnitude of damper effort would need to be constantly adjusted till the recorded acceleration corresponded to the magnitude of damper effort existing, to give optimum results under continuous excitation. Although such is thought to be feasible, it is suggested that a more practical system would be based on the maximum conceivable force requirement. Thus, having selected the worst road conditions which the vehicle is to encounter, a maximum required damper effort is fixed from consideration of transient and harmonic responses. Then, whenever the recorded axle acceleration exceeds the critical value, this damping effort would be immediately inserted into the system. If the disturbance were in fact transient, the high damping effort would be removed as soon as the axle acceleration decayed, but if the disturbance were harmonic, high damping would be required to persist, and this could be achieved by interlocking in such a way that high damping effort was not eliminated until the envelope of maximum axle accelerations fell considerably below the critical initiation value.

Thus, with this simplified system, it is seen that any excitation condition which demands a high value of damper force to control axle motions, does in fact automatically introduce the maximum possible damper effort, and consequently is overdamped. However, it must be realized that the compromise between the optimum damper effort and that required for the worst possible excitation does not represent a great decrease in performance, as both conditions require high damping effort.

The system suggested would thus still possess the great advantage of separating body and axle control, and would have close to the ideal performance under all conditions while still remaining a practical proposition.

Finally, it must be emphasized that the relation between axle acceleration and optimum damping effort presented in this thesis, is that determined from an assumed linear tyre characteristic having zero damping, and merely proves the feasibility of the method. It would thus be essential to determine similar relationships for an actual tyre, so that variations in spring rate, damping, and tyre curvature could be allowed for prior to introducing axle acceleration control in a practical suspension.

(3) With reference to non-linear springs, the analogue shows that both symmetric hardening-hardening and unsymmetric hardening-softening spring characteristics of the suspension give worse transient and harmonic responses than does a linear suspension of the same basic rate.

Furthermore, although such characteristics do not exhibit a true resonant frequency, tests indicated that damping was essential to prevent high amplitudes of harmonic oscillation from developing. The critical damping effort needed to ensure unique oscillation amplitudes throughout the frequency spectrum, proved to be equal to the damping effort developed in a strictly linear suspension having the same basic rate.

Consequently a non-linear spring system requires even greater damping than a linear suspension if it is to control body motions to the same degree.

Also it was noticed that although the degree of non-linearity introduced had considerable influence on the motions of the body, the effect on axle motions was practically insignificant. This must of course be true so long as the tyre maintains road contact, but it does indicate that damping effort provides the best control of axle motion.

Thus it is obvious that the most satisfactory suspension characteristic, from the point of view of transient and harmonic responses, is a linear one, and although hardening non-linearity must be introduced to prevent shock loading when "ride clearances" are overcome, it should be done in such a way that the greatest possible range of linear suspension characteristic is available for the absorption of normal ride motions.

With respect to the hardening -softening characteristic, it must be realized that although the performance of the non-linear spring is worse than that of a linear one of the same base rate, the primary object of this characteristic is to reduce the dependence of performance on load, im vehicles having large load variation. Thus as a comparison of vehicle performance, the response of the non-linear spring should be compared with that of a linear one having the same maximum rate, and in this case, the performance of the "constant natural frequency " spring is considerably better, but it is absolutely essential to provide the same magnitude of damping effort in order to ensure that this is so.

Thus, if a "constant natural frequency" spring is employed as a suspension characteristic, there is no relaxation of the damping requirements whatsoever, and the optimum controlled form in this case would be the same as that for a linear spring having the same maximum rate.

# APPENDIX. 1 .

## REFERENCES.


ABRAMSON, H.N. "Response Curves for a System with Softening Restoring Force"
A.S.M.E. J.of A.M. 1955 p434.

ALEXANDER, J "Effects of Various Accelerations on Sickness Rate"
J. of Psychology July 1945 .

ARNOLD, F.R. "Steady State Behaviour of Systems provided with Non-Linear Dynamic Vibration Absorbers"
A.S.M.E. J.of A.M. 1955 p457.

AULT, N. and GRAFF, G. "Numerical Analysis of Damped Vibration Systems"
Machine Design May 1952

BASTOW, D. "Independent Rear Suspension"
I.Mech.E Proc.A.D. 1950-51

BonVALLET, G.L. "Levels and Spectra of Transportation Vehicle Noise"
J. Acoustic Soc. America March 1950

BOWYER, D and GOODALE, P "Magnetic Liquid Dampers for Vehicle Suspension"
University of Adelaide. Honours Thesis 1956

BRISTOW, J.R. "Noise in Private Cars"
I.Mech,E. Proc. A.D. 1951

BROCK, J. "An Iterative Numerical Method for Non-Linear Vibrations"
A.S.M.E. J.of A.M. March 1951

BULL, A.W. "Tyre Behaviour in Steering"
S.A.E. Journal 1939

CAIN, B.S. "Vibration of Road and Rail Vehicles"
Pitman Publications 1940 .

CAMPBELL, I. "Sports Car"
Chapman-Hall Publication 1954

COX, H.L. "Riding Qualities of Wheeled Vehicles"
I.Mech.E. Proc.A.D. 1955-56 p265

CREDE, C.E. "Vibration and Shock Isolation"
Wiley Publication 1951

DEAN-AVERNS, R. "Vehicle Vibration - A Study Relating Mainly to Suspensions"
Auto. Eng. Aug. 1956

DEN HARTOG, J.P. "Mechanical Vibrations"
McGraw-Hill Publication 1947 3rd Edition

DEN HARTOG, J.P. "Forced Vibration with Combined Viscous and Coulomb Friction "
A.S.M.E. Trans. 1931 APM -53 - 9

DEN HARTOG, J.P. and HEILES, R.M. "Forced Vibration in Non-Linear Systems with Various Combinations of Linear Springs"
A.S.M.E. Trans. J.of A.M. 1936 pA127

DEN HARTOG, J.P. and MIKINA, S.J. "Forced Vibration with Non-Linear Spring Constants"
A.S.M.E. Trans. 1932 APM-54-15

EBERHORST, R.E. "Roll Angles"
Auto. Eng. Oct. 1951

ENGLEBERG, E. "The Analysis of Transient Vibrations by Means of a Mechanical Analogue"
Mechanical Engineering Oct. 1950

ERGIN, E.I. "Transient Response of a Non-Linear System by a Bi-Linear Approximation Method"
A.S.M.E. J.of A.M. 1956 p635

FLETCHER, H and MUNSON, W.A. "Loudness, its Definition, Measurement, and Calibration"
J. Acoustic Soc. America. Oct. 1933.

FOGG, A. "The Motor Industry Research Association Proving Ground"
I.Mech.E. Proc.A.D. 1955-56 p89

GALLAGHER, J and VOLTERRA, E. "A Mathematical Analysis of the Relaxation Type of Vehicle Suspension"
A.S.M.E. J.of A.M. Sept.1952 and March 1953

GELDHARD, E. "The Perceptibility of Mechanical Vibration"
J. of General Psychology 1940

HANSEN, H. and CHENEA, P. "Mechanics of Vibration"
Wiley Publication 1952.

HICKS, H.A. and PARKER, G.H. "Harshness in the Automobile"
S.A.E. Journal 1939

HUANG, F.C. "Harmonic Oscillation of Non-Linear Two Degrees of Freedom Systems"
A.S.M.E. J.of A.M. 1954 and 1955

JACKLIN, H.M. "Human Reaction to Vibration"
S.A.E. Journal. 1936 .

JACKLIN, H.M. and LIDDELL, G.L. — "Riding Comfort Analysis" Purdue University Research Bulletin No. 44.

JACOBSEN, L.S. — "On the General Method of Solving Second Order Ordinary Differential Equations by Phase-Plane Displacements" A.S.M.E. J.of A.M. Vol. 19. 1952 p543.

JACOBSEN, L.S. — "Steady Forced Vibrations as Influenced by Damping" A.S.M.E. J.of A.M. 1936

JACOBSEN, L.S. and JESPERSEN, H.J. — "Steady Forced Vibration of a Single Mass System with a Non-Linear Restoring Force" J.of Franklin Institute Vol.220 1935.

JAEGER, J.C. — "Applied Mathematics" Oxford University Press. 1951

JANEWAY, R.N. — "Passenger Vibration Limits" S.A.E. Journal 1948 .

JOY, T.J.P. and HARTLEY, H.C. — "Tyre Characteristics as Applicable to Vehicle Stability Problems" I.Mech.E. Proc.A.D. 1953-54 p 113 .

KELVIN, LORD. — "On the Graphical Solution of Dynamical Problems" Phil. Mag. Vol.34. 1892

KIMBALL, A.L. — "Vibration Damping" A.S.M.E. Trans. Vol.51. 1929

KINCHLEY, J.W. and STOCK, C.R. — "Shock Absorbers" I.Mech.E. Proc.A.D. 1951.

KLOTTER, K. "Free Oscillation of Systems having Quadratic Damping, and Arbitrary Restoring Forces" A.S.M.E. J.of A.M. 1955 p.493 .

KLOTTER, K. and PINNEY, E. "A Comprehensive Stability Criterion for the Forced Vibration of Non-Linear Systems" A.S.M.E. J.of A.M. Vol.20. 1953. p.91.

KOFFMANN, J.F. "Vibration and Noise" Auto. Eng. Feb.1957.

LANCHESTER, D. "Rolling and Lateral Oscillations" I. Auto. Eng. Vol.X.L.

LIPPERT, S. "Human Response to Vertical Vibration" S.A.E. Journal 1947.

LAY, W.E. and LEFT? P.W. "Wind Effect on Car Stability" S.A.E. Journal 1953.

LEFSCHETZ, S. "Contributions to the theory of Non-Linear Oscillations" Purdue University Press. 1950. 1952 and 1956

LEWIS, F.M. "Extended Theory of the Viscous Vibration Damper" A.S.M.E. J.of A.M. 1955 p 377 , 1956 p 153

LUDEKE, C.A. "An Experimental Investigation of Forced Vibration in Mechanical Systems Having Non-Linear Restoring Force. J.of Applied Physics. Vol.17. 1946 p 603

LAY, W.E. and FISCHER, L.C. "Riding Comfort and Seat Cushions" S.A.E. Journal 1940

LIND-WALKER, G.E. "Directional Stability"
Auto. Eng. 1950

MAHALINGHAM, G. "Forced vibration of Systems with Non-linear Non-Symmetric Characteristics"
A.S.M.E. J.of A.M. Sept. 1957.

McLACHLAN, N.W. "Ordinary Non-Linear Differential Equations in Engineering and Physical Sciences"
Oxford University Press. 1950

MERCIER, P.E. "Vehicle Suspensions"
Auto. Eng. Oct.1942.

MINORSKY, N. "Non-Linear Mechanics"
J.W.Edwards Publication 1947.

MYKLESTADT, N.O. "Fundamentals of Vibration Analysis"
McGraww-Hill Publication 1956 .

MYKLESTADT, N.O. "Concept of Complex Damping"
A.S.M.E. J.of A.M. 1953 p 284.

NICKELSEN, J.M. "Shock Absorbers"
S.A.E. Journal 1940

O'CONNOR, B.E. "Damping in Suspensions"
S.A.E. Journal Vol.54. 1946 .

OLLEY, M. "Independent Suspension - its Why's and Wherefores"
S.A.E. Journal 1934 .

OLLEY, M. "National Influences on American Passenger Car Design"
I. Auto. Eng. Proc. 1939 Vol. XXXII

OLLEY, M. "Road Manners of the Modern Car"
I. AUTO. ENG. Proc. 1947-48 Vol. XLI

PARKER, G.H. and HICKS, L "Harshness in the Automobile"
S.A.E. Journal 1939

PATON, C.R. "Ride Controls and Calibration"
S.A.E. Journal 1938

PATON, C.R., PICKARD, E.C. and HOEHM, V.H. "Seat Cushions and the Ride Problem"
S.A.E. Journal 1940

PETERS, J. "Magnetic Liquid Damping"
University of Adelaide, Graduation Thesis 1957.

PIPES, L.A. "Analysis of a Non-Linear Dynamic Vibration Absorber"
A.S.M.E. J. of A.M. Vol 20 Dec. 1953
" " Vol 21 Nov. 1954

POLHEMUS, V.D. "Secondary Vibrations in Rear Suspensions"
S.A.E. Journal 1950

POSTLETHWAITE, F. "Human Susceptibility to Vibration"
Engineering 1944

RAUSCHER, M. "Steady Oscillations of Systems with Non-Linear and Unsymmetrical Elasticity"
A.S.M.E. Trans. Vol 60. 1938. pA169.

ROBERTSON, R.E. "Synthesis of a Non-Linear Dynamic Vibration Absorber"
Journal of Franklin Institute Vol 254. 1952.

ROBERTSON, R.E. "An Iterative Method for Non-Linear Vibrations" A.S.M.E. J.of A.M. 1953 p 237.

SAG, N. "Comparative Tests on Gregoire versus Conventional Suspension on A.E.C. Chassis." Tech. Note. 133/56 S.A. M.T.T.

SAG, N. "Comparative Tests on Dual Rate versus Conventional Suspension on Leyland Chassis" Tech. Note. 259/57 S.A. M.T.T.

SCHILLING, R. and FUCHS, H. "Modern Passenger Car Ride Characteristics" A.S.M.E. J.of A.M. Vol. 63. 1941.

SOROKA, W. "Mechanical and Electrical Analogies of Vibration Isolation Problems" Product Engineering Feb. 1950

STONEX, K.A. "Car Control Factors and their Measurement" S.A.E. Trans. March 1941 .

SUSSHOLZ, B. "Forced and Free Motion Of a Mass on an Air Spring" A.S.M.E. Trans. 1944 pA101
" " 1945 pA175

TEA, C.A. "Secondary Vibrations in the Front Suspension" S.A.E. Journal 1950.

TIMOSHENKO, S. and YOUNG, D.M. "Vibration Problems in Engineering" Van Nostrand 1955.

WHEATON, G. "Wear Characteristics of Magnetic Liquids" University of Adelaide Graduation Thesis Dec. 1955.

WEBBER, I.E. "Initial Testing and Construction of an Experimental Trailer "
University of Adelaide Graduation Thesis
Dec. 1956.

WIEBEL, E.E., COKYUCEL, N.M. and BLAU, R.E. "A Mechanical Analogue for the Solution of Vibration Problems of a Single Degree of Freedom"
A.S.M.E. J.of A.M. June 1948

WOLF, A.M. "Individual Rear Suspension"
S.A.E. Journal 1947.

WOLF, A.M. "Experimental Suspension System"
Auto. and Aviation Industries. July 1948.

WOOD, R.H. "Vibration Limits"
J.Royal Inst.British Architects 1948

WYLIE, C.R. "On the Forced Vibration of Non-Linear Springs"
J. Franklin Inst. Vol. 236 1943.

## APPENDIX 2.

### MAGNET LIQUID DAMPERS.

The characteristics of practical dampers,employing a magnetic liquid as the working medium, were investigated under two conditions, firstly in the development of the mechanical analogue in which such dampers were used to simulate suspension damping as described in section 7.3, and secondly in the development of full size prototype dampers for use on the experimental trailer described in Appendix 5.

(1) MECHANICAL ANALOGUE DAMPER.

In the original design of this unit, it was realized that every effort was required to concentrate the generated magnetic flux in the magnetic liquid so that the maximum effeciency and minimum physical size of the unit could be obtained. Furthermore, in order to assure the maximum range of control of damping effort it was desirable to reduce the retentivity of the remainder of the magnetic circuit to the lowest possible value. To these ends the unit represented diagrammatically in Fig.7.6. was devised. The main body of the unit was manufactured from wrought iron and the magnetic path directed by brass insulating pieces to pass almost entirely through the magnetic liquid.

The magnetic liquid used was a mixture of the iron carbonyl powder normally used in the production of radio component cores in S.A.E. 10 oil in the proportions 7:1 by weight. This ratio was found to represent the most satisfactory compromise between high fluid density and long settling time which could be achieved with the cammon lubricating oils as a suspension medium. Also with these proportions, it was noticed,,that although the iron particles did separate out of suspension quite rapidly on standing, they did not form into a tightly packed layer for a considerable time, and, provided the standing time was not excessive, the mixture could be rapidly re-stirred into a homogeneous suspension. In fact, the damper design used proved to be insensitive to any settling of the iron particles which may have occured from time to time, and consequently gave consistent performance throughout the duration of the project. This is attributed to the "pure shear" action of the damper in developing resistance to motion, and although not the most effecient, would seem to be the most reliable form of damper if settling troubles cannot be otherwise eliminated.

Preliminary experiments with this magnetic liquid mixture indicated that its magnetic permeability u was in the region of 6, and this value has since been confirmed in more refined experiments by Peters (1957). Such a low value of permeability indicates that the mixture is very much less dense than the compacted powder for which values of u = 10,000 are quoted. Consequently, the major factor in determining the reluctance of the magnetic circuit by which the damping effort is controlled, is the magnetic liquid itself, and thus the flux path in this should be kept to the minimum consistent with the required capacity of the unit. With this in view, a series of experiments were carried out on the mechanical analogue unit to evaluate the effects of material and construction of the torque disc, as this provided the most convenient method of changing the effective flux path length in the magnetic liquid. Four different discs were used, these being of slotted and plain design, and of wrought iron and duralumin materials. The graph showing the overall results of these tests for constant current excitation of the damper is given in Fig.A.2.1, and this shows that when operating under identical conditions of amplitude and frequency, and with the same magnetising current, the maximum effort is developed by the damper which has a torque disc of high permeability. This is undoubtedly due to the far greater magnetic flux which is developed in the liquid under this condition, as the substitution of a duralumin disc decreases the capacity of the unit to approx. 25% of that achieved with a wrought iron disc, while the increase in magnetic reluctance causedby this substitution is 4 times as the disc contributes one third of the low permeability path length of the circuit. Consequently the maximum effort developed in the damper under particular operating conditions is directly proportional to the magnetic flux existing in the liquid itself, and the main importance of the material of the torque disc is the influence it has on the electromagnet effeciency.

It is interesting to note that with slotted torque discs, the damper effort developed exceeds that available with plain discs. In the case of the duralumin disc this is most pronounced and arises mainly from the increase in the average flux passing through the liquid, but any slotting of the wrought iron disc must introduce a reduction in this average flux when operating with the same energizing currents, as the permeability of the liquid is far less than that of the wrought iron. Thus there is a secondary effect arising in the shape of the torque disc which more than compensates for the reduction in the average magnetic flux in the fluid, and this is thought to be an improvement in the shearing action caused by flux concentration at the sharp edges of the slots. There will thus be an optimum shape of torque disc to give the greatest damping effort for a particular energizing current, but this optimum was not sort in these experiments, as the damper effort available was considered satisfactory.

The major features of this damper were,

(a) The resisting effort developed under constant current excitation was almost independent of the frequency of operation.

(b) Both the magnitude and shape of the Work Diagram were largely dependent on the amplitude of operation.

The first of these was particularly noticeable for the damper with the wrought iron torque disc, for which negligible increase in effort was detected over an operating frequency range of 10:1. With the duralumin discs, the dependence of damper effort on frequency was far more pronounced, but still much less than would be experienced with a linear damper, there being only a 60% increase in effort over the 10 : 1 speed range.

Typical examples of the damper work diagrams recorded during these tests are given in Fig.A.2.2. These show that when fitted with a wrought iron torque disc and operating at high amplitudes, the damper characteristic is very close to the mathematical ideal of Coulomb damping, the only deviation being a slight rounding of the characteristic after change in the direction of relative motion of the damper elements.
The physical distance over which this deviation from constant developed effort exists, is independent of the amplitude of operation of the damper, and consequently as this decreases, the rounding becomes relatively more and more apparent in the shape of the diagram until finally, at low amplitudes,of operation, the characteristic shape is distorted to a parallelipiped as shown in Fig.A.2.2. It thus appears that this form of damper requires a definite magnitude of relative motion between the elements after any change in direction, before the full effort can be developed. This necessary motion proved to be the same whether the disc were slotted or not, and consequently must be a characteristic of the magnetic flux path through the iron carbonyl mixture which requires re-orientation after every change in the direction of motion.

Basically, the same characteristics apply when the damper is fitted with a duralumin torque disc, but the work diagram is generally far more rounded both at changes of direction and throughout the stroke, indicating that the damper is dependent to some extent on the velocity of operation, and this is borne out by the increase in effort with speed of operation as mentioned above.

(2) PROTOTYPE SUSPENSION DAMPERS.

Two units were developed for full scale testing on the experimental trailer by Bowyer and Goodale (1956). The first of these was based essentially on the analogue unit, developing a torque to resist relative motion of its elements by a pure shear action, while the second was similar to a positive displacement vane type damper which developed resisting effort by pumping the liquid through a small clearance. The two units were developed separately by the experimenters, as it was feared that the pure shear type might not be capable of developing sufficient damping resistance, although from both settling and wear characteristics, which were shown to be quite appreciable by Wheaton (1955), it was superior in principle to the vane-type damper.

A damper testing machine was used during the development of the prototype dampers, to enable the performance under strictly controlled conditions to be investigated. The damper work diagram of full size units was obtainable on this machine by moving the elements relative to each other with simple harmonic motion of adjustable amplitude and frequency, and instantaneously recording the resistance developed and the position in the oscillation cycle as with the analogue units, Fig.A.2.3. shows the damper testing machine mounted with the magnetic liquid vane type damper.

The results of these experiments showed that the pure shear type damper was the most satisfactory as it was possible to arrange its magnetic circuit to be very effecient, and the capacity of the unit was ample when operated under constant energizing currents. On the other hand, the vane type damper proved to be most susceptible to both wear and settling of the iron particles from suspension, and could not be arranged to possess a good magnetic circuit so that the overall effeciency of the unit was far lower.

Under test the shear type damper proved to have identical characteristics with those of the analogue unit operating at small amplitudes of motion. Thus the shape of the work diagram was a parallelipiped for all the amplitudes of motion which could be achieved with the unit.

Although the capacity of the unit was satisfactory for operation under constant energizing currents, the same unit was later employed by Peters (1957) who attempted road tests with controlled damping as suggested by the results of the analogue, and under these conditions, the high electrical impedance of the circuit to rapid changes in energizing current seriously reduced the instantaneous capacity, and the necessary damping forces were not developed.

Consequently further development work is required on the magnetic liquid damper to ensure that the maximum capacity is achieved under constant energizing currents, and also to reduce to a minimum the overall time constant of the unit.

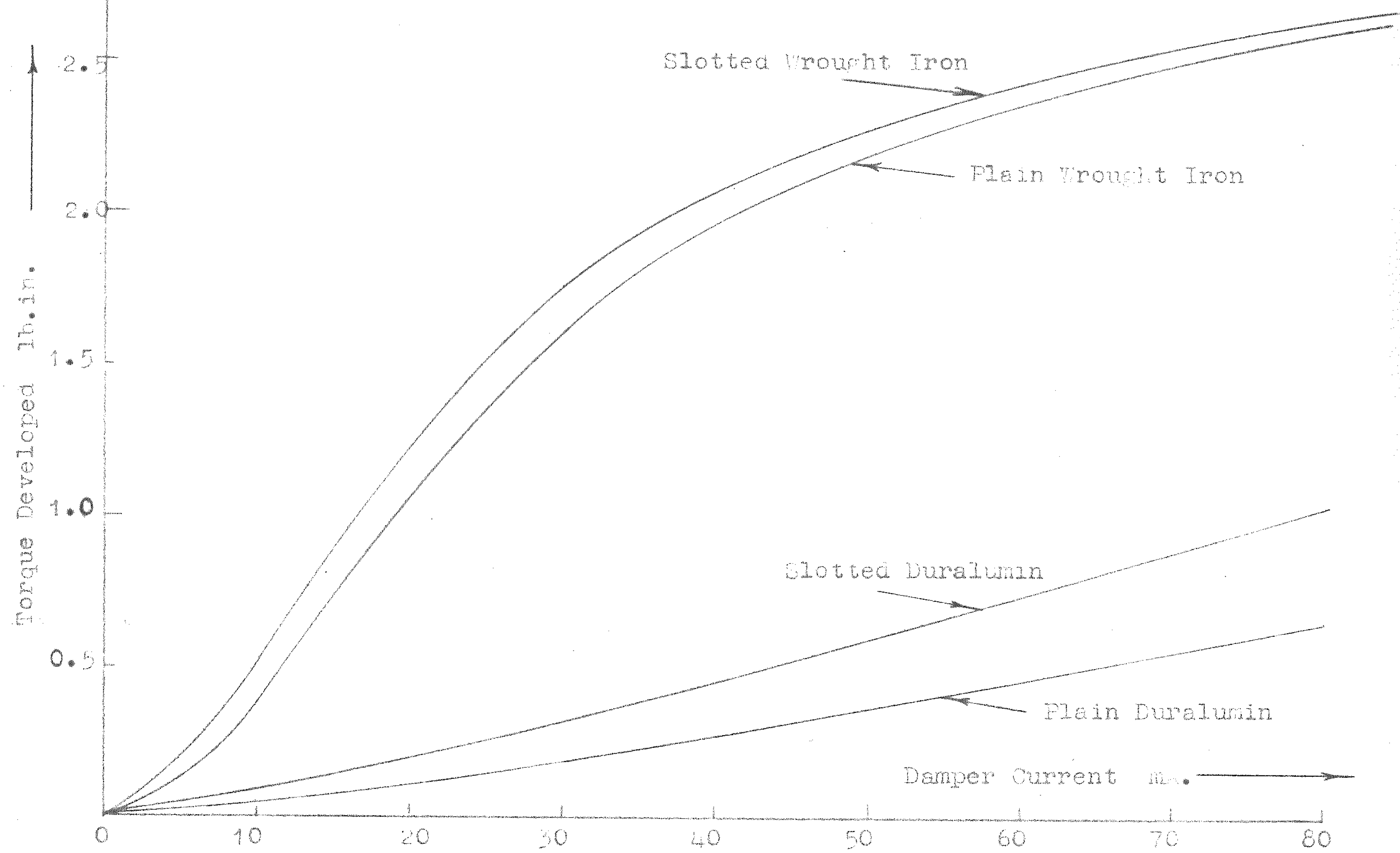



FIG. A.?.1. EFFECT OF TORQUE DISC ON THE PERFORMANCE OF MECHANICAL ANALOGUE DAMPER.

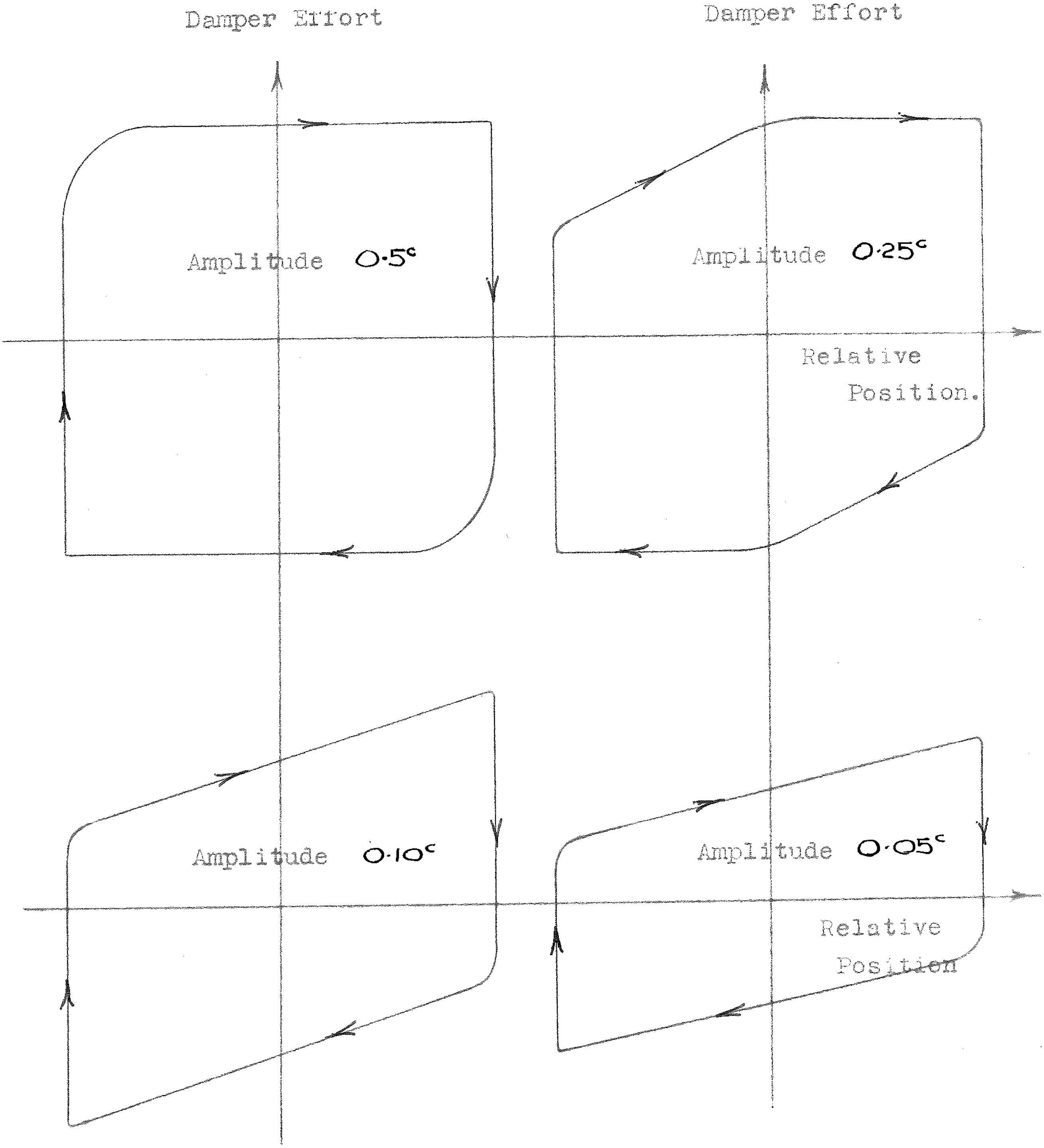



FIG.A.2.2. EFFECT OF AMPLITUDE OF MOTION ON THE "WORK DIAGRAM" OF MAGNETIC LIQUID DAMPERS WITH CONSTANT EXCITATION.

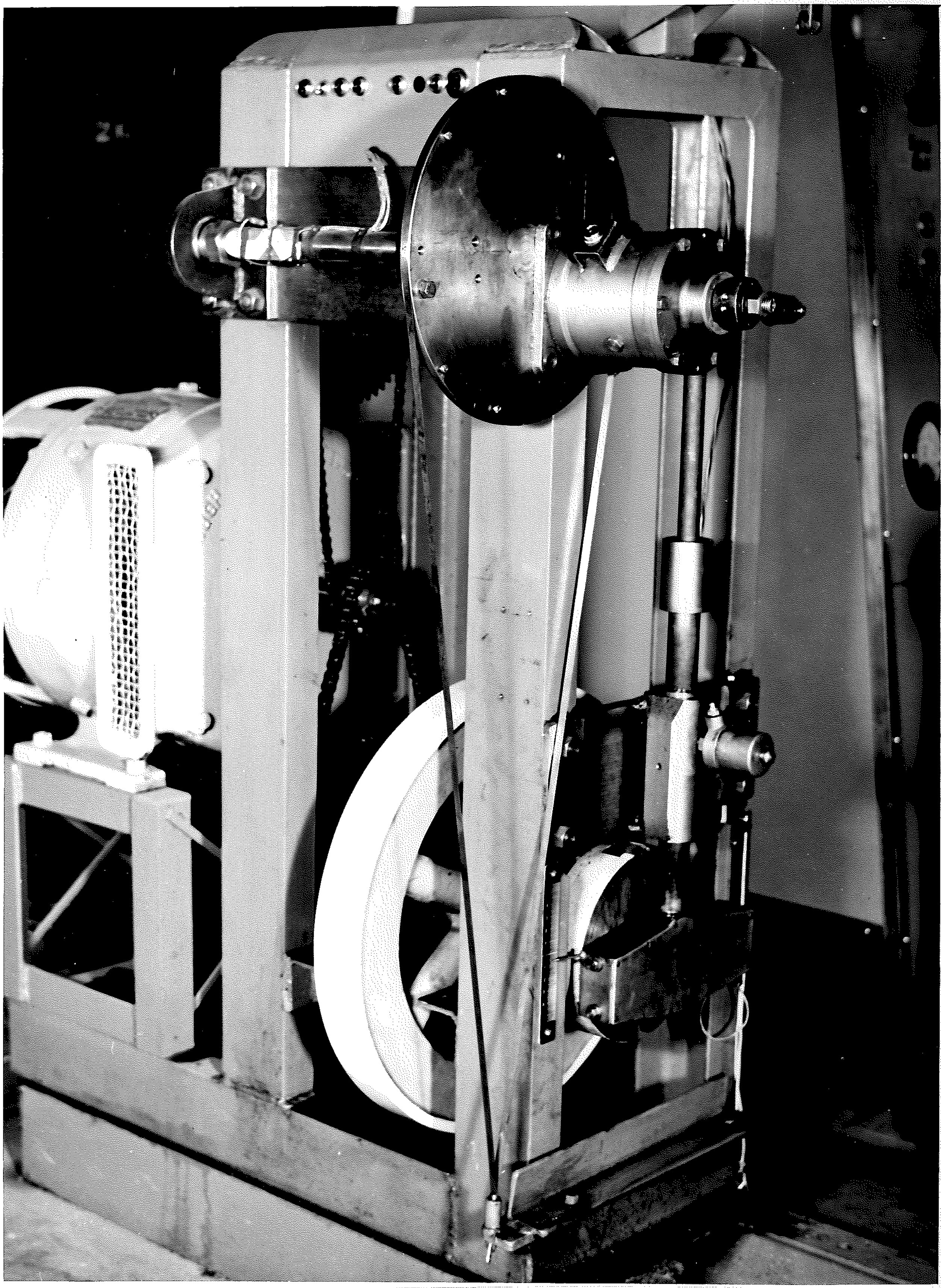


FIG.A.2.3. PROTOTYPE DAMPER TESTING MACHINE.

# APPENDIX. 3 .

## CALCULATED RESPONSE OF A SIMPLIFIED SYSTEM CAPABLE OF WHEELHOP VIBRATION.

The severity of oscillation amplitudes which can develop in the motions of the axle mass of a vehicle when the tyre loses contact with the road surface for a finite period of the oscillation cycle, may best be illustrated by analysis of the simplified system shown in Fig.A.3.1(a). The effective spring rate applied to motions of the mass in this system has the characteristic shown in Fig.A.3.1(b).

This implies that ;

(a) The tyre and body springs are basically linear.

(b) The motion of the body is negligible in the frequency ranges considered.

(c) The system is undamped.

(d) No energy is dissipated in the system when the tyre spring regains contact with the road surface.

Thus for deflections of the mass in one direction from the equilibrium position, the effective spring rate is consistently linear and of rate $K_1$, while for deflections of opposite sign, this spring rate is maintained only up to a value "a", and thereafter is reduced to a lower linear rate $K_2$.

The spring rates

$K_1$ = Combined stiffness of the tyre and suspension spring in series.

$$= K_A + K_B$$

$K_2$ = Stiffness of the suspension spring alone.

$$= K_B$$

## A.3.1. TRANSIENT RESPONSE .

The frequency and amplitude of oscillation of the system are independent only if the excursions into the linear region are less than or equal to "a" . If the double amplitude of the oscillation exceeds 2a, the excursion "x" into the softened region may be determined from considerations of energy balance assuming a motion A of the mass in the linear region.

Thus

$$x = \sqrt{(K_1/K_2)^2 . a^2 + (K_1/K_2)(A^2 - a^2)} - (K_1/K_2).a \qquad \text{.................(A.3.1.)}$$

Giving a total double amplitude of motion

$$A + a + x = \sqrt{(K_1^2/K_2^2) .a^2 + (K_1/K_2)(A^2 - a^2)} + a(1 - (K_1/K_2)) \qquad \text{..................(A.3.2.)}$$

Furthermore the period of oscillation can be determined from the equations of simple harmonic motion during the separate intervals A, a, and x . Thus, during the motion from position A to zero the time taken

$$t_1 = \frac{\pi}{2}\sqrt{\frac{m}{K_1}} \qquad \text{......................(A.3.3.)}$$

From 0 to "a" in overshoot takes time

$$t_2 = \sqrt{\frac{m}{K_1}} . \operatorname{arsin} \frac{a}{A} \qquad \text{...................(A.3.4.)}$$

during which time the mass acquires a velocity

$$v = \sqrt{(K_1/m) . (A^2 - a^2)} \qquad \text{.................(A.3.5.)}$$

The motion controlled by spring rate $K_2$ commences with this velocity and comes to rest in a distance "x" determined from Equation A.3.1.

This corresponds to a simple harmonic motion of amplitude

$$B = \frac{K_1}{K_2} \cdot \frac{(A^2 - a^2)}{2x} + \frac{x}{2} \qquad \text{.................(A.3.6.)}$$

Thus the time taken in travelling through the interval "a" to "x" is ;

$$t_3 = \frac{\pi}{2}\sqrt{\frac{m}{K_2}} - \sqrt{\frac{m}{K_2}} \text{ arsin. } \frac{K_1(A^2 - a^2) - K_2 . x^2}{K_1(A^2 - a^2) + K_2 . x^2}$$

$$\text{.................(A.3.7.)}$$

Consequently the complete period of oscillation is given by

$$T = (t_1 + t_2 + t_3) . 2$$

$$= 2\left( \frac{\pi}{2}\sqrt{\frac{m}{K_1}} + \sqrt{\frac{m}{K_1}} \text{ arsin } \frac{a}{A} + \frac{\pi}{2}\sqrt{\frac{m}{K_2}} - \sqrt{\frac{m}{K_2}} \text{ arsin } \frac{K_1(A^2 - a^2) - K_2x^2}{K_1(A^2 - a^2) + K_2x^2} \right)$$

$$\text{..........................(A.3.8.)}$$

The response of the system to a step in the road profile, given by the equations A.3.1 and A.3.8. is given graphically in Fig.A.3.2., the values of "a", m , $K_1$ , and $K_2$ , being selected from the analogue equivalence section 7.6. as 0.54 inches, 77 lb, 1240 lb per inch, and 60 lb per inch respectively.

Fig.A.3.2. shows that road contact is lost for some period of the ensuing vibration cycle whenever the road step exceeds the initial static tyre deflection. For road steps higher than this value, the period of lost road contact increases very quickly at first, but tends to reach a limit at the higher bump sizes. At the same time, the period of road contact decreases rapidly at first and then tends ass-ymptotically to a minimum value, so that the total period of the osc-illations rises rapidly at first, but does tend to a maximum value .

The excursions of the axle into the softened region also increase rapidly as the magnitude of the bump increases, thus if the bump is 1" the wheelhop is ½", but for a 2" bump the hop is 3", and for a 3" bump the wheelhop has risen to 6".

These results are consistent with an effective decrease in spring rate with increasing road bump caused by a greater percentage of the motion occuring in the softer spring. As this is in itself linear, the amplitude of motion will always increase faster than the magnitude of the step, but the oscillation period can never exceed that represented by the rate $K_2$ alone, and hence will tend to this value.

Although these results are somewhat pessemistic applied to an actual vehicle suspension in that they neglect both the curvature of the tyre which would be significant with steps of such magnitude, and the damping of the system, they do illustrate that ride clearances may easily be disrupted and that wheelhop may be caused by relatively small road irregularities.

A.3.2. HARMONIC RESPONSE.

The response of the system Fig.A.3.1. to a regular road surface undulation may best be determined graphically by the method of Mahalingham (1957). To do this the equation of motion of the mass in a similar linear system must be first derived.

Thus

$$m D^2 x_A = K_A(x_0 - x_A) - K_B x_A \quad \text{.......................(A.3.9.)}$$

$$\text{or} \quad m D^2 x_A + K_B x_0 = (K_A + K_B)(x_0 - x_A) \quad \text{................(A.3.10.)}$$

If the road surface is regular then

$$x_0 = X_0 . \cos pt \qquad \text{and} \qquad x_A = X_A . \cos pt \quad \text{........(A.3.11.)}$$

Thus

$$\frac{K_B}{m} \cdot X_0 - p^2 \cdot X_A = \frac{K_A + K_B}{m} \cdot (X_0 - X_A) \quad \ldots\ldots\ldots\ldots(A.3.12.)$$

In the non-linear case represented by Fig.A.3.1, the composite spring characteristic $(K_A + K_B)$ becomes a function of the relative motion between the axle and ground i.e. $(X_0 - X_A) = x$. The right-hand side of equation A.3.12. is thus replaced by a "frequency function" $F(x)$ after Mahalingham, where $F(x) = w_x^2 \cdot x$, the value $w_x$ being the free vibration circular frequency having a double amplitude of 2x as determined from Equation A.3.8.

Thus

$$(K_B/m).X_0 - p^2.(X_0 - x) = w_x^2 \cdot x \quad \ldots\ldots\ldots\ldots(A.3.13)$$

$$\text{or} \quad (K_B/m - p^2).X_0 + p^2.x = w_x^2 \cdot x \quad \ldots\ldots\ldots\ldots(A.3.14)$$

The solution of this equation is obtained graphically by the intersection of the two curves

$$\left.\begin{aligned} y &= (K_B/m - p^2)\,X_0 + p^2 x \\ \text{and} \quad y &= w_x^2 . x \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(A.3.15)$$

This procedure is illustrated in Fig.A.3.3. for an excitation amplitude of 0.25 inches, and the resultant harmonic responses are plotted in Fig.A.3.4. This indicates immediately the typical softening non-linear characteristic and the tremendous magnification of axle motion which can result if the tyre and road surface lose contact during the vibration cycle. As both of the road amplitudes investigated are in fact far less than the static tyre deflection, it is thus obvious that damping forces are essential in the suspension to keep the axle oscillation amplitudes to a reasonable level.

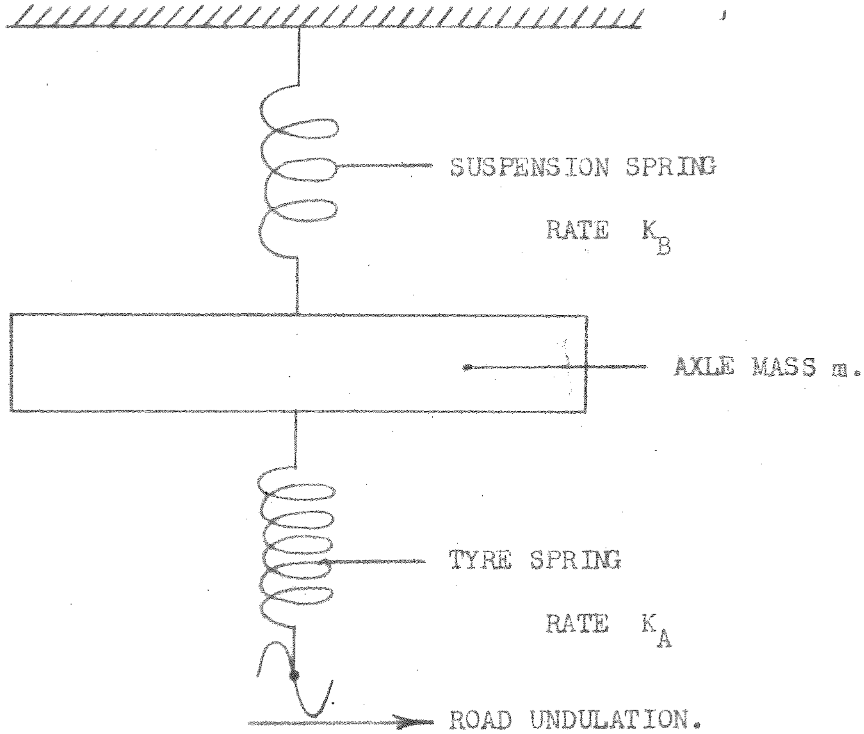



Fig. A.3.1.(a)

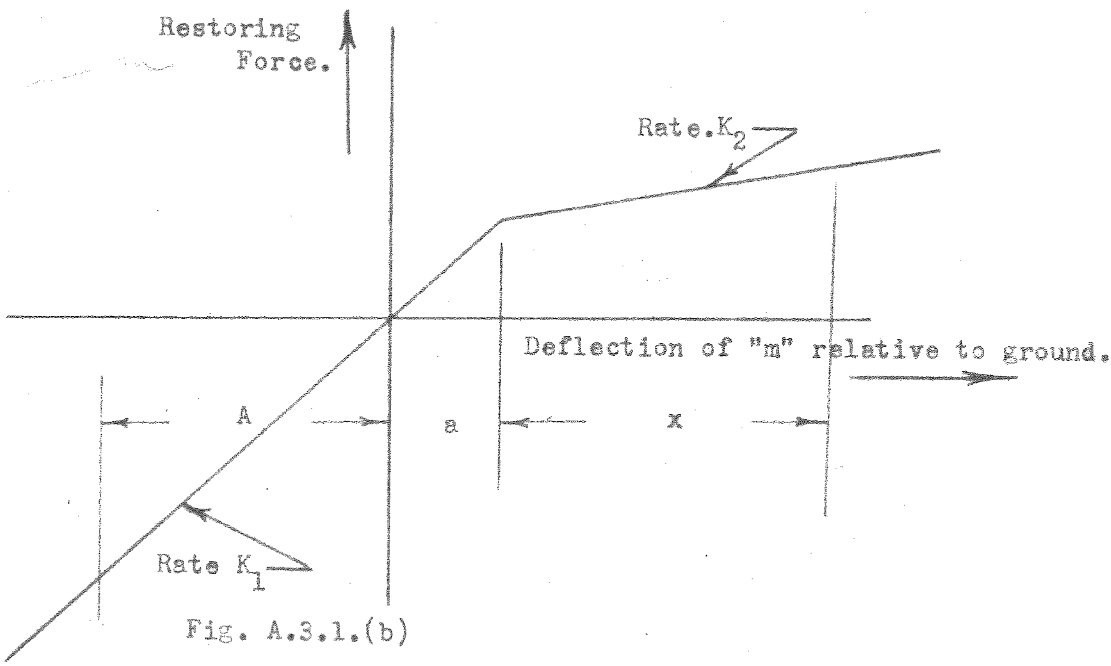



Fig. A.3.1.(b)

FIG.A.3.1. BASIC CHARACTERISTICS OF SIMPLIFIED SYSTEM CAPABLE OF WHEELHOP.

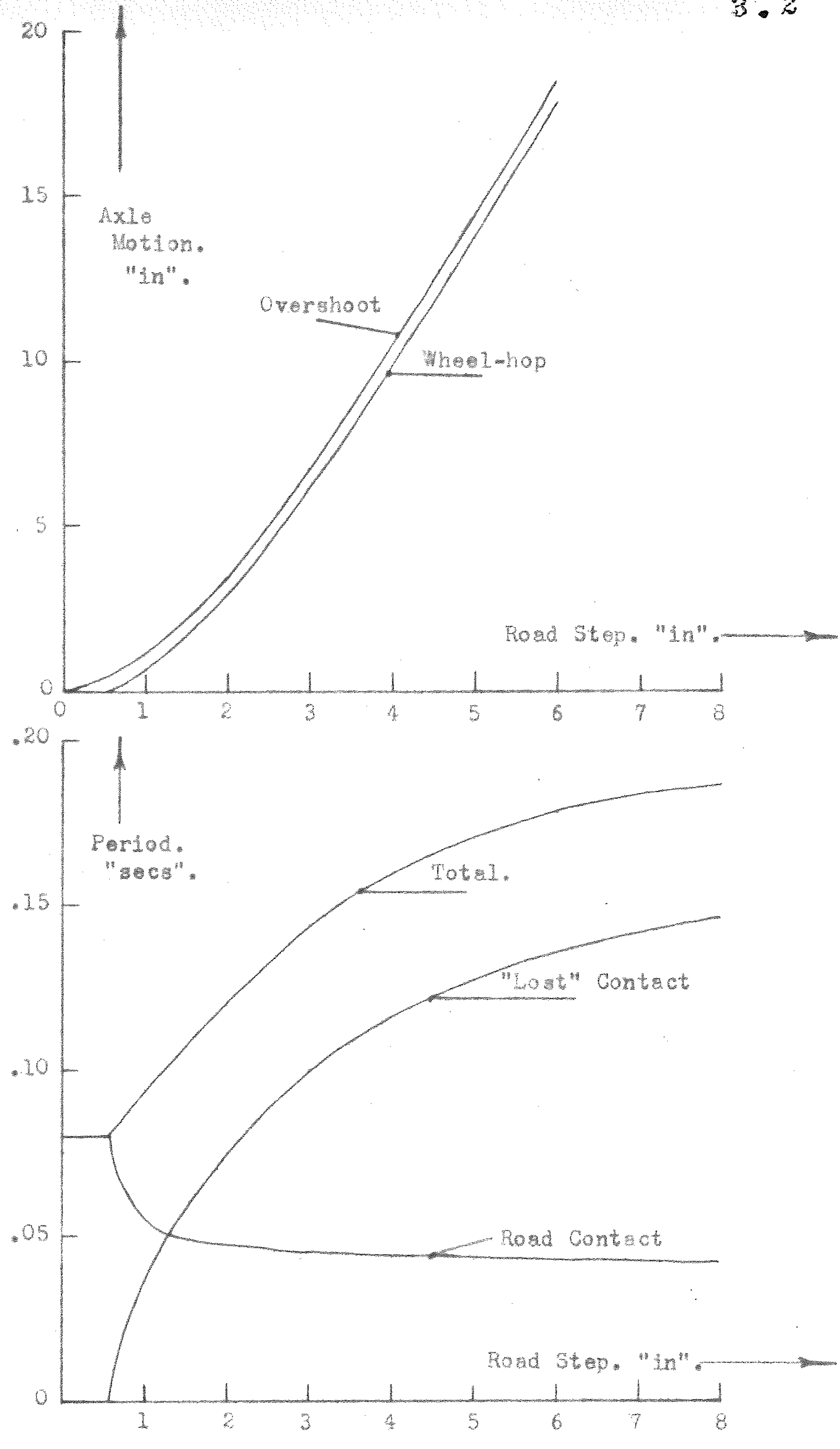



FIG.A.3.2. TRANSIENT RESPONSE TO A ROAD STEP - WITH WHEEL-HOP.

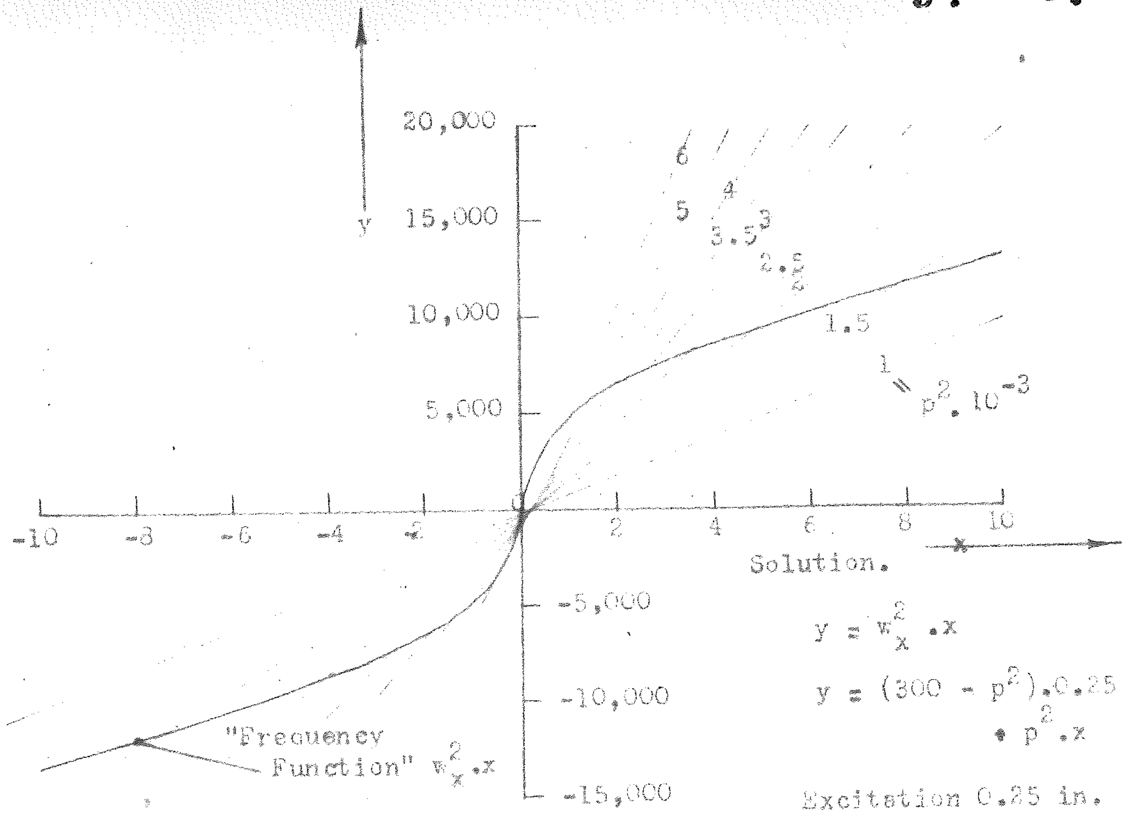



FIG.A.3.3. GRAPHICAL SOLUTION OF HARMONIC RESPONSE -WITH WHEEL-HOP.

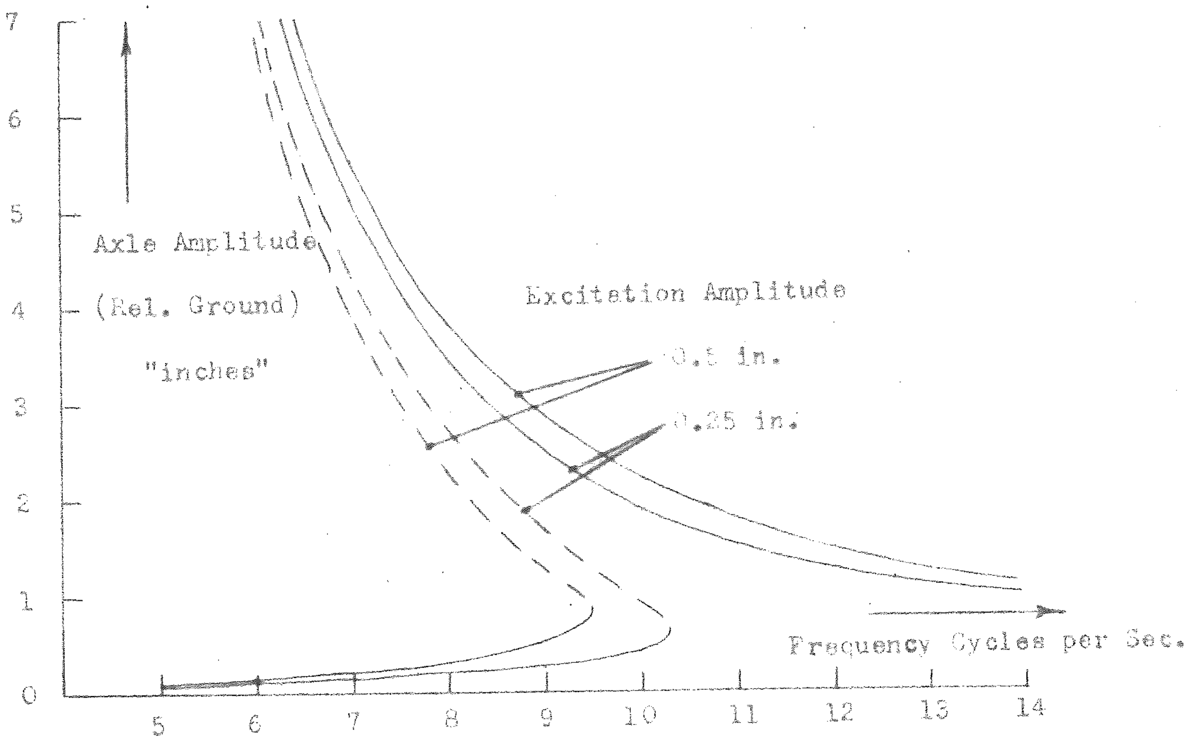



FIG.A.3.4. HARMONIC RESPONSE TO REGULAR ROAD UNDULATIONS - WITH WHEEL

## APPENDIX 4.

## ELECTRICAL RESISTANCE STRAIN GAUGE ACCELEROMETERS.

Electrical resistance strain gauge accelerometers were developed during the course of the project for the instantaneous recording of the accelerations of various components. Although initially designed for use on the mechanical analogue to measure the angular accelerations of the axle and body inertias, the design was later modified to facilitate measurement of vertical accelerations in vehicles under road test when it became obvious that the commercially available units had serious limitations.

### A.4.1. MECHANICAL ANALOGUE ACCELEROMETERS.

A typical unit is illustrated in Fig.A.4.1, and consists of a small mass which is flexibly supported in one plane from a base frame, the whole being magnetically shielded to reduce external pick-up of electrical signal. The accelerometer mass is suspended on a flexible steel shim so that it is free to oscillate about a vertical axis only when the unit is in its designed horizontal mounting position, but oscillation is restrained by two cantilever strips which bear on pivot points machined on to the ends of the accelerometer mass. These outer strips have Phillips PR9212 - 600 ohm electrical resistance strain gauges glued to both their inner and outer surfaces, and the leads from these are connected to plug points in the insulated back piece.

The accelerometer is initially set up with a balanced pre-load in both the outer strips so that the mass is in equilibrium and can only move relative to the frame by relaxation of the preload in one strip, and a corresponding increase in load on the other strip. As the two outer strips are identical, the inner gauge on one strip suffers exactly the same strain as the outer gauge on the other strip and vice versa so that the four gauges can be wired in bridge circuit to give the maximum signal to acceleration ratio with complete temperature compensation.

In operation on the analogue, the tangential forces on the mass caused by changes in angular velocity are also accompanied by changes in radial force which are malphased, but these are taken by the central shim supporting the mass, and thus do not distort the angular acceleration signal provided by the gauges.

Damping of the accelerometers was achieved by an oil film connection between the flat bottom of the mass and the frame of the unit. The magnitude of the damping was adjusted by selecting an oil viscosity and a spacing between the mass and frame such that the recorded overshoot after a step displacement of the mass was between 10 and 20 %, the damping then being in the range 0.5 to 0.7 critical as recommended by DenHartog (1947).

The initial experimentation on the analogue indicated that two units of different natural frequency were required to provide satisfactory electrical signals throughout the wide range of working frequencies. This is consistent with the general feature of accelerometers that they require a natural frequency higher than the highest frequency to be accurately measured, but the higher this value, the smaller is the signal to acceleration ratio. Two units were thus included on the analogue on each of the oscillating inertias, the accelerometers being physically similar but one having 0.007 inch thick DURAL strain gauge strips giving a natural frequency of approximately 40 cycles per sec while the second having 0.012 inch thick steel strain gauge strips had a natural frequency of about 150 cycles per sec. The low frequency accelerometers were suitable for body resonance oscillations, but the stiffer units were essential at the axle resonant speeds especially on the axle inertia where the accelerations were high. As it was necessary to permanently build the accelerometers onto the analogue elements to establish constant parameters for the system, the low frequency accelerometers were also provided with locks to prevent accelerometer mass motions relative to their frames during operation at speeds approaching their resonant frequencies or under conditions of high acceleration, as such could easily damage these units.

The accelerometers gave satisfactory results on the analogue, but they did not give the accuracy anticipated in the harmonic excitation conditions because of distortion in the signals arising from the complete frame vibrations of the welded pedestal to which the analogue was attached. Although this was greatly reduced by stiffening the frame, it could not be completely eliminated, and this feature, coupled with the difficulty of indicating damper performance by acceleration measurement, resulted in the units being superceded by the damper torquemeter described in section 7.4.2. Nevertheless their use in transient tests was most satisfactory, as under these conditions frame vibrations excited by the drive motor were absent, and the electrical signals from the accelerometer were free from distortion.

A.4.2. <u>ROAD VEHICLE ACCELEROMETER.</u>

This unit was developed from the analogue accelerometers after early road tests had been carried out with a commercially available accelerometer, and it became obvious that the natural frequency of this unit was too low in comparison with the vibration frequencies which could exist in road vehicles.

Fig.A.4.1. also shows the vehicle accelerometer. In this case, the accelerometer has a vertical axis, but is pre-loaded between 0.015 inch thich steel strain gauge strips as before, the unit having a natural frequency of approximately 100 cycles per second.
To ensure adequate fixing of the mass, and to prevent "stiction", the preload reaction is taken through pivot bearings fixed to the flexible strain guage strips and hardened steel points on the accelerometer mass.

Damping is provided by an oil dashpot whose cup is machined into the baseplate of the unit with the plunger made as an extension to the lower pivot bearing. Optimum damping was selected as for the analogue units by adjusting oil viscosity to give 10-20% overshoot in response to a step displacement of the mass. A close tolerance cover was provided for the dashpot so that the unit can be lain on its side for short periods without losing the damping oil. This facilitates static calibration of the unit, as it enables the gravity loading on the accelerometer mass to be relieved from the strain gauge strips, and hence provides an effective plus and minus 1g calibration .

As for the analogue units, the entire accelerometer was enclosed in an iron shield for protection and electromagnetic pickup shielding, so that the maximum signal to noise ration can be achieved.

This unit was used in numerous road tests as described in Appendices 5 and 6, and gave satisfactory results under all conditions to which it was subjected, so that it does present a robust and accurate accelerometer design, its greatest drawback being the electrical amplifiers and pen-recorders necessary to convert the signal to a visible trace of acceleration versus time. In this respect, the transistor D.C. Pre-Amplifiers developed in the Adelaide University Mechanical Engineering Department were invaluable, as they provided the required high magnification of signal with adequate stability, and possessed the robustness essential in a portable unit used for road testing.

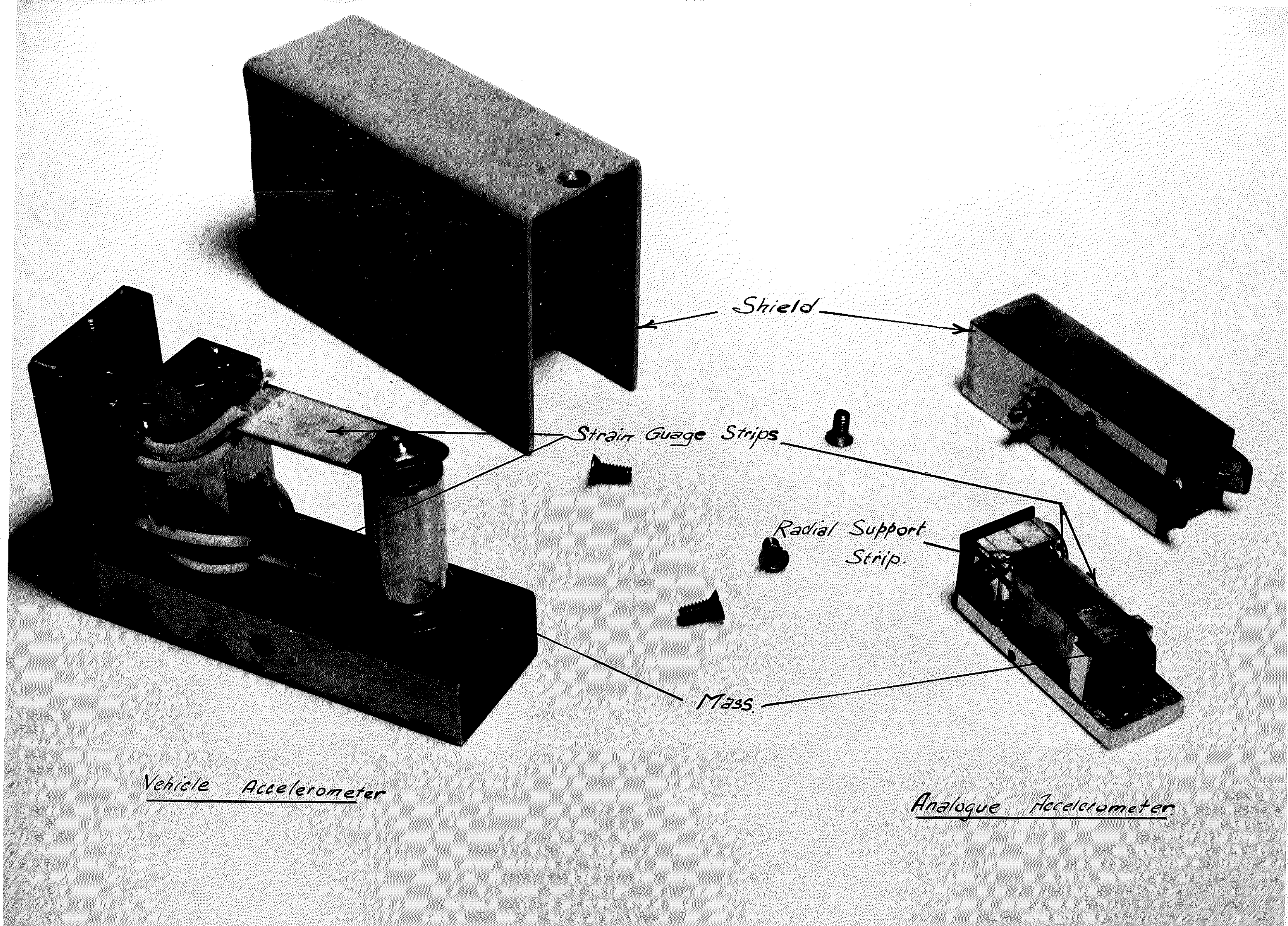



FIG. A.4.1. ELECTRICAL RESISTANCE STRAIN GAUGE ACCELEROMETERS.

## APPENDIX. 5 .

## ROAD TESTS .

Although the mechanical analogue facilitated investigation of the effects of non-linear springing and damping in the suspension of road vehicles under strictly controlled conditions, it did require simplification of the problem by assuming tyre characteristics and concentrated elements while neglecting primary interaction and wheel-hop and considering only mathematical amenable excitation conditions.

Propotype testing of units with characteristics suggested by the results of the analogue under actual conditions of operation encountered by a road vehicle suspension was thus desirable.

### A.5.1. EXPERIMENTAL TRAILER.

In view of the great number of interacting factors in the behaviour of a vehicle suspension, it was decided to minimise the number of parameters of the system employed, and to separate the sources of damping and springing as far as possible. To these ends an experimental trailer was finally selected, and this was fitted with a Morris Oxford type independent torsion bar suspension by Webber (1956). By this means the interaction of bounce and pitch was eliminated, and the effect of vertical motion of the hitch point on the towing vehicle could be minimised by recording accelerations immediately above the trailer axles.

Furthermore, the system allowed great flexibility in the characteristics employed for damping and springing as the torsion bars could be uncoupled and subsidiary spring systems substituted, while magnetic liquid dampers could be inserted in the upper wishbone link as in the original suspension and thus did not disturb its geometry. When actually testing the various suspension characteristics, every effort was made to present strictly comparable systems, so tyre pressure and wheel load were adjusted to the design value of the basic suspension. To reduce the manufacturing costs of prototype suspensions, and also to facilitate rapid comparison of road tests, only one wheel of the trailer was used for the experimental units, while the other was maintained with the original torsion bar and conventional damper. The procedure during road test was thus to run first the conventionally sprung wheel over the prepared track, and then the experimentally sprung wheel over the same track to compare the responses.

A general view of the experimental trailer is given in Fig.A.5.1 which shows the loaded unit mounted with recording instrumentation and a magnetic liquid damper experimental suspension.

A.5.2. <u>EXCITATION CONDITIONS.</u>

In view of the comparative nature of road testing, it is essential that the conditions of excitation be known exactly, and be capable of exact reproduction. There was unfortunately no test track available where the road surface conditions could be guaranteed to remain constant, so all road tests were conducted on a specially prepared section of flat bitumen road. In general the standard tests were achieved by driving the trailer over wooden obstacles of known dimensions which were fixed to the road surface. These included a singular bump, positive and negative steps followed by considerable settling lengths with slow lead-off, and a series of regularly spaced planks simulating a corrugated road surface. The long wave length of road undulations required to excite body resonances of the trailer rendered impractical any attempt to reproduce such a surface by this means, so body resonance tests were not included, but it is hoped that a suitable track will be available in the future.

The procedure during road testing was to tow the trailer at various speed in the normal range over the standard obstacles while recording the resultant performance of the suspension. By this means, the complete speed response of the various systems was determined and overall comparisons could be made.

### A.5.3. INSTRUMENTATION FOR ROAD TESTING.

The basic instrumentation for road testing was an accelerometer for measuring the resultant accelerations of the body mass during passage of the trailer over the standard obstacles. From the accelerometer records, the magnitude of the force transmission to the body could be determined, and a comfort index derived by the method suggested by Jacklin (1936). Initially the unit used was a Cambridge Two Channel Accelerometer which was used to measure the vertical and for and aft accelerations of the body, but analysis of the resulting traces indicated that much of the record was composed of frequencies in excess of the natural frequency of the instrument i.e. 36 cycles per sec. and consequently these were not true measures of the accelerations of the trailer body.

The predominant vibration frequency in these records proved to be the the general panel oscillation of the resilient trailer body and thus could not be eliminated, but satisfactory accelerometer traces could be obtained by isolating the instrument from the trailer chassis by a special rubber pad. Although this naturally did modify the form of the recorded accelerations, the results were quite satisfactory for a comparative series of tests, and adequate for the determination of a comfort index, but it was felt that such results did not give a true enough indication of force transmission, so the high frequency strain gauge accelerometer described in Appendix.4.was constructed and thereafter used for direct indication of the panel accelerations as it proved most satisfactory.

The only additional instrumentation which was found to be essential in the preliminary road tests was a relative position indicator between the axle and body which served to indicate possible wheel-hop as none of the tests attempted resulted in large body movements.

A.5.4. <u>PROTOTYPE MAGNETIC LIQUID DAMPERS.</u>

Road tests of these units were first carried out by Bowyer and Goodale (1956) after a development programme to produce satisfactory magnetic liquid dampers for vehicle use. Their investigations were limited to constant current excitation of the dampers, but even with this limitation they were able to indicate regions of improved performance over the conventional damper with which comparison was made. A later attempt was made by Peters (1957) to control the damper diagram shape and magnitude in the method suggested by the analogue results by using the same damper unit, but this proved unsuccessful as the high impedance of the electrical circuit to rapid changes in the damper current seriously reduced the resultant torque output of the unit below the required values. Further development work on the magnetic liquid damper prototype is thus required before conclusive results on this aspect can be verified in road tests.

A.5.5. NON-LINEAR SPRINGING.

The effect of non-linear spring arrangements mounted in the trailer suspension on the response to the standard obstacles was investigated by Lord (1957) and Chartres (1957).

Lord employed an André-Niedhart system using rubber in combined compression and shear to give a constant natural frequency spring, while Chartres used an air-cell type suspension having characteristics far closer to linear under static test. Unfortunately the road testing of these suspensions was only supplementary to the development of prototype suspension units capable of use on the trailer, so the results were not sufficiently comprehensive to be regarded as conclusive but further development work on these units is progressing .

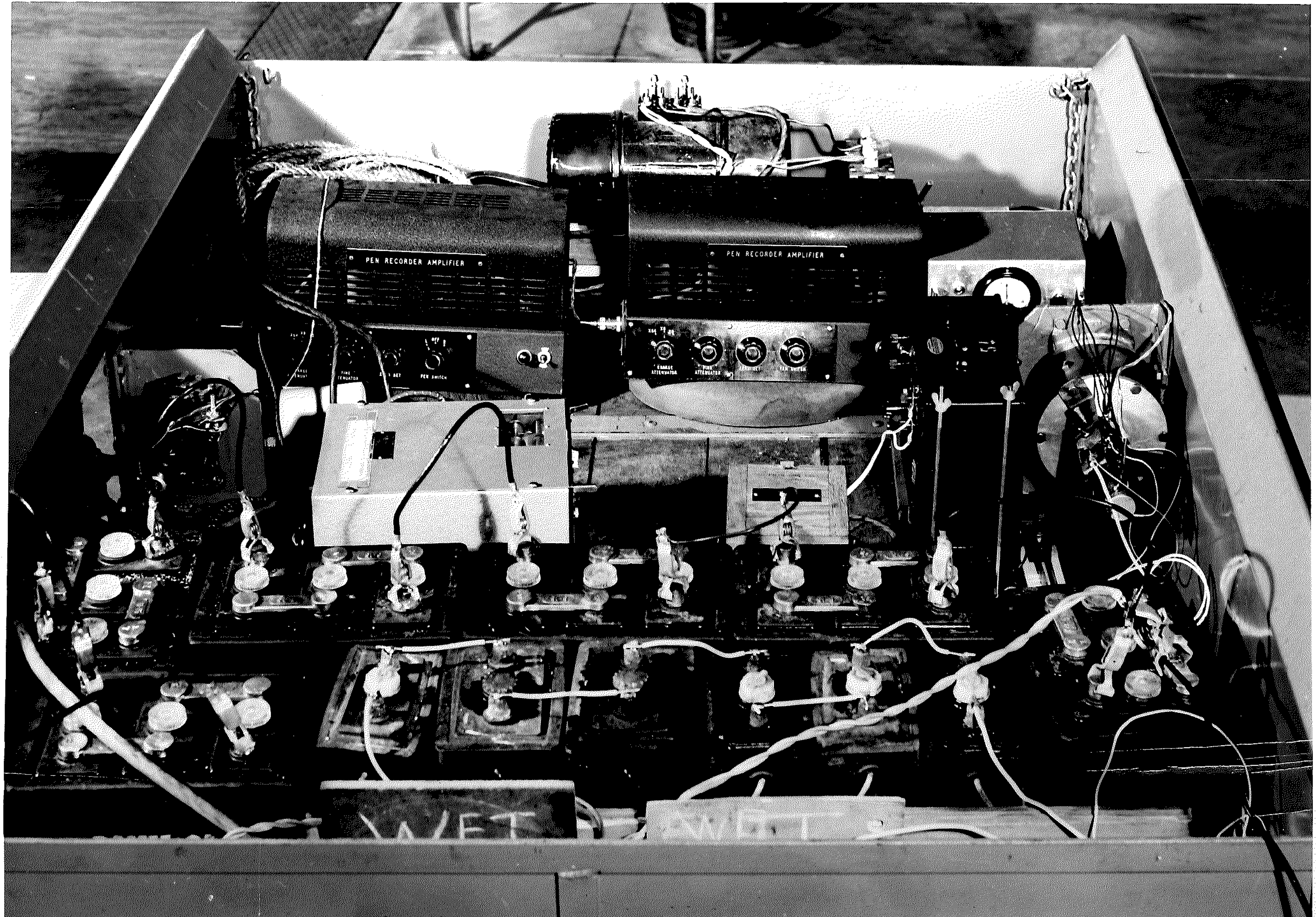



FIG.A.5.1. EXPERIMENTAL TRAILER MOUNTED WITH MAGNETIC LIQUID DAMPER, SHOWING ALSO THE RECORDING INSTRUMENTATION FOR ROAD TEST.

## APPENDIX. 6.

### MUNICIPAL TRAMWAYS TRUST BUS TESTS.

During 1956 and 1957, road tests were carried out by the author for the South Australian Municipal Tramways Trust, in order to evaluate the improvements in performance available in single decker diesel buses when fitted with non-linear suspension springs. The spring characteristics tested were of the unsymmetric hardening-softening type, and aimed at producing a basic body oscillation frequency independent of the live passenger load carried by the buses, thereby improving part load performance. With this in view, non-linear springs were only considered for the rear suspensions of these vehicles.

Two mechanical systems for achieving such characteristics, suggested by different chassis manufacturers, were tested under strictly similar conditions to aid in a final selection. These arrangements, the Gregoire type fitted to an A.E.C. chassis, and a Dual Rate type fitted to a Leyland chassis are shown diagrammatically in Fig.A.6.1. together with their basic characteristics.

The criteria of performance were selested as the force transmission to the body structure, and the passenger comfort, so accelerometry was the only instrumentation required. The Cambridge two channel accelerometer was mounted above the rear suspension of the test vehicles on the standard seat cushions and served to evaluate the passenger comfort conditions by the method suggested by Jacklin (1936), while the electrical resistance strain gauge accelerometer was mounted directly on the vehicle floor to measure the magnitude of force transmission directly.

The tests were conducted on a stretch of specially prepared track having a 3 inch deep channel 20 inches wide to similate a pot-hole or negative bump, and a 2 inch board 12 inches wide representing a positive protrusion in the road surface. To assess the overall performance, each bus was driven over this track at 10, 15, 20, 25, and 30 miles per hour at each of two loading conditions representing unladen and half load capacities. The responses recorded during these tests were then compared with those obtained on identical buses fitted with conventional semi-elliptic springs having linear characteristics.

Detailed reports of these tests were issued by Sag (1956 and 1957) but the overall comparison based on the mean of the values recorded throughout the test speed ranges is given in the following Table.A.6.1.

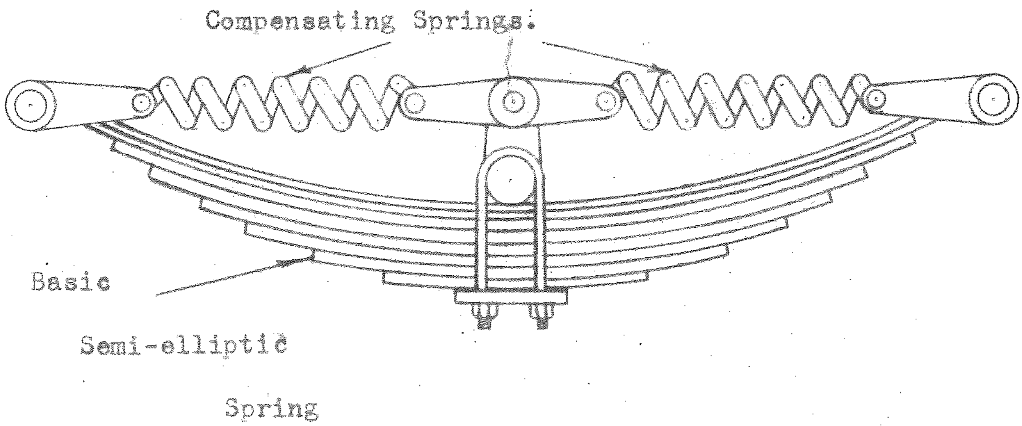
TABLE. A.6.1. AVERAGE PERCENTAGE IMPROVEMENT OF NON-LINEAR SUSPENSIONS OVER STANDARD SUSPENSIONS FITTED TO M.T.T. BUSES.

| | Passenger Comfort | | Force Transmission | |
|---|---|---|---|---|
| | Neg. Bump | Pos. Bump | Neg. Bump | Pos. Bump |
| Gregoire Unladen | 105% | 90% | 23% | 34% |
| Gregoire 50% Load | 24% | 82% | 11% | 38% |
| Dual Rate Unladen | 34% | 11% | 8.5% | 11% |
| Dual Rate 50% Load | 26% | 22% | 8.5% | 13% |

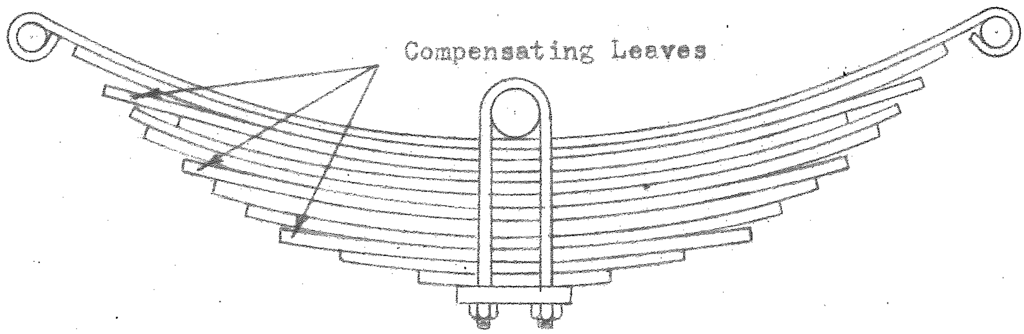
These results confirmed that the "Constant Frequency" type of non-linear spring does provide a considerable improvement in performance over that of the linear rate spring when the vehicle operates under lightly loaded conditions, and indicates that this improvement decreases with increasing load as is to be expected.

The table also indicates that the reduction in force transmission and comfort indices was far more pronounced with the Gregoire type suspension than with the dual rate type, and this was recognised to be a function of the reduced interleaf friction which is a feature of this particular arrangement.

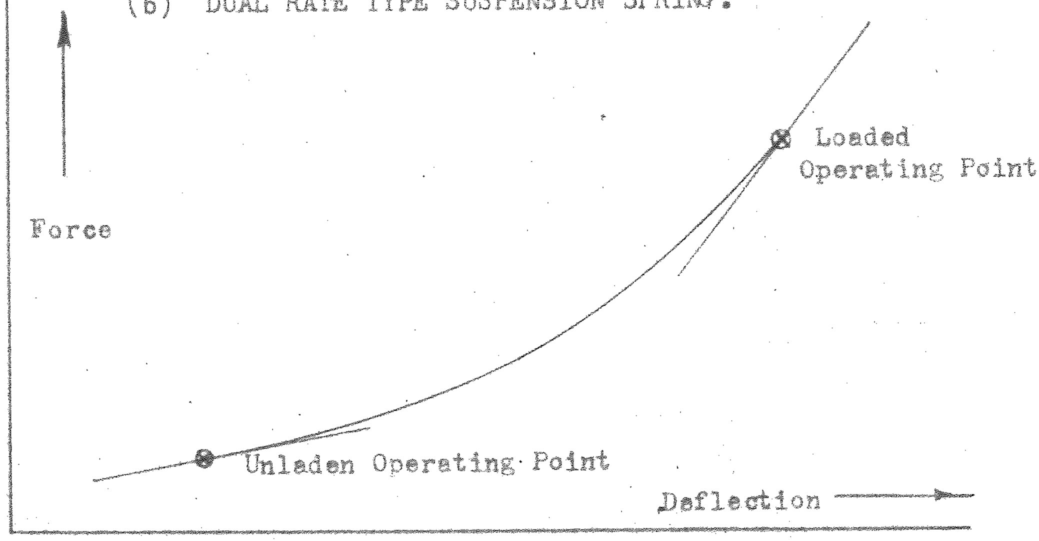
Nevertheless, the improvements in performance available with the Leyland Dual Rate springs were considered worthwhile, and in view of their cheapness and simplicity of changeover, it was recommended that these units be introduced as standard replacements for the conventional springs as these became due for renewal, while they were fitted as standard equipment on all new vehicles.




(a) GREGOIRE TYPE SUSPENSION SPRING.




(b) DUAL RATE TYPE SUSPENSION SPRING.




(c) CONSTANT FREQUENCY SPRING CHARACTERISTIC.

FIG.A.6.1. NON-LINEAR SPRING SYSTEMS TESTED FOR THE MUNICIPAL TRAMWAYS TRUST.