

On Open Venturis

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Abstract

Flow through an open converging-diverging channel, aimed at achieving maximum velocity at the throat.

Introduction

Consider a finite open converging-diverging tube or channel as in Figure 1 placed in an unbounded uniform stream, the purpose of which is to induce an internal flow with a maximum speed V that is significantly in excess of the stream speed U . If this Venturi had been part of a closed tube carrying a fixed volume flux, in principle there would have been no limit to the velocity V achievable at the throat, by making the constriction there smaller and smaller relative to the main tube's cross-section. However, as this happens, the drag increases and more force via pressure gradient is required to maintain constant volume flux in the tube.

On the other hand, if the Venturi is finite and open at each end, and is placed in an unbounded uniform stream U of incompressible viscous fluid, such large velocities V through the throat are not possible. We hope that a significant volume of fluid enters the Venturi and then must speed up as it passes through its throat. However, as the throat becomes more constricted, the drag increases and it is harder for fluid to pass through it. Hence less fluid actually enters the Venturi, and eventually it chokes and simply becomes an obstacle, around which all of the fluid flows; so $V = 0$. This choking phenomenon begins with separation of the interior wall boundary layer in the adverse zone of increasing pressure aft of the throat, and V/U can be increased significantly only so long as such separation does not occur.

So it would appear that an open Venturi tube has an upper bound on the achievable velocity augmentation ratio V/U at the throat, and it is the purpose of the present paper to discuss that ratio, and provide preliminary results toward its theoretical determination. We use tools of potential flow, and of laminar and turbulent boundary-layer theory. In fact, very little flow augmentation is achievable in laminar flow, and only the separation-delaying effect of turbulence in the boundary layer allows V/U to take values significantly greater than 1.

In the present paper, we treat only two-dimensional flow in channels, although the work extends readily to three-

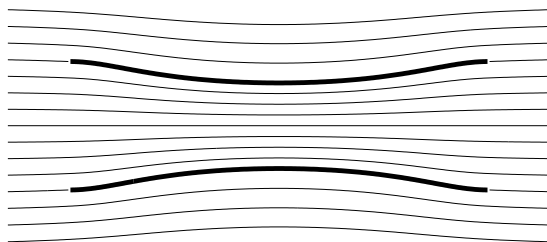


Figure 1: An example of a symmetric open Venturi with end width/throat width of 1.5 and zero end slopes.

dimensional flow in tubes. We assume that the walls of the channel are $y = \pm h(x)$, $0 < x < L$. We denote the minimum (throat) half-width as h_0 , the maximum half-width $h(L)$ being at the exit. Our examples are generally fore-aft symmetric with $h_0 = h(L/2)$, although this is not a necessary assumption.

1D Inviscid Flows

If the channel is long compared to its width, with $h(x) \ll L$, in the absence of viscous effects the flow within the channel will be uniform across the channel, with x -wise velocity $u(x)$ inversely proportional to $h(x)$ by conservation of mass. At the same time, the whole Venturi being slender will negligibly disturb the external stream U , so that to ensure pressure continuity between interior and exterior flows, the fluid must emerge at the exit $x = L$ with the stream speed U . Hence (c.f. [8],[9])

$$u(x) = U \frac{h(L)}{h(x)}. \quad (1)$$

On this inviscid basis, as large a throat velocity ratio $V/U = h(L)/h_0$ as we please is achievable by making the throat half-width h_0 sufficiently small.

Although this simple one-dimensional theory by itself therefore gives no immediate answer for the maximum value of V/U , it can form the basis for subsequent boundary-layer computations that do, by supplying the forcing pressure coefficient (based on the throat velocity V), namely

$$C_p(x) = 1 - \frac{u^2}{V^2} = 1 - \frac{h_0^2}{h(x)^2}. \quad (2)$$

The pressure gradient is positive whenever the channel width is increasing, and in general remains finite up to the exit. If we demand zero exit pressure gradient, the channel wall must be locally parallel to the stream, with zero end slope $h'(L) = 0$.

Figure 1 is an example of a symmetric channel of this type with a thickness ratio $h(L)/h_0 = 1.5$, namely

$$\frac{h(x)}{h_0} = \left[1 - \frac{10}{9} \left(1 - \frac{2x}{L} \right)^2 + \frac{5}{9} \left(1 - \frac{2x}{L} \right)^4 \right]^{-1/2} \quad (3)$$

which is such that the pressure coefficient $C_p(x)$ is a quartic polynomial. Within the one-dimensional approximation, this channel then also has a velocity ratio $V/U = 1.5$.

We now turn to two-dimensional computations for channels which are not necessarily long compared to their width, when there is no such immediate relationship between these two ratios.

2D Inviscid Flows

The full 2D potential flow about and through a finite-length channel is equivalent to that for a cambered airfoil (representing the wall) in ground effect (representing

the symmetry plane at the channel centreline), and can be determined by a number of numerical procedures borrowed from aerodynamics.

We have developed code for a general boundary curve $y = h(x)$ of zero thickness, by distributing vortices along that curve, together with their images in the symmetry plane $y = 0$. The vortex strength is found so that the flow is tangent to the boundary, subject to the Kutta condition of zero vortex strength at the trailing edge. This leads to a singular integral equation for the vortex strength function, which is converted to linear algebraic equations by discretisation assuming stepwise-constant vorticity strength on each of N panels into which the wall curve is divided. Adequate (about 3-figure) accuracy is achieved with N values of the order of a few hundred.

Figure 1 shows streamlines computed from such a computation for the special wall shape given by equation (3), and Figure 2 shows corresponding velocities along the interior and exterior walls as functions of x . This is for an aspect ratio $L/(2h_0) = 5$ which is only moderately slender, and the exit (and entrance) velocity at the wall trailing edge is $1.053U$, about 5% above the free-stream value assumed by the 1D theory. Meanwhile the throat velocity is $1.627U$ at the wall, about 8% higher than the value predicted by the 1D theory. Of course, in this 2D theory the velocity profile is no longer uniform across the channel, and for example the centreline exit velocity is $1.088U$ and the centreline throat velocity is $1.591U$. Results in closer agreement with the 1D theory would be obtained by using larger aspect ratios $L/(2h_0)$.

The program also computes velocities exterior to the channel, e.g. on the outside of the wall, and for example, exterior to the throat we find a (minimum) velocity of $0.857U$, see Figure 2. The present program assumes zero wall thickness, but could easily be extended to permit the exterior wall to be shaped in such a way as to disturb the uniform stream as little as possible.

The wall boundary pressures computed by this program can be used as input to a boundary-layer computation. If the wall is of generally positive curvature, with a minimum width at the throat rising to a maximum at the exit, the interior pressure similarly rises from its minimum at the throat, to a maximum value close to atmospheric pressure at the exit.

Although the assumed Kutta condition guarantees that the pressure itself approaches a finite limit at the trailing edge, it does so (so long as the wall curvature remains positive) with infinite positive pressure gradient. Hence

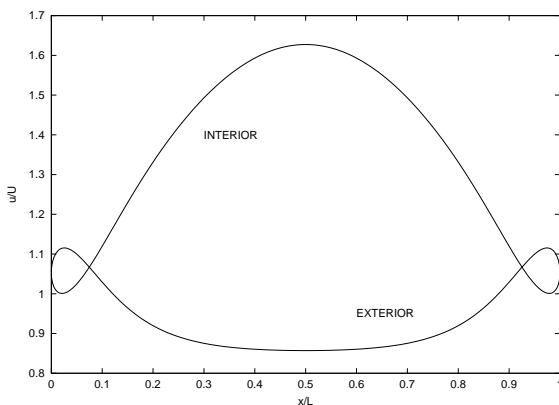


Figure 2: Velocity distributions along interior and exterior walls, for the flow of Figure 1.

for such a channel wall, the interior-wall boundary-layer flow must separate prior to reaching the trailing edge. Although this separation can be delayed to very close to the trailing edge, it is unlikely that shapes with such an end singularity will be optimal for maximum V/U . Hence it is desirable either to cancel the trailing-edge singularity, or to make it negative with a small exit region of favourable pressure gradient. It is always possible to achieve this by reversing the wall curvature locally, but (unlike in the 1D theory above) this does not necessarily mean that the exit pressure gradient is zero when the trailing-edge channel slope $h'(L)$ vanishes.

For example, the special channel of Figure 1 with $h'(L) = 0$ actually allows the interior wall pressure to rise to a local maximum, i.e. the interior velocity has a local minimum (see Figure 2), just ahead of the trailing edge, so that the actual trailing edge pressure gradient takes a favourable negative-infinite value. Thus the negative curvature of this channel near the trailing edge somewhat over-compensates, and in order to yield zero exit pressure gradient, it would be necessary to retain a small positive end slope, of magnitude about $h'(L) = 0.1$. Again, if we increase the aspect ratio $L/(2h_0)$, this end singularity in the pressure gradient would reduce toward zero for walls with zero end slope, in conformity with the 1D theory.

In any case, this 2D inviscid theory still allows arbitrarily large velocity augmentation ratios V/U , and we must turn to viscous effects, incorporating drag mechanisms and separation, to establish bounds on that ratio.

Laminar Viscous Flows

There are a number of ways to study laminar flows of a viscous fluid about a fixed body of the type of interest here, ranging up to full Navier-Stokes solution simultaneously exterior (on a grid extending to infinity) and interior to the channel, in effect generalising the 2D potential flow of the previous section. Here however we are content to describe two simpler methods providing more direct insight, and concentrating on the interior flow.

Lubrication-like theory

The first method is one previously used by Bentwich and Tuck [1] for a generalised lubrication problem. This is a slender-channel interior-flow model in the spirit of the 1D inviscid solution described earlier, in that it holds only for $h(x) \ll L$, and takes account of the exterior flow only by demanding that the pressure in the interior flow return to atmospheric at the exit from the channel.

An important feature of this method is that the entrance velocity to the channel is determined as part of the solution to the problem, and is adjusted until the exit pressure condition is satisfied. Since the resulting entrance velocity is generally significantly less than the free-stream speed, this feature thus models the “choking” tendency due to drag, as described in the introduction.

The actual equation solved in the interior flow is Prandtl’s boundary-layer equation. However, this equation is satisfied not just close to the walls, but throughout the channel width, with a pressure distribution that is uniform across the channel but unknown as a function of x along the channel. This is similar to “inverse” boundary-layer problems, as described by Keller [4].

This is a “moderate Reynolds number” theory, with a low Reynolds number limit corresponding to lubrication theory and an essentially parabolic velocity profile, and

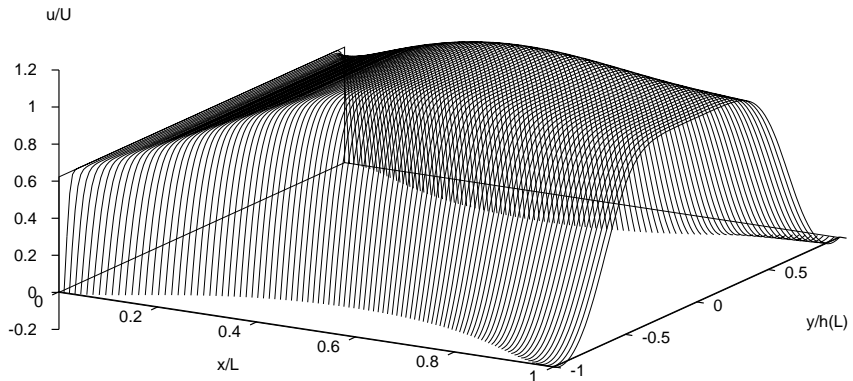


Figure 3: Velocity profiles for laminar viscous flow at $R_h = 80$ in the channel of Figure 1.

a high Reynolds number limit where the viscous effects are confined to the walls and the interior flow is uniform. The theory interpolates between these limits, effectively allowing the boundary layers to extend across the whole width of the channel.

Most computations were performed for parabolic-arc walls at (lubrication-scaled) Reynolds numbers $R_h = Uh(L)^2/\nu L$ up to several hundreds. The output velocity ratio V/U increases with R_h , but becomes essentially Reynolds-number independent once R_h exceeds about 60, asymptoting to a maximum value which is a function only of the width ratio $h(L)/h_0$. Furthermore, V/U could not be made to exceed about 1.095 for parabolic-arc channels, no matter how much we constrict the throat, the optimum occurring at about $h(L)/h_0 = 2$, with little effect of this ratio between 1.5 and 3.0.

Separation as defined by zero wall shear, followed by a low-velocity near-wall backflow, occurs near the exit for all R_h above about 30, but this does not prevent computations continuing to considerably higher Reynolds numbers. Note that a moderate value for the Reynolds number R_h based on the lubrication lengthscale $h(L)^2/L$ corresponds to a much higher value for the conventional Reynolds number $R_L = UL/\nu$ based on the channel length L ; for example $R_h = 100$ implies $R_L = 10000$ for a 10:1 aspect-ratio channel.

The program was also run for the special channel of Figure 1, and a 3D representation of the velocity profile $u(x, y)$ at $R_h = 80$ is given in Figure 3. In this case the entrance velocity is found to be $u_0 = 0.627U$ and the maximum (centreline) velocity is $V = 1.089U$, occurring at $x = 0.58L$, after which the centreline velocity returns to very close to the free stream speed U at the exit.

Zero wall shear occurs at $x = 0.84L$, followed by a very weak vortex close to the walls, with maximum backflow velocity about $-0.012U$. Figure 3 displays the evolution of the velocity profile across the channel, from its uniform entrance value, with initial Blasius boundary layers at the walls eventually growing to fill a significant fraction of the channel, but still with a quite flat profile over the central half of the channel at the exit station.

The blocking effect of viscosity and of the wall vortex is significant in reducing the effective entrance velocity, and this is why the maximum centreline velocity only just exceeds the free stream speed even though the channel width is reduced by two-thirds. Less blockage (and eventually no separation) occurs for a more gentle channel with a lower $h(L)/h_0$ ratio, and then the entrance velocity approaches the free stream speed, but the subse-

quent maximum velocity still hardly exceeds it. Among all symmetric wall shapes tried, the maximum velocity V could never be made to exceed $1.12U$.

Stratford laminar estimation

A very simple estimate for laminar boundary-layer separation is available via the formula of Stratford [5] as modified by Curle and Skan [3], see Rosenhead [10], p. 328. Namely, to avoid separation we must have

$$F(x) = x\sqrt{C_p(x)}C_p'(x) < 0.102 \quad (4)$$

This estimate has been shown ([10], p. 331) to be remarkably accurate compared to experiment and other more sophisticated laminar boundary layer computational procedures. Strictly x in equation (4) is arclength, but this is correctly approximated by the Cartesian x -coordinate for slender channels. Then we use the 1D inviscid theory as in equation (2) to determine $C_p(x)$, and in order to find the maximum value of V/U , demand that $F(L) = 0.102$, i.e. assume that separation is about to occur at the exit to the channel.

In this way, for example, we can easily show that for a symmetric parabolic-arc wall, the maximum value of $V/U = h(L)/h_0$ to avoid separation is only 1.049. This is a lot worse than the lubrication-like theory predicted for a parabolic-arc wall, but is explained by the fact that the latter allows weak separation.

In fact we can use the Stratford formula (4) to design the best possible pressure distribution, and hence within the 1D model, the best possible wall shape, by requiring that the boundary layer be on the verge of separation everywhere in the adverse region. In that case the equation $F(x) = 0.102$ is an ODE to determine $C_p(x)$. For example, assuming a fore-aft symmetric design, the solution in $x \geq L/2$, subject to $C_p = 0$ at the throat $x = L/2$, is

$$C_p(x) = \left[\frac{3}{2}(0.102) \log \frac{2x}{L} \right]^{2/3} \quad (5)$$

But at the exit $x = L$ we must have $C_p(L) = 0.224 = 1 - U^2/V^2$; hence $V/U = 1.135$. No symmetric open Venturi can achieve more than a 13.5% speed-up in unseparated laminar flow!

Turbulent Flow

Clearly if an open Venturi is to be able to accelerate the flow significantly, this must be as a result of the better ability of turbulent boundary layers to remain attached within adverse pressure gradients, and we need to take turbulence into account. Again there are many possible

computational tools, ranging up to full Navier-Stokes solution in the whole infinite domain, coupled with one of a myriad of CFD turbulence models of increasing complexity. However, given that our present concern is mainly with the boundary layers on the interior walls, simpler models for the present problem are worth exploring first.

In particular, Stratford [6] has also provided a simple separation criterion for turbulent boundary layers, namely

$$F(x) = C_p(x) \sqrt{(x - x_0) C_p'(x)} < 0.35 \quad (6)$$

for Reynolds numbers (based on length L) of the order of 10^6 , which we assume. Cebeci and Smith [2] have demonstrated that the formula (6) is in good agreement with other more sophisticated methods, and with experiment.

In contrast to the corresponding laminar formula (4), there is a complicating feature of the turbulent Stratford formula (6) in that there is a “virtual origin” $x = x_0$ from which an idealised flat-plate turbulent boundary layer is assumed to start, rather than the actual leading edge $x = 0$. In reality as we move from the leading edge toward the throat, there occurs a sequence of (not always precisely defined) events involving an evolving laminar boundary layer, then transition to turbulence, etc. Stratford [6] gives empirical methods for estimating x_0 (providing the transition point is known, which is not usually the case), but since our aim here is to find upper bounds, we can simply allow x_0 to vary until the best result is achieved.

If we again use as our first model a parabolic-arc wall, and write down as with laminar flow the requirement that the boundary layer is about to separate at the exit, i.e. $F(L) = 0.35$, we find that the maximum value of V/U is a quite promising 1.363, and it occurs when $x_0 = L/2$. That is, the most favourable situation for maximising the throat velocity is to place the virtual origin right at the throat itself. This is, as Stratford [6] cautions, “... only roughly correct, as the turbulent flat plate profile assumed to exist at $x = x_0$ would not be properly formed ...”, but nevertheless the nearer we can get to this situation, the higher the achievable V/U .

An even better result is achievable once again by exploiting reversed curvature near the exit. For example we can achieve a 50% acceleration, i.e. $V/U = 1.5$, with the symmetric shape of Figure 1 as given by equation (3), which satisfies $h'(L) = 0$. It is notable that for this shape, the separation crisis is not at the actual exit, the function $F(x)$ reaching a maximum slightly less than the critical value 0.35 of equation (6) at $x \approx 0.9L$. Within a family like that of equation (3) having a quartic polynomial pressure coefficient, this value $V/U = 1.5$ seems to be about as high as one can go.

However, it is not necessarily the best V/U achievable for any $h(x)$. Again we can seek the optimum by requiring the turbulent boundary layer to be on the verge of separation for all $x > L/2$, setting $F(x) = 0.35$ and solving the resulting ODE for $C_p(x)$. This would produce an optimum “diffuser” shape, like that proposed by Stratford [7], and some preliminary computations suggest the possibility of $V/U = 2$ or greater.

However, there is now the extra complication of the choice of the virtual origin x_0 . Choosing $x_0 = L/2$ is no longer acceptable for these optimal shapes, which are singular in that limit. Virtual origins close to the throat seem to have advantages, but Stratford’s [6] caution now becomes more critical, and perhaps a better computa-

tional model is needed.

Conclusions

Velocity augmentation ratios of the order of $V/U = 1.5$ are predicted by the simple but usually reliable Stratford turbulent boundary layer separation criterion. This value was achieved for a special but not necessarily optimal channel shape having zero end slope. Even higher values of V/U may be possible without separation for optimised channel shapes, whereas laminar flow allows no higher than $V/U = 1.135$ before separation occurs.

There are a number of possible extensions and improvements in computational techniques for this class of problem. Full Navier-Stokes solution in the whole flowfield, inside and outside the channel, with suitable turbulence modelling, is the most obvious, though computationally challenging procedure. One intermediate possibility is that of constructing a turbulent equivalent of the lubrication-like theory of [1], which would only require solving boundary-layer equations inside the channel. There are a number of appealing features of such a theory, such as the determination of the entrance velocity, and the ability to compute a weakly separated flow near the exit, potentially allowing somewhat higher V/U ratios than those for unseparated flow.

The present paper has concerned itself only with theoretical and computational techniques. In no way does this emphasis imply that such techniques are more significant than actual observation, and systematic experimental measurements of V/U for families of open Venturis would be highly desirable.

References

- [1] Bentwich, M. and Tuck, E.O., Sliding Sheets: Lubrication with Comparable Viscous and Inertia Forces, *J. Fluid Mech.*, **135**, 1983, 51–69.
- [2] Cebeci, T. and Smith, A.M.O., *Analysis of Turbulent Boundary Layers*, Academic Press, 1974.
- [3] Curle, N. and Skan, S.W., Approximate Methods for Predicting Separation Properties of Laminar Boundary Layers, *Aero. Quart.*, **8**, 1957, 257–268.
- [4] Keller, H.B., Numerical Methods in Boundary-Layer Theory, *Ann. Rev. Fluid Mech.*, **10**, 1978, 417–433.
- [5] Stratford, B.S., The Prediction of Separation of the Laminar Boundary Layer, *Rep. and Mem. Aero. Res. Council, London*, **3002**, 1954.
- [6] Stratford, B.S., The Prediction of Separation of the Turbulent Boundary Layer, *J. Fluid Mech.*, **5**, 1959, 1–16.
- [7] Stratford, B.S., An Experimental Flow with Zero Skin Friction Throughout its Region of Pressure Rise, *J. Fluid Mech.*, **5**, 1959, 17–35.
- [8] Tuck, E.O., A Nonlinear Unsteady One-Dimensional Theory for Wings in Extreme Ground Effect, *J. Fluid Mech.*, **98**, 1980, 33–47.
- [9] Tuck, E.O., Steady Flow and Static Stability of Airfoils in Extreme Ground Effect, *J. Eng. Math.*, **15**, 1981, 89–102.
- [10] Rosenhead, L. (editor), *Laminar Boundary Layers*, Oxford, Clarendon Press, 1963.