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1. Introduction

International comparison of living standards is a central issue in economics. Rankings of per capita GDP are widely used for such comparisons. However, many problems arise since (i) countries use their own currencies, (ii) have their own price levels, and (iii) consumers' preferences could differ across countries. These and other factors lead to biased estimation of national income. The present paper is part of a literature which intends to remedy such biases.

Market exchange rates can be used for standardizing international incomes to a common measure, e.g. US dollars. This approach is simple to apply and widely used. However, even if the purchasing power parity doctrine (PPP) is assumed to hold for tradable goods, PPP may not hold for the prices of untradable goods converted by the market exchange rate. As a result, this approach turns out to be biased, because it ignores the price differences in non-tradables. In other words, it cannot give an accurate measure of differences in living standards across countries.

This approach has been substantially improved by the introduction of the constant international price measure, which has been adopted by some countries since the 1980's. This method is employed by United Nations International Comparison Project (ICP), through the United Nations and the Commission of European Communities (1987). This commission provides ICP prices and quantities for 38 categories of expenditure across 60 countries. The Geary-Khamis (G-K) method is used to aggregate the quantities under international prices; thus, a PPP based measure is derived to compare real GDP in these countries. The ICP data have been further extended across countries and time by Summers and Heston (1991); the resulting ranking is known as the Penn World Table (PWT). This measure can to some extent avoid the flaws brought about by the exchange-rate-based approach, because it ensures expenditures are calculated under the same price level. However, another problem arises: substitution bias. Particularly, higher expenditure on some commodities may be caused by a lower price level rather than higher income. Since this point cannot be reflected by constant international price measures, this approach could be utility-inconsistent and sensitive to the reference price vector. Moreover, this approach requires data with a high level of disaggregation, thus placing considerable demands on data collection. In fact, only 60 countries are involved in ICP (1987), while the incomes of non-benchmark countries are predicted through a regression model[4]. This leads to a larger bias. Consequently, the development of unbiased measures of living standards is a key objective of the field of measurement economics. Such measures should be applicable to a majority of countries in the world and assist economic research and policy in a wide range of applications.

The present paper is closely related to Dowrick and Akmal 2003 (henceforth referred to as DA). As observed by DA, Afriat[5] income can be calculated directly from detailed price data from the United Nations International Comparison Project (ICP), but only for benchmark country/year combinations. For these country/year combinations, DA find a log-linear relationship between foreign-exchange-rate-based (FX) income and Afriat income with a slope significantly different from one. The resulting parameter estimates can be used to estimate Afriat income for non-

[4]This procedure was developed by Summers and Heston (1991). This paper suggests a method for estimating true incomes for non-benchmark countries, and thus opened up a new area in measurement economics. However, it lacks a theoretical foundation.
[5] The relevant references to Sydney Afriat's work can be found in Dowrick and Quiggin (1994).

benchmark country/year combinations on the basis of observable FX income. Thus, DA point the way toward improved retroactive measurement of national income.

The present paper focuses on the theoretical foundations of this relationship between the FX income, GK income and true (Afriat) income. DA use a model that arrives at a log-linear relationship between these income measures. The present paper employs an alternative model, which in contrast to DA's model arrives at a full general equilibrium solution for the endogenous variables in terms of the exogenous variables. Using a Ricardian trade model with Cobb-Douglas preferences, the present paper generalizes several theoretical propositions from DA's model. In our baseline model, we confine ourselves to the two-country case and ignore barriers to trade. An additional advantage of the model used in the present paper is that it can serve as the basis for a variety of extensions by adding certain features that may affect the relationship between FX income, GK income and true income.

Some extensions of the Ricardian general equilibrium trade model are also employed. First, instead of modelling trade barriers that are considered in the traditional literature, such as transport costs and tariffs, we take into account distribution costs, ascribed to local distribution services, to measure the wedge between the prices of tradable goods. We also generalize the "Balassa-Samuelson (BS) effect", i.e. we allow that a rich country could have a lower price level than a poor country. In addition, the two countries become small open economies, trading in a global market under world prices. We then show that market exchange rates will overstate real international income gaps if and only if the price level of services is higher in the relatively rich country. On the other hand, constant price comparisons will overstate true international income gaps if the constant price vector corresponds to that of a country where the productivity is lower (the poor country) and understate true international income gaps if the constant price vector corresponds to that of a country where the productivity is higher (the rich country), regardless of the price level of services. Based on this theoretical foundation, estimation of true (Afriat) GDP for non-benchmark countries could be improved, by adding other variables to the regression.

The paper is organized as follows. The next section presents a brief review of the theoretical and empirical literature. The baseline model is stated and the relevant propositions are derived in section 3. Some extensions are modeled in section 4. The empirical estimation of true (Afriat) GDP for non-benchmark countries is discussed in section 5. The final section summarizes the results and suggests the directions for further research.

2. Literature review

The bias of foreign-exchange-rate measures was noted by Balassa (1964), building on earlier work by Samuelson. This principal contribution focuses on non-traded goods to explain the differences in real exchange rates among countries. It suggests that if the international productivity gap is larger in traded goods than in non-traded goods, the purchasing power of the currency used in the country with higher productivity would be overestimated based on the market exchange rate. This proposition is explained by introducing a non-traded good to the traditional Ricardian trade model. With free trade, the prices for traded goods should be identical. However, the wage rate of producers in the tradable goods sector should be different across countries because of differences in productivity and the wage rate in the traded goods sector will determine the wage rate in the non-traded goods sector, where the productivity difference is not as large. Therefore, the country with lower productivity has a lower price for non-traded goods and the average price is also lower. This explanation is interesting but it raises several problems: it is yet to be proven by general equilibrium analysis, and Balassa's assumption of a single-factor production function is particularly unrealistic. We would also expect the productivity gap to be larger in the non-traded goods sector, which would result in very different implications.

Bhagwati (1984) provides another explanation why services are cheaper in poor countries. He proposes a general equilibrium model with multi-factor production functions, which focuses on comparative endowments differences across countries, while Balassa (1964) is concerned with comparative productivity gaps. Bhagwati (1984) argues that even if productivity differences across sectors between countries are ignored, labor-intensive services are relatively cheaper in the poor country because poor countries seem to have lower capital labor ratios. Furthermore, he shows that countries with similar GDP per capita may violate the Balassa-Samuelson (BS) effect. It is one of the earliest papers to explain why the BS effect may be absent in certain samples of countries.

The substitution bias of constant international price measures was in fact detected by ICP researchers. However, this kind of approach is still being used, probably because it is hard to construct an unbiased index, but also in part because unbiased indices may face a sub-aggregation problem. Dowrick and Quiggin (1994) illustrate substitution bias by testing the detailed ICP price data set including 60 countries. The Samuelsonian revealed-preference approach is viewed as a basis for welfare comparisons. The basic idea is simple: if A can afford B's consumption bundle at A's prices while B cannot afford A's bundle at B's prices, then the wellbeing of A is greater than that of B. Dowrick and Quiggin (1994) show that constant price measures could be a reliable means for welfare comparisons between two countries with a large difference in living standards. However, these rankings may not be in accordance with revealed-preference criteria if the two countries have similar levels of development. Additionally, these indices are sensitive to the reference price vector. For instance, if the relative price of one good in a country is lower than that in the reference price vector, it will lead to substitution towards this locally cheaper good and overestimation of the expenditure on this good.

Although Dowrick and Quiggin (1994) demonstrate the substitution bias of constant international price measures, the paper's discussion of theoretical foundations is limited, and it fails to construct an unbiased measure. Dowrick and Quiggin (1997) address this point and show that by applying the same data base as used in the ICP surveys, it is feasible to find an unbiased income index, which is utility-consistent and independent of the reference price or quantity vector, defined as the Afriat index. Seventeen OECD countries are selected from ICP data to construct this ideal index. It is found that ICP measures are often outside these bounds, which has further justified the existence of substitution bias in ICP income index. The finding of Afriat index represents another step forward in the measurement of national income. However, it is yet to be widely used so far. The reason may be that separate components of GDP, such as consumption and investment, cannot be obtained from this approach.

Dowrick and Akmal (2003) apply this ideal index to analyse the inequality in international income distribution. This paper also reveals that macroeconomic modelling can shed light on the relationship between the FX income, GK income and true (Afriat) income. Using a simple trade model of two countries, this paper explains that market exchange rates will overestimate the real international income gaps and a constant price comparison will underestimate the real income gaps if the reference price vector corresponds to a relatively rich country. Using the unbiased Afriat income measure, real-world inequality between countries is shown to have barely changed over time. Thus the marked *increase* in global inequality indicated by the FX measure and the marked *decrease* indicated by the GK measure are both spurious. In addition, to avoid the pitfall that Afriat incomes for non-benchmark countries cannot be calculated directly from ICP data, Dowrick and Akmal (2003) run a regression based on their theoretical model, which reflects the log-linear relationship between Afriat income and FX income.[6] Using this relationship, they estimate the true income for non-benchmark countries.

However, their trade model does not provide a general equilibrium solution. With an endogenous trade pattern, there should be at least three cases, including complete specialization by both countries and one country specializing while the other does not. Our alternative trade model will yield a unique general equilibrium solution. We will also generalize and confirm several of Dowrick and Akmal's (2003) theoretical results. In addition, our model paves the way for further extensions. Other factors influencing the relationship between the FX income, GK income and true (Afriat)

[6] This regression model constitutes an improvement over the procedure by Summers and Heston (1991) in that it has a theoretical basis. However, a lot more could be done to improve the macroeconomic model.

income could be considered, making the theoretical foundation more realistic, more meaningful, and empirically applicable.

Several extensions are modeled in the present paper. In the previous literatures, it is usually assumed that PPP holds for traded goods because of free trade. However, evidence shows that relative PPP does not hold for tradable goods (Isard 1977, Giovannini 1988), and Engel (1999) finds that changes in prices of tradable goods can explain the most part of the movements in the US real exchange rate. Most economists are inclined to attribute this fact to trade barriers, which can cause a price wedge between countries. However, Burstein, Neves, and Rebelo (2002) argue that trade barriers are not the main reason for the price difference. They find that distribution costs constitute a large part of the consumer's basket, representing more than 40% of the retail price of the average consumer good in the US and about 60% in Argentina. They further suggest that because distribution services are labor-intensive and labor is untradable, a wedge between the prices of tradable goods in different countries may occur. Therefore, the model can be improved by addressing distribution costs, as is done in the present paper.

Secondly, preceding papers usually assume that the service sector has the same productivity in different countries so the price level of services will be higher in the rich country. However, as shown by Bhagwati (1984), no formal theory can show that non-traded sectors have productivity parity. Besides, Bergin, Glick and Taylor (2004) imply that if the technology gap is larger in the non-traded sector, then the price level in a poor country will be higher than a rich country. We incorporate this idea in the present paper by endogenizing the BS effect.

In addition, previous papers consider a world with only two countries. In part 4 of this paper, the two countries are assumed to be small open economies, trading in a global market under world prices. This assumption may be more realistic and also generalizes the pattern of trade.

3. The baseline model

3a. The baseline model's assumptions and conventions

Consider a world with two countries, 1 and 2; three goods, *A*, *B* and *S*; and only one factor of production, labor, denoted *L*. Production functions are assumed to have fixed coefficients. Units of *A*, *B* and *S* are standardized so that the fixed coefficients in country 1 are all equal to 1. Furthermore, both countries are assumed to be equally productive in *S*. Letting L_{ij} denote country *i*'s labor input into the production of good *j*, and Y_{ij} country *i*'s output of good *j*, we thus have

$$Y_{1A} = L_{1A},\ Y_{1B} = L_{1B},\ Y_{1S} = L_{1S},\ Y_{2S} = L_{2S}.$$

A and *B* are traded while *S*, which can be thought of as locally provided services, is nontradable. Country 2 is assumed to have an absolute advantage in *A* and *B*, and a comparative advantage in *B*. This can be expressed as follows, letting λ_j denote country 2's fixed labor productivity coefficient in output *j*:

$$Y_{2A} = \lambda_A L_{2A},\ Y_{2B} = \lambda_B L_{2B},\ \lambda_B > \lambda_A > 1.$$

Producers and consumers are assumed to be profit- and utility-maximizers respectively. Labor is assumed to be perfectly immobile internationally but perfectly mobile between domestic sectors, so that wages are equalized intersectorally but not internationally. The labor market and all other markets are assumed to be in a long-run perfectly competitive equilibrium. Thus full employment obtains: $\sum_j L_{ij} = L_i;\ i = 1,2;\ j = A,B,S$. Country i (=1,2) is assumed to be inhabited by L_i persons with internationally identical Cobb-Douglas utility functions; letting c_{ij} denote *per capita* consumption of good *j*,

$U_i = c_{iA}^{\alpha} c_{iB}^{\beta} c_{iS}^{1-\alpha-\beta};\ \alpha > 0, \beta > 0, \alpha + \beta < 1;\ i = 1,2$.

For the time being, we assume zero transportation costs and no barriers to trade. The structural equations provided so far completely describe the model. Taking country 1 labor to be the numeraire, so that the wage rate there, w_1, satisfies $w_1 \equiv 1$, we are ready to solve this general equilibrium model.

3b0. The 3 relevant cases

The case where both countries incompletely specialize can be ruled out. Letting *p* denote the terms of trade, the domestic price ratios are equalized through trade: $p \equiv p_{1A} / p_{1B} = p_{2A} / p_{2B}$. Because domestic producers face different rates of transformation in each country, $\lambda_B / \lambda_A > 1$ (where 1 is the rate of transformation in country 1), there exists no p such that an interior point in (Y_A, Y_B) space can be optimal in both countries simultaneously. Thus one of the following 3 trade patterns must apply:

Case 1: Country 1 specializes in *A*, while country 2 produces both *A* and *B*;

Case 2: Country 2 specializes in *B*, while country 1 produces both *A* and *B*;

Case 3: Country 1 specializes in *A*, and country 2 specializes in *B*.

In addition to goods A and B, both countries produce positive amounts of S in any equilibrium, due to the Cobb-Douglas preferences. Because $p_{iS} = w_i;\ i = 1,2$, per capita consumption of services always equals $c_{iS} = 1 - \alpha - \beta$. Our solution strategy is to first solve the model for each case separately, and then to find out when each case occurs.

3b1. Case 1:Only country 1 specializes

In this case, for producers in country 2 to be maximizing profits at an interior point in (Y_{2A}, Y_{2B}) space, the terms of trade must equal country 2's transformation rate:

$$p = \lambda_B / \lambda_A. \qquad (1)$$

Due to zero profits, $p_{1A} = p_{1S} = 1$ (note that zero profit conditions for a country only apply to goods that are actually produced in that country), and, by (1), $p_{1B} = \lambda_A / \lambda_B$. Due to the Cobb-Douglas utility specification, α, β, and $1-\alpha-\beta$ are the budget shares of each good in each country. Hence

$$c_{1A} = \alpha;\ c_{1B} = \beta\lambda_B / \lambda_A.$$

Letting *E* denote the exchange rate, i.e. the price of country 1's currency in terms of country 2's currency, price equalization of tradables requires $p_{2A} = E;\ p_{2B} = E\lambda_A / \lambda_B$; and, due to zero profits, $p_{2j} = w_2 / \lambda_j, j = A, B$; hence $p_{2S} = w_2 = E\lambda_A$. Having found consumer prices and per capita income, it follows that per capita consumption levels in country 2 are $c_{2A} = \alpha\lambda_A;\ c_{2B} = \beta\lambda_B$.

3b2. Case 2: Only country 2 specializes

Except for country-1-based normalizations, this case is symmetrical to case 1. Due to zero profits, $p_{1A} = p_{1B} = p_{1S} = 1$ and $p_{2A} = p_{2B} = E$. Again by zero profits, $p_{2S} = w_2 = E\lambda_B$. Based on these prices and wages, per capita consumption levels are $c_{1A} = \alpha;\ c_{1B} = \beta;\ c_{2A} = \alpha\lambda_B;\ c_{2B} = \beta\lambda_B$.

3b3. Case 3: Both countries specialize

Due to zero profits, $p_{1A} = p_{1S} = 1$ (thus $p_{2A} = E$) and $p_{2B} = w_2 / \lambda_B$. It follows that $c_{1A} = \alpha;\ c_{1B} = \beta p;\ c_{2B} = \beta\lambda_B$. Furthermore, Cobb-Douglas utility dictates that $c_{2A} = \alpha c_{2B} / \beta p$; hence $c_{2A} = \alpha\lambda_B / p$. For producers in both countries to be maximizing profits, we must have the terms of trade lie between both transformation rates:

$$1 \le p \le \lambda_B / \lambda_A. \qquad (2)$$

To solve this case, we need to find the equilibrium p. Let $C_{ij} = L_i c_{ij}$ denote country i's total consumption of good j and X_{ij} its quantity of exports. Country 2 employs $L_{2B} = L_2 - L_{2S} = L_2 - C_{2S}$ workers in the production of good B, which therefore equals $Y_{2B} = \lambda_B (L_2 - C_{2S})$. Similarly, $Y_{1A} = L_1 - C_{1S}$. Trade equilibrium requires $p_{1A} X_{1A} = p_{1B} X_{2B}$; thus $p_{1A}(L_1 - C_{1A} - C_{1S}) = p_{1B}(\lambda_B (L_2 - C_{2S}) - C_{2B})$. Rearranging, we have $p(1 - c_{1A} - c_{1S}) = (L_2 / L_1)(\lambda_B (1 - c_{2S}) - c_{2B})$. Now substitute the per capita consumption values, and combine with (2), to obtain (defining the key variable ϕ in the process):

Necessary condition for case 3: $1 \ \leq \ p = \left(\frac{L_2}{L_1}\right)\left(\frac{\alpha}{\beta}\right)\lambda_B \equiv \phi \ \leq \ \frac{\lambda_B}{\lambda_A}$. (3)

This yields the remaining price and consumption solutions. Prices for good B are $p_{1B} = 1/\phi;\ p_{2B} = E/\phi$. Recalling that $p_{2B} = w_2/\lambda_B$, we have $p_{2S} = w_2 = E\left(L_1/L_2\right)\left(\beta/\alpha\right)$. Plugging (3) into the consumption expressions that have *p* in them, we have $c_{1B} = \left(L_2/L_1\right)\alpha\lambda_B$ and $c_{2A} = \beta L_1/L_2$.

3c. The baseline model's solution

In cases 1 and 2, the output of tradables by the non-specializing country is determined by the specializing country's demand and supply conditions. The key feasibility conditions here are:

Case 1: $C_{1B} = X_{2B} < \ \lambda_B(L_2 - C_{2S}) - C_{2B}$ and (4)

Case 2: $C_{2A} = X_{1A} < \ L_1 - C_{1S} - C_{1A}$ (5)

These inequalities are strict for the following reason. Take, for example, inequality (4). If country 2's production capacity in good *B* (taking into account domestic consumption of services), $\lambda_B(L_2 - C_{2S})$, were to be exhausted, then we would no longer have case 1, because country 2 would be specializing in good *B*.

Now divide both sides of inequality (4) by L_1 and substitute the per capita consumption solutions to obtain

Necessary condition for case 1: $\left(\frac{L_2}{L_1}\right)\left(\frac{\alpha}{\beta}\right)\lambda_B \equiv \phi \ > \frac{\lambda_B}{\lambda_A}$ (6)

Similarly, we obtain:

Necessary condition for case 2: $\phi < 1$. (7)

PROPOSITION 1: The baseline model has a unique equilibrium (i.e. unique equilibrium values for all endogenous prices and quantities) for all values of $L_i \in (0,\infty)$; $\alpha, \beta \in (0,1)$, where $\alpha + \beta < 1$; and $\lambda_j \in (0,\infty)$.

SKETCH of PROOF: Individual optimality of solutions for consumers and producers has been demonstrated. Production and consumption feasibility and equilibria in all markets for those solutions are easily verified. Uniqueness is proven by (3), (6), and (7), which are mutually exclusive, and are (by optimality, market equilibria, and feasibility) not only necessary but also sufficient conditions for each of the respective cases to occur.

3d. International comparisons in the baseline model

3d1. Comparisons of true incomes

As pointed out by DA (p.5), cardinality in welfare comparisons is achieved by the money-metric Allen-welfare index (letting *e* denote the expenditure function):

$$A_r^{2:1} \equiv \frac{e[U_2, \mathbf{p}_r]}{e[U_1, \mathbf{p}_r]}, \tag{8}$$

which normally requires specification of a reference price vector, $\mathbf{p}_r$. However, in the special case of a utility function which is homogeneous of degree one – as in our model – the Allen index is independent of the reference price vector and is equal to the ratio of utilities:

$$A^{2:1} = U_2 / U_1. \tag{9}$$

The Allen index thus provides, in this case, a unique measure of the true per capita income ratio between country 2 and country 1. Evaluating utilities for the per capita consumption solutions yields the following true income ratios:

PROPOSITION 2:

$$A^{2:1} = \begin{cases} \lambda_A^{\alpha+\beta} \\ \left(\dfrac{L_1\beta}{L_2\alpha}\right)^{\alpha+\beta} \\ \lambda_B^{\alpha+\beta} \end{cases} \text{ iff } \left(\frac{L_2\alpha}{L_1\beta}\right)\lambda_B \equiv \phi \begin{cases} > & \lambda_B / \lambda_A \\ \in & [1, \lambda_B / \lambda_A]. \\ < & 1 \end{cases} \qquad (10)$$

In words, the true income ratio depends on country 2's absolute advantage in good A in case 1; on its absolute advantage in good B in case 2; and depends on relative factor scarcity and consumer preferences for outputs in case 3. Remarkably, in case 3, the precise magnitude of the true income ratio is independent of the countries' relative productivity. Still, we do know that it must be greater than one. Due to the bounds on ϕ that are necessary and sufficient for this case to occur, in this case (as in both other cases) true income is higher in country 2:

Corollary of PROPOSITION 2: The country with absolute advantage in both tradables has higher per capita real income.

PROOF: Follows from $\lambda_A > 1$ and $\lambda_B > 1$ in all three cases.

2d2. Foreign Exchange Rate comparisons of incomes

As in DA (p.5), "in this model, per capita National Income and Gross Domestic Product are identical and, measured in local currencies, are simply equal to the wage. So the GDP or income ratio that is obtained from exchange rate comparison is simply the ratio of wage levels expressed in a common currency." Hence we obtain the following solution for the FX-based income ratio:

$$FX^{2:1} = \frac{w_2}{Ew_1} = \begin{cases} \lambda_A \\ \left(\dfrac{L_1\beta}{L_2\alpha}\right) \\ \lambda_B \end{cases} \text{ iff } \quad \phi \begin{cases} > & \lambda_B / \lambda_A \\ \in & [1, \lambda_B / \lambda_A] \\ < & 1 \end{cases}. \tag{11}$$

It follows that, regardless of which case obtains, FX-based relative income is the same log-linear function of true relative income, and thus consistently biased upwards.

PROPOSITION 3: Non-Traded Sector Bias in FX comparisons[7]

(i) Market exchange rates overstate true international income differentials.

(ii) The magnitude of this bias is an increasing function of the true income differential (or, alternatively, of the FX-based differential) and the domestic expenditure share of the non-traded sector.

PROOF: Follows from (10) and (11), recalling that the expenditure share of nontradables in either country is $0 < 1 - \alpha - \beta < 1$, and noting that by the Corollary of Proposition 2, $A^{2:1} > 1$:

$$\frac{FX^{2:1}}{A^{2:1}} = \left(FX^{2:1}\right)^{1-\alpha-\beta} = \left(A^{2:1}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} > 1 . \tag{12}$$

This positive bias is due to the fact that FX-based income measures do not take into account the prices actually faced by consumers. While prices of tradables are being equalized, the price of nontradables is relatively lower in the low-productivity country – an effect not captured by the FX measure.

[7] This proposition and its phrasing follow closely Proposition 2 (DA, p.6). However, note the difference in part (ii).

PROPOSITION 4: The relative price of nontradables is higher in country 2 than in country 1; the entire FX bias is attributable to this fact.

Proof: The price ratio of nontradables to tradable good *j* in country *i* is p_{iS} / p_{ij}.

Recalling that $p_{2j} / p_{1j} = E$ and that $p_{iS} = w_i$, we have $\frac{p_{2S} / p_{2j}}{p_{1S} / p_{1j}} = \frac{w_2}{Ew_1} = FX^{2:1}$.

The fact $FX^{2:1} > 1$ (from Proposition 3) proves the first part of Proposition 4; the second part follows from $FX^{2:1}$ being equal to the ratio of relative domestic prices. Note that since this ratio is independent of $j = A, B$, this result holds no matter which price index is chosen for tradables.

3d3. Purchasing Power Parity comparisons of incomes[8]

Next, we turn to the measurement of the international income ratio by the Geary-Khamis (GK) method of calculating purchasing power parity. This method values each country's GDP at 'international prices'. Applied to the present model, this method requires specification of one scalar element of the international price vector only. As the price ratio of tradables, *p*, is equal for both countries, it does not matter which country we use for the prices of tradables; thus we need to introduce only one additional variable: the international price of nontradables, *g* (which can of course be scaled up or down by *E*). We represent the GK price vector as:

$$\mathbf{P}^{GK}(g) \equiv [g, p_{1A}, p_{1B}]. \tag{13}$$

The Geary-Khamis measure of real GDP *per capita* for country *i* is the *per capita* consumption bundle evaluated at international prices. Evaluating the consumption

[8] The wording of some passages of this paragraph follows closely that of the corresponding paragraph in DA (p.6-7).

bundles derived above at international prices, the GK income ratio between countries 2 and 1 is:

$$GK^{2:1}(g) \equiv \frac{GK_2(g)}{GK_1(g)} = \frac{(1-\alpha-\beta)g+(\alpha+\beta)FX^{2:1}}{(1-\alpha-\beta)g+(\alpha+\beta)}. \tag{14}$$

Remarkably, (14) holds for each of our three cases, as can be verified with equation (11) – even though in each case $FX^{2:1}$ depends on different parameters. Whether the Geary-Khamis comparison under- or over-states the true income ratio depends on the value of g. We summarize the relationship in Proposition 5, which is analogous to Proposition 3 of DA (p.7):

PROPOSITION 5: Substitution bias in Geary-Khamis comparisons

(i) A bilateral international comparison of per capita income which value expenditure at constant prices will understate the true income differential if the constant price vector corresponds to that of the high-productivity country, or the prices of an even richer country.

(ii) A constant price comparison will overstate the true income differential if the constant price vector corresponds to that of the low-productivity country, or the prices of an even poorer country.

(iii) The bias is greater, the less similar is the reference price vector with respect to the comparison country prices.

(iv) Where (i) or (ii) holds, the magnitude of the bias is an increasing function of the income differential (whether measured through FX or by true income) between the two countries.

PROOF: Letting $\gamma \equiv 1-\alpha-\beta$ and noting again that $FX^{2:1} > 1$, it can be verified that (14) is equivalent to equation (14) of DA. Thus, their proof applies.

4. Extensions of the baseline model

Most of the assumptions in this part are the same as our baseline model. However, we extend the model by assuming the two countries become small open economies, trading in a global market under world prices. Besides, since we aim to endogenize the BS effect, the production functions for two countries become

$Y_{1A} = L_{1A}, Y_{1B} = L_{1B}, Y_{1S} = L_{1S}$,

$Y_{2A} = a\lambda_A L_{2A}, Y_{2B} = a\lambda_B L_{2B}, Y_{2S} = aL_{2S}$, $\left(\lambda_B > \lambda_A > 1/a;\ a > 1\right)$,

where L_{ij} and Y_{ij} respectively denote labor input and output of good *j* in country *i*, and λ_A, λ_B and a are fixed labor productivity coefficients. This implies that it is uncertain which country has the higher price level because this depends on which sector has the larger productivity gap. In other words, the positive BS effect is not guaranteed to exist. We further introduce distribution costs by assuming that consuming a unit of good A and B requires ϕ_A and ϕ_B units of untradable distribution services respectively in country 1, and ϕ_A / a and ϕ_B / a units in country 2, because the service productivity is a times as high as that in country 1. Without loss of generality, we assume that good S does not require distribution services. It is also assumed that labor cannot move between countries but is perfectly mobile between local sectors. Thus, the wage rate in different sectors of a country is equal, while it is unequal across countries. Two countries trade in a world market under exogenous world prices nominated by a world currency. PPP is assumed to hold for tradable goods at producer level. E_i denotes the world market exchange rate between the currency of country i and the world currency, which is defined as the price of country i's currency in terms of the world currency. Let $\overline{p}_{ij}$ and p_{ij} denote the producers price and retail price for tradable goods respectively, p denote the terms of trade in the world market, implying 1 unit good A changes for p units B, and w_i denote the nominal wage rate in country i. Without loss of generality again, let $\overline{p}_{1A} = 1$. When individuals face different terms of trade in the world market, competitive equilibrium results in one of five different trade patterns:

Case 1: if $1<p<\lambda_B / \lambda_A$, country 1 specializes in A and country 2 specializes in B;

Case 2: if $p<1$, both countries specialize in B;

Case 3: if $p>\lambda_B / \lambda_A$, both countries specialize in A;

Case 4: if $p=1$, country 1 produces both goods while country 2 specializes in B;

Case 5: if $p=\lambda_B / \lambda_A$, country 1 specializes in A while country 2 produces both goods.

Let us discuss case 1 first. In this case, since there is no profit for the producer and PPP is assumed to hold at the producer level, we obtain

$\overline{p}_{1A} = 1$, $\overline{p}_{1B} = 1/p$, $\overline{p}_{2A} = E_1 / E_2$, $\overline{p}_{2B} = E_1 / E_2 p$.

The retail prices are given by:

$p_{1A} = 1 + w_1\phi_A$, $p_{1B} = 1/p + w_1\phi_B$, $p_{2A} = E_1 / E_2 + w_2\phi_A / a$, $p_{2B} = E_1 / E_2 p + w_2\phi_B / a$.

Since $w_1 = 1$ and $w_2 = a\lambda_B E_1 / E_2 p$, then

$p_{1A} = 1 + \phi_A$, $p_{1B} = 1/p + \phi_B$, $p_{1S} = w_1 = 1$;

$p_{2A} = E_1 / E_2 + \lambda_B\phi_A E_1 / E_2 p$, $p_{2B} = E_1 / E_2 p + \lambda_B\phi_B E_1 / E_2 p$, $p_{2S} = w_2 / a = \lambda_B E_1 / E_2 p$.

It is obvious that PPP does not typically hold for retail prices and it is uncertain which country has more expensive untradable services.

The Cobb-Douglas preferences imply that α, β and γ always represent the share of income spent on *A*, *B* and *S*, respectively. Hence,

$$c_{1A} = \frac{\alpha}{1+\phi_A},\ c_{1B} = \frac{\beta}{1/p+\phi_B},\ c_{1S} = \gamma;$$

$$c_{2A} = \frac{a\alpha\lambda_B}{p(1+\phi_A\lambda_B/p)},\ c_{2B} = \frac{a\beta\lambda_B}{p(1/p+\phi_B\lambda_B/p)},\ c_{2S} = a\gamma,$$

where *p* is exogenous.

Similarly, we obtain solutions for the other cases. The results are as follows.

In case 2,

$$c_{1A} = \frac{\alpha/p}{1+\phi_A/p},\ c_{1B} = \frac{\beta/p}{1/p+\phi_B/p}, c_{1S} = \gamma;$$

$$c_{2A} = \frac{a\alpha\lambda_B}{p(1+\phi_A\lambda_B/p)}, c_{2B} = \frac{a\beta\lambda_B}{p(1/p+\phi_B\lambda_B/p)},\ c_{2S} = a\gamma.$$

In case 3,

$$c_{1A} = \frac{\alpha}{1+\phi_A},\ c_{1B} = \frac{\beta}{1/p+\phi_B},\ c_{1S} = \gamma;$$

$$c_{2A} = \frac{a\alpha\lambda_A}{1+\phi_A\lambda_A},\ c_{2B} = \frac{a\beta\lambda_A}{1/p+\phi_B\lambda_A},\ c_{2S} = a\gamma.$$

In case 4,

$$c_{1A} = \frac{\alpha}{1+\phi_A}, c_{1B} = \frac{\beta}{1+\phi_B},\ c_{1S} = \gamma;$$

$$c_{2A} = \frac{a\alpha\lambda_B}{1+\phi_A\lambda_B},\ c_{2B} = \frac{a\beta\lambda_B}{1+\phi_B\lambda_B},\ c_{2S} = a\gamma.$$

In case 5,

$$c_{1A} = \frac{\alpha}{1+\phi_A},\ c_{1B} = \frac{\beta}{\lambda_A / \lambda_B + \phi_B},\ c_{1S} = \gamma;$$

$$c_{2A} = \frac{a\alpha\lambda_A}{1+\phi_A\lambda_A},\ c_{2B} = \frac{a\beta\lambda_A}{\lambda_A / \lambda_B + \phi_B\lambda_A},\ c_{2S} = a\gamma.$$

Using these results, we obtain the money-metric Allen-welfare index for each case in equilibrium,

$$A^{2:1} = \begin{cases} a(\frac{\lambda_B}{p})^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_B / p})^{\alpha}(\frac{1/p+\phi_B}{1/p+\phi_B\lambda_B / p})^{\beta}, \\ a\lambda_B^{\alpha+\beta}(\frac{1+\phi_A / p}{1+\phi_A\lambda_B / p})^{\alpha}(\frac{1/p+\phi_B / p}{1/p+\phi_B\lambda_B / p})^{\beta}, \\ a\lambda_A^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{1/p+\phi_B}{1/p+\phi_B\lambda_A})^{\beta}, \\ a\lambda_B^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_B})^{\alpha}(\frac{1+\phi_B}{1+\phi_B\lambda_B})^{\beta}, \\ a\lambda_A^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{\lambda_A / \lambda_B+\phi_B}{\lambda_A / \lambda_B+\phi_B\lambda_A})^{\beta}. \end{cases} \quad . \tag{15}$$

Since $\lambda_B > \lambda_A > 1$ and $a > 1$, it is straightforward to prove that $A^{2:1}$ is larger than 1. This result is not surprising because it just verifies that the country with higher productivity will be wealthier. What is of significance is that we can next ascertain the relationship between international comparisons of FX income, GK income and true (Afriat) income.

4a. Foreign exchange rate comparisons of incomes

As in Dowrick and Akmal (2003), per capita national income and GDP are assumed to be identical and equal to the wage rate. Therefore, the FX-based per capita income ratio is given by:

$$FX^{2:1} = \frac{E_2 w_2}{E_1 w_1} = \begin{cases} a\lambda_B / p, \\ a\lambda_B, \\ a\lambda_A, \\ a\lambda_B, \\ a\lambda_A. \end{cases} \tag{16}$$

PROPOSITION 6: Market exchange rate comparisons will overstate true international income gaps if and only if the price level of services is higher in the relatively rich country.

PROOF: From equation (15) and (16), we obtain

$$\frac{FX^{2:1}}{A^{2:1}} = \begin{cases} (\frac{\lambda_B}{p})^{1-\alpha-\beta} (\frac{1+\phi_A \lambda_B / p}{1+\phi_A})^{\alpha} (\frac{1/p+\phi_B \lambda_B / p}{1/p+\phi_B})^{\beta}, \\ \lambda_B^{1-\alpha-\beta} (\frac{1+\phi_A \lambda_B / p}{1+\phi_A / p})^{\alpha} (\frac{1/p+\phi_B \lambda_B / p}{1/p+\phi_B / p})^{\beta}, \\ \lambda_A^{1-\alpha-\beta} (\frac{1+\phi_A \lambda_A}{1+\phi_A})^{\alpha} (\frac{1/p+\phi_B \lambda_A}{1/p+\phi_B})^{\beta}, \\ \lambda_B^{1-\alpha-\beta} (\frac{1+\phi_A \lambda_B}{1+\phi_A})^{\alpha} (\frac{1+\phi_B \lambda_B}{1+\phi_B})^{\beta}, \\ \lambda_A^{1-\alpha-\beta} (\frac{1+\phi_A \lambda_A}{1+\phi_A})^{\alpha} (\frac{\lambda_A / \lambda_B+\phi_B \lambda_A}{\lambda_A / \lambda_B+\phi_B})^{\beta}. \end{cases}$$

Since $p_{1S} = \begin{cases} 1, \\ 1/p, \\ 1, \\ 1, \\ 1. \end{cases}$ and $\frac{p_{2S} E_2}{E_1} = \begin{cases} \lambda_B / p, \\ \lambda_B / p, \\ \lambda_A, \\ \lambda_B, \\ \lambda_A. \end{cases}$, we can obtain $\frac{FX^{2:1}}{A^{2:1}} > 1$ iff $\frac{p_{2S} E_2}{E_1} > p_{1S}$, as was to be shown.

This proposition generalizes the results from Dowrick and Akmal (2003) and the baseline model. In particular, if the non-traded labor services are assumed to be more expensive in the rich country, our result coincides with theirs. Moreover, this result may help improve measures of national income for non-benchmark countries: variables such as productivity for untradable goods and the terms of trade should be included as explanatory variables in the regression equation. This will be shown in part 4.

4b. Purchasing-Power-Parity comparisons of incomes

Next, we turn to Purchasing-Power-Parity-based income comparisons. The Geary-Khamis (G-K) method is used by ICP (1987) to aggregate the quantities under constant international prices, which are calculated as weighted arithmetic averages of prices prevailing in the system. Therefore, in our model, the per capita GK income ratio could be constructed as

$$GK^{2:1} = \frac{p_A.c_{2A} + p_B.c_{2B} + p_S.c_{2S}}{p_A.c_{1A} + p_B.c_{1B} + p_S.c_{2S}},$$

where p_i denotes the constant international price for good i.

PROPOSITION 7: Constant price comparisons will overstate true international income gaps if the constant price vector corresponds to that of a country where the productivity is lower (the poor country) and understate true international income gaps if the constant price vector corresponds to that of a country where the productivity is higher (the rich country), regardless of the service price relationship between these two countries.

PROOF: We again consider case 1 first. Suppose the country 2's prices are used, then

$$GK_2^{2:1} = \frac{\frac{a\alpha\lambda_B}{p(1+\phi_A\lambda_B/p)}(1+\phi_A\lambda_B/p) + \frac{a\beta\lambda_B}{p(1/p+\phi_B\lambda_B/p)}(1/p+\phi_B\lambda_B/p) + a\gamma\lambda_B/p}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_B/p) + \frac{\beta}{1/p+\phi_B}(1/p+\phi_B\lambda_B/p) + \gamma\lambda_B/p}$$

$$= \frac{a\lambda_B/p}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_B/p) + \frac{\beta}{1/p+\phi_B}(1/p+\phi_B\lambda_B/p) + \gamma\lambda_B/p}.$$

Thus,

$$\frac{GK_2^{2:1}}{A^{2:1}} = \frac{\dfrac{a\lambda_B / p}{\dfrac{\alpha}{1+\phi_A}(1+\phi_A\lambda_B / p) + \dfrac{\beta}{1/p+\phi_B}(1/p+\phi_B\lambda_B / p) + \gamma\lambda_B / p}}{a(\dfrac{\lambda_B}{p})^{\alpha+\beta}(\dfrac{1+\phi_A}{1+\phi_A\lambda_B / p})^{\alpha}(\dfrac{1/p+\phi_B}{1/p+\phi_B\lambda_B / p})^{\beta}}$$

$$= \frac{(\dfrac{\lambda_B}{p})^{\gamma}}{\left(\dfrac{\alpha(1+\phi_A\lambda_B / p)}{1+\phi_A} + \dfrac{\beta(1/p+\phi_B\lambda_B / p)}{1/p+\phi_B} + \gamma\lambda_B / p\right)(\dfrac{1+\phi_A}{1+\phi_A\lambda_B / p})^{\alpha}(\dfrac{1/p+\phi_B}{1/p+\phi_B\lambda_B / p})^{\beta}}.$$

We want to show that $\frac{GK_2^{2:1}}{A^{2:1}} < 1$. Let $\frac{1+\phi_A\lambda_B / p}{1+\phi_A} = x$, $\frac{1/p+\phi_B\lambda_B / p}{1/p+\phi_B} = y$ and $\lambda_B / p = z$.

We just need to show that $x^{\alpha}y^{\beta}z^{\gamma} < \alpha x + \beta y + \gamma z$. Since $x = y = z$ does not hold, we can obtain the result simply from Jensen's inequality.[9]

Suppose country 1's prices are used, then

$$GK_1^{2:1} = \frac{\dfrac{a\alpha\lambda_B}{p(1+\phi_A\lambda_B / p)}(1+\phi_A) + \dfrac{a\beta\lambda_B}{p(1/p+\phi_B\lambda_B / p)}(1/p+\phi_B) + a\gamma}{\dfrac{\alpha}{1+\phi_A}(1+\phi_A) + \dfrac{\beta}{1/p+\phi_B}(1/p+\phi_B) + \gamma}$$

$$= \frac{a\alpha\lambda_B}{p(1+\phi_A\lambda_B / p)}(1+\phi_A) + \frac{a\beta\lambda_B}{p(1/p+\phi_B\lambda_B / p)}(1/p+\phi_B) + a\gamma.$$

Therefore,

$$\frac{GK_1^{2:1}}{A^{2:1}} = \frac{\dfrac{a\alpha\lambda_B}{p(1+\phi_A\lambda_B / p)}(1+\phi_A) + \dfrac{a\beta\lambda_B}{p(1/p+\phi_B\lambda_B / p)}(1/p+\phi_B) + a\gamma}{a(\dfrac{\lambda_B}{p})^{\alpha+\beta}(\dfrac{1+\phi_A}{1+\phi_A\lambda_B / p})^{\alpha}(\dfrac{1/p+\phi_B}{1/p+\phi_B\lambda_B / p})^{\beta}}$$

$$= \frac{\dfrac{\alpha(1+\phi_A)}{(1+\phi_A\lambda_B / p)} + \dfrac{\beta(1/p+\phi_B)}{(1/p+\phi_B\lambda_B / p)} + \gamma(\dfrac{\lambda_B}{p})^{-1}}{(\dfrac{\lambda_B}{p})^{-\gamma}(\dfrac{1+\phi_A}{1+\phi_A\lambda_B / p})^{\alpha}(\dfrac{1/p+\phi_B}{1/p+\phi_B\lambda_B / p})^{\beta}}.$$

[9] We are grateful to Ludovic Renou for his help with this proof.

Let $\frac{1+\phi_A}{1+\phi_A\lambda_B/p}=x$, $\frac{1/p+\phi_B}{1/p+\phi_B\lambda_B/p}=y$ and $(\frac{\lambda_B}{p})^{-1}=z$, then we can get $\frac{GK_1^{2:1}}{A^{2:1}}>1$.

Correspondingly, the same conclusion can be derived for the other cases (see the appendix).

This proposition is consistent with the one suggested by Dowrick and Akmal (2003) and our baseline model if it is assumed that nontraded labor services are more expensive in the rich country. However, when the BS effect is endogenized, our extensions are significant. We found that the relationship between GK income and true income only depends on the productivity of the country of which the price vector is used as the constant price vector, while it does not depend on whether the higher price vector or the lower one is used. Accordingly, it is possible that even if the relatively low price vector is chosen as the constant one, the true national income gaps will be underestimated as long as it occurs in the country with higher productivity. This outcome is opposite to the one in the previous literature.

5. Empirical result

As mentioned in the earlier sections, Afriat income could be calculated only for the countries included in ICP surveys, while a lot of countries did not participate in the program. For that reason, we have to predict true income for non-benchmark countries. Dowrick and Akmal (2003) suggest a regression model with a log-linear relationship between True income and FX income. Inspired by their work, the theoretical model explored in the present paper implies that the regression could be improved by augmenting the number of variables. From equation (15) and (16), if we just ignore the influence of distribution costs for the moment, and normalise true income per capita in country 1 to unity, the equation could be stated as

$$\ln(A^i) - \ln(a^i) = (\alpha + \beta)(\ln(FX^i) - \ln(a^i)).$$

Our regression model is based on this equation. We select the cross-country data for the year 1980. The source of FX income (GDP per capita) data is "World Bank Global Development Network Growth Database", and Afriat income is directly taken from Dowrick and Quiggin (1997). Since Afriat incomes have been normalised in their paper, we need to add an intercept term in our regression. Thus, the regression model could be specified as

$$\ln(A^i) - \ln(a^i) = c + (\alpha + \beta)(\ln(FX^i) - \ln(a^i))$$ [10],

where c is a constant. There are no direct data on Labor productivity for untradable goods (a^i). Hence, it is calculated as value added (volume) for the total service industry, divided by total employment in this sector, the data of which are presented in "The OECD STAN database for Industrial Analysis". We plot the data of $\ln(A^i) - \ln(a^i)$ against $\ln(FX^i) - \ln(a^i)$ in Figure 1, which clearly displays the log-linear relationship. Although the available sample size is small, including only 12 countries, the regression is still run applying OLS and the results are reported in the table 1.

Figure 1

$\ln(A^i) - \ln(a^i)$ against $\ln(FX^i) - \ln(a^i)$,1980
actual and predicted values for 12 OECD countries

[10] We are thankful to Tin Nguyen for his help with this empirical model specification.

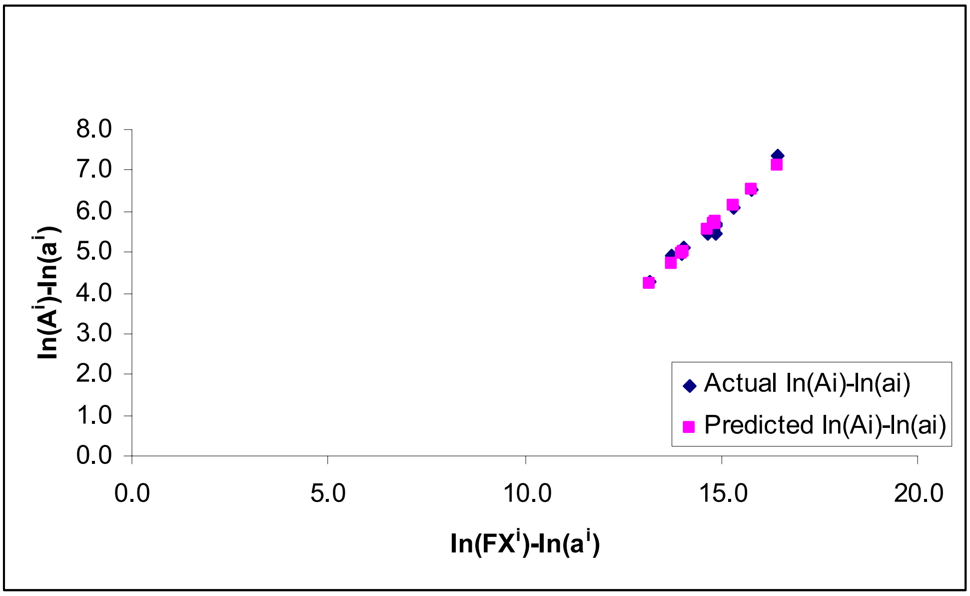

Table 1

The dependent variable is $\ln(A^i) - \ln(a^i)$.

	Coefficients	Standard Error	t stat	p > |t|	Lower 95%	Upper 95%
Intercept: c	-7.429845	0.921119	-8.06611	0.000000	-9.235205	-5.624486
$\ln(FX^i) - \ln(a^i) : \alpha + \beta$	0.884564	0.063584	13.91163	0.000000	0.759940	1.009187

R-Squared	0.967081
Adjusted R-Squared	0.963789
Standard Error	0.153931
Sample size	12

We find that the term of $\ln(FX^i) - \ln(a^i)$ is highly significant – as expected – and the value of the coefficient fits our theoretical model. Dowrick and Akmal's (2003) result is broadly similar, although they suggest that the coefficient should be between 0.6 and 0.8. It is also found that the coefficient is significantly different from 1 at the 5% level by one-tailed test, which confirms that FX income is biased. Nevertheless, there are several problems which merit our attention because they may affect the estimation results. Firstly, the sample size is too small to satisfy the requirement for a regression. Secondly, as mentioned above, Afriat incomes have been normalized, which might lead to a biased estimation especially since the sample size is small. Moreover, all observations in the sample involve OECD countries; thus the basic assumption of the theoretical model that labor cannot move between countries may be violated.

6. Conclusion and further research

The main point of the present paper is not the demonstration that true relative income is a function of foreign-exchange based relative income, as this merely confirms DA's result. The main contribution of our baseline model is, instead, the result that this function holds regardless of the relative importance of income determinants. Whether it is factor productivity, input scarcity, or demand for outputs that determine income differentials, the same functional relationship between the FX income ratio and the true income ratio holds. This insight strengthens the case for empirical estimation of the relationship between FX income and Afriat income for benchmark year-country combinations, in order to estimate Afriat income for non-benchmark years/countries. The result may be improved measurement of national incomes for non-benchmark years/countries.

We incorporated some extensions to make the model more general and more realistic. Some results from the existing literature are confirmed. However, using several new assumptions, we derive some more general propositions, which are not entirely consistent with the traditional ones. For example, by endogenizing the BS effect, we show that the rich country could have a lower price level. If this case really occurs, the relationship between FX income, GK income and true (Afriat) income will be different from the results obtained from previous studies.

In addition, based on our theoretical foundations, regression used to predict true income for non-benchmark countries is improved by adding a variable. Although the sample size is unsatisfactory, the approach taken and the results should be acceptable. We look ahead to replicating this analysis using a better data set.

Future work can follow the direction taken in the present paper. Estimation of Afriat income can be improved by further generalizing the theoretical foundations. The following aspects may be worth considering in further research. Firstly, multi-factor production functions with a continuum of differentiated goods in a monopolistically competitive market could be analysed. Secondly, the two-country model could be extended to an *n*-country model. Thirdly, we can endogenize tradability by introducing trade barriers. For example, if the comparative advantage of producing a good is so great that it can cover the total cost including costs attributed to trade barriers, it becomes tradable, while others remain untradable. Fourth, while DA obtain a log-linear empirical relationship between Afriat income and FX income, they do not take into account off-equilibrium movements in exchange rates. Their method could be improved by finding a way to smooth time series of exchange rates. Moreover, DA and this paper do not consider financial markets. These may need to be considered, because financial variables affect both exchange rates (used for computing FX income) and prices (used for computing the GK-PPP and Afriat measures). A dynamic model could explore this in further detail.

Another promising question for future research is how to appropriately estimate national incomes and living standards. In fact, even if the Afriat index is an unbiased estimate of GDP per capita, could it really measure proper "well-being" or, perhaps more to the point, “economic well-being” of a country? Firstly, environment and natural resource depletion and degradation are ignored when national income is measured, so apparent economic growth may in fact be unsustainable and hence illusory. Secondly, national income accounts ignore nonmarket output, which includes in particular household activity. Thirdly, informal and illegal transactions are not adequately reflected in official GDP. In addition, national income should not be the sole measure of living standards. Other social indicators, like life expectancy, literacy rates and distribution of individual welfare should be incorporated in the analysis of well-being.

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APPENDIX

PROOFS OF PROPOSITION 7 IN THE OTHER FOUR CASES

In case 2,

$$GK_2^{2:1}=\frac{\frac{a\alpha\lambda_B}{p(1+\phi_A\lambda_B/p)}(1+\phi_A\lambda_B/p)+\frac{a\beta\lambda_B}{p(1/p+\phi_B\lambda_B/p)}(1/p+\phi_B\lambda_B/p)+a\gamma\lambda_B/p}{\frac{\alpha/p}{1+\phi_A/p}(1+\phi_A\lambda_B/p)+\frac{\beta/p}{1/p+\phi_B/p}(1/p+\phi_B\lambda_B/p)+\gamma\lambda_B/p}$$

$$=\frac{a\lambda_B/p}{\frac{\alpha/p}{1+\phi_A/p}(1+\phi_A\lambda_B/p)+\frac{\beta/p}{1/p+\phi_B/p}(1/p+\phi_B\lambda_B/p)+\gamma\lambda_B/p}.$$

Thus,

$$\frac{GK_2^{2:1}}{A^{2:1}}=\frac{\frac{a\lambda_B/p}{\frac{\alpha/p}{1+\phi_A/p}(1+\phi_A\lambda_B/p)+\frac{\beta/p}{1/p+\phi_B/p}(1/p+\phi_B\lambda_B/p)+\gamma\lambda_B/p}}{a\lambda_B^{\alpha+\beta}(\frac{1+\phi_A/p}{1+\phi_A\lambda_B/p})^{\alpha}(\frac{1/p+\phi_B/p}{1/p+\phi_B\lambda_B/p})^{\beta}}$$

$$=\frac{\lambda_B^{\gamma}}{\left(\frac{\alpha(1+\phi_A\lambda_B/p)}{1+\phi_A/p}+\frac{\beta(1/p+\phi_B\lambda_B/p)}{1/p+\phi_B/p}+\gamma\lambda_B\right)(\frac{1+\phi_A/p}{1+\phi_A\lambda_B/p})^{\alpha}(\frac{1/p+\phi_B/p}{1/p+\phi_B\lambda_B/p})^{\beta}}.$$

Let $\frac{1+\phi_A\lambda_B/p}{1+\phi_A/p}=x$, $\frac{1/p+\phi_B\lambda_B/p}{1/p+\phi_B/p}=y$ and $\lambda_B=z$. Since $x^{\alpha}y^{\beta}z^{\gamma}<\alpha x+\beta y+\gamma z$, we obtain $\frac{GK_2^{2:1}}{A^{2:1}}<1$.

On the other hand,

$$GK_1^{2:1}=\frac{\frac{a\alpha\lambda_B}{p(1+\phi_A\lambda_B/p)}(1+\phi_A/p)+\frac{a\beta\lambda_B}{p(1/p+\phi_B\lambda_B/p)}(1/p+\phi_B/p)+a\gamma/p}{\frac{\alpha/p}{1+\phi_A/p}(1+\phi_A/p)+\frac{\beta/p}{1/p+\phi_B/p}(1/p+\phi_B/p)+\gamma/p}$$

$$=\frac{a\alpha\lambda_B}{1+\phi_A\lambda_B/p}(1+\phi_A/p)+\frac{a\beta\lambda_B}{1/p+\phi_B\lambda_B/p}(1/p+\phi_B/p)+a\gamma.$$

Therefore, $$\frac{GK_1^{2:1}}{A^{2:1}}=\frac{\frac{a\alpha\lambda_B}{1+\phi_A\lambda_B/p}(1+\phi_A/p)+\frac{a\beta\lambda_B}{1/p+\phi_B\lambda_B/p}(1/p+\phi_B/p)+a\gamma}{a\lambda_B^{\alpha+\beta}(\frac{1+\phi_A/p}{1+\phi_A\lambda_B/p})^{\alpha}(\frac{1/p+\phi_B/p}{1/p+\phi_B\lambda_B/p})^{\beta}}$$

$$=\frac{\frac{\alpha}{1+\phi_A\lambda_B/p}(1+\phi_A/p)+\frac{\beta}{1/p+\phi_B\lambda_B/p}(1/p+\phi_B/p)+\gamma\lambda_B^{-1}}{(\frac{1+\phi_A/p}{1+\phi_A\lambda_B/p})^{\alpha}(\frac{1/p+\phi_B/p}{1/p+\phi_B\lambda_B/p})^{\beta}\lambda_B^{-\gamma}}.$$

Let $\frac{1+\phi_A/p}{1+\phi_A\lambda_B/p}=x$, $\frac{1/p+\phi_B/p}{1/p+\phi_B\lambda_B/p}=y$ and $\lambda_B^{-1}=z$. It follows that $\frac{GK_1^{2:1}}{A^{2:1}}>1$.

In case 3,

$$GK_2^{2:1}=\frac{\frac{a\alpha\lambda_A}{1+\phi_A\lambda_A}(1+\phi_A\lambda_A)+\frac{a\beta\lambda_A}{1/p+\phi_B\lambda_A}(1/p+\phi_B\lambda_A)+a\gamma\lambda_A}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_A)+\frac{\beta}{1/p+\phi_B}(1/p+\phi_B\lambda_A)+\gamma\lambda_A}$$

$$=\frac{a\lambda_A}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_A)+\frac{\beta}{1/p+\phi_B}(1/p+\phi_B\lambda_A)+\gamma\lambda_A}.$$

Thus,

$$\frac{GK_2^{2:1}}{A^{2:1}}=\frac{\frac{a\lambda_A}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_A)+\frac{\beta}{1/p+\phi_B}(1/p+\phi_B\lambda_A)+\gamma\lambda_A}}{a\lambda_A^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{1/p+\phi_B}{1/p+\phi_B\lambda_A})^{\beta}}$$

$$=\frac{\lambda_A^{\gamma}}{\left(\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_A)+\frac{\beta}{1/p+\phi_B}(1/p+\phi_B\lambda_A)+\gamma\lambda_A\right)(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{1/p+\phi_B}{1/p+\phi_B\lambda_A})^{\beta}}.$$

Let $\frac{1+\phi_A\lambda_A}{1+\phi_A}=x$, $\frac{1/p+\phi_B\lambda_A}{1/p+\phi_B}=y$ and $\lambda_A=z$; hence $\frac{GK_2^{2:1}}{A^{2:1}}<1$.

On the other hand,

$$GK_1^{2:1}=\frac{\frac{a\alpha\lambda_A}{1+\phi_A\lambda_A}(1+\phi_A)+\frac{a\beta\lambda_A}{1/p+\phi_B\lambda_A}(1/p+\phi_B)+a\gamma}{\frac{\alpha}{1+\phi_A}(1+\phi_A)+\frac{\beta}{1/p+\phi_B}(1/p+\phi_B)+\gamma}$$

$$=\frac{a\alpha\lambda_A}{1+\phi_A\lambda_A}(1+\phi_A)+\frac{a\beta\lambda_A}{1/p+\phi_B\lambda_A}(1/p+\phi_B)+a\gamma$$

Thus,

$$\frac{GK_1^{2:1}}{A^{2:1}}=\frac{\frac{a\alpha\lambda_A}{1+\phi_A\lambda_A}(1+\phi_A)+\frac{a\beta\lambda_A}{1/p+\phi_B\lambda_A}(1/p+\phi_B)+a\gamma}{a\lambda_A^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{1/p+\phi_B}{1/p+\phi_B\lambda_A})^{\beta}}$$

$$=\frac{\frac{\alpha}{1+\phi_A\lambda_A}(1+\phi_A)+\frac{\beta}{1/p+\phi_B\lambda_A}(1/p+\phi_B)+\gamma\lambda_A^{-1}}{\lambda_A^{-\gamma}(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{1/p+\phi_B}{1/p+\phi_B\lambda_A})^{\beta}}.$$

Let $\frac{1+\phi_A}{1+\phi_A\lambda_A}=x$, $\frac{1/p+\phi_B}{1/p+\phi_B\lambda_A}=y$ and $\lambda_A^{-1}=z$; hence $\frac{GK_1^{2:1}}{A^{2:1}}>1$.

In case 4,

$$GK_2^{2:1} = \frac{\frac{a\alpha\lambda_B}{1+\phi_A\lambda_B}(1+\phi_A\lambda_B) + \frac{a\beta\lambda_B}{1+\phi_B\lambda_B}(1+\phi_B\lambda_B) + a\gamma\lambda_B}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_B) + \frac{\beta}{1+\phi_B}(1+\phi_B\lambda_B) + \gamma\lambda_B}$$

$$= \frac{a\lambda_B}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_B) + \frac{\beta}{1+\phi_B}(1+\phi_B\lambda_B) + \gamma\lambda_B}.$$

Thus,

$$\frac{GK_2^{2:1}}{A^{2:1}} = \frac{\dfrac{a\lambda_B}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_B) + \frac{\beta}{1+\phi_B}(1+\phi_B\lambda_B) + \gamma\lambda_B}}{a\lambda_B{}^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_B})^{\alpha}(\frac{1+\phi_B}{1+\phi_B\lambda_B})^{\beta}}$$

$$= \frac{\lambda_B{}^{\gamma}}{\left(\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_B) + \frac{\beta}{1+\phi_B}(1+\phi_B\lambda_B) + \gamma\lambda_B\right)(\frac{1+\phi_A}{1+\phi_A\lambda_B})^{\alpha}(\frac{1+\phi_B}{1+\phi_B\lambda_B})^{\beta}}.$$

Let $\frac{1+\phi_A\lambda_B}{1+\phi_A} = x$, $\frac{1+\phi_B\lambda_B}{1+\phi_B} = y$ and $\lambda_B = z$; hence $\frac{GK_2^{2:1}}{A^{2:1}} < 1$.

On the other hand,

$$GK_1^{2:1} = \frac{\frac{a\alpha\lambda_B}{1+\phi_A\lambda_B}(1+\phi_A) + \frac{a\beta\lambda_B}{1+\phi_B\lambda_B}(1+\phi_B) + a\gamma}{\frac{\alpha}{1+\phi_A}(1+\phi_A) + \frac{\beta}{1+\phi_B}(1+\phi_B) + \gamma}$$

$$= \frac{a\alpha\lambda_B}{1+\phi_A\lambda_B}(1+\phi_A) + \frac{a\beta\lambda_B}{1+\phi_B\lambda_B}(1+\phi_B) + a\gamma.$$

Therefore,

$$\frac{GK_1^{2:1}}{A^{2:1}} = \frac{\frac{a\alpha\lambda_B}{1+\phi_A\lambda_B}(1+\phi_A) + \frac{a\beta\lambda_B}{1+\phi_B\lambda_B}(1+\phi_B) + a\gamma}{a\lambda_B{}^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_B})^{\alpha}(\frac{1+\phi_B}{1+\phi_B\lambda_B})^{\beta}}$$

$$= \frac{\frac{\alpha}{1+\phi_A\lambda_B}(1+\phi_A) + \frac{\beta}{1+\phi_B\lambda_B}(1+\phi_B) + \gamma\lambda_B{}^{-1}}{(\frac{1+\phi_A}{1+\phi_A\lambda_B})^{\alpha}(\frac{1+\phi_B}{1+\phi_B\lambda_B})^{\beta}\lambda_B{}^{-\gamma}}.$$

Let $\frac{1+\phi_A}{1+\phi_A\lambda_B} = x$, $\frac{1+\phi_B}{1+\phi_B\lambda_B} = y$ and $\lambda_B{}^{-1} = z$; hence $\frac{GK_1^{2:1}}{A^{2:1}} > 1$.

In case 5,

$$GK_2^{2:1} = \frac{\frac{a\alpha\lambda_A}{1+\phi_A\lambda_A}(1+\phi_A\lambda_A) + \frac{a\beta\lambda_A}{\lambda_A/\lambda_B+\phi_B\lambda_A}(\lambda_A/\lambda_B+\phi_B\lambda_A) + a\gamma\lambda_A}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_A) + \frac{\beta}{\lambda_A/\lambda_B+\phi_B}(\lambda_A/\lambda_B+\phi_B\lambda_A) + \gamma\lambda_A}$$

$$= \frac{a\lambda_A}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_A) + \frac{\beta}{\lambda_A/\lambda_B+\phi_B}(\lambda_A/\lambda_B+\phi_B\lambda_A) + \gamma\lambda_A}.$$

Thus, $$\frac{GK_2^{2:1}}{A^{2:1}} = \frac{\dfrac{a\lambda_A}{\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_A) + \frac{\beta}{\lambda_A/\lambda_B+\phi_B}(\lambda_A/\lambda_B+\phi_B\lambda_A) + \gamma\lambda_A}}{a\lambda_A^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{\lambda_A/\lambda_B+\phi_B}{\lambda_A/\lambda_B+\phi_B\lambda_A})^{\beta}}$$

$$= \frac{\lambda_A^{\gamma}}{\left(\frac{\alpha}{1+\phi_A}(1+\phi_A\lambda_A) + \frac{\beta}{\lambda_A/\lambda_B+\phi_B}(\lambda_A/\lambda_B+\phi_B\lambda_A) + \gamma\lambda_A\right)(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{\lambda_A/\lambda_B+\phi_B}{\lambda_A/\lambda_B+\phi_B\lambda_A})^{\beta}}.$$

Let $\frac{1+\phi_A\lambda_A}{1+\phi_A} = x$, $\frac{\lambda_A/\lambda_B+\phi_B\lambda_A}{\lambda_A/\lambda_B+\phi_B} = y$ and $\lambda_A = z$; hence $\frac{GK_2^{2:1}}{A^{2:1}} < 1$.

On the other hand,

$$GK_1^{2:1} = \frac{\frac{a\alpha\lambda_A}{1+\phi_A\lambda_A}(1+\phi_A) + \frac{a\beta\lambda_A}{\lambda_A/\lambda_B+\phi_B\lambda_A}(\lambda_A/\lambda_B+\phi_B) + a\gamma}{\frac{\alpha}{1+\phi_A}(1+\phi_A) + \frac{\beta}{\lambda_A/\lambda_B+\phi_B}(\lambda_A/\lambda_B+\phi_B) + \gamma}$$

$$= \frac{a\alpha\lambda_A}{1+\phi_A\lambda_A}(1+\phi_A) + \frac{a\beta\lambda_A}{\lambda_A/\lambda_B+\phi_B\lambda_A}(\lambda_A/\lambda_B+\phi_B) + a\gamma.$$

Therefore, $$\frac{GK_1^{2:1}}{A^{2:1}} = \frac{\frac{a\alpha\lambda_A}{1+\phi_A\lambda_A}(1+\phi_A) + \frac{a\beta\lambda_A}{\lambda_A/\lambda_B+\phi_B\lambda_A}(\lambda_A/\lambda_B+\phi_B) + a\gamma}{a\lambda_A^{\alpha+\beta}(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{\lambda_A/\lambda_B+\phi_B}{\lambda_A/\lambda_B+\phi_B\lambda_A})^{\beta}}$$

$$= \frac{\frac{\alpha}{1+\phi_A\lambda_A}(1+\phi_A) + \frac{\beta}{\lambda_A/\lambda_B+\phi_B\lambda_A}(\lambda_A/\lambda_B+\phi_B) + \gamma\lambda_A^{-1}}{(\frac{1+\phi_A}{1+\phi_A\lambda_A})^{\alpha}(\frac{\lambda_A/\lambda_B+\phi_B}{\lambda_A/\lambda_B+\phi_B\lambda_A})^{\beta}\lambda_A^{-\gamma}}.$$

Let $\frac{1+\phi_A}{1+\phi_A\lambda_A} = x$, $\frac{\lambda_A/\lambda_B+\phi_B}{\lambda_A/\lambda_B+\phi_B\lambda_A} = y$ and $\lambda_A^{-1} = z$; hence $\frac{GK_1^{2:1}}{A^{2:1}} > 1$.