Iterative Rationality in the Dirty Faces Game

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April 26, 2007

ABSTRACT

The Dirty Faces game requires players to perform iterative reasoning in order to arrive at equilibrium play. The game is dominance solvable with a unique equilibrium when it is correctly specified. The particular payoff structure has significant implication on whether the reasoning process leads to equilibrium play. This paper illustrates that the traditional specification - as used by Weber (2001) - leads to multiple equilibria and the game loses its dominance solvability. We modify the payoff structure and restore uniqueness. The resulting game, which is dominance solvable, is implemented in an experiment to test the depth of iterative reasoning in humans. Our data analysis suggests that some deviation from equilibrium play is due to limited depth of iteration. Additionally, we find evidence that the lack of confidence in other players' iterative abilities also induces deviations from equilibrium play.

Keywords: Game Theory, Iterative Reasoning, Experimental Economics.
JEL Classification Numbers: C91, C92, C72.

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Acknowledgment

I would like to thank my supervisor, `Ralph C. Bayer`, who has given me many valuable suggestions and assistance in producing this thesis. This will be impossible without his support. `Rupert Sausgruber` involved in the preparation of the experimental sessions, getting our hands (but not faces) dirty with programming. Thank you to the two anonymous examiners who have provided some ideas to make it a better thesis. Thanks also go to the Behaviour Economics class of 2004, in no particular order, `Bin Guan`, `Jerry "Seinfeld" Shen`, `Mark "Cappa" Caputo`, and `"Bonny" Bonnici` who have assisted in conducting the experiment - from "Big Moose". Taking regular caffeine shots and making random noises ϵ with `Changxia`, `Nopphawon`, `"Tiger" Zhang`, and `Cramey` have kept me from becoming a mad game theorist - thanks for your support. Last but not the least, to my wife `Frances`, who came into my life at a time when this paper was plain white, then put up with my absence as this paper get wet with ink - Thank you.

Hong Kong and Adelaide - April 2007

Declaration

NAME: Mickey Chi Yung Chan PROGRAM: Master of Economics

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1. INTRODUCTION

Dominance solvable games are characterised by having a unique equilibrium. Players may derive their best responses through iterative deletion of strictly dominated strategies. The reasoning process of iterative deletion for every player requires the iteration of the following steps:

`step 1` A player should delete all strictly dominated strategies from his set of strategies, as they can never be best responses.

`step 2` At the same time, this player knows that other players understand his reasoning. Therefore, other players only play best responses to his non-dominated strategies. So this player should delete the dominated strategies of the other players, given that they do not consider his dominated strategies.

The iteration process may not yet end here. `step 1` has reduced the player's own strategy space given that the other players might play any strategies. `step 2` reduced the other players' strategy space given that they do not consider him to play dominated strategies. Now previously undominated strategies can become strictly dominated. Then the iterating process repeated with `step 1` and `step 2` on the now smaller strategy space. In a dominance solvable game, this process goes on until only one strategy profile (a collection of one strategy per player) remains. This profile is the unique equilibrium of the game. The equilibrium is reached when every player has behaved rationally by performing the required steps of iteration. Therefore, these games are often used in experiments to examine the depth of iterative reasoning of human subjects.

One of the most influential approaches to measure people's depth of iterative reasoning is the experimental implementations of the "'p-beauty contests"', which was first studied experimentally by Nagel (1995). Similar to guessing what average readers may pick as the most beautiful faces in a page 3 beauty contest in a tabloid[1], players in the p-beauty contest are asked to pick a number such that it is closest to the average of all players' numbers multiplied by a fraction p.

In Nagel's game, players select a number between 0 and 100. The average of the numbers picked by the players are multiplied by a fraction $p \in (0, 1)$. Profits are paid according to how close the guesses are to this target.[2] In the first step of iteration, the p-average cannot be higher than $100p$, since 100 is the largest number. Therefore, selecting any numbers in $(100p, 100]$ is strictly dominated by $100p$. All players should realise this under the assumption of rationality. It follows in the second step of iteration that selecting any numbers in $(100p^2, 100p]$ are strictly dominated by $100p^2$. Similarly, after k-steps of iteration, the players should deduce that any numbers in $(100p^k, 100p^{k-1}]$ are strictly dominated by $100p^k$. When this iteration process continues infinitely, the Nash Equilibrium is reached. Everyone selects zero in equilibrium.[3] The numbers selected by players reveal how many steps of iteration have been performed. For example, a number between $(100p^2, 100p]$ indicates the player has performed one step of iteration.

[1] The name is called the beauty contests after the famous passage in Keynes (1936) *General Theory of Employment, Interest, and Money*. It draws an analogy between picking companies in the stock market, and guessing the winner in the newspaper beauty contest.

[2] There are multiple payout rules, which all lead to the same equilibrium. The closest person wins a prize, or the payoff is negatively proportional to the distance from the target, are possible rules.

[3] The Nash equilibrium is always 0 for $p < 1$, and 100 for $p > 1$.

Nagel (1995) experiment has used groups of fourteen to sixteen German students as subjects. Her results from games with $p = 2/3$ showed spikes at 33 and 22 (one and two steps of iteration from the midpoint 50, respectively), with very few picking zero. This was replicated and reported by Ho, Camerer, and Weigelt (1998). Bosch-Domnech, Montalvo, Nagel, and Satorra (2002) examined the newspaper contests which drew a larger sample population (over 3000 participants in one contest), and there are also spikes at 33, 22, and 0.[4] They have concluded that the typical beauty contest results are robust, as both the lab and the larger field experiment gave the same picture. Sutter (2005) and Kocher and Sutter (2006) used the beauty contest to compare individual and group learning behaviour.

However, there are problems with the use of experimental beauty contests, as researchers can not discriminate between steps of iteration and focal points. An example would be 50, which is a midpoint between 0 and 100, but it is also one step of iteration for $p = 1/2$. In addition, the results show that the starting point of iteration seems to be some arbitrary focal points, like 50 in the above-mentioned papers.

The dirty faces game (Littlewood 1953) provides a different approach. Given the game has been set up properly, it requires players to perform a precise number of iterations to reach equilibrium. The number of iterations is necessarily finite, depends on the parameters of the game and it can therefore be manipulated in an experiment. The basic story is the following: players know whether other players' faces are dirty, and whether there is at least one dirty face in the group. Their task is to find out as quickly as possible if their own faces are dirty, since

[4] The winning numbers were between 13 and 17 in three contests. Playing the equilibrium strategy is detrimental when other players do not obey dominance. In fact, players who picked the winning numbers may either have anticipated the irrational responses and therefore deviated from equilibrium, or they might have just performed a limited number of iterations.

they do not know the state of their own faces. They may need to observe the reaction of other players in order to find it out.

Depending on how complicated the situation is, more or less steps of iteration are necessary to find out the condition of their own faces. Particularly, consider there are unspecified number of players with at least one dirty face among players. One step of iteration is required for whom has not observed any dirty faces. Two steps of iteration are required for whom has observed exactly one dirty face. Three steps of iteration are required for whom has observed two dirty faces. Generally, n step of iteration is required for the player who has observed $n-1$ dirty faces. In addition, the iterative reasoning of the player becomes dependent on the reaction of other players whenever he has observed any dirty faces.

Weber (2001) implemented the game experimentally. He found that most subjects can perform the simplest level of iteration. However, the number of best response plays drops significantly if more than one step of iteration are necessary. Unfortunately, Weber overlooked that his formulation allowed for multiple equilibria in weakly dominated strategies. The existence of multiple equilibria may have been the reason for his results. Additionally, it is not really possible to talk of steps of iteration if a game has multiple equilibria, as the deletion of strictly dominated strategies does not necessarily lead to the desired equilibrium.

The cause of multiple equilibria in Weber's implementation comes from the payoff structure. The culprit is the payoffs that are the same at all stages in the game. So announcing the state of their own faces as soon as possible does not strictly dominate waiting - it only weakly dominates. Hence, the relatively poor performance of Weber's subjects could be due to the fact that the iterated deletion of dominated strategies breaks down, since dominance is not strict. We propose a refinement of the game structure by introducing a waiting cost. The waiting cost reduces the payoffs progressively with time. Therefore it establishes strict dominance. Waiting unnecessarily becomes strictly dominated. With this "refinement", the previously weakly dominated strategies become strictly dominated and hence they can be deleted iteratively. There remains only one single surviving strategy for all players. It follows that there is a unique equilibrium. This makes the game truly dominance solvable.

We have designed a series of experiments that are based on the refined game. The experiment aims to look at the frequencies of agreement with the theoretical best responses on the equilibrium path. The experimental results show that most subjects succeed when one iteration is required. However, the frequencies of agreement with equilibrium play fall by half when more than one iteration is required. We look at the factors that may explain the fall in the frequencies of agreement by employing a linear random-intercept logit model. We find that the number of players in a cohort, and the required number of iteration are important factors in explaining the frequencies of agreement. This supports the limited computation hypothesis, which may be seen as a kind of bounded rationality. Human subjects make decisions under the constraints of limited computational ability, resources and time. However, these constraints are largely ignored under our assumptions in our theoretical analysis. We also find support that some deviation from equilibrium play originates from the subjects' doubts about the rationality of the other players.

We discuss the background of the dirty faces game and some related literature in chapter 2. The notation and settings are stated formally for the dirty faces game in chapter 3. In the subsequent two chapters, we will show the existence of multiple best responses and hence multiple equilibria in the dirty faces game.

We will then propose a refinement to the payoff structure, which makes the game dominance solvable. After that, we discuss the design and the results of our experiment. Finally, we conclude this paper with suggestions for future experiment.

2. THE DIRTY FACES GAME

The dirty faces game is a logical problem involving iterative reasoning. It was described by Littlewood (1953):

> "Three ladies, A, B, C, in a railway carriage all have dirty faces and are all laughing. It suddenly flashes on A: why doesn't B realize C is laughing at her? Heavens! I must be laughable. (Formally: If I, A, am not laughable, B will be arguing: If I, B, am not laughable, C has nothing to laugh at. Since B does not so argue, I, A, must be laughable.)"

The ladies know the state of others' faces, but not their own. The state of their own faces may be revealed through others' reactions. In this case, someone must have a dirty face for others to laugh at. If each lady has an incentive to take a different action (like stop laughing and go to the restroom) once she is certain that her face is dirty, others will be able to deduce the state of their own faces from the observation of other faces and actions. We will later explain this reasoning in more detail. Note that the deduction relies on everyone knowing that there is at least one dirty face. In addition, the chain of reasoning breaks down if one of the ladies does not draw the correct inferences from what she is seeing. The lady who keeps laughing in all situations spoils the day for the other ladies, as other ladies might draw the wrong conclusions from her indifferent behaviour. It is because other ladies cannot make sense of the behaviour of this lady. There needs to be common knowledge of the incentives and rationality for the deduction to work. Everyone has to understand the appropriate behaviour for given situations, and must also believe that others behave in the same manner.

In what follows we will model the dirty faces situation in extensive form, similar to Weber (2001). Formally, the dirty faces game consists of n rational players. Players are assigned one of the two types randomly: O for the clean face, or X for the dirty face, with the priors for each type being p and $1-p$ respectively. The assignments are distributed identically and independently for all players. Each player can observe the other players' types, but not his own. This is followed by an announcement about whether there is at least one dirty face among the players.

After all players have been provided with this information, they start choosing actions in stages, with the maximum of n stages in an n-player game. We define the actions available to players as *up* and *down* - *up* is claiming ignorance (maybe keep laughing), whereas *down* is claiming to have a dirty face. The game ends either when any player has chosen *down*, or after n stages have already passed.

The payoff for players is solely dependent on their actions at the end of the game and their types: Claiming ignorance by choosing *up* always gets zero for any types; On the other hand, claiming to have a dirty face by choosing *down* is rewarded differently for different types. Correctly claiming to have a dirty face receives a positive payoff, while a false claim leads to a negative payoff. The payoff structure is a crucial part for allowing players to properly interpret the meaning of one another's types and actions. The payoff structure should provide incentives to claim having a dirty face immediately, once this can be established from the types of other players and from past plays. When this is true, the players will be able to correctly find out their types in equilibrium. Behaviourally, it is necessary that players are able to iterate far enough and believe that the other players can do the same. If the game and its payoffs are properly specified, then this game can be used in the laboratory to test a) if humans can iterate far enough and b) if they believe that the others can do the same. We will define and analyse this game in the next three chapters.

We will show that in the original setting, as used by Weber, the payoff structure does not provide an incentive such that anyone wants to reveal the state of his face through his action immediately once he knows it. Therefore, besides the equilibrium where everybody can learn his type, there are equilibria where this is not true. So we conclude that Weber's experimental setting is not appropriate for testing iteration depth and rationality. We will provide a payoff "refinement", which establishes the uniqueness of equilibrium. This refined game can then be used to answer the question originally posed by Weber.

3. SETTINGS AND NOTATION

In this section we will define the game more precisely and introduce our notation.

The n-player dirty faces game proceeds as follow:

1. There are n players, $i = 1 \ldots n$. Player $-i$ denotes the partner(s) of player i. We call the collection of all players, $\forall i \in \{1, \ldots, n\}$, a cohort in a game.

2. Nature draws types θ_i, $i = 1 \ldots n$, from the distribution that is identical for and independent between all players. There are two possible types, O and X, with the prior probabilities of $1 - p$ and p, respectively.

3. An announcement takes place, which provides common knowledge among the players if there is at least one type X player in the cohort. The announcement is denoted by the boolean variable ρ, and declares whether:

   (a) no one has drawn type X, $\rho = false$, or

   (b) there is at least one type X player, $\rho = true$.

4. Players observe the types of their partners θ_{-i}, but cannot observe their own type θ_i.

5. Players make decisions and these are evaluated in the following sequence:

   `Stage counter`: Starting with $t = 1$.

   `Decision stage`: Each player chooses an action, either *up* or *down*.

`Evaluation stage`: The game ends if either

(a) any players has chosen *down*, or

(b) n stages have passed, i.e. when $t = n$.

Otherwise, the game continues with players returning to the `decision stage`. The stage counter t is advanced by one and the players learn the actions taken by all players in the previous stage.

6. Payoffs are realised.

The payoffs $u_i\left(a_i, \theta_i\right)$ are dependent on the action and the type of the player. When *down* has been chosen, a type X player receives α, and a type O player receives $-\beta$:

$$u_i\left(down, X\right) = \alpha$$

$$u_i\left(down, O\right) = -\beta$$

On the other hand, the player receives zero payoff whenever choosing *up*, regardless of his type:

$$u_i\left(up, \theta_i\right) = 0$$

The payoffs are significant in determining whether we have multiple equilibria. Weber (2001) chose payoffs such that:

$$p\alpha - (1-p)\beta < 0 \tag{3.1}$$

This condition ensures the expected payoff to be negative when players choose *down*, given that they hold the prior beliefs. It follows that playing *down* is strictly dominated by playing *up* when the prior beliefs are held. However, this is not sufficient to render the game dominant solvable, as it will be shown in the

Tab. 3.1: Payoff matrix table for dirty faces game

		Own Type	
		X	O
Own	*up*	0	0
Actions	*down*	α	$-\beta$

following chapter.

Overall, the payoffs for different actions given a particular type are shown in Table 3.1.

In the `decision stage`, the basis on which players choose a particular action is their beliefs. The belief of player i, μ_i, is his believed probability of being type X, or having a dirty face in the original language. Obviously, μ_i has all the properties of a standard probability measure.

The action and the belief of the player i at stage t, given the observation of the partners' types and the announcement, are denoted by $a_i^{(t)}(\theta_{-i},\rho)$ and $\mu_i^{(t)}(\theta_{-i},\rho)$, respectively. The relevant history of play from the previous stages is invariant given that the game is still going, since all players must have played *up* in previous stages in order to advance the game or the game must be in stage one. Therefore we do not need to carry the history in the notation. For example, $a_i^{(2)}(\theta_{-i},\rho)$ contains the fact that all players $\forall i \in \{1\ldots n\}$ must have played $a_i^{(1)}(\theta_{-i},\rho) = up$.

A pure strategy is a profile of actions at every information set. Thus a pure strategy is expressed by an action vector $\mathbf{a}$. In an equilibrium, we also need a belief vector μ which assigns a believed probability at every information set.

$$\mathbf{a_i}(\theta_{-i},\rho) = \left(a_i^{(1)}(\theta_{-i},\rho), \cdots, a_i^{(n)}(\theta_{-i},\rho)\right)$$

$$\mu_{\mathbf{i}}(\theta_{-i},\rho) = \left(\mu_i^{(1)}(\theta_{-i},\rho), \cdots, \mu_i^{(n)}(\theta_{-i},\rho)\right)$$

The dirty faces game is a Bayesian game. The actions and the beliefs of players form parts of the equilibrium solution.

4. BEST RESPONSES

The following derivation will focus on the two-player game, $n = 2$. The logic easily extends to more players. In a later section, we will generalise our findings to more players. Players are assumed to be rational in maximising their expected payoffs and to be risk-neutral.[1] These assumptions are common knowledge among players within a cohort.

When a player observes θ_{-i}, the type of his partner, the uncertainty for this player is reduced from all permutations of two player types to only being uncertain of his own type. So initially, the probability of being type X is the prior probability. Each player is in an information set containing two histories. This happens for all players in the cohort. More importantly, every player knows this is true for his partner and himself, and every player knows his partner knows that this is true for his partner and himself, and so on. This common knowledge allows players to find the best response, given the beliefs are updated using all information rationally. Common knowledge also allows each player to foresee the best response of their partner on the equilibrium path, which reveals additional information that can be used for updating his belief.

Nevertheless, the first additional information for players is the announcement. Following the observation, the announcement allows players to reassess their beliefs.

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[1] The choice of payoffs is sufficient to ensure that guessing is dominated, as long as players are not overly risk-loving.

We can group the observation of partner's type and the announcement into three disjoint observable events[2] that players will encounter:

- $\rho = false$
- $\theta_{-i} = O \wedge \rho = true$
- $\theta_{-i} = X \wedge \rho = true$

4.1 No dirty face ($\rho = false$)

The trivial case $\rho = false$ provides certainty that everyone has drawn type O. This reduces the information set to a singleton node. Choosing *down* at any stages yields a certain negative payoff $-\beta$. This action is strictly dominated by choosing *up*. Therefore the best response[3] is

$$\begin{aligned} \mathbf{a_i^*}\left(false\right) &= \left(up, up\right) \\ \mu_\mathbf{i}^*\left(false\right) &= \left(0, 0\right) \end{aligned} \tag{4.1}$$

4.2 At least one dirty face and I see none ($\theta_{-i} = O \wedge \rho = true$)

$\rho = true$ is the more interesting case. The usefulness of this information depends on what type of partner that player i has observed. For player i having observed a type O partner must lead to the belief of having drawn type X with certainty. Correct updating should go like this: "There is at least one dirty face and my parnter does not have it. So I must have it." The information set is reduced to

[2] There are $2n - 1$ observable events for an n-player game.

[3] We have omitted θ_{-i} in the argument when $\rho = false$ in order to simplify the notation, since $\theta_{-i} = O$ is effectively declared by the announcement. We have already impose the equilibrium requirement that players do not make mistakes while updating their beliefs.

a singleton. Therefore, it is a best response to choose *down* immediately in the first stage, end the game and receive the positive payoff α.

$$a_i^{(1)*}(O, true) = down \tag{4.2}$$

$$\mu_i^{(1)*}(O, true) = 1 \tag{4.3}$$

Had the game advanced to the second stage, the best response would be

$$a_i^{(2)*}(O, true) = down \tag{4.4}$$

$$\mu_i^{(2)*}(O, true) = 1 \tag{4.5}$$

Only one step of iterative reasoning is required in this scenario. Hence, player i can determine that he has drawn type X with certainty in the first stage. Actions (4.2) and (4.4) form a viable strategy for this subgame with beliefs (4.3) and (4.5):

$$\mathbf{a_i^*}(O, true) = (down, down) \tag{4.6}$$
$$\mu_{\mathbf{i}}^*(O, true) = (1, 1)$$

However, it is possible that player i intentionally chooses *up* in the first stage, and waits to choose *down* in the second stage, with the same set of beliefs:

$$\mathbf{a_i^*}(O, true) = (up, down) \tag{4.7}$$
$$\mu_{\mathbf{i}}^*(O, true) = (1, 1)$$

Playing the strategy (4.6) ensures player i receives α with certainty. With the strategy (4.7), on the other hand, the payoff for player i becomes dependent on what action his partner has chosen in the first stage. The payoff is zero if his partner chooses *down*, and the payoff is α if his partner chooses *up*. Therefore, strategy (4.7) is weakly dominated by strategy (4.6). However, if player i is certain that his partner will choose *up*, then playing $(up, down)$ is also a best response. Then strategies (4.6) and (4.7) are payoff equivalent for player i. Therefore, both strategies are part of the best responses in this subgame, given player i is certain that his partner must choose *up* in the first stage. We will show in the next section that choosing *up* is the unique best response of his partner in this situation. So both strategies are potential equilibrium strategies for this realisation of types.

In this subgame, we shall refer to player i employing the separating strategy if his best response is (4.6), and the pooling strategy if his best response is (4.7). As we will see in the following section, the unique best response for $\theta_{-i} = X \wedge \rho = true$ is choosing *up* in the first stage. In case of strategy (4.6), player i would choose different actions for different observations in the first stage, hence the term separating strategy. In the other case, strategy (4.7) would see player i chooses *up* in the first stage regardless of his observation, hence the pooling strategy.

4.3 At least one dirty face and I see one ($\theta_{-i} = X \wedge \rho = true$)

The information "there is at least one dirty face" contains no additional information if the player has already seen one dirty face. Therefore, player i must still hold the prior belief about himself being type X. This means player i is still in an information set with two histories.

For the continuation in this situation[4], the choice of payoffs ensures that

[4] Player i is in an information set that contains two histories: nature may either have drawn OX or XX - where the second X is his partner's type. It cannot be called a subgame as he is

choosing *down* yields an expected negative payoff and is therefore not sequentially rational[5]. Regardless of the strategy of his partner, the best response in the first stage in this situation is given by

$$a_i^{(1)*}(X, true) = up \tag{4.8}$$

$$\mu_i^{(1)*}(X, true) = p \tag{4.9}$$

If the game proceeds to the second stage, player i must have observed his partner's action as $a_{-i}^{(1)} = up$. This may signal the private information of his partner, namely $\theta_i = X$, which would have forced his partner to play *up*. However, this signaling depends on whether his partner employs a separating strategy or the pooling strategy.

Suppose his partner has employed a separating strategy, then observing $a_{-i}^{(1)} = up$ leads player i to the updated belief to be type X with certainty. This is because the separating strategy provides a unique signal for different types. Hence, player i will be able to update his belief. Since advancing to the second stage implies his partner must have played *up*, this indicates his partner must have observed $\theta_i = X$. Therefore, player i may conclude that he has drawn type X and choosing *down* is the best response in the second stage.

$$a_i^{(2)*}(X, true) = down \tag{4.10}$$

$$\mu_i^{(2)*}(X, true) = 1 \tag{4.11}$$

not in a singleton node in this decision period.

[5] The action taken by the player at an information set must be optimal, given his belief at that information set and the other players' strategies in the continuation game. If this is true for this player at each information set, then his strategy is sequentially rational.

Tab. 4.1: Best Responses and Classification

Event (θ_{-i}, ρ)	*Best Response* $\mathbf{a_i}$	*Belief* $\mu_\mathbf{i}$	*Characterisation*
$O, false$	(up, up)	$(0, 0)$	Trivial
$O, true$	$(down, down)$	$(1, 1)$	Separating
$O, true$	$(up, down)$	$(1, 1)$	Pooling
$X, true$	$(up, down)$	$(p, 1)$	Response to Separating
$X, true$	(up, up)	(p, p)	Response to Pooling

On the other hand, the pooling strategy does not signal any useful information. The belief of player i remains unchanged, and the best response is playing *up* in the second stage:

$$a_i^{(2)*}(X, true) = up \tag{4.12}$$

$$\mu_i^{(2)*}(X, true) = p \tag{4.13}$$

The best responses with consistent beliefs for all the continuation games are summarized in Table 4.1.

5. EQUILIBRIUM

The dirty faces game can be modeled as a Bayesian extensive game with observable actions. The only uncertainty is the assignment of types O and X in our game, which is the initial move of chance. A candidate for an equilibrium consists of a pair: i) a profile of behavioral strategies[1] and ii) a probability measure called a belief on one's own type[2] for every player. The solution concept we use is a perfect Bayesian equilibrium, which is defined as:

Sequential rationality The behavioral strategy chosen at a given decision node should give an expected payoff that is at least as good as all other possible strategies in the continuation game. In short, the chosen strategy cannot be strictly stochastically dominated.

Correct initial belief The beliefs for players are determined by the priors initially. In the two-player game, this means players should form the following initial beliefs accordingly before they make their first move in the game: a) $\mu_i^{(1)}(\rho = false) = 0$; b) $\mu_i^{(1)}(\theta_{-i} = O \wedge \rho = true) = 1$; and c) $\mu_i^{(1)}(\theta_{-i} = X \wedge \rho = true) = p$.

Action-determined beliefs There are two parts in this requirement. Firstly, players' beliefs on any partners' private information should remain unchanged if that partner has not moved at that history. Secondly, the observable

[1] This degenerates to pure strategies if players do not randomise.
[2] Recall that the belief is defined as the assessed probability of being of type X oneself.

actions from different partners influence the update of players' beliefs on different partners' privately held information independently.

Bayesian updating Players' beliefs should be derived via Bayes' rule upon their observations of others' actions. On top of the consideration of any new information, the new belief must also be consistent with priors and previously held beliefs.

See Osborne and Rubinstein (1994) for a more formal treatment on the Bayesian extensive game with observable actions and the definition of the perfect Bayesian equilibrium.

Strategies $s_i\left(\mathbf{a_i}, \mu_\mathbf{i} \middle| \theta_{-i}, \rho\right)$ may be constructed by using the best responses for the proper subgames and the sequential rational continuation if there are no proper subgames. Both were developed in the previous chapter. After our analysis in the previous chapter there remain only four pure strategies which are potential best responses for the whole game.

$$S1 = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (down, down) & (1,1) & |\theta_{-i} = O \\ (up, down) & (p,1) & |\theta_{-i} = X \end{pmatrix} \quad S2 = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (up, down) & (1,1) & |\theta_{-i} = O \\ (up, down) & (p,1) & |\theta_{-i} = X \end{pmatrix}$$

$$S3 = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (down, down) & (1,1) & |\theta_{-i} = O \\ (up, up) & (p,p) & |\theta_{-i} = X \end{pmatrix} \quad S4 = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (up, down) & (1,1) & |\theta_{-i} = O \\ (up, up) & (p,p) & |\theta_{-i} = X \end{pmatrix}$$

$S1$ and $S3$ are separating strategies, since the player differentiates his actions for different observed θ_{-i}. $S2$ and $S4$ are pooling strategies as the player chooses *up* in the first round regardless of his observation.

Another characterisation of the strategies is through the response in the continuation game after $\theta_{-i} = X$ and $\rho = true$. $S1$ and $S2$ are best responses if the partner plays the separating strategy, while $S3$ and $S4$ are best responses if the partner plays a pooling strategies.

5.1 Pure strategy equilibria

There are sixteen permutations of pure strategy pairs when the two players choose one of these four remaining strategies each. However, not every strategy pair satisfies the requirements for equilibrium. For example, the pair $(S2, S4)$ cannot be an equilibrium; player 1's response to $\theta_2 = X$ yields a negative expected playoff since player 2 is playing the pooling strategy. Player 1 can improve his expected payoff by deviating to $S4$.

Only four of these permutations having consistent beliefs and actions which are sequentially rational given the beliefs. It is easy to check that we have two symmetric pure strategy equilibria, $(S1, S1)$ and $(S4, S4)$, and two asymmetric pure strategy equilibria, $(S2, S3)$ and $(S3, S2)$. We don't elaborate on these equilibria, as they can be obtained as limiting cases of equilibria in partially mixed strategies, which are developed in the following section.

5.2 Hybrid strategy equilibria

For player i who has observed $\rho = true$ and $\theta_{-i} = O$, it is possible to mix between the subgame perfect continuations characterised in (4.6) and (4.7).[3] Suppose player i randomises between the strategy $S1$ and $S2$ with probabilities $1-\sigma_i$ and σ_i respectively, $\sigma_i \in [0, 1]$. In other words, when he sees a type O partner (given $\rho = true$) in stage one, he chooses up with probability $Pr\left(a_i^{(1)*} = up|\theta_{-i} = O\right) = {\sigma_i}^4$. His sequentially rational partner must choose up in the first stage. The game

[3] The resulting mixed strategy is weakly dominated by the strategy (4.6).

[4] This can be seen as being identical to the behavioural strategy, as player i chooses the same continuation action ($down$) in the second stage in both pure strategies.

will advance to the second stage. His partner updates his belief using Bayes' rule:

$$\mu_{-i}^{(2)*} = Pr\left(X|up\right) = \frac{Pr(up|X)Pr(X)}{Pr(up)},$$

where:

- $Pr(up|X) = 1$ since playing *down* is strictly dominated for the prior.
- $Pr(X) = p$ is the prior probability.
- $Pr(up)$ is the joint probability of playing *up* in two disjoint events[5]:

$$\begin{aligned} Pr(up) &= Pr(up, X) + Pr(up, O) \\ &= Pr(up|X)Pr(X) + Pr(up|O)Pr(O) \\ &= p + (1-p)\sigma_i \end{aligned}$$

Therefore $\mu_{-i}^{(2)*} = p/(p+(1-p)\sigma_i)$. The best response for player $-i$ to the mixed strategy σ_i is $a_{-i}^{(2)*}(X, true) = down$ if and only if $\mu_{-i}^{(2)*}\alpha \geq (1-\mu_{-i}^{(2)*})\beta$, or iff

$$\sigma_i \leq \frac{p\alpha}{(1-p)\beta} \tag{5.1}$$

Otherwise, player $-i$ should play *up*. Therefore we have the following hybrid strategy equilibria $\left(S_i^*, S_{-i}^*\right)$, where $\sigma_i \in [0,1]$, $a_{-i}^{(2)*} = down$ if (5.1) holds and $a_{-i}^{(2)*} = up$:

$$S_i^* = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (\sigma_i, down) & (1,1) & |\theta_{-i} = O \\ (up, down) & (p,1) & |\theta_{-i} = X \end{pmatrix} \quad S_{-i}^* = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (down, down) & (1,1) & |\theta_i = O \\ (up, a_{-i}^{(2)*}) & (p, \mu_{-i}^{(2)*}) & |\theta_i = X \end{pmatrix}$$

and

[5] The case $\rho = false$ does not have to be considered since $Pr(X|up) = 0$ when both players are told that they have drawn type O.

$$S_i^* = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (\sigma_i, down) & (1,1) & |\theta_{-i} = O \\ (up, up) & (p,p) & |\theta_{-i} = X \end{pmatrix} \quad S_{-i}^* = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (up, down) & (1,1) & |\theta_i = O \\ (up, a_{-i}^{(2)*}) & (p, \mu_{-i}^{(2)*}) & |\theta_i = X \end{pmatrix}$$

The two forms of equilibria differ in what strategy the non-mixing partner plays in the other subgame where $\theta_i = O \wedge \rho = true$; one plays separating and the other plays pooling strategies. It follows that the best responses are different for player i in order to be sequentially rational given the correct beliefs.

By symmetry, we can obtain the following mixed strategy equilibria when both players use mixed strategies:

$$S_i^* = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (\sigma_i, down) & (1,1) & |\theta_{-i} = O \\ (up, a_i^{(2)*}) & (p, \mu_i^{(2)*}) & |\theta_{-i} = X \end{pmatrix} \quad S_{-i}^* = \begin{pmatrix} (up, up) & (0,0) & |\rho = false \\ (\sigma_{-i}, down) & (1,1) & |\theta_i = O \\ (up, a_{-i}^{(2)*}) & (p, \mu_{-i}^{(2)*}) & |\theta_i = X \end{pmatrix}$$

where $a_{-i}^{(2)*}$ and $a_i^{(2)*}$ are determined by σ_i and σ_{-i}, respectively.

Since these equilibria are true $\forall \sigma_i, \sigma{-}i \in [0,1]$, so there are in fact an infinite number of mixed equilibria. The limiting cases gives rise to the four pure strategy equilibria[6], as discussed in the previous section.

If we take all equilibria in the 2-player version, we see that observing the second-period play of a player who observed X after (up, up) does not help to determine the level of iteration, as both *up* or *down* can be on the equilibrium path. The multiplicity of equilibria renders this version of the dirty faces game inappropriate for empirical testing.

5.3 Generalisation

It can be shown that there are also multiple equilibria in the three player game. Suppose the cohort has drawn XOO, the first player i has observed $\theta_{-i} = OO$

[6] $(S1, S1)$ for $\sigma_i, \sigma_{-i} = 0$; $(S4, S4)$ for $\sigma_i, \sigma_{-i} = 1$; $(S2, S3)$ for $\sigma_i = 1, \sigma_{-i} = 0$; and $(S3, S2)$ for $\sigma_i = 0, \sigma_{-i} = 1$. In addition, if one of the $\sigma's$ takes on 0 or 1 while the other $\sigma \in\in (0,1)$, we have the hybrid strategies.

and $\rho = true$. This player can immediately infer being type X. He can choose *down* to end the game, and claim the payoff immediately. This action is similar to the separating strategy $S1$ in the two player game. Specifically, this strategy is $\mathbf{a_i} = (down, down, down)$ with $\mu_{\mathbf{i}} = (1, 1, 1)$, and it signals clearly to his partners that he chooses *down* as soon as he knows his own type and plays *up* when he does not. This signaling provides useful information for his partners to deduce their own types.

Yet player i can also wait by choosing *up* in the first stage, then chooses *down* at either one of the latter stages to claim the same payoff, since his partners must choose *up* given that they hold the prior belief. Such pooling strategies as $(up, up, down)$ and $(up, down, down)$ with the belief (1,1,1) are also best responses. Choosing *up* by this player does not indicate to his partners whether he has worked out his own type already. His partners are unable to iterate further.

Another case is a draw of XXO where the best responses in the first stage for all players is *up* with prior belief p. Remember that two type X players, called player 1 and player 2, must have observed $\theta_{-i} = XO$; another player, 3, who is type O must have observed $\theta_{-i} = XX$. For player 1, a best response in the second stage is $a_1^{(2)*}(XO) = down$ with $\mu_1^{(2)*}(XO) = 1$, *a*s long as he believes player 2 to play the separating strategy $(down, down, down)$ whenever $\mu_{\mathbf{2}} = (1, 1, 1)$. This is because player 1 knows that player 2 would have played *down*, had player 2 observed $\theta_{-i} = OO$. Since player 2 has played *up*, however, player 1 can infer that player 2 holds the belief $\mu_2^{(1)*} = p$ and playing *up* is sequentially rational. The uncertainty arising from having observed a type X player remains. Since player 1 can observe player 3 being of type O, he can infer that he must be type X. This is dependent on player 1 believes that player 2 would end the game if player 2 knows himself to be type X. On the other hand, if player 1 believes player 2 to play the pooling strategy $(up, down, down)$ if $\mu_{\mathbf{2}} = (1, 1, 1)$, player 2's action of *up* does not signal to player 1 whether $\mu_2^{(1)*} = 1$ or $\mu_2^{(1)*} = p$. Hence, the best response for player 1 has become $a_1^{(2)*}(XO) = up$ with $\mu_1^{(2)*}(XO) = p$. The argument is along the same line for player 2.

We see that the same problem of multiple equilibria remains in the three-player version. Moreover, there are many situations when more than just one player have to rely on the signals of other players. This reduces the discriminatory power of the game for empirical tests even further. It is easy to see how the same logic applies to games with more than 3 players. Basically, the only observable behaviour (given $\rho = true$), which cannot be part of an equilibrium is for a player choosing *down* in any period t, if the number of type X partners is larger than t.[7] Everything else can be equilibrium play.

5.4 Week dominance and separation

We can see a common thread: actions of a player signal his belief clearly to other players, if the player chooses *down* as soon as he is certain of being type X. This signals to other players that he chooses *up* if and only if he is uncertain of his type. This is crucial for other players to update their beliefs. Formally, all players must match their actions to their beliefs on a one on one basis: $a_i^{(t)*} = up$ for $\mu_i^{(t)*} = p$, and $a_i^{(t)*} = down$ for $\mu_i^{(t)*} = 1$ in order to achieve separation. Separation works in this game when players delete (do not play) weakly dominated strategies.

Table 5.1 shows the expected payoffs for correct beliefs and strategies after a draw of OX. Player X may delete all strategies which are weakly dominated by $(down, \bullet)$. This yields the separating strategy to be the equilibrium strategy.

[7] In other words, suppose $k \in [0, n]$ is the number of type X partner that player i has observed, then $a_i^{(t)}$ $(\theta = ”k” \wedge \rho = true) = down$ is always a strictly dominated action $\forall k > t$.

Tab. 5.1: Strategic Interaction if OX is drawn

		Player X†	
		$(up, down)$	$(down, \bullet)$
	(up, up)	$(0, \alpha)$	$(0, \alpha)$
Player O §	$(up, down)$	$(-, \alpha)$	$(0, \alpha)$
	$(down, \bullet)$	$(-, 0)$	$(-, \alpha)$

† (up, up) is strictly dominated for Player X, not shown.

§ - means negative expected payoff for player O.

In contrast, we get another picture if player O starts the deletion ahead of player X. Choosing *down* in the first round is *strictly* dominated ex ante for $\theta_{-i} = X$. Deleting $(down, \bullet)$ from player O's strategy sets results in player X being indifferent in his strategic choices. It follows that $a_i^{(t)*} = up$ for $\mu_i^{(t)*} = 1$ is also a best response. It destroys the one-to-one relationship between actions and beliefs, hence preventing others from interpreting the observed actions. This gives rise to multiple equilibria, as then there are many best responses. *We have encountered the general problem that the outcome from iterated deletion of weakly dominated strategies is order-dependent.*

This result can be further generalised for an n-player game. When $\rho = true$ and a player has observed that all his partners have drawn type O, he can immediately deduce his type to be X. It follows that there are $(n-1)$ pooling strategies, and one separating strategy which are mutual best responses. These strategies form parts of the different equilibria. This gives rise to multiple equilibria in the traditional dirty faces game analysed and experimentally implemented by Weber (2001). Consequently, this setting is not appropriate for measuring the depth of iteration and testing for consistent believes of human subjects. In what follows we slightly refine the payoffs of the game, such that the separating equilibrium becomes unique. This allows for a proper experimental study of the depth of iteration and the consistency of beliefs.

6. A REFINED VERSION OF THE DIRTY FACES GAME

The classical formulation of the dirty faces game cannot be used to measure the level of iterative reasoning due to multiple equilibria. This arises from equal payoffs for different strategies in the subgame, following a player observing the event $\rho = true \wedge \theta_{-i} = O$ at stage one. Iterative deletion cannot eliminate the weakly dominated strategy from the set of strategies. This casts doubt on results obtained from experiments, which use designs based on the classical game specification.

In the following, we introduce a discount factor δ to remedy the existence of multiple equilibria in the game. The payoffs for all players are discounted by this factor whenever the game advances to a later stage. The payoff matrix for a two-player game is shown in Table 6.1. The effect of this waiting cost turns the pooling strategies from being weakly dominated to being *strictly* dominated. This allows players to delete the pooling strategies from the set of best responses iteratively. The separating strategy will be the sole surviving strategy at the end of the iterative deletion process.

Tab. 6.1: Payoff table for the dirty faces game with discount in the k^{th} stage

		Own Type	
		X	O
Actions	*up*	0	0
	down	$\alpha\delta^{k-1}$	$-\beta\delta^{k-1}$

It is easy to derive the equilibrium in the refined game. It is unique and separating. We show how the process of iterated deletion works in the two-player example. It certainly generalises to games with more players.

- When player i observes $\rho = false$, his best response is $\mathbf{a}_i^* = (up, up)$ with $\mu_i^* = (0, 0)$ - this is the same as in the game without discounting.

- When player i observes $\rho = true \wedge \theta_{-i} = O$, the player should choose *down*, end the game and claim the payoff α immediately. Claiming the payoff at latter stages results in receiving a lower payoff due to discounting. Hence the best response in this case is $\mathbf{a}_i^* = (down, down)$ with $\mu_i^* = (1, 1)$.

- When player i observes $\rho = true \wedge \theta_{-i} = X$, the best response at stage one is choosing the action $a_i^{(1)*}(X, true) = up$ with the belief $\mu_i^{(1)*}(X, true) = p$. The game may advance to stage two if and only if his partner has also chosen *up* at stage one. This can only mean that his partner must have formed the belief $\mu_{-i}^{(1)*} = p$ after observing $\theta_i = X$, as choosing *up* after observing $\theta_i = O$ is now strictly dominated and can be deleted. He therefore knows his partner would have chosen *down* to end the game at stage one if $\theta_i = O$ after forming the belief $\mu_{-i}^{(1)*} = 1$. It follows that the best response at stage two is $a_i^{(2)*}(X, true) = down$.

In summary, the sole surviving strategy for the refined dirty faces game is simply the strategy $S1$.[1] Given symmetry, the pure-strategy Perfect Bayesian equilibrium is $(S1, S1)$, $\forall \theta_{-i} \in \{O, X\}$ and $\forall \rho \in \{true, false\}$.

[1] This strategy can be summarized for an n-player game as *keep choosing up as many times as many X-type partners you have seen, and after that, choose down immediately if the game still continues.* When the game ends with type X players play *down*, those who plays *up* should realise that they are type O.

Hybrid strategies are also never part of any equilibrium, as they are all strictly dominated by the pure strategy $S1$, too. Any mixed strategy will involve pooling strategies which have lower payoffs due to discounting when delaying playing *down* under certainty.

Therefore we conclude that $S1$ is the strategy that survives iterated elimination of strictly dominanted strategies for every player in the refined dirty faces game. We have a unique Perfect Bayesian Equilibrium $(S1, S1)$.

This result can be extended to an n-player game. Any delay in claiming the positive payoff incurs a waiting cost, when certainty is established, i.e. $\theta_i = X$ and $\mu_i = 1$ for player i. This effectively renders all pooling strategies strictly dominated by the separating strategy. In turn, players' responses signal useful information for others to perform iterative reasoning. Introducing the waiting cost does not only resolve the issue of multiple equilibria, but it also dismisses any hybrid strategies as equilibrium candidates by strict dominance.

7. EXPERIMENT

This paper is motivated by the experiments in Weber (2001). The procedural design and session parameters in our experiment are the same as in those experiments. The only difference is the introduction of the discount factor on delayed payoffs. Our aim is to see how removing the multiple equilibria may affect the experimental outcomes. This purpose leaves little room for design choices, since we would like to compare the results with those from Weber's experiment. In particular, we stick with the use of the neutral language labeling types O and X, instead of using framed language like "clean" and "dirty faces".

There are - as in Weber's paper - two treatments with cohort sizes of $n = 2$ and $n = 3$. So the necessary level of iterated reasoning for equilibrium play is relatively low. The prior probabilities for drawing type O and type X are 1/3 and 2/3, respectively. These priors reduce the occurrence of the trivial case of $\rho = false$ sufficiently. The payoff parameters are $\alpha = 100$ and $\beta = 400$ points, which together with the prior probabilities should prevent gambling. The expected return is -67 points when subjects choose *down* if they hold the prior beliefs. Meanwhile, choosing *up* at the end of the game always yields zero. The parameters are in line with one of the treatments in Weber's experiments.

The points earned during the experiment are converted to cash at the end of the game. The conversion ratio is one cent per point, or AUD 1 for every 100 points. An endowment of 900 points, or AUD 9, is provided to every subject at the beginning of the session. The main purpose of the endowment is to prevent early bankruptcy in the session. An injection of points (treated like a loan) will be provided to the subject when bankruptcy occurs.[1]

The discount factor is set at $\delta = 0.8$. The reduction in the payoff is noticeable as each stage passes, but not so significant that gambling behaviour at latter stages might be induced. The incentive condition against gambling is maintained throughout the game, since the discounting is applied to all payoffs.

Each treatment consists of fourteen playing periods. Each period consists of a single dirty faces game that is independent from other periods, i.e. the types drawn in the earlier period have no influence on the types drawn in latter periods. The profits are tallied at the end of each period, and payments are paid at the end of the session. Subjects are matched together for all periods of the session; we use a partner treatment.[2]

Each game starts with the computer randomly and independently assigning types to subjects according to the priors. An announcement is made on the screen whether there is at least one type X player in the cohort. It is followed by revealing the types of partners - in the 3-player case, the two partners are identified as Left and Right. Subjects enter the first stage and are asked to simultaneously choose their actions - either *up* or *down*. After everyone in the cohort has chosen his action, the actions chosen by their partners are revealed. If

[1] The average/median profits after fourteen periods are AUD 11.74/12.4 in the 2-player treatment, and AUD 7.98/9 in the 3-player treatment. The number of people who make losses (negative profit) are 1(out of 42 subjects) in the 2-player game and 8(out of 48) in the 3-player game.

[2] A partner treatment matches a subset of players from the subject pool for the entire session in a multi-period experiment. This contrasts a stranger treatment when all players are re-matched randomly within the subject pool in different periods within a session. The different of these two type of treatment is in the statiscal interpretation of the data. In partner treatments, each match up is considered as one *independent* observation. On the other hand, the entire subject pool is a single independent observation in the stranger treatment. This is because any interaction between players creates a correlation in data generation.

the game advances to the next stage, subjects are asked to choose actions again. The game continues until someone has chosen *down* or n stages have passed. The period payoff is displayed at the end of the period, but subjects are never told their own type at the end of the game.

The sessions were conducted in a computer lab at the University of Adelaide. The subjects were recruited from a pool of students. The experimental sessions were programmed and run using the software z-tree (Fischbacher 1999). Communication among subjects were controlled for, as they were seated in self-contained booths. The seating order was randomly assigned. No communication was allowed during the session. Subjects received all the relevant information in written instructions and interactively on screen. The instructions for the 2-player game can be found in the appendix A.

8. RESULTS

There were two sessions for each treatment, 2-player and 3-player games. The experimental sessions were conducted in October, 2004. The sessions provided a total of 1260 observations from 90 subjects who played in 14 periods - 42 subjects formed 21 groups in the 2-player game, and the other 48 subjects formed 16 groups in the 3-player game. Subjects were primarily undergraduate students from the University of Adelaide. The distribution of students from various disciplines is shown in table 8.1.

8.1 Descriptive Statistics

Table 8.2 reports the individual level data over the 14 playing periods. For a given event, each entry states the number of subjects choosing an action in agreement with the predicted best response. The percentage of these subjects

Tab. 8.1: Distribution of courses enrolled by the student subjects

course	number	percent
Arts	4	4.44
Commerce	17	18.89
Economics	30	33.33
Engineering	24	26.67
Finance	4	4.44
Sciences	9	10.00
Others	2	2.22
Total	90	100.00

Tab. 8.2: Individual Rationality across periods for 2 player game

	Number of players								
	2				3				
Obs.	$\rho = false$	O	X		$\rho = false$	OO	XO	XX	
pred.[a]	UU	D	UD	Total	UUU	D	UD	UUD	Total
Period	Agreements:n(freq.)								
1	2(1.00)	11(0.92)	17(0.61)	30(0.71)		2(0.67)	8(0.44)	12(0.44)	22(0.46)
2	5(0.83)	7(0.88)	15(0.54)	27(0.64)		2(1.00)	13(0.59)	15(0.63)	30(0.63)
3	5(0.83)	8(0.89)	15(0.56)	28(0.67)	5(0.83)	5(0.83)	11(0.55)	10(0.63)	31(0.65)
4	8(1.00)	10(1.00)	16(0.67)	34(0.81)		3(0.75)	9(0.50)	12(0.46)	24(0.50)
5	6(1.00)	8(1.00)	14(0.50)	28(0.67)	5(0.83)	1(1.00)	8(0.57)	15(0.56)	29(0.60)
6	2(1.00)	10(0.83)	19(0.68)	31(0.74)		4(1.00)	10(0.42)	11(0.55)	25(0.52)
7	2(1.00)	9(1.00)	20(0.65)	31(0.74)		7(1.00)	15(0.58)	8(0.53)	30(0.63)
8	6(1.00)	10(1.00)	17(0.65)	33(0.79)		2(1.00)	7(0.35)	15(0.58)	24(0.50)
9	9(0.90)	6(0.86)	14(0.56)	29(0.69)		1(0.50)	9(0.50)	19(0.68)	29(0.60)
10	7(0.88)	9(0.90)	16(0.67)	32(0.76)	3(1.00)	3(1.00)	9(0.56)	16(0.62)	31(0.65)
11	8(1.00)	9(1.00)	17(0.68)	34(0.81)		2(1.00)	12(0.55)	13(0.54)	27(0.56)
12	4(1.00)	7(1.00)	19(0.61)	30(0.71)	2(0.67)	2(1.00)	9(0.45)	11(0.48)	24(0.50)
13	2(1.00)	7(1.00)	23(0.70)	32(0.76)	5(0.83)	6(1.00)	15(0.63)	7(0.58)	33(0.69)
14	4(1.00)	5(1.00)	22(0.67)	31(0.74)		4(1.00)	10(0.56)	12(0.46)	26(0.54)
Agg.	70(0.95)	116(0.94)	244(0.62)	430(0.73)	20(0.83)	44(0.92)	145(0.52)	176(0.55)	385(0.57)

[a] U means *up* and D means *down*.

out of all subjects who were in the same situation is stated in the parentheses. For example, 9 subjects chose the predicted best response *down* in the tenth period of the 2-player game. This is 90% (or 0.90) of the 10 subjects (not stated) who have observed a type O partner.

There is clearly a high frequency of agreement with predicted responses in the events where the required number of iterations is low. These are cases where one level of iteration is required when subjects have observed $\rho = true$ and all partners are type O - $\theta_{-i} = O$ for $n = 2$ or $\theta_{-i} = OO$ for $n = 3$. Beyond that, the frequency of agreement with the predicted response gets much lower as the required level of iteration increases.

Holding the required number of iteration constant, it seems more players are able to perform two levels of iteration in the 2-player game (observed X, 0.62) than in the 3-player game (observed OX, 0.52). This might suggest that a higher number of players in a cohort complicates the matters further for subjects and hinders their iterative process.

In the 3-player game, there does not seem to be any significant difference in the frequency of agreement with the predicted responses between the two highest levels of iteration. The frequencies are 0.52 for two levels of iteration (observed OX) and 0.55 for three levels of iteration (observed XX). It is reasonable to assume that if subjects are unable to perform two levels of iteration then they should be unable to perform any higher levels of iteration. A typical picture for such an assertion would have been a higher frequency of agreement in the simpler case. However, the closeness of the two frequencies may also suggest that once subjects are able to perform two levels of iteration, they can also perform a higher level of iteration. They might slide down the slippery slope of iteration and go all the way.

The following should be kept in mind when looking at these summary statistics:

1. We consider players in the following occasions as rational, even though their chosen actions have led to losses. Suppose the cohort has drawn OX in the 2-player game. The player with type O has observed a player of type X as partner. The best responses are *up* for the player of type O and *down* for the player of type X in the first stage. The game should have ended there. However, if partner X has chosen the strictly dominated action *up* in the first stage, player O should choose *down* in the second stage. Player O logically believes that his partner should only have chosen *up* if he had observed a type X player. Nevertheless, player O is wrong by logically playing *down* and suffers losses. His mistake is due to the irrational signaling from his partner. Similar situations may occur in the 3-player game. In these situations, we consider player O as rational, even though his play led to losses due to relying on wrong signals.

2. Players play the dominant strategy when they believe their partners play the dominant strategy, i.e. they believe their partners are rational. However, if they believe that their partners are not rational, i.e. not playing dominant strategies, they might themselves deviate from the equilibrium strategy and play a "rationalizable" strategy,[1] such as $S4$ in the 2-player game. A rationalizable strategy is a strategy that is a best response to some strategies of the opponent, but not necessarily to the equilibrium strategy. So a player who does not believe in his partner behaving rationally may consider the signaling from this partner as useless, since following the signal will lead to losses, as in the situations mentioned above. Nevertheless, we consider

[1] Bernheim (1984); also Pearce (1984)

their rationalizable actions as irrational, even though that the deviation from equilibrium can be rational if they believe that the other player(s) in the cohort are not rational.

3. Consider a cohort that has drawn XX in the 2-player game. A risk averse player in this game will play the strategy (up, up) if he is not able to make two steps of iteration[2], whereas a rational player will play the best response $(up, down)$. However, the partners may have chosen *down* beforehand, which ended the game in the first stage. Then we will not observe the actions in the second stage. Moreover, we are not able to distinguish a totally rational player and a risk-averse player with limited iterative ability if other players make mistakes. So we don't know if the player is able to do the second step of iteration.

Our results are not too different from Weber's finding in the 2-player treatment, in which 87% made rational response in the XO condition, and 53% in the second round of the XX condition. In our experiments, the results are 86% (211 from 246 occasions)and 56% (149 from 268) respectively. It seems to suggest that removing the multiple equilibria had no influence.[3]

8.2 Influence of Cohort Sizes

Differences in group size may have an influence on how often subjects actually choose the best response regardless of the the level of iteration needed. This is

[2] This risk averse player is *rational* by playing this strategy, in the sense that choosing *up* yields zero return comparing to an expected return of -67 with choosing *down*. However, he is not entirely rational, since he cannot iterate correctly.

[3] Subjects in Weber's study were either UCLA or Caltech graduate and undergraduate students. We expected them ceteris paribus to do better than our students, as admission in these universities should be more selective. So our prior that removing the multiple equilibria should lead to a better performance might still be true.

Tab. 8.3: Influence of group sizes on individual rationality

	Situations by Levels of Iteration					
	1: No type X observed			2: One type X observed		
n	obs	ranksum	expected	obs	ranksum	expected
2	123	10656	10578	391	137188.5	131376
3	48	4050	4128	280	88267.5	94080
total	171	14706	14706	671	225456	225456
p-value		0.528			0.006	

because the theoretical best responses require the rationality of all players and the common knowledge. This really means that all players understand how to play the game and know the best responses, and they also know and believe that the other players also understand how to play the game, and so on. The common knowledge argument breaks down when there is even a slight hint of doubt among any one of the players. Under this circumstance, players believe that the signaling from partners does not provide any new information. This leads players to play rationalizable strategies instead of the equilibrium strategies. So we believe that when more players are in the cohort, the common knowledge assumption is more likely to break down.

A two-sample Wilcoxon rank-sum (Mann-Whitney) test is used to examine the cases, where one and two steps of iteration are required[4]. The null hypothesis is the frequency of best responses for given steps of iteration are the same for different cohort sizes n.

The results are tabulated in table 8.3. They show that cohort sizes have an influence on subjects ability to choose best responses. In cases when one step of iteration is required (only type O's are observed), subjects are able to deduce the actual situation and form beliefs without requiring any signals from others. This

[4] Given $\rho = true$, one step of iteration is needed for subjects who have observed only type O partners - i.e. $\theta_{-i} = O$ for $n = 2$ and $\theta_{-i} = OO$ for $n = 3$. Two steps of iteration are required after having observed exactly one type X partner, i.e. $\theta_{-i} = X$ for $n = 2$ and $\theta_{-i} = OX$ for $n = 3$.

is consistent with our hypothesis that the number of players makes no difference. On the other hand, subjects become dependent on the signals from partners in order to deduce their own types if they observe one other player being of type X. The signaling complicates the matters because players must also decide how to interpret the signals. Players choose the predicted responses on the equilibrium path if they believe others will choose the predicted responses on the equilibrium path by obeying dominance. Moreover, players will also have to believe that others do know that they obey dominance themselves, and so on. Only then can the signals be interpreted as predicted in a Perfect Bayesian Equilibrium. This chain of reasonings about whether one can believe that others play the predicted responses and obeying dominance does not allow any doubts. However, subjects may be cautious and may not believe others to obey dominance, or they do not believe others to believe them to obey dominance, and so on. Even believing others to be cautious, and so on, may be a sufficient cause for deviating from the equilibrium path. Worse still, only one player is required for causing this deviation. Under these circumstances, rational players may play rationalizable strategies instead.[5] So if the suspicion that others will get it wrong is a strong driving force for equilibrium deviation, then we should expect that deviation happens more often when more players are involved. This is because players should have a higher believed probability that at least one of the other players does not behave according to dominance. Our statistical test confirms this intuition. The number of correct deductions is greater in the treatment with two players ($p - value = 0.006$).

In summary, it seems reasonable to suggest that subjects choose the best responses consistently over different cohort sizes when signaling is not required

[5] The strategy $S1$, $S2$, $S3$ & $S4$, and the hybrid strategies σ_i are all rationalizable.

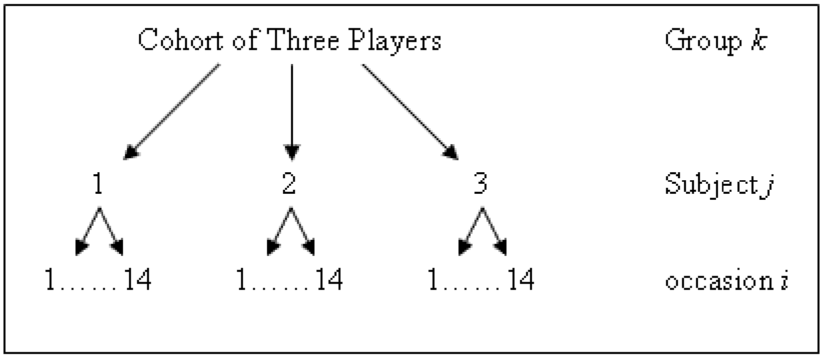

Fig. 8.1: Nested Factors of the Dirty Faces Game

from partners for them to deduce their own types. However, once signaling is involved, increasing the number of players in the cohort increases the likelihood of subjects deviating from the equilibrium path.

8.3 Serial Correlation

Standard regression modeling usually assumes that the errors have zero means and are mutually independent. However, we would expect residuals of the data collected from the experiment for individuals and cohorts to be correlated. This is inherent in our experimental settings - Individuals generated data over 14 playing periods, while they also interacted with other players within their cohorts. Instead of treating this as a nuisance, we can extract more information from the data by using a linear random intercept model (Rabe-Hesketh and Skrondal 2006).

Note that each data point in the experiment is generated by individual j from cohort k in the period i. The data can be classified in a nested hierarchical structure as shown in figure 8.1: starting with occasions (or playing periods) at level 1, individuals at level 2, and cohorts at level 3. In the random intercept model, we assume that each level has its own noise that is independent from other levels, but serial correlation exists for data points within the levels. This serial correlation is model by level specific random effects. This suggests a model with three error terms: the within-subject noises for each observation ϵ_{ijk}, the between-subject noises $\zeta_{jk}^{(2)}$, and the between-cohort noises $\zeta_{k}^{(3)}$.

8.4 GLLAMM Modeling

We are interested in the likelihood of subjects making decisions that match our theoretical rational responses. Our proposed linear predictor uses logit as the link function with m covariates. We estimate the following equation:

$$logit\left\{Pr(y=1|x_{ijk})\right\} = \beta_0 + \beta_1 x_{1ijk} + \cdots + \beta_m x_{mijk} + \zeta_{jk}^{(2)} + \zeta_{k}^{(3)} + \epsilon_{ijk}$$

We have obtained some individual characteristics of the participating subjects through a questionnaire at the end of the experimental sessions. The questionnaire asked for gender, age group, and courses attended at university. We incorporate these information as covariates in our analysis. The regression includes the following dependent and independent variables.

`IR` is a dichotomous dependent variable that denotes whether subjects have played the rational response for the information they have received. It should be noted that a response is considered as rational as long as the subjects act according to what they have observed of others' actions, even if these actions are not rational.

`n` is the number of players in the cohort. This variable is ordinal, hence we treat this as a dummy variable with the baseline being $n = 2$.

steps is the number of iteration required to work out one's own type. This parameter is also ordinal in nature. We do not want to impose any specific functional relationship with a dependent variable. Therefore we use a set of dummy variables to represent different number of iteration. The more iterations the problem requires, the higher is the order of the problem. In the regression, the variables are `step1, step2 and step3`, with `step0` (when $\rho = false$) being the baseline case. Although the variable `step3` is perfectly correlated with $n = 3$, it does not bias the estimators.

courses are a set of dummy variables for different courses enrolled by subjects. There were eight choices for subjects to choose from in the questionaire: `Arts`, `Commerce`, `Economics`, `Engineering`, `Finance`, `Science`, `Others, and N/A`. Most subjects were recruited from these schools and hence the questionaire was designed accordingly. In the regression, we have treated `Other and N/A` as one group and is the baseline.

gender is a dummy variable represents the sex of the subject. The baseline is female.

maturity is a dummy variable showing whether the subject is over 25.[6,7] The baseline is under 25.

The regression results are shown in table 8.4. A detailed regression output can be found in the appendix.

[6] The insurance underwriters and car rental companies seem to have realised something long ago. They require drivers under 25 to pay excess and higher premium.

[7] The first half of Gruber and Yurgelun-Todd (2006) documents the psychology literatures related to maturation. While physical development seems to complete at around 18, frontal-lobe development may continue well into the third decade of life (Sowell et al. 1999). Centers for Disease Control (CDC) define adolescence from age 10 to 24 (Virginia Department of Health - Office of Family Health Service 2006).

Tab. 8.4: regression results

IR	Coef.	Std. Err.	z	$P > \|z\|$	[95% Conf.	Interval]
n	-.845	.367	-2.30	0.021	-1.564	-.126
step1	.177	.527	0.34	0.737	-.857	1.210
step2	-2.572	.423	-6.08	0.000	-3.401	-1.743
step3	-2.276	.444	-5.13	0.000	-3.146	-1.407
Arts	1.795	.821	2.19	0.029	.186	3.403
Commerce	1.318	.746	1.77	0.077	-.144	2.780
Economics	1.437	.689	2.09	0.037	.087	2.788
Engineering	2.282	.744	3.07	0.002	.824	3.741
Finance	.607	1.519	0.40	0.690	-2.371	3.584
Science	2.167	.753	2.88	0.004	.690	3.643
male	.094	.345	0.27	0.784	-.581	.770
maturity	-.405	.565	-0.72	0.473	-1.513	.702
cons	1.712	.830	2.06	0.039	.084	3.340

`n` is a significant factor as we have discussed in the previous section. More players in a cohort increases the likelihood for a breakdown of the rationality and the common knowledge chain. Thus, subjects are more likely to play other rationalizable strategies and less likely to choose the equilibrium actions if the number of players increases.

`step1` is not statistically significantly different from the baseline case. It seems that subjects have found the one step of iteration problem as simple as the trivial announcement. Beyond that, the difficulty has jumped, as `step2` and `step3` have negatively impacted on subjects in choosing the rational responses. A Wald test rejects the null hypothesis that `step1` and `step2` are equal. The increased complexity of the problem is one of the main reasons. In addition, the break down in rationality and common knowledge may again be a reason. This follows from subjects requiring signals from partners in order to solve multi-step problems.

One of the interesting observations is the proximity in the size of the estimators for `step2` and the `step3`. It seems to suggest that higher order problems do not poise additional difficulties to players. The Wald test cannot reject the null hypothesis that they are equal. A simple explanation is that subjects cannot solve a three step problem if they cannot solve a two step problem. However, it does not explain the closeness of the two estimates. An alternative explanation is that once subjects have worked out any methods for the two-step problem, these methods of logical deduction or these rules of thumb[8] can also be applied for the higher order problems.

In regard to the influence of `courses`, it seems `Engineering` and `Science` are the standouts, followed by `Arts`, `Economics` and `Commerce` students. We have also run the regression with the baseline being the `Engineering` students. The results suggest that `Engineering`, `Science` and `Arts` students perform similarly. Our results certainly does not settle the rivalry claims of being smarter between the Engineering students and Arts students. Nevertheless, the interpretation should only focus on courses that have larger sample sizes (see table 8.1). Although subjects from some disciplines seem to be more adaptive to this game than others, it is difficult to tell whether it is due to training in university courses or the self-selection bias of students enrolling into these courses.

The sign of the `male` dummy is positive but far away from being statistically significant. The "battle of the sexes" stays undecided. Also `maturity` seems to play no role in the dirty faces game.

The constant term is significantly different from zero. This is reasonable, since our baseline scenario has included the trivial announcement when most players can easily choose the theoretical best response.

[8] The rule of thumb has been explained in the footnote 1 from chapter 6.

9. CONCLUSION

In this paper, we have shown that the dirty faces game studied by Weber (2001) has multiple equilibria, because of the payoffs being constant for all stages. The multiple equilibria consist of permutations of a separating strategy, and $n-1$ pure pooling strategies for an n-player game. Additionally, there is a continuum of semi-separating equilibria. We cannot rule out the pooling and hybrid strategies, as they are only weakly dominated. This is because players, who are certain to be of type X before they make their first choice, are indifferent between immediately securing a positive payoff and delaying. Since there are more than one sequentially rational strategy for some players to choose from, we cannot use this version of the dirty faces game to measure the level of iterative reasoning. So we refined the payoffs of the original game. By introducing a waiting cost through a discount factor, the (partially) pooling strategies become strictly dominated by the separating strategy. There remains a single strategy that survives deletion of strictly dominated strategies.

By establishing that players have only one strictly dominant strategy, the dirty faces game with discounting can be used as a measurement of the level of iterative reasoning in experiments. So we implemented our refined dirty-faces game in the laboratory. Our experimental results have found that majority of players are able to deduce at the simplest level. The frequencies of agreement with predicted response falls to about half, once correct signaling from other players and correct interpretation of these signals are required for deduction. However, these results are not very different from those in Weber's experiments. So the existence of multiple equilibria does not seem to explain the decline in the number of subjects playing the theoretical best responses once the number of necessary iterations increases.

We further examined the data by using a linear multilevel random intercept model of the logit family. The results agree with the explanations of limited computational ability of the subjects. There seems to be a threshold level of iterative reasoning where the frequency of best responses drops. However, players are able to perform higher level of iterative reasoning once they have passed that threshold. This is probably due to the similar nature of the reasoning process for all levels of iteration.

The number of players in the cohort is another important factor influencing the frequency of best responses. This may be explained by the limited computation, since the level of iterative reasoning increases in some situations. Players may also be more likely to play rationalizable strategies, as larger cohort size increases doubt on other players obeying dominance. So additionally to limited computational ability, the fear of other player's inability plays an important role in explaining the drop of equilibrium play.

Besides the insights gained, our results also give some direction for the design of future experiments. Firstly, one might introduce framing to see if this will increase the frequency of equilibrium play by assisting players to visualise the nature of the problem. The neutral language used in the experiment may have been confusing, as players may find it hard to logically connect the actions and the types in the game. Secondly, it might be worthwhile to introduce computer partners who will always obey dominance. This should eliminate the non-equilibrium rationalizable strategies that arise from the fear of other players behaving irrationally. Finally, the cohort size may be increased beyond three players. One could see whether the frequency of best responses still remains constant if more than two steps of iterations are needed.

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APPENDIX

A. INSTRUCTION FOR THE 2-PLAYER GAME

INSTRUCTIONS

Welcome to our experiment. Please read these instructions carefully. Understanding the instructions is crucial for earning money.

This is an experiment in decision-making. You will be paid for your participation. The exact amount you will receive will be determined during the experiment and will depend on your decisions. This amount will be paid to you in cash after the conclusion of the experiment. If you have any questions during the experiment, raise your hand and the experimenter will assist you. It is strictly forbidden to talk, exclaim or to communicate with other participants during the experiment. It is very important for us that you obey these rules. Otherwise the data generated in this session are useless.

In this experiment, you will play a series of 14 identical games in which you can earn or lose money based on your choices. You start with an endowment of AUD 9. Wins and losses during the 14 games will be added to or deducted from this endowment. At the end of the experiment you will be paid the resulting amount in cash.

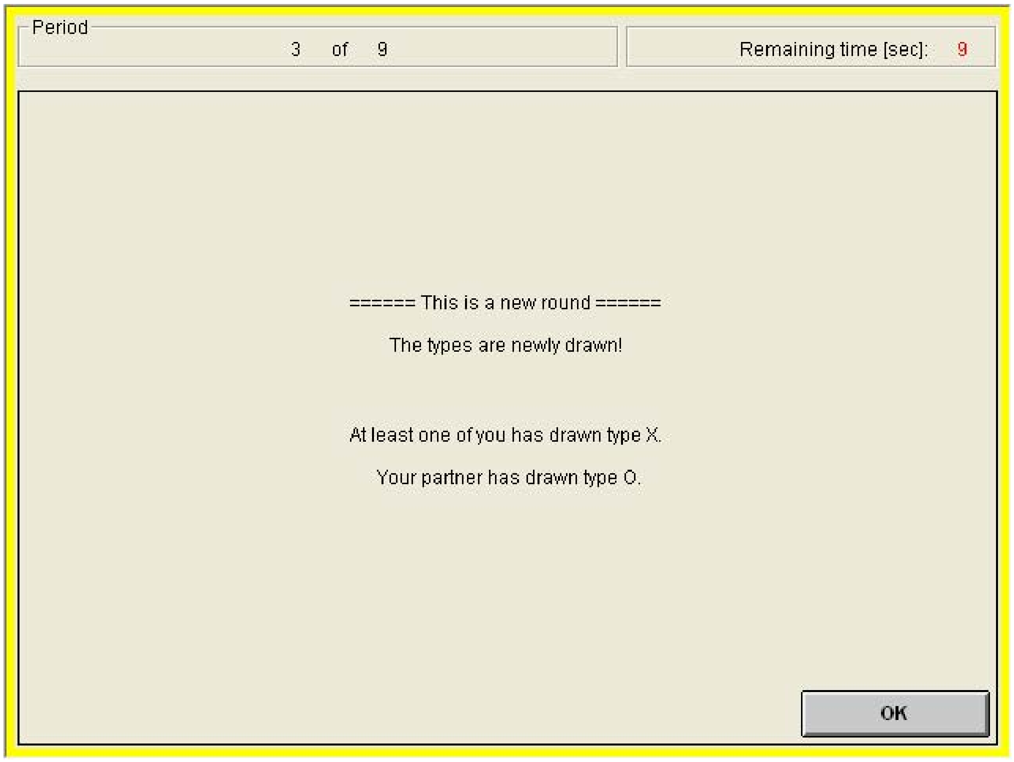
You are paired with one other participant (called partner) throughout all the 14 games. You will not know the identity of this other person, either during or after the experiment, just the other person does not know your identity.

Types

At the start of each of the 14 games, the computer will randomly draw a type for you and a type for the person you are paired with. The possible types are "X" and "O". The computer always draws from an urn with two balls of type "X" and one ball of type "O". So the probability that you are of type "X" is 2/3 while the probability that you are of type "O" is 1/3. Note that the draw for each person will be from a **different** urn. This means that the likelihood of you being of a certain type does **not** depend on what type the other person is.

Information

Each participant will only be told the type of the partner, but no his/her own type. So you will know the type of the person you are paired with, but not your own type. Your partner will know your type, but not his/her own. Additionally, you will be told if at least one person (you or/and your partners) is of type ”X”. Below you have an example of how the information you will get may look like. In this case you see that at least one person is of type ”X” and that your partner is of type ”O”.



Decisions (maximum of two rounds per game)

Round 1
After you have seen the type of the other player in your group and the information whether at least one player (you and/or your partner) is of type ”X” you are asked to choose one of two actions: ”Up” or ”Down”. The combination of **your** type *and* your decision will determine how much money you earn. Note that your payoff does only depend on **your** type and **not** on the type of your partner. The money you earn or lose is determined in the following table:

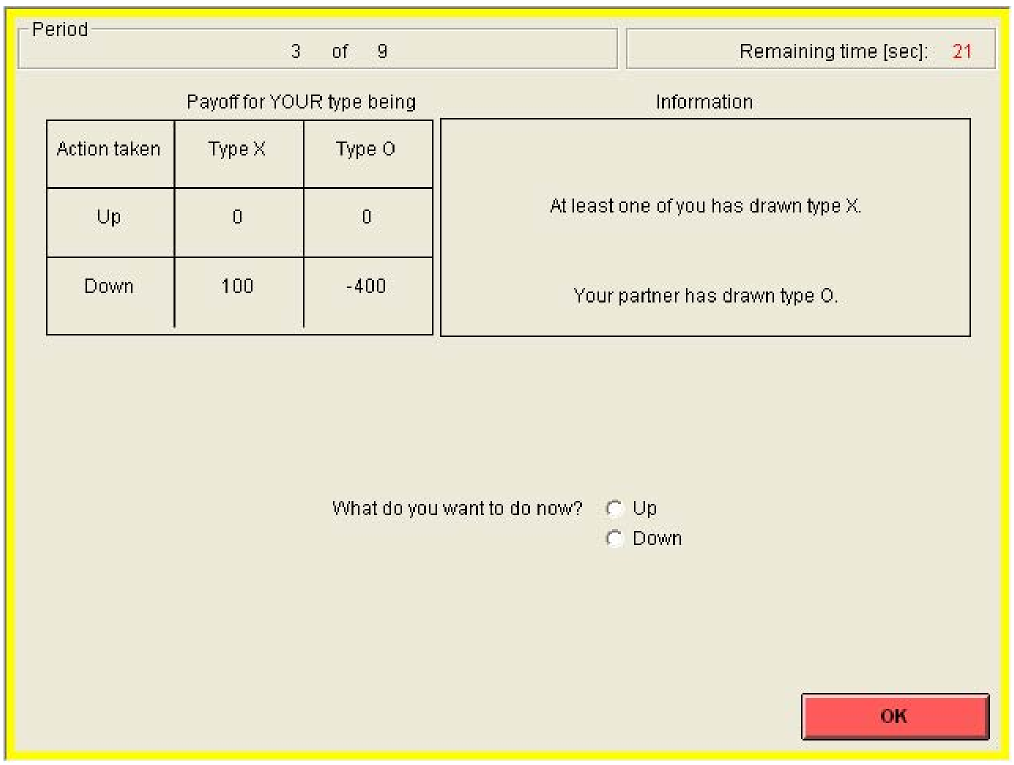
		Your Type	
		”X”	”O”
Your choice	”up”	0	0
	”down”	win 100 cents	lose 400 cents

1. If you choose ”Up” your current earnings will not change.

2. If you choose "Down" and your type is "X" one dollar is added to your earnings.

3. If you choose "Down" and your type is "O" then four dollars will be deducted from your account.

Note again that the type that determines the payoffs is **your** type and not the type of your partner. An example of a decision screen is shown below. The payoffs in the table below are given in cents.



After you made your decision the following happens: If either you or your partner has chosen "Down" the current game ends. Your payoff will be calculated and shown on the screen. Then a new game begins with a new draw of the types. However, if both you and your partner have decided to play "Up" the game enters a second round.

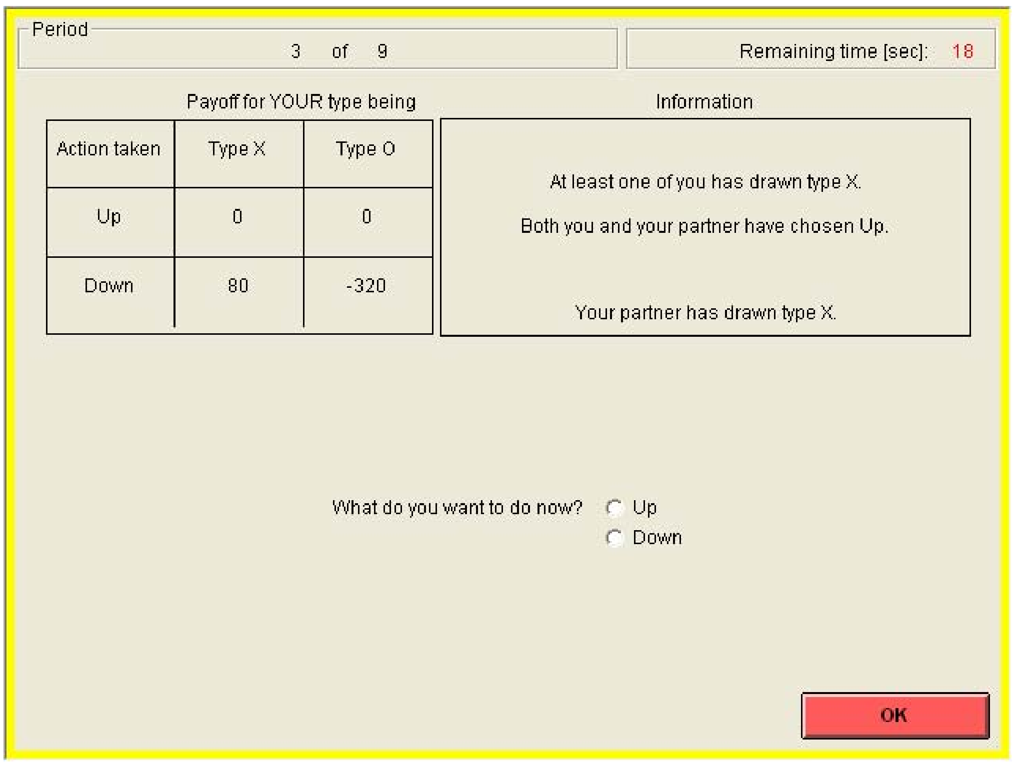
Round 2 (only if both players chose "Up" in round 1)
Round two of the game practically works the same way as round one does. Note that you and your partner keep the types that were drawn before decision round 1.

The only difference in round two is that the payoffs for choosing down are multiplied by a factor of 0.8. The payoffs in round two are the following:

		Your Type	
		"X"	"O"
Your	"up"	0	0
choice	"down"	win 80 cents	lose 320 cents

So now you win 80 cents (instead of 1 Dollar in round 1) if you are of type "X" and you choose "Down". If you choose "Down" and it turns out that you are of type "O" you lose 320 cents (instead of 4 Dollars in round 1). Choose "Up" once again does not cause any gains or losses regardless of your type.

An example of the decision screen is given below:



After round 2 the game ends no matter of the actions previously taken. Your payoff will be calculated and shown on the screen. Then a new game starts with a new draw of types (as explained above).

In total you will play 14 of these games. At the end of the experiment (after 14 games) you will be given a little questionnaire where you have to fill in your details. The questionnaire is only used to make sure that you get the money you have earned.

Thank you very much for your participation.

B. REGRESSION OUTPUTS

```
. xi: gllamm IR n i.steps Arts Commerce Economics Engineering Finance
> Science male maturity, family(binomial) link(logit) i(subject group)
i.steps           _Isteps_0-3         (naturally coded; _Isteps_0 omitted)

Iteration 0:   log likelihood = -674.43401
Iteration 1:   log likelihood = -647.34639
Iteration 2:   log likelihood = -646.73987
Iteration 3:   log likelihood = -646.57504
Iteration 4:   log likelihood = -646.51678
Iteration 5:   log likelihood = -646.51591
Iteration 6:   log likelihood =  -646.5159

number of level 1 units = 1260
number of level 2 units = 90
number of level 3 units = 37

Condition Number = 23.719967

gllamm model

log likelihood = -646.5159

------------------------------------------------------------------------------
          IR |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           n |  -.8448701   .3670312    -2.30   0.021    -1.564238   -.1255021
   _Isteps_1 |    .176697    .527176     0.34   0.737    -.8565489    1.209943
   _Isteps_2 |  -2.572258   .4229972    -6.08   0.000    -3.401317   -1.743198
   _Isteps_3 |  -2.276276   .4436332    -5.13   0.000    -3.145781   -1.406771
        Arts |   1.794869   .8206749     2.19   0.029     .1863761    3.403362
    Commerce |   1.317823   .7460061     1.77   0.077    -.1443224    2.779968
   Economics |    1.43732   .6889629     2.09   0.037     .0869775    2.787663
 Engineering |    2.28248   .7439945     3.07   0.002     .8242772    3.740682
     Finance |   .6067253   1.519068     0.40   0.690    -2.370594    3.584045
     Science |   2.166613   .7531743     2.88   0.004     .6904186    3.642808
        male |   .0944371   .3445848     0.27   0.784    -.5809367    .7698108
    maturity |  -.4053203   .5651966    -0.72   0.473    -1.513085    .7024447
       _cons |   1.711725   .8303512     2.06   0.039     .0842661    3.339183
------------------------------------------------------------------------------

Variances and covariances of random effects
------------------------------------------------------------------------------

***level 2 (subject)

    var(1): 2.0055548 (.48605149)

***level 3 (group)

    var(1): 3.251e-17 (2.465e-09)
------------------------------------------------------------------------------

. test  _Isteps_1==_Isteps_2      //Check how big a jump it was
```

```
 ( 1)  [IR]_Isteps_1 - [IR]_Isteps_2 = 0

           chi2(  1) =   59.12
         Prob > chi2 =    0.0000

. test  _Isteps_2==_Isteps_3
 ( 1)  [IR]_Isteps_2 - [IR]_Isteps_3 = 0

           chi2(  1) =    2.35
         Prob > chi2 =    0.1252
```

```
. xi: gllamm IR n i.steps Arts Commerce Economics Finance Science miscstudy male
>  maturity, family(binomial) link(logit) i(subject group)
i.steps           _Isteps_0-3         (naturally coded; _Isteps_0 omitted)

Iteration 0:   log likelihood = -674.43401
Iteration 1:   log likelihood = -647.34984
Iteration 2:   log likelihood = -646.78204
Iteration 3:   log likelihood = -646.67599
Iteration 4:   log likelihood = -646.53877
Iteration 5:   log likelihood = -646.51609
Iteration 6:   log likelihood =  -646.5159
Iteration 7:   log likelihood =  -646.5159

number of level 1 units = 1260
number of level 2 units = 90
number of level 3 units = 37

Condition Number = 19.207853

gllamm model

log likelihood = -646.5159

------------------------------------------------------------------------------
          IR |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           n |  -.8448512   .3670746    -2.30   0.021    -1.564304   -.1253982
   _Isteps_1 |   .1766994   .5271761     0.34   0.737    -.8565468    1.209946
   _Isteps_2 |  -2.572252   .4229986    -6.08   0.000    -3.401314    -1.74319
   _Isteps_3 |  -2.276272   .4436333    -5.13   0.000    -3.145778   -1.406767
        Arts |   -.487609   .6189338    -0.79   0.431    -1.700697     .725479
    Commerce |  -.9646519   .4970924    -1.94   0.052    -1.938935    .0096313
   Economics |  -.8451514   .4342355    -1.95   0.052    -1.696237    .0059345
     Finance |   -1.67558   1.448232    -1.16   0.247    -4.514062    1.162902
     Science |   -.115863   .5061963    -0.23   0.819     -1.10799    .8762635
   miscstudy |  -2.282471   .7440584    -3.07   0.002    -3.740799   -.8241434
        male |   .0944573   .3446336     0.27   0.784    -.5810122    .7699267
    maturity |  -.4053227   .5651984    -0.72   0.473    -1.513091    .7024457
       _cons |   3.994168   .6736466     5.93   0.000     2.673845    5.314491
------------------------------------------------------------------------------

Variances and covariances of random effects
------------------------------------------------------------------------------

***level 2 (subject)

    var(1): 2.0055407 (.48603886)

***level 3 (group)

    var(1): 1.061e-18 (4.454e-10)
------------------------------------------------------------------------------

. test  _Isteps_1==_Isteps_2      //Check how big a jump it was
 ( 1)  [IR]_Isteps_1 - [IR]_Isteps_2 = 0

           chi2(  1) =   59.12
         Prob > chi2 =    0.0000

. test  _Isteps_2==_Isteps_3
 ( 1)  [IR]_Isteps_2 - [IR]_Isteps_3 = 0

           chi2(  1) =    2.35
         Prob > chi2 =    0.1252
```