

# Iterative Rationality in the Dirty Faces Game

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## ABSTRACT

The Dirty Faces game requires players to perform iterative reasoning in order to arrive at equilibrium play. The game is dominance solvable with a unique equilibrium when it is correctly specified. The particular payoff structure has significant implication on whether the reasoning process leads to equilibrium play. This paper illustrates that the traditional specification - as used by Weber (2001) - leads to multiple equilibria and the game loses its dominance solvability. We modify the payoff structure and restore uniqueness. The resulting game, which is dominance solvable, is implemented in an experiment to test the depth of iterative reasoning in humans. Our data analysis suggests that some deviation from equilibrium play is due to limited depth of iteration. Additionally, we find evidence that the lack of confidence in other players' iterative abilities also induces deviations from equilibrium play.

**KEYWORDS:** Game Theory, Iterative Reasoning, Experimental Economics.

*JEL Classification Numbers:* C91, C92, C72.

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## Declaration

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