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Landau gauge ghost and gluon propagators and the Faddeev-Popov operator spectrum

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In this talk we report on a recent lattice investigation of the Landau gauge gluon and ghost propagators in pure $SU(3)$ lattice gauge theory with a special emphasis on the Gribov copy problem. In the (infrared) region of momenta $q^2 \leq 0.3\ \mathrm{GeV}^2$ we find the corresponding MOM scheme running coupling $\alpha_s(q^2)$ to rise in q. We also report on a first $SU(3)$ computation of the ghost-gluon vertex function showing that it deviates only weakly from being constant. In addition we study the spectrum of low-lying eigenvalues and eigenfunctions of the Faddeev-Popov operator as well as the spectral representation of the ghost propagator.

1. INTRODUCTION

The infrared behavior of the Landau gauge gluon and ghost propagators in QCD is closely related to the confinement phenomenon formulated in terms of the Gribov-Zwanziger horizon condition [1] and the Kugo-Ojima criterion [2]. Assuming a constant ghost-gluon vertex function the propagators also provide a nonperturbative determination of the running QCD coupling $\alpha_s(q^2)$ in a MOM scheme. They are a subject of intensive studies within the framework of truncated systems of Dyson-Schwinger (DS) equations and within the lattice approach.

In the infinite 4-volume limit the DS approach has led to an intertwined infrared power behavior of the gluon and ghost dressing functions [3,4,5]

$$\begin{aligned} Z_D(q^2) &\equiv q^2 D(q^2) \propto (q^2)^{2\kappa}, \\ Z_G(q^2) &\equiv q^2 G(q^2) \propto (q^2)^{-\kappa} \end{aligned} \quad (1)$$

with the same $\kappa \approx 0.59$, i.e. a vanishing gluon propagator $D(q^2)$ in the infrared occurs in intimate connection with a diverging ghost propagator $G(q^2)$. As a consequence, the MOM scheme running coupling

$$\alpha_s(q^2) = \frac{g^2}{4\pi}\, Z_D(q^2)\, Z_G^2(q^2) \quad (2)$$

is turning to a finite fix point in the infrared limit [5]. As we shall show below, our lattice simulations do not agree with this expectation. $\alpha_s(q^2)$ seems to tend to zero in this limit.

Possible solutions of the discrepancy have been discussed from different points of view in the recent literature. In [6] the DS approach has been studied on a finite 4-torus with the same truncated set of equations as for the infinite volume. $\alpha_s(q^2)$ was shown to tend to zero for $q^2 \to 0$ in one-to-one correspondence with what one finds on the lattice. This would indicate very strong finite-size effects and a slow convergence to the infinite-volume limit. However, we do not find any other indication for such a strong finite-size effect in our lattice simulations, except the convergence to the infinite-volume limit would be extremely slow. An alternative resolution of the problem has been proposed by Boucaud *et al.* [7]. These authors argue the ghost-gluon vertex in the infrared might contain q^2-dependent contributions which could modify the DS results for the mentioned propa-

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gators[4]. However, recent detailed DS studies of the ghost-gluon vertex did not provide hints for such a modification [8]. Thus, at present there is no solution of the puzzle. In any case more thorough lattice investigations are desirable.

For $SU(2)$ extensive lattice investigations can be found in [9]. Unfortunately, the authors did not reach the interesting infrared zone where the mentioned inconsistencies become visible. The same holds for $SU(3)$ lattice computations of the ghost and gluon propagators as reported in [10,11]. We have been pursuing an analogous study for the $SU(3)$ case with the special emphasis of the Gribov copy problem [12]. Moreover, we have investigated the spectral properties of the Faddeev-Popov operator [13]. The low-lying eigenmodes of the latter are expected to be intimately related to a diverging ghost propagator. In addition, in [14] we have reported on a first $SU(3)$ lattice computation of the ghost-ghost-gluon vertex at zero gluon momentum. We confirm, what has been found already for $SU(2)$, namely the data are quite consistent with a constant vertex function [15].

In this talk we will give a short review of these investigations. In the meantime larger lattice sizes and better statistics have been reached confirming our previous computations and conclusions.

2. GHOST AND GLUON DRESSING FUNCTIONS

We have simultaneously studied the gluon and ghost propagators in the quenched approximation. $SU(3)$ gauge field configurations $U = \{U_{x,\mu}\}$ thermalized with the standard Wilson gauge action have been put into the Landau gauge by iteratively maximizing the gauge functional

$$F_U[g] = \frac{1}{4V} \sum_x \sum_{\mu=1}^{4} \mathrm{Re}\,\mathrm{Tr}\; {}^gU_{x,\mu} \qquad (3)$$

with $g_x \in SU(3)$. It has numerous local maxima (Gribov copies), each satisfying the lattice Landau gauge condition $\partial_\mu A_\mu = 0$. To explore to what extent this ambiguity has a significant influence on gauge dependent observables, we have gauge-fixed each thermalized configuration several times using the *over-relaxation* algorithm, starting from random gauge copies. For each configuration U, we have selected the first (`fc`) and the best (`bc`) gauge copy (that with the largest functional value) for subsequent measurements. For details we refer to [12].

It turns out that the Gribov ambiguity has no systematic influence on the infrared behavior of the gluon propagator. This holds as long as one restricts to periodic gauge transformations on the four-torus. In [16] the gauge orbits have been extended to the full gauge symmetry including also non-periodic $\mathbf{Z}(N)$ transformations. As a consequence the Gribov problem appears to be enhanced even for the gluon propagator. Here we restrict ourselves to the standard case of periodic gauge transformations. In marked contrast to the weak Gribov copy dependence of the gluon propagator the ghost propagator at lower momenta depends on the selection of gauge copies. (For a study of the influence of Gribov copies on the ghost propagator in the $SU(2)$ case see [17].) The effect of the Gribov copies is illustrated in Fig. 1. There we have plotted the `fc`-to-`bc` ratios of the ghost and gluon dressing functions. Obviously for the ghost propagator the Gribov problem can cause $O(5\%)$ deviations in the low-momentum region ($q < 1$ GeV). However, a closer inspection of the data for the ghost propagator indicates that the influence of Gribov copies becomes weaker for increasing lattice size. This is in agreement with a recent claim by Zwanziger according to which in the infinite volume limit averaging over gauge copies in the Gribov region should lead to the same result as over copies restricted to the fundamental modular region [18]. In [16] similar indications have been found for $SU(2)$. Fig. 2 shows the ghost and gluon dressing functions versus q^2. In contrast to our previous papers [12,14] we have plotted only `fc`-copy results, where we have included some new data for lattices $24^3 \times 48$ and 32^4 at $\beta = 6.0$. Both propagators have been renormalized separately for each β with the normalization condition $Z = 1$ at $q = \mu = 3$ GeV. Note the deviation of the data for the asymmet-

[4]We thank A. Lokhov for bringing the arguments in [7] to our attention.

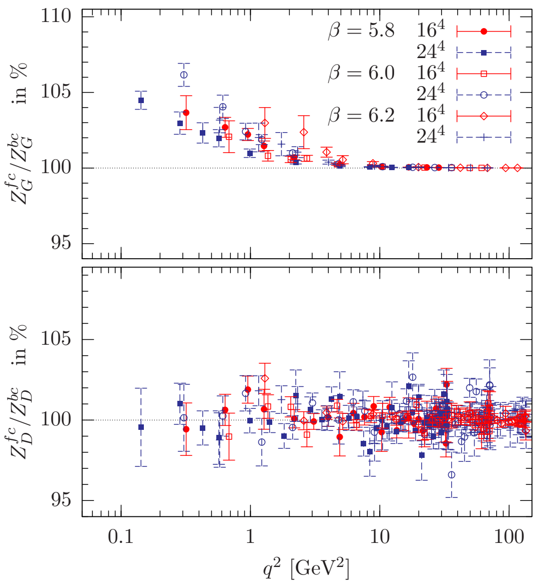

Figure 1. The ratios $Z^{\rm fc}/Z^{\rm bc}$ for the dressing functions Z_G (upper panel) and Z_D (lower panel) determined on first (`fc`) and best (`bc`) gauge copies.

ric lattice $24^3 \times 48$. In principle, for better gauge copies the curve can become only less singular. Still it is very difficult to extract a possible power behavior at low q^2 in comparison with Eq. (1).

3. THE GHOST-GLUON VERTEX AND THE RUNNING COUPLING

In Fig. 3 we present the combined result for the running coupling (2). Fits to the 1-loop and 2-loop running coupling are also shown. Our new data points confirm our previous observation that the running coupling monotonously decreases with decreasing momentum in the range $q^2 < 0.4$ GeV^2. We have plotted `fc` copy results, showing that this behavior is not a matter of selecting better gauge copies. The nice coincidence of the data for various lattice sizes (except the points refering to the asymmetric lattice) makes it hard to see how finite-volume effects can be blamed for the disagreement with the DS continuum result Eq. (1). Thus, the arguments as given in [7] should be checked in order hopefully to resolve the conflict.

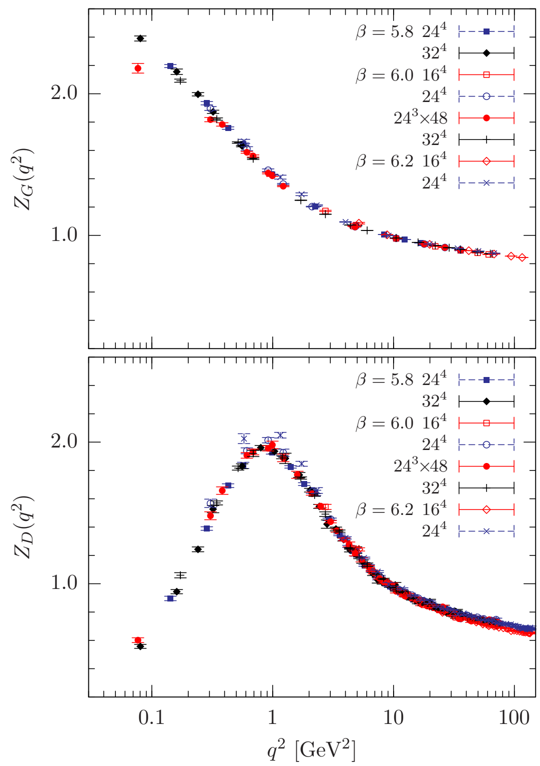

Figure 2. The ghost dressing function Z_G (above) and the gluon dressing function Z_D (below) *vs.* momentum q^2 both measured on `fc` gauge copies.

One can ask, whether the ghost-ghost-gluon vertex renormalization function $Z_1(q^2)$ is really constant at lower momenta. A recent investigation of this function defined at vanishing gluon momentum for the $SU(2)$ case [15] supports that $Z_1(q^2) \approx 1$ at least for momenta larger than 1 GeV. We have performed an analogous study for $Z_1(q^2)$ in the case of $SU(3)$ gluodynamics. Our (still preliminary) results are presented in Fig. 4. There is a slight variation visible in the interval 0.3 GeV$^2 \le q^2 \le 5$ GeV^2. But this weak deviation from being constant will not have a dramatic influence on the running coupling.

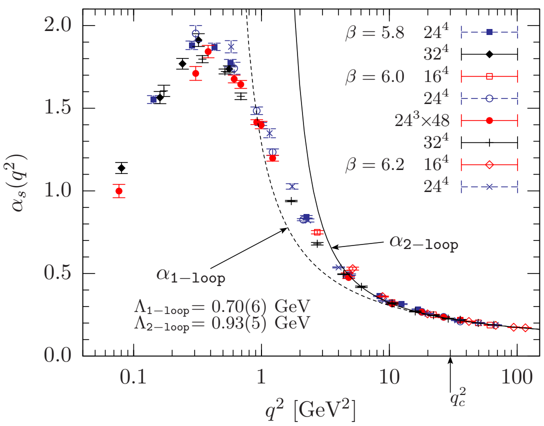

Figure 3. The momentum dependence of the running coupling $\alpha_s(q^2)$ measured on `fc` gauge copies.

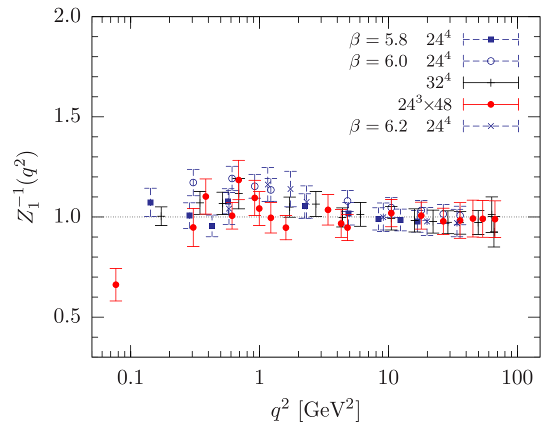

Figure 4. The inverse ghost-ghost-gluon vertex renormalization function $Z_1^{-1}(q^2)$ measured on `fc` gauge copies and normalized to unity at $q = 3$ GeV.

4. LOW-LYING MODES OF THE FADDEEV-POPOV OPERATOR

Now let us discuss the spectrum of the low-lying eigenvalues λ_i of the Faddeev-Popov (F-P) operator. For each maximum of the gauge functional $F_U[g]$, besides of its discarded eight trivial zero modes, the eigenvalues of the F-P operator are positive. The corresponding gauge field configuration is said to lie within the Gribov region. If the lowest non-trivial eigenvalue tends to zero the configuration approaches the so-called Gribov horizon.

Having the lowest n eigenvalues and eigenvectors determined, one can construct an approximation to the ghost propagator

$$G_n(q^2) = \sum_{i=1}^{n} \frac{1}{\lambda_i} \vec{\Phi}_i(k) \cdot \vec{\Phi}_i(-k) \qquad (4)$$

with $\vec{\Phi}_i(k)$ being the Fourier transform of the i-th eigenmode (k denotes the lattice momentum). If all $n = 8V$ eigenvalues and eigenvectors were known, the ghost propagator $G(q^2)$ would be completely determined. In the recent literature [19,20] it is stated that the F-P operator acquires very small eigenvalues in the presence of vortex excitations. In any case, according to a popular belief it is an enhanced density of eigenvalues near zero which causes the ghost propagator to diverge stronger than $1/q^2$ near zero momentum [18].

At $\beta = 6.2$ we have extracted the 200 lowest (non-trival) eigenvalues and their corresponding eigenfunctions for lattice sizes 12^4, 16^4 as well as 50 modes for 24^4. Additionally, 90 eigenvalues have been extracted on a 24^4 lattice at $\beta = 5.8$. This allows us to check how the low-lying eigenvalues are shifted towards $\lambda = 0$ as the physical volume is increased. In order to clarify how the choice of gauge copy influences the spectrum, the eigenvalues and eigenvectors have been extracted separately on our sets of `fc` and `bc` gauge-fixed configurations.

In Fig. 5 the distributions of the lowest λ_1 and second lowest λ_2 eigenvalues of the F-P operator are shown for different (physical) volumes. There $h(\lambda, \lambda + \Delta)$ represents the average number (per configuration) of eigenvalues found in the intervall $[\lambda, \lambda + \Delta]$. Open bars refer to the distribution on `fc` gauge copies while full bars to that on `bc` copies. From this figure it is quite obvious that both eigenvalues, λ_1 and λ_2, are shifted to lower values as the physical volume is enlarged. In conjunction the spread of λ values decreases. We have found that the center of

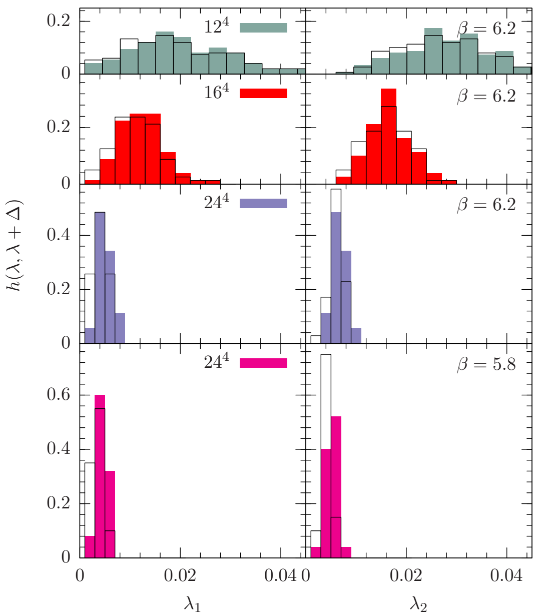

Figure 5. The distribution $h(\lambda)$ of the lowest (left panels) and second lowest (right panels) Faddeev-Popov eigenvalues λ_1 and λ_2. Filled columns refer to `bc` gauge copies, while open columns refer to `fc` copies.

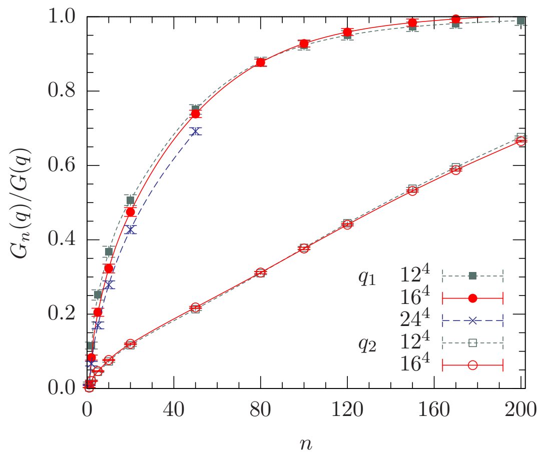

Figure 6. The ratio of the ghost propagator $G_n(q^2)$ approximated in accordance with Eq. (4) to the corresponding full estimator $G(q^2)$ as a function of n for the lowest q_1 and second lowest momentum q_2 ($\beta = 6.2$, lattice sizes 12^4, 16^4 and 24^4).

those distributions tends towards zero faster than $1/L^2$. Here L refers to the linear lattice extension in physical units. For example, we have found $\langle\lambda_1\rangle(L) \propto 1/L^{2+\varepsilon}$ with $\varepsilon \approx 0.16(4)$. It is also visible that the eigenvalues λ_1 and λ_2 on `fc` gauge copies are on average lower than those obtained on `bc` copies.

Eq. (4) suggests that low-lying eigenvalues and eigenvectors have a major impact on the ghost propagator at the lowest momenta. Actually, this is not so easy to predict. We have studied, to what extent the lowest n modes saturate the estimator of the ghost propagator $G_n(q^2)$ on a given set of gauge copies. We show the result in Fig. 6 for the lowest (q_1) and the second lowest momentum (q_2) available on lattices of different size at $\beta = 6.2$. The estimates for $G_n(q^2)$ have been divided by the full (conjugate gradient) estimator $G(q^2)$ in order to compare the saturation for different volumes. Considering first the lowest momentum q_1 we observe from Fig. 6 that the rates of convergence differ, albeit slightly, for the three different lattice sizes. The relative deficit rises with the lattice volume. In order to reproduce the ghost propagator within a few percent, $150 \ldots 200$ eigenmodes turn out to be sufficient. For the second lowest momentum q_2 even 200 eigenmodes are far from saturating the result.

5. CONCLUSIONS

We have studied the low-momentum region of Landau gauge gluodynamics using Monte Carlo simulations with the Wilson plaquette action and various lattice sizes from 16^4 to 32^4. For the inverse bare coupling we haven chosen $\beta = 5.8$, 6.0 and 6.2. In this way we have reached momenta down to $q^2 \simeq 0.1\text{GeV}^2$. We have presented data mainly for the `fc` gauge copies, but we have shown the ghost propagator to become less singular within a $O(5\%)$ deviation, when better gauge copies are taken. We have found indications that the Gribov effect weakens as the volume is increasing. Towards $q \to 0$ in the infrared momentum region, the gluon dressing function was shown to decrease, while the ghost dressing function turned out to rise. However, the connected power laws predicted by the infinite-volume DS approach cannot be confirmed from our data. Correspondingly, the behavior of the running coupling $\alpha_s(q^2)$ in a suitable momentum subtraction scheme (based on the ghost-ghost-gluon vertex) does not approach the expected finite limit. Instead, this coupling has been found to decrease for lower momenta after passing a turnover at $q^2 \approx 0.4$ GeV^2.

In addition, we have investigated the spectral properties of the F-P operator and its relation to the ghost propagator for various lattice sizes. As argued in [18] we have found the low-lying eigenvalues to be shifted towards zero (the configurations approaching the Gribov horizon) as the volume is increased. The low-lying eigenvalues extracted on `bc` gauge copies (those with the largest functional value) are larger on average than those on `fc` copies. Thus, for finite volumes better gauge-fixing (in terms of the gauge functional) keeps the configurations at larger distance from the Gribov horizon. The study of the contributions to the ghost propagator coming from the lowest eigenmodes shows that even at the lowest momentum one needs a large number of modes in order to saturate the propagator. For increasing volumes and larger lattice momenta the convergence of the eigenmode expansion becomes even slower.

All simulations have been done on the IBM pSeries 690 at HLRN. We are grateful to Hinnerk Stüben for providing parts of his code. One of us (M. M.-P.) acknowledges useful discussions with V.K. Mitrjushkin.

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