

An Extended Mumford-Shah Model and an Improved Region Merging Algorithm for Image Segmentation



THE UNIVERSITY OF ADELAIDE
Department of Applied Mathematics

Trevor Tao

*A thesis submitted for the degree of
Doctor of Philosophy.*

Primary Supervisor: Dr John van der Hoek (University of Adelaide)

*Secondary Supervisor: Dr David James Crisp (Defence Science and
Technology Organization)*

October 25, 2005

Contents

Abstract	ix
Signed Statement	xi
Acknowledgements	xiii
1 Introduction	1
1.1 Low-Level Methods	2
1.2 High-Level Methods	4
1.3 The Variational Formulation	6
1.3.1 The Mumford-Shah Model and Region Merging	7
1.3.2 The Snake Model, Active Contour Model and Level Set Methods	9
1.3.3 The Level Set Method	12
1.3.4 The Topological Snake Model	14
1.3.5 Stochastic Models	15
1.4 Advantages of the Variational Formulation	16
1.5 Outline of the Thesis	17
2 The Basic Mumford-Shah Functional and Region Merging Algorithms	19
2.1 Introduction	19
2.2 Discussion of the Mumford-Shah Functional	21

2.3	Koepfler's Algorithm	25
2.4	An Improvement of Koepfler's Algorithm	28
2.5	Conclusions	34
3	Mathematical Analysis of an Extended Mumford-Shah Model for Image Segmentation	35
3.1	Introduction	35
3.2	Limitations of the Mumford-Shah Model	38
3.3	Image Segmentation in a Bayesian Setting	38
3.4	Our Main Result for the Extended Model	43
3.5	Experimental Results	64
3.6	Conclusions	65
4	Computation of a Unique Minimizer of the Energy Functional for the Extended Mumford-Shah Model.	67
4.1	Introduction	67
4.2	The Image g and its Unique Minimizer of the Energy Functional . . .	70
4.3	Conclusions	83
5	A Solution to the Small Sample Problem for Region Merging Al- gorithms	85
5.1	Introduction	85
5.2	Solution to the Small Sample Problem	88
5.3	The Basic Algorithm	95
5.4	Test Images and Experiments	96
5.5	The Accuracy Measure	97
5.6	Experimental results	98
5.6.1	The SHAPE Image	98
5.6.2	The CONTRAST Image	99
5.6.3	The NOISE Image	100

5.6.4	The RADIUS Image	101
5.6.5	The HOUSE Image	102
5.7	Conclusions	103
6	The Modelling of Images with Texture	111
6.1	Introduction	111
6.2	Modifying the EMS Model to Account for Textures	116
6.3	Experimental Results	117
6.3.1	Synthetic Image	118
6.3.2	The Cameraman Image	120
6.3.3	A Brodatz Mosaic	121
6.4	Conclusions	122
7	Selection of an Optimal Value of the Scale Parameter for the Extended Piecewise Constant Mumford-Shah Functional	123
7.1	Introduction	123
7.2	Significance of Merges	127
7.3	Experimental Results for the Significance of Merges	129
7.4	Modelling the Merge Cost	134
7.5	Experimental Results for Modelling the Merge Cost	135
7.6	Conclusions	140
8	Conclusions	141
A	The Distance Between Two Error Functions for the Metric d Defined in Chapter 5	145
	Bibliography	149

List of Tables

2.3.1 Best segmentation for different values of λ	27
2.4.1 Segmentation obtained by the FLSA for different values of λ	30
5.7.1 Time taken for Image SHAPE	105
5.7.2 Accuracy results/No. of ellipses lost for Image SHAPE	105
5.7.3 Time taken for Image CONTRAST	106
5.7.4 Accuracy results/No. of circles lost for Image CONTRAST	106
5.7.5 Time taken for Image NOISE	107
5.7.6 Accuracy results /No. of circles lost for Image NOISE	107
5.7.7 Accuracy results/No. of circles lost for Image NOISE2	108
5.7.8 Time taken for Image RADIUS . $\sigma_0^2 =$ variance offset, var = variance of added noise.	109
5.7.9 Accuracy results /No. of circles lost for Image RADIUS	109

List of Figures

1.2.1 An edge separating two regions.	5
1.2.2 The optimal edge computed for a simple image for Martelli's problem.	6
1.3.1 An example segmentation of an image in the discrete domain.	8
1.3.2 Adding and deleting markers for the Topological Snake model.	15
2.3.1 Four different segmentations obtainable by region merging.	26
3.5.1 Segmentation of the Boat image.	65
3.5.2 Segmentation of a synthetic image	65
4.2.1 Definition of g with parameters $L = 6, s = 5$	71
4.2.2 An example of decomposing a region into horizontal strips.	80
5.7.1 Four different synthetic images to be segmented.	108
5.7.2 Comparison of algorithms FLSA-MAP and FLSA-CDF for Image NOISE.	110
5.7.3 Comparison of algorithms FLSA-MAP and FLSA-CDF for Image RADIUS.	110
5.7.4 Comparison of algorithms FLSA-MAP and FLSA-CDF for Image HOUSE.	110
6.1.1 The difference between the EMS model and the use of transform domain for mean and variance.	116
6.2.1 Seven masks used for texture segmentation.	116

6.3.1 An image with striped texture with strong noise.	119
6.3.2 An image with striped texture with weak noise.	119
6.3.3 A two-dimensional image with striped texture with weak noise.	119
6.3.4 The Cameraman image.	120
6.3.5 A Brodatz mosaic.	121
7.3.1 Four images to be segmented.	129
7.3.2 Graphs for merge cost and significance of merges for the House image.	131
7.3.3 Graphs for merge cost and significance of merges for Gaussian Noise image.	131
7.3.4 Graphs for merge cost and significance of merges for the Multiscale image.	132
7.3.5 Graphs for merge cost and significance of merges for the Boat image.	132
7.3.6 Optimal segmentations obtained using Significance of merges for the images in Figure 7.3.1.	133
7.3.7 Segmentations at different scales for the Multiscale image.	133
7.5.1 Graph of $\lambda/R^{-1/2}$ for the House image.	137
7.5.2 Graph of $\lambda/R^{-1/2}$ for the Gaussian Noise image.	138
7.5.3 Graph of $\lambda/R^{-1/2}$ for the Multiscale image.	138
7.5.4 Graph of $\lambda/R^{-1/2}$ for the Boat image.	139
7.5.5 Optimal segmentations obtained using Modelling the merge cost for the images in Figure 7.3.1.	139

Abstract

In this thesis we extend the Mumford-Shah model and propose a new region merging algorithm for image segmentation. The segmentation problem is to determine an optimal partition of an image into constituent regions such that individual regions are homogenous within and adjacent regions have contrasting properties. By optimal, we mean one that minimizes a particular energy functional. In region merging, the image is initially divided into a very fine grid, with each pixel being a separate region. Regions are then recursively merged until it is no longer possible to decrease the energy functional.

In 1994, Koepfler, Lopez and Morel developed a region merging algorithm for segmenting an image. They consider the piecewise constant Mumford-Shah model, where the energy functional consists of two terms, accuracy versus complexity, with the trade-off controlled by a scale parameter. They show that one can efficiently generate a hierarchy of segmentations from coarse to fine. This algorithm is complemented by a sound theoretical analysis of the piecewise constant model, due to Morel and Solimini.

The primary motivation for extending the Mumford-Shah model stems from the fact that this model is only suitable for “cartoon” images, where each region is uncontaminated by any form of noise. Other shortcomings also need to be addressed. In the algorithm of Koepfler et al., it is difficult to determine the order in which the regions are merged and a “schedule” is required in order to determine the number

and fine-ness of segmentations in the hierarchy. Both of these difficulties mitigate the theoretical analysis of Koepfler's algorithm. There is no definite method for selecting the "optimal" value of the scale parameter itself. Furthermore, the mathematical analysis is not well understood for more complex models. None of these issues are convincingly answered in the literature.

This thesis aims to provide some answers to the above shortcomings by introducing new techniques for region merging algorithms and a better understanding of the theoretical analysis of both the mathematics and the algorithm's performance. A review of general segmentation techniques is provided early in this thesis. Also discussed is the development of an "extended" model to account for white noise contamination of images, and an improvement of Koepfler's original algorithm which eliminates the need for a schedule. The work of Morel and Solimini is generalized to the extended model. Also considered is an application to textured images and the issue of selecting the value of the scale parameter.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

SIGNED: DATE:

Acknowledgements

I would like to acknowledge my supervisors John van der Hoek and David Crisp for their valuable guidance and much good advice.

I also wish to thank my family and friends for support and encouragement during my PhD program, and my mum for the endless proofreading.

The support of the Adelaide University Scholarship and the CSSIP Supplementary Scholarships is also gratefully appreciated, as are the friendly staff and excellent facilities at Adelaide University.