An Extended Mumford-Shah Model and an Improved Region Merging Algorithm for Image Segmentation

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Abstract

In this thesis we extend the Mumford-Shah model and propose a new region merging algorithm for image segmentation. The segmentation problem is to determine an optimal partition of an image into constituent regions such that individual regions are homogenous within and adjacent regions have contrasting properties. By optimal, we mean one that minimizes a particular energy functional. In region merging, the image is initially divided into a very fine grid, with each pixel being a separate region. Regions are then recursively merged until it is no longer possible to decrease the energy functional.

In 1994, Koepfler, Lopez and Morel developed a region merging algorithm for segmentating an image. They consider the piecewise constant Mumford-Shah model, where the energy functional consists of two terms, accuracy versus complexity, with the trade-off controlled by a scale parameter. They show that one can efficiently generate a hierarchy of segmentations from coarse to fine. This algorithm is complemented by a sound theoretical analysis of the piecewise constant model, due to Morel and Solimini.

The primary motivation for extending the Mumford-Shah model stems from the fact that this model is only suitable for "cartoon" images, where each region is uncomtaminated by any form of noise. Other shortcomings also need to be addressed. In the algorithm of Koepfler et al., it is difficult to determine the order in which the regions are merged and a "schedule" is required in order to determine the number

and fine-ness of segmentations in the hierarchy. Both of these difficulties mitigate the theoretical analysis of Koepfler's algorithm. There is no definite method for selecting the "optimal" value of the scale parameter itself. Furthermore, the mathematical analysis is not well understood for more complex models. None of these issues are convincingly answered in the literature.

This thesis aims to provide some answers to the above shortcomings by introducing new techniques for region merging algorithms and a better understanding of the theoretical analysis of both the mathematics and the algorithm's performance. A review of general segmentation techniques is provided early in this thesis. Also discussed is the development of an "extended" model to account for white noise contamination of images, and an improvement of Koepfler's original algorithm which eliminates the need for a schedule. The work of Morel and Solimini is generalized to the extended model. Also considered is an application to textured images and the issue of selecting the value of the scale parameter.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

SIGNED: .... DATE: ....

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