D-Brane Charge Groups and Fusion Rings in Wess-Zumino-Witten Models

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Abstract

This thesis presents the computation and investigation of the charges and the corresponding charge groups for untwisted symmetry-preserving D-branes in a Wess-Zumino-Witten model over a compact, connected, simply-connected, simple Lie group. First, some general ideas from conformal field theory are reviewed and applied to Wess-Zumino-Witten models. Boundary conformal field theory is then introduced with the aim of deriving the Cardy constraint relating the consistent boundary conditions to fusion. This is used to justify certain dynamical processes for branes, called condensation, which lead to a conserved charge and constraints on the corresponding charge group (following Fredenhagen and Schomerus). These constraints are then used to determine the charge groups for untwisted symmetry-preserving branes over all compact, connected, simply-connected, simple Lie groups. Rigorous proofs are detailed for the Lie groups $\mathsf{SU}(r+1)$ and $\mathsf{Sp}(2r)$ for all ranks r, and the relevance of these results to K-theory is discussed. These proofs rely on an explicit presentation of the corresponding fusion rings (over $\mathbb{Z}$), which are also rigorously derived for the first time.

This computation is followed by a careful treatment of the Wess-Zumino-Witten model actions; the point being that the consistent quantisation paradigm developed can also be applied to brane charges to determine the charge groups. The usual (string-theoretic) D-brane charges are introduced, and are proved to exactly reproduce the charges of Fredenhagen and Schomerus when certain quantisation effects are brought into play. This is followed by a detailed investigation of the constraints induced on the corresponding charge groups by insisting that the string-theoretic charges be well-defined. These constraints are demonstrated to imply those of Fredenhagen and Schomerus except when the Wess-Zumino-Witten model is over a symplectic Lie group, $\mathsf{Sp}(2r)$. In the symplectic case, numerical computation shows that these constraints can be strictly stronger than those of Fredenhagen and Schomerus. A possible resolution is offered indicating why this need not contradict the K-theoretic interpretation.

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