Very Viscous Flows Driven by Gravity

with particular application to

Slumping of Molten Glass

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Abstract

This thesis examines the flow of very viscous Newtonian fluids driven by gravity. It is written with concern for specific applications in the optics industry, with emphasis on the slumping of molten glass into a mould, as in the manufacture of optical components, which are in turn used to manufacture ophthalmic lenses. This process is known as *thermal replication*. However, the work has more general applicability, and disc viscometry, used to determine the viscosity of very viscous fluids, is also considered. In addition, one chapter of the thesis is devoted to the flow of dripping honey, as another example of a very viscous flow to which the model can be applied.

The Stokes creeping-flow equations are used to model the very viscous flows of interest. The main solution method is finite elements, and a purpose-written computer program has been developed to solve the creeping-flow equations by this method. The present program is restricted to solving for either two-dimensional or axisymmetric flows but is extendible to three dimensions. In addition, semi-analytic series and asymptotic methods are used for some small portions of the work.

The optical applications of this work demand consideration of the topic of computing surface curvature, and therefore second derivatives, from inexact and discrete numerical and experimental data. For this purpose, fitting of B-splines by a least-squares method to coordinate data defining the surface has been used.

Much of the work assumes isothermal conditions, but in the context of the accuracy required in optical component manufacture it is also possible that non-isothermal effects will be important. Consequently, this restriction is eventually relaxed and some consideration given to non-isothermal conditions.

In order to validate the creeping-flow model and finite-element program, comparisons of numerical simulations with experimental results are performed. A preliminary assessment of the importance of non-isothermal conditions to the thermal-replication process is also made by comparing isothermal and non-isothermal simulations with experimental results. The isothermal model is found to best match the experimental data.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

Signed: ....

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