

# IDENTIFICATION AND MODELLING OF HYDROLOGICAL PERSISTENCE WITH HIDDEN MARKOV MODELS

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# Table of contents

<b>Table of contents</b> .....	<b>i</b>
<b>List of figures</b> .....	<b>v</b>
<b>List of tables</b> .....	<b>xi</b>
<b>Abstract</b> .....	<b>xvii</b>
<b>Statement of Originality</b> .....	<b>xix</b>
<b>Acknowledgements</b> .....	<b>xxi</b>
<b>List of Symbols</b> .....	<b>xxiii</b>
<b>Chapter 1 Introduction</b> .....	<b>1</b>
1.1 The influence of climatic persistence .....	1
1.2 Objectives of Thesis .....	3
1.3 Outline of Thesis .....	4
<b>Chapter 2 Identifying persistence within the global climate</b> .....	<b>7</b>
2.1 Observed modes of climatic persistence .....	7
2.1.1 El Niño Southern Oscillation (ENSO) .....	8
2.1.2 Low frequency climate variability and abrupt climate changes .....	10
2.1.3 Antarctic Circumpolar Wave (ACW).....	13
2.1.4 Indian Ocean Dipole (IOD).....	15
2.2 Measuring changes in the Pacific climate .....	16
2.2.1 ENSO indices .....	16
2.2.2 Quantifying multi-decadal Pacific variability .....	19
2.2.3 Stable climate states .....	21
2.3 Hydrological impact of climatic persistence .....	22
2.3.1 Relationships between ENSO and rainfall variability.....	23
2.3.2 Relationships between ENSO and streamflow variability .....	25
2.3.3 Low frequency modulation of ENSO impacts .....	27
2.4 Summary of chapter .....	30
<b>Chapter 3 Hydrological persistence</b> .....	<b>31</b>
3.1 Estimating persistence in hydrological time series .....	31
3.1.1 Statistical tests to identify persistence.....	32
3.1.2 Hurst phenomenon .....	33
3.1.3 Previous studies of persistence in hydrology .....	37
3.2 Stochastic modelling of hydrological time series.....	37
3.2.1 Univariate time series models .....	39
3.2.2 Modelling the Hurst phenomenon.....	42
3.2.3 Shifting level (SL) models .....	46

---

3.2.4	Segmentation models.....	47
3.3	Hidden Markov models (HMMs) .....	49
3.3.1	Fundamentals of HMMs .....	50
3.3.2	Moments of HMMs .....	53
3.3.3	HMM modelling assumptions .....	55
3.3.4	Evaluating HMM likelihood.....	56
3.3.5	Global optimisation routines.....	59
3.4	Bayesian modelling framework.....	60
3.4.1	Markov chain Monte Carlo (MCMC) methods .....	61
3.4.2	Convergence of MCMC algorithms.....	64
3.4.3	Bayesian model selection.....	66
3.5	Summary of chapter.....	68
<b>Chapter 4</b>	<b>Identifying hydroclimatic relationships.....</b>	<b>71</b>
4.1	Hydrologic data.....	71
4.1.1	Spatially-averaged rainfall data .....	71
4.1.2	Streamflow data .....	75
4.2	Relationships between hydrology and climate indices .....	79
4.2.1	Influence of ENSO upon rainfall .....	79
4.2.2	Influence of IPO upon rainfall .....	83
4.2.3	Influence of non-Pacific climate modes upon rainfall .....	84
4.2.4	Influence of ENSO and IPO upon streamflow .....	88
4.3	Relationships between arid zone hydrology and climate indices .....	91
4.3.1	Flows in the Todd River .....	91
4.3.2	Flows in African arid zone rivers .....	95
4.4	Summary of chapter.....	100
<b>Chapter 5</b>	<b>Identifying hydrological persistence .....</b>	<b>103</b>
5.1	Using runs analysis to identify hydrological persistence.....	103
5.1.1	Persistence within monthly rainfall .....	103
5.1.2	Persistence within monthly streamflows .....	107
5.2	Identifying evidence for the Hurst phenomenon .....	110
5.2.1	Sampling bounds for estimates of the Hurst exponent .....	110
5.2.2	Estimating the Hurst exponent in monthly rainfall data .....	111
5.3	Summary of chapter.....	114
<b>Chapter 6</b>	<b>Problems with the calibration of Gaussian HMMs to annual rainfall.....</b>	<b>115</b>
6.1	Statistics of annual rainfall series .....	115
6.2	Calibration of two-state Gaussian HMMs to annual rainfall .....	118
6.3	Calibration of two-state Gaussian HMMs to simulated data .....	126

---

6.4	HMMs degenerating to mixture distributions .....	128
6.5	Summary of chapter .....	131
<b>Chapter 7</b>	<b>Using parametric HMMs to identify persistence in monthly rainfall .....</b>	<b>133</b>
7.1	Statistics of monthly rainfall data.....	133
7.2	Calibration of two-state HMMs to monthly rainfall .....	136
7.2.1	Calibration of two-state lognormal HMMs.....	136
7.2.2	Calibration of two-state gamma HMMs.....	142
7.2.3	Relationships between HMM states and measures of climate variability .....	146
7.3	Using HMMs to identify possible three-state monthly persistence.....	147
7.4	Problems in the calibration of incorrect parametric HMMs.....	155
7.4.1	Using monthly rainfall for District 66 to illustrate calibration problems .....	155
7.4.2	Calibration of incorrect HMMs to simulated data.....	158
7.5	Summary of chapter .....	160
<b>Chapter 8</b>	<b>Non-parametric hidden Markov models.....</b>	<b>163</b>
8.1	Model structure .....	163
8.1.1	Background to the NP HMM .....	163
8.1.2	Partitioning the unit square .....	164
8.1.3	Approximating the partition.....	166
8.1.4	Likelihood function for the NP HMM .....	167
8.1.5	Identification and estimation of state conditional distributions .....	169
8.2	Calibration of NP HMMs to various simulated data series.....	169
8.2.1	Time series simulated from two-state Gaussian HMMs .....	170
8.2.2	Time series simulated from a two-state lognormal HMM .....	171
8.2.3	Time series simulated from a two-state HMM having one Gaussian and one lognormal conditional distribution.....	172
8.2.4	Time series simulated from a two-state Poisson HMM .....	173
8.2.5	Time series simulated from a three-state Gaussian HMM .....	174
8.3	Calibration of NP HMMs to Sydney and District 66 rainfall.....	176
8.3.1	Calibration of two-state NP HMMs to annual rainfall data .....	177
8.3.2	Calibration of two-state NP HMMs to monthly rainfall data.....	179
8.3.3	Calibration of three-state NP HMMs to monthly rainfall data.....	186
8.4	Calibration of NP HMMs to Australian hydrologic data .....	189
8.4.1	Spatially-averaged monthly rainfall for New South Wales.....	189
8.4.2	Spatially-averaged monthly rainfall across Australia .....	191
8.4.3	Monthly streamflows.....	197
8.5	Investigating persistence in alternative hydrological variables.....	199
8.5.1	Analysing persistence within the number of monthly rain-days .....	199
8.5.2	Analysing monthly persistence in rainfall extremes .....	202

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8.6	Summary of chapter.....	208
<b>Chapter 9</b>	<b>Extensions to the standard HMM formulation.....</b>	<b>211</b>
9.1	Incorporating explicit state durations.....	211
9.1.1	Hidden semi-Markov model (HSMM) structure .....	211
9.1.2	Calibration of a two-state lognormal HSMM to Sydney monthly rainfall .....	213
9.2	Incorporating temporal dependence in observations .....	215
9.2.1	Autoregressive hidden Markov model (ARHMM) structure.....	215
9.2.2	Calibration of a two-state lognormal ARHMM to Sydney monthly rainfall.....	216
9.3	Analysing persistence at multiple time scales simultaneously .....	218
9.3.1	Hierarchical hidden Markov model (HHMM) structure.....	218
9.3.2	Calibration of a two-state lognormal hierarchical HMM to Sydney monthly rainfall.....	220
9.4	Summary of chapter.....	222
<b>Chapter 10</b>	<b>Utilising NP HMMs to identify appropriate parametric models for persistent data.....</b>	<b>223</b>
10.1	Developing HMMs to model hydrological persistence in spatial rainfall .....	223
10.2	Developing HMMs to model hydrological persistence in streamflow .....	231
10.3	Summary of chapter.....	236
<b>Chapter 11</b>	<b>Simulating persistent hydrologic data with HMMs: Three case studies..</b>	<b>237</b>
11.1	Background to hydrologic simulation.....	237
11.2	Simulating monthly rainfall for District 71 .....	238
11.3	Simulating monthly streamflows for the Burdekin River.....	242
11.4	Simulating catchment-scale rainfall for the Warragamba Reservoir .....	246
11.5	Summary of chapter.....	254
<b>Conclusions</b>	.....	<b>255</b>
<b>Future work</b>	.....	<b>259</b>
<b>References</b>	.....	<b>261</b>

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## List of figures

Figure 1.1 Deviations from median values of reconstructed natural flows in the River Murray evaluated for annual totals ( $\times 10^3$ Gigalitres, top) and deseasonalised monthly variates (bottom).....	2
Figure 2.1 Mechanisms of ENSO (after McPhaden et al., 1998) .....	9
Figure 2.2 Schematic diagram of the ACW (after Baines, 1998) .....	14
Figure 2.3 Schematic diagrams of the positive phase (on left) and negative phase (on right) of the Indian Ocean dipole (after Rao, 2005) .....	15
Figure 2.4 Power spectrum for monthly MEI totals (solid line), with 95% confidence limits (dashed lines), using averaged periodogram method.....	17
Figure 2.5 Time series of five-month running means of monthly Niño-3.4 values.....	18
Figure 2.6 Time series of seasonal IPO values (blue) and monthly PDO values (red).....	20
Figure 2.7 Rainfall deficiencies across Australia during 2002/3 .....	24
Figure 2.8 Global streamflow anomalies during El Niño (after Viles and Goudie, 2003) .....	27
Figure 3.1 Variation in Hurst exponent with variation in autocorrelation from Monte Carlo simulations of AR(1) models showing median and 90% of samples.....	43
Figure 3.2 Schematic diagram of a binary HMM (adapted from Elliott <i>et al.</i> , 1995) .....	56
Figure 4.1 Map of Australian meteorological districts .....	72
Figure 4.2 Distributions for monthly rainfall in selected meteorological districts .....	74
Figure 4.3 Map of the Murray-Darling basin (source: Murray-Darling Basin Commission).....	76
Figure 4.4 Location of the Lake Eyre basin (after Parliament of Australia Library, 2001).....	77
Figure 4.5 Time series of monthly flows for the seven rivers analysed, shown over 10-year periods.....	78
Figure 4.6 Linear correlations between NINO3 values and deseasonalised rainfall for the four districts calculated for each calendar month (multiplied by -1), with solid line indicating the magnitude of statistically significant correlation .....	80
Figure 4.7 Districts that show statistically significant linear correlation between deseasonalised monthly rainfall and monthly NINO3 values.....	81
Figure 4.8 Districts that show a statistically significant difference in monthly mean rainfall between La Niña and El Niño months .....	82
Figure 4.9 Districts that show a statistically significant difference in monthly mean rainfall between IPO+ and IPO- months .....	84
Figure 4.10 Districts in which the correlation between monthly DMI values and deseasonalised monthly rainfall from which NINO3 has been partialled exceeds the correlation between monthly NINO3 values and deseasonalised monthly rainfall from which DMI has been partialled .....	86
Figure 4.11 Districts in which the correlation between deseasonalised monthly rainfall and monthly PC <sub>1</sub> values exceeds the correlation between rainfall and monthly NINO3 .....	88
Figure 4.12 Daily flows in the Todd River (1994-1996) .....	92
Figure 4.13 Average monthly flows in the Todd River expressed as percentages of annual average.....	92

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Figure 4.14 Locations of the Omatako and Omaruru rivers, together with their sampling sites shown as triangles (after Cigizoglu <i>et al.</i> , 2002).....	95
Figure 4.15 Daily flows in the Omatako and Omaruru Rivers (1983-1985).....	96
Figure 5.1 Districts showing statistically significant persistence in deseasonalised monthly rainfall using the length of runs test (LORT) .....	105
Figure 5.2 Districts showing statistically significant persistence in annual rainfall using the length of runs test (LORT) .....	105
Figure 5.3 Districts showing statistically significant persistence in deseasonalised monthly rainfall using each of the four runs statistics .....	106
Figure 5.4 Correlograms for deseasonalised monthly flows in the seven rivers analysed .....	108
Figure 5.5 Partial correlograms for deseasonalised monthly flows in the seven rivers analysed .....	109
Figure 5.6 Sampling distribution of estimates for Hurst exponents for 1000 simulations of independent series of length 90 (left) and length 1080 (right) .....	111
Figure 5.7 Districts showing estimates of Hurst exponents for deseasonalised monthly rainfall series that exceed the 95 <sup>th</sup> percentile of multiple simulations of independent series.....	112
Figure 5.8 Districts showing estimates of Hurst exponents for annual rainfall that exceed the 95 <sup>th</sup> percentile of multiple simulations of independent series.....	112
Figure 6.1 Time series of annual rainfall for the Sydney and District 66 data.....	116
Figure 6.2 Gaussian probability plots showing the distribution of annual totals of Sydney (on left) and District 66 (on right).....	117
Figure 6.3 Posterior distributions from the calibration of a two-state Gaussian HMM to the annual rainfall data for Sydney.....	122
Figure 6.4 Time series of annual rainfall for Sydney shown alongside the posterior state series associated with parameter MLEs after the calibration of a two-state Gaussian HMM .....	123
Figure 6.5 Posterior state series associated with parameter MLEs after the calibration of a two-state Gaussian HMM to the annual rainfall for both Sydney and District 66.....	123
Figure 6.6 Median state series and 90% credibility interval from the calibration of a two-state Gaussian HMM to the annual rainfall for Sydney.....	125
Figure 6.7 Median state series and 90% credibility interval from the calibration of a two-state Gaussian HMM to the annual rainfall for Alice Springs.....	125
Figure 6.8 Median state series and 90% credibility interval from the calibration of a two-state Gaussian HMM to series of length 142 simulated from a two-state Gaussian HMM.....	127
Figure 6.9 Median state series for 142 values and 90% credibility interval from the calibration of a two-state Gaussian HMM to series of length 1000 simulated from a two-state Gaussian HMM .....	128
Figure 6.10 Posterior distribution for the sum of transition probabilities from the calibration of a two-state Gaussian HMM to the annual rainfall data for Sydney with median shown .....	130
Figure 7.1 Time series of monthly rainfall observations from Sydney and District 66 over the period 1980-1989.....	134
Figure 7.2 Mean monthly rainfall totals with bars showing 90% of monthly values for Sydney and District 66 .....	134
Figure 7.3 Lognormal probability plots showing scaled deseasonalised monthly rainfall for Sydney (left) and District 66 (right) .....	136

---

Figure 7.4 Time series of scaled deseasonalised monthly rainfall for Sydney and District 66 over the period 1980-1989 .....	136
Figure 7.5 Posterior distributions, with median values, from the calibration of a two-state lognormal HMM to deseasonalised monthly rainfall for Sydney, with the state conditional means and standard deviations of the natural logarithms of monthly data shown.....	138
Figure 7.6 Median state series and 90% credibility interval from the calibration of a two-state lognormal HMM to the deseasonalised monthly rainfall for Sydney .....	139
Figure 7.7 Time series of deseasonalised monthly rainfall for Sydney (red line) shown alongside the median state series following the calibration of a two-state lognormal HMM.....	140
Figure 7.8 Posterior distribution of the sum of transition probabilities from the calibration of a two-state lognormal HMM to deseasonalised monthly rainfall for Sydney with median shown .....	140
Figure 7.9 Posterior distribution of the sum of transition probabilities for the calibration of a two-state Gaussian HMM to deseasonalised monthly rainfall for Sydney with median shown .....	142
Figure 7.10 State conditional distributions estimated from the calibration of a two-state lognormal HMM (left) and a two-state gamma HMM (right) to the deseasonalised monthly rainfall for Sydney .....	145
Figure 7.11 State conditional distributions estimated from the calibration of a three-state lognormal HMM (left) and three-state gamma HMM (right) to the deseasonalised monthly rainfall for Sydney .....	149
Figure 7.12 Posterior distributions of transition probabilities from the calibration of a three-state lognormal HMM to deseasonalised monthly rainfall for Sydney with medians shown ....	150
Figure 7.13 Posterior distributions of transition probabilities from the calibration of a three-state gamma HMM to deseasonalised monthly rainfall for Sydney with medians shown.....	150
Figure 7.14 Posterior state probabilities over a 10-year period from the calibration of a three-state gamma HMM to the deseasonalised monthly rainfall for Sydney .....	151
Figure 7.15 Sequence of most likely states for a 10-year period from the calibration of a three-state lognormal HMM to the deseasonalised monthly rainfall for Sydney, colours as used in Figure 7.14.....	151
Figure 7.16 Posterior distributions for the sums of self-transition probabilities from the calibration of a three-state lognormal HMM (left) and a three-state gamma HMM (right) to the deseasonalised monthly rainfall for Sydney with medians shown.....	153
Figure 7.17 Median state series and 90% credibility interval over a 10-year period from the calibration of a two-state lognormal HMM to deseasonalised monthly rainfall for District 66 .....	157
Figure 7.18 Median state series and 90% credibility interval for 120 values from the calibration of a two-state Gaussian HMM to the simulated time series of length 1080 .....	158
Figure 7.19 Median state series and 90% credibility interval for the same 120 values as Figure 7.18 from the calibration of a two-state lognormal HMM to the simulated series of length 1080.....	159
Figure 8.1 An example of the partition of a unit square into wet and dry model states.....	164
Figure 8.2 The estimated separation of unit squares from three simulated HMMs into wet and dry distributions, in equal proportion, that are a) well separated, b) identical, and c) overlapping .....	165
Figure 8.3 A two-point division of a unit square for a two-state NP HMM.....	166

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Figure 8.4 A two-point division of a unit square for a three-state NP HMM.....	166
Figure 8.5 A nine-point division of a unit square for a two-state NP HMM.....	167
Figure 8.6 Gaussian probability plot showing estimates for conditional model states for a simulation of length 1000.....	170
Figure 8.7 Gaussian probability plot showing estimates for conditional model states for a simulation of length 100.....	171
Figure 8.8 Lognormal probability plot showing estimates for conditional model states for a simulation of length 1000.....	172
Figure 8.9 Estimated conditional distributions for dry state shown on Gaussian probability plot (left) and wet state shown on lognormal probability plot (on right).....	173
Figure 8.10 Marginal distribution of a series simulated from a two-state Poisson HMM.....	173
Figure 8.11 Histograms showing state conditional distributions for the wet state (on left) and dry state (on right).....	174
Figure 8.12 Gaussian probability plot showing estimates for conditional model states from the calibration of a three-state NP HMM to a simulated series of length 1500.....	176
Figure 8.13 Gaussian probability plot showing estimates for conditional model states from the calibration of a two-state NP HMM to the annual rainfall for Sydney.....	177
Figure 8.14 Median state series and 90% credibility interval from the calibration of a two-state NP HMM to the annual rainfall for Sydney .....	178
Figure 8.15 Median state series and 90% credibility interval for a 10-year period from the calibration of a two-state NP HMM to the deseasonalised monthly rainfall for Sydney ..	180
Figure 8.16 Lognormal probability plot showing estimates for conditional model states from the calibration of a two-state NP HMM to the deseasonalised monthly rainfall for Sydney ..	181
Figure 8.17 Median state series and 90% credibility interval for a 10-year period from the calibration of a two-state NP HMM to the deseasonalised monthly rainfall for District 66 .....	183
Figure 8.18 Lognormal probability plot showing estimates for conditional model states from the calibration of a two-state NP HMM to the deseasonalised monthly rainfall for District 66 .....	184
Figure 8.19 Estimates of state conditional distributions from the calibration of a three-state NP HMM to the deseasonalised monthly rainfall for Sydney .....	187
Figure 8.20 Estimates of state conditional distributions from the calibration of a three-state NP HMM to the deseasonalised monthly rainfall for District 66 .....	187
Figure 8.21 Meteorological districts in New South Wales, with arrow showing the transect from Sydney to northwestern corner of the state.....	190
Figure 8.22 Spatial distribution of the magnitude of persistence from the calibration of two-state NP HMMs to the deseasonalised monthly rainfall for each district.....	193
Figure 8.23 Rank correlations between monthly rainfall for each district and monthly rainfall of District 66 (blue), and rank correlations between the median state series for each district and the median state series for District 66 (orange) from the calibrations of two-state NP HMMs to the deseasonalised monthly rainfall for each district .....	195
Figure 8.24 Districts in which three-state NP HMMs are superior models for monthly rainfall than two-state NP HMMs using Bayes Factors.....	196

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Figure 8.25 Time series of monthly rainfall observations for Sydney (blue line with points) and the number of days on which rainfall was recorded each month (red line) over a ten-year period .....	199
Figure 8.26 Seasonal variations in the ratio of monthly averages to annual averages for total monthly rainfall and number of rain-days in each calendar month for Sydney .....	200
Figure 8.27 Median state series from the calibrations of two-state NP HMMs to the time series of deseasonalised monthly rainfall totals for Sydney (blue line with points) and deseasonalised variates of the number of monthly rain-days (red line).....	201
Figure 8.28 Seasonal variations in the ratio of monthly averages to annual averages for maximum rainfall intensities for Sydney recorded over various durations, and total monthly rainfall .....	203
Figure 8.29 Median state series from the calibrations of two-state NP HMMs to the time series of deseasonalised monthly rainfall totals for Sydney (blue line with points) and deseasonalised maximum rainfall intensities for each month recorded over 6-minute durations (red line).....	204
Figure 8.30 Median state series from the calibrations of two-state NP HMMs to the time series of deseasonalised monthly rainfall totals for Sydney (blue line with points) and deseasonalised maximum rainfall intensities for each month recorded over 1-hour durations (red line).....	204
Figure 8.31 Median state series from the calibrations of two-state NP HMMs to the time series of deseasonalised monthly rainfall totals for Sydney (blue line with points) and deseasonalised maximum rainfall intensities for each month recorded over 24-hour durations (red line).....	205
Figure 9.1 Probability mass function for negative binomial and geometric distributions .....	212
Figure 9.2 Conditional distributions from the calibrations of a two-state lognormal HMM and a two-state lognormal HSMM to the deseasonalised monthly rainfall for Sydney .....	214
Figure 9.3 Posterior distributions for probabilities associated with negative binomial duration densities from the calibration of a two-state lognormal HSMM to the deseasonalised monthly rainfall for Sydney, with medians shown .....	214
Figure 9.4 Probability mass functions for duration densities from the calibrations of a lognormal HMM and a lognormal HSMM with negative binomial duration densities to the deseasonalised monthly rainfall for Sydney .....	215
Figure 9.5 Bayesian net diagram for first-order ARHMM .....	216
Figure 9.6 Posterior distributions for lag-1 autocorrelations estimated from the calibration of a two-state lognormal ARHMM to the deseasonalised monthly rainfall for Sydney, with medians shown.....	217
Figure 9.7 Posterior distribution for the sum of transition probabilities from the calibration of a two-state lognormal ARHMM to the deseasonalised monthly rainfall for Sydney, with median.....	218
Figure 9.8 Framework for the hierarchical HMM .....	219
Figure 10.1 Gaussian probability plot showing estimates for state conditional distributions from the calibration of a two-state NP HMM to the deseasonalised monthly Murray flows .....	231
Figure 10.2 Lognormal probability plot showing scaled deseasonalised monthly Murray flows .....	233
Figure 10.3 Correlogram for natural logarithms of deseasonalised monthly Murray flows.....	234

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---

Figure 10.4 Partial correlogram for natural logarithms of deseasonalised monthly Murray flows .....	234
Figure 10.5 Correlogram for residuals after fitting an AR(3) model to the natural logarithms of deseasonalised monthly Murray flows .....	234
Figure 11.1 Partial correlogram for the time series of natural logarithms of deseasonalised monthly rainfall for District 71.....	239
Figure 11.2 Gaussian probability plot showing residuals from fitting an AR(2) model to the natural logarithms of the deseasonalised monthly rainfall for District 71.....	239
Figure 11.3 Gaussian probability plots showing deseasonalised monthly rainfall for District 71, together with annual and five-year aggregates of this series, alongside median values and intervals that contain 90% of values from 1000 simulations using a two-state lognormal HSMM (left column) and an AR(2) model with lognormal residuals (on right).....	241
Figure 11.4 Histogram of scaled deseasonalised monthly Burdekin flows.....	243
Figure 11.5 Lognormal probability plot showing scaled deseasonalised monthly Burdekin flows .....	243
Figure 11.6 Gaussian probability plots showing deseasonalised monthly flows for the Burdekin River, together with annual and five-year aggregates of this series, alongside median values and intervals that contains 90% of values from 1000 simulations using a three-state NP HMM (left column) and an AR(3) model with lognormal residuals (on right).....	245
Figure 11.7 Locations of the Warragamba Reservoir catchment and adjacent smaller catchments, together with rain gauges that provide long-term rainfall data (after Thyer and Kuczera, 1999b) .....	247
Figure 11.8 Lognormal probability plot showing scaled deseasonalised monthly values for the Warragamba composite rainfall series .....	248
Figure 11.9 Gaussian probability plots showing deseasonalised monthly rainfall for the Warragamba composite rainfall series, together with annual and five-year aggregates of this series, alongside median values and intervals that contains 90% of values from 1000 simulations using a three-state lognormal HMM (left column) and an AR(3) model with lognormal residuals (on right) .....	250
Figure 11.10 Relationship between the annual composite rainfall data for Warragamba and annual runoff (blue points) together with a piecewise linear regression over the period (1961-1993).....	252

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## List of tables

Table 2.1 Comparison of the strength of the most recent 10 El Niño periods and their impacts upon the hydrology of Australia (source: Bureau of Meteorology).....	25
Table 4.1 Sample statistics for selected district-averaged monthly rainfall series.....	73
Table 4.2 Anderson-Darling statistics for various probability distributions fitted to deseasonalised monthly rainfall.....	74
Table 4.3 Summary statistics for the monthly flow records of the seven rivers analysed.....	77
Table 4.4 Summary statistics for the seven flow records from January to June.....	78
Table 4.5 Summary statistics for the seven flow records from July to December.....	79
Table 4.6 Results for 2-sample t-tests and linear correlations between deseasonalised monthly rainfall and monthly NINO3, with associated probabilities, for four district-averaged rainfall series.....	80
Table 4.7 Results for 2-sample t-tests and linear correlations between deseasonalised monthly rainfall and monthly IPO values, with significance probabilities.....	83
Table 4.8 Partial correlations between deseasonalised monthly rainfall and monthly NINO3 when allowing for DMI and deseasonalised monthly rainfall and monthly DMI values when allowing for NINO3.....	85
Table 4.9 Linear correlations (and significance probabilities) between deseasonalised monthly rainfall and monthly PC <sub>1</sub> and NINO3 values.....	87
Table 4.10 Coefficients and standard errors for regression of deseasonalised monthly streamflows on climate indices, with R <sup>2</sup> values, standard deviation of residuals and significance.....	89
Table 4.11 Monthly climate indicator categories based on NINO3 and IPO indices.....	90
Table 4.12 Mean difference in deseasonalised monthly streamflows between opposite ENSO phases, with 90% interval for the difference and associated significance level.....	91
Table 4.13 Summary flow statistics for the Todd River.....	92
Table 4.14 Coefficients and standard errors (SE) for regression of total annual Todd flows on climate indices, with R <sup>2</sup> values, standard deviation of residuals (S) and overall P-values.....	93
Table 4.15 Comparison of optimum Poisson regression models for number of annual spates in the Todd using six-month averages of MEI and IPO values as indicative of annual totals, and category indicator variables (shown in bold) as predictors.....	94
Table 4.16 Summary flow statistics for the Omatako and Omaruru Rivers.....	96
Table 4.17 Coefficients and standard errors (SE) for regression of annual flows in the Omatako and Omaruru on climate indices, with R <sup>2</sup> values, standard deviation of residuals (S) and P-values.....	97
Table 4.18 Coefficients and standard errors (SE) for optimum regression model of bivariate flow records, with bivariate normal distribution fitted to residuals for Omatako and Omaruru rivers.....	97
Table 4.19 Comparison of optimum Poisson regression models for number of annual spates in the Omatako and Omaruru Rivers using six-month averages of MEI and IPO values as indicative of annual totals, and category indicator variables (shown in bold) as predictors.....	99

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Table 4.20 Coefficients and standard errors (SE) for optimum bivariate Poisson models, using climate indices as predictors, and Bayes Factors from comparing to fixed parameter models .....	100
Table 4.21 Coefficients and standard errors (SE) for optimum bivariate Poisson models, using climate categories as predictors, and Bayes Factors from comparing to fixed parameter models.....	100
Table 5.1 Persistence statistics for four district-averaged monthly rainfall series and NINO3.	104
Table 5.2 Rank correlations between the four runs statistics for deseasonalised monthly rainfall of the 107 districts across Australia.....	106
Table 5.3 Rank correlations between the four runs statistics and the linear correlations between deseasonalised monthly rainfall in the 107 districts across Australia and monthly NINO3 values.....	107
Table 5.4 Persistence statistics for deseasonalised monthly flows for the seven rivers .....	110
Table 5.5 Rank correlations between estimates of the Hurst exponents, runs statistics and linear correlations between deseasonalised monthly rainfall and monthly NINO3 values for the 107 districts across Australia.....	113
Table 5.6 Hurst exponents for deseasonalised monthly flows for the seven rivers analysed....	113
Table 6.1 Statistics for the eight annual rainfall series.....	116
Table 6.2 Anderson-Darling goodness-of-fit statistics from calibrating Gaussian distributions to the various annual rainfall records, with values significant at a 5% level shown in bold .	118
Table 6.3 Convergence diagnostics from using the variance ratio method to analyse 6,000 samples from 10 Adaptive Metropolis chains to obtain estimates of posterior distributions for a two-state Gaussian HMM calibrated to annual Sydney rainfall.....	119
Table 6.4 Convergence diagnostics from using the variance ratio method to analyse 10,000 samples from 6 Adaptive Metropolis chains to obtain estimates of posterior distributions for a two-state Gaussian HMM calibrated to annual Sydney rainfall.....	119
Table 6.5 Medians of posterior distributions for HMM transition probabilities, with 90% credibility intervals from the calibration of two-state Gaussian HMMs .....	120
Table 6.6 Medians of posterior distributions for parameters of HMM conditional distributions, with 90% credibility intervals.....	121
Table 6.7 Parameter estimates for simulated series of lengths 142 and 1000, with posterior median and 90% credibility intervals shown.....	126
Table 6.8 Posterior medians with 90% credibility intervals for the sums of transition probabilities from the calibration of two-state Gaussian HMMs to the various annual rainfall series .....	130
Table 7.1 Statistics for the eight series of monthly rainfall totals .....	133
Table 7.2 Anderson-Darling goodness-of-fit statistics from the calibration of various distributions to the scaled positive anomalies of each monthly rainfall record.....	135
Table 7.3 Medians of posterior distributions for HMM transition probabilities, with 90% credibility intervals from the calibration of two-state lognormal HMMs .....	137
Table 7.4 Posterior medians with 90% credibility intervals for the sums of transition probabilities from the calibration of two-state lognormal HMMs to the deseasonalised monthly rainfall series .....	141
Table 7.5 Medians of posterior distributions for HMM transition probabilities and their sum, with 90% credibility intervals from the calibration of two-state gamma HMMs.....	143

---

Table 7.6 Medians of posterior distributions for parameters of gamma distributions, with 90% credibility intervals, from the calibration of a two-state gamma HMM to the deseasonalised monthly rainfall for Sydney .....	144
Table 7.7 Estimated means (with standard deviations) for state conditional distributions from the calibrations of HMMs to the deseasonalised monthly rainfall for Sydney .....	144
Table 7.8 Model selection results from the calibrations of two-state lognormal (LN) and two-state gamma HMMs to deseasonalised monthly rainfall series .....	146
Table 7.9 Numbers of months in which most probable HMM states from the calibration of a two-state lognormal HMM to the deseasonalised monthly rainfall for Sydney coincide with different ENSO phases.....	147
Table 7.10 Numbers of months in which most probable HMM states from the calibration of a three-state gamma HMM to the deseasonalised monthly rainfall for Sydney coincide with different ENSO phases.....	152
Table 7.11 Medians of posterior distributions for the sums of self-transition probabilities, with 90% credibility intervals, from the calibrations of three-state lognormal HMMs and three-state gamma HMMs to deseasonalised monthly rainfall series .....	154
Table 7.12 Bayes factors from comparing calibrations to deseasonalised monthly rainfall series of two-state lognormal HMMs, to three-state lognormal HMMs and gamma HMMs .....	155
Table 7.13 Comparison of posterior distributions showing median and 90% credibility intervals, from the calibration of two-state lognormal HMMs to deseasonalised monthly rainfall in Sydney and District 66.....	156
Table 7.14 Comparison of posterior distributions from the calibration of a two-state Gaussian HMM to a time series simulated from an identical model.....	158
Table 7.15 Median and 90% credibility intervals for parameters of a two-state lognormal HMM calibrated to the simulated time series of length 1080.....	159
Table 8.1 Posterior medians and 90% credibility intervals for parameters of a two-state Gaussian HMM fitted to the series simulated from a two-state Gaussian HMM .....	171
Table 8.2 Posterior medians and 90% credibility intervals for transition probabilities of a three-state Gaussian HMM calibrated to the series simulated from a three-state Gaussian HMM .....	175
Table 8.3 Posterior medians and 90% credibility intervals for other parameters for a three-state Gaussian HMM calibrated to the series simulated from a three-state Gaussian HMM.....	175
Table 8.4 Posterior medians and 90% credibility intervals for transition probabilities of a three-state NP HMM calibrated to the series simulated from a three-state Gaussian HMM .....	175
Table 8.5 Posterior medians and 90% credibility intervals for transition probabilities from calibrating two-state Gaussian HMMs and NP HMMs to annual rainfall series .....	178
Table 8.6 Convergence diagnostics from using the variance ratio method to analyse 6,000 samples from 10 Adaptive Metropolis chains to obtain estimates of posterior distributions for a two-state NP HMM calibrated to deseasonalised monthly Sydney rainfall .....	179
Table 8.7 Comparison of posterior distributions for transition probabilities, showing medians and 90% credibility intervals, from the calibration of a two-state lognormal HMM and two-state NP HMM to the deseasonalised monthly rainfall of Sydney .....	180
Table 8.8 Comparison of posterior distributions for transition probabilities, showing medians and 90% credibility intervals, from the calibration of a two-state lognormal HMM and two-state NP HMM to the deseasonalised monthly rainfall for District 66.....	182

---

Table 8.9 Numbers of months in which most probable HMM states from the calibration of a two-state NP HMM to the deseasonalised monthly rainfall for Sydney coincide with ENSO phases .....	185
Table 8.10 Numbers of months in which most probable HMM states from the calibration of a two-state NP HMM to the deseasonalised monthly rainfall for District 66 coincide with ENSO phases.....	185
Table 8.11 Medians of posteriors for HMM transition probabilities and their sum, with 90% credibility intervals from the calibration of two-state NP HMMs to deseasonalised monthly rainfall.....	185
Table 8.12 Numbers of months in which most probable HMM states from the calibration of a three-state NP HMM to the deseasonalised monthly rainfall for Sydney coincide with ENSO phases.....	188
Table 8.13 Numbers of months in which most probable HMM states from the calibration of a three-state NP HMM to the deseasonalised monthly rainfall for District 66 coincide with ENSO phases.....	188
Table 8.14 Medians of posterior distributions for the sums of self-transition probabilities, with 90% credibility intervals, from the calibrations of three-state NP HMMs to deseasonalised monthly rainfall series, with Bayes Factors comparing these calibrations to two-state NP HMMs.....	189
Table 8.15 Mean and median monthly rainfall for various meteorological districts in New South Wales, together with the posterior median and 90% credibility intervals for the sum of transition probabilities from calibrating two-state NP HMMs to deseasonalised monthly rainfall.....	190
Table 8.16 Linear correlations between the monthly rainfall in each district and the monthly rainfall in District 66, and between the median state series for each district and the median state series in District 66 from calibrating two-state NP HMMs to the deseasonalised monthly rainfall .....	191
Table 8.17 Rank correlations (with p-values in brackets) between various runs statistics and posterior medians for the sums of transition probabilities from the calibration of two-state NP HMMs to deseasonalised monthly rainfall in each district .....	194
Table 8.18 Comparison of posterior distributions for transition probabilities from the calibration of two-state NP HMMs to time series of deseasonalised monthly streamflows, with posterior medians and 90% credibility intervals shown.....	197
Table 8.19 Posterior distributions for the sums of self-transition probabilities from the calibration of three-state NP HMMs to time series of deseasonalised monthly streamflows, with medians and 90% credibility intervals shown, and Bayes Factors comparing the calibrations of two-state NP HMMs to the calibrations of three-state NP HMMs.....	198
Table 8.20 Comparison of posterior distributions for transition probabilities, showing medians and 90% credibility intervals, from the calibration of two-state NP HMMs to the deseasonalised monthly rainfall of Sydney and the deseasonalised number of monthly rain-days.....	201
Table 8.21 Sample statistics for maximum rainfall intensities for Sydney over selected durations .....	202
Table 8.22 Linear correlations between time series of maximum rainfall intensities for Sydney over selected durations and total monthly rainfall for the period (1921-2000).....	203
Table 8.23 Linear correlations between median state series obtained from calibrating two-state NP HMMs to the time series of deseasonalised maximum monthly rainfall intensities for	

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Sydney over selected durations and deseasonalised monthly rainfall totals for the period (1921-2000).....	205
Table 8.24 Posterior distributions for transition probabilities, showing medians and 90% credibility intervals, from the calibration of two-state NP HMMs to the time series of deseasonalised maximum monthly rainfall intensities for Sydney over selected durations and deseasonalised monthly rainfall totals for the period (1921-2000).....	206
Table 8.25 Linear correlations between series of most likely states from calibrating three-state NP HMMs to the time series of deseasonalised maximum monthly rainfall intensities for Sydney over selected durations and deseasonalised monthly rainfall totals for the period (1921-2000).....	206
Table 8.26 Posterior medians and 90% credibility intervals for the sums of self-transition probabilities from the calibrations of three-state NP HMMs, and Bayes Factors comparing the calibrations of two-state NP HMMs to the calibrations of three-state NP HMMs .....	207
Table 9.1 Comparison of posteriors for parameters of conditional distributions for two-state lognormal HSMM and two-state lognormal HMM, with median and 90% credibility interval .....	213
Table 9.2 Comparison of posteriors for parameters of conditional distributions for two-state lognormal ARHMM and two-state lognormal HMM, with median and 90% credibility intervals.....	217
Table 9.3 Summary of posterior distributions for parameters of wet and dry years from a two-state lognormal HHMM, showing medians and 90% credibility intervals .....	221
Table 10.1 Posterior medians and 90% credibility intervals for transition probabilities from calibrating two-state NP HMMs to deseasonalised monthly rainfall.....	224
Table 10.2 Anderson-Darling goodness-of-fit statistics for estimates of state conditional distributions from the calibration of two-state NP HMMs to deseasonalised monthly rainfall .....	224
Table 10.3 Posterior medians and 90% credibility intervals for transition probabilities from the calibration of two-state lognormal HMMs to deseasonalised monthly rainfall.....	225
Table 10.4 Linear correlations between median state series from calibrating various HMMs to deseasonalised monthly rainfall .....	225
Table 10.5 Numbers of months in which most probable HMM states from the calibrations of two-state NP, lognormal and gamma HMMs to the deseasonalised monthly rainfall for District 27 coincide with ENSO phases .....	226
Table 10.6 Posterior medians and 90% credibility intervals for transition probabilities from the calibration of three-state NP HMMs to the deseasonalised monthly rainfall.....	227
Table 10.7 Posterior medians and 90% credibility intervals for the sums of self-transition probabilities from the calibration of three-state NP HMMs, and Bayes Factors comparing the calibrations of two-state NP HMMs to the calibrations of three-state NP HMMs .....	227
Table 10.8 Anderson-Darling goodness-of-fit statistics for estimates of state conditional distributions from the calibration of three-state NP HMMs to the deseasonalised monthly rainfall .....	228
Table 10.9 Posterior medians and 90% credibility intervals for transition probabilities from the calibration of three-state lognormal HMMs to the deseasonalised monthly rainfall .....	228
Table 10.10 Posterior medians and 90% credibility intervals for the sum of self-transition probabilities from the calibration of various three-state HMMs.....	229



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Table 10.11 Bayes Factors ( $\ln BF_{M, LN}$ ) comparing the calibrations of various parametric models ( $M$ ) to the calibrations of two-state lognormal HMMs to the deseasonalised monthly rainfall.....	229
Table 10.12 Anderson-Darling goodness-of-fit statistics for estimates of state conditional distributions from the calibration of a two-state NP HMM to deseasonalised monthly Murray flows .....	231
Table 10.13 Numbers of months in which most probable HMM states from the calibration of two-state NP HMMs to the deseasonalised monthly Murray flows coincide with ENSO phases .....	232
Table 10.14 Anderson-Darling goodness-of-fit statistics for estimates of state conditional distributions from calibrating three-state NP HMMs to deseasonalised monthly Murray flows .....	232
Table 10.15 Numbers of months in which most probable HMM states from the calibrations of three-state NP, lognormal and gamma HMMs to the deseasonalised monthly Murray flows coincide with ENSO phases .....	233
Table 10.16 Model selection results from calibrations to the deseasonalised monthly Murray flows .....	236
Table 11.1 Statistics for the time series of deseasonalised monthly totals from District 71 over various aggregations calculated with one-month moving windows, along with the median and 90% confidence interval of statistics from 1000 simulations using a two-state lognormal HSMM and an AR(2) model with lognormal residuals .....	240
Table 11.2 Run statistics for the time series of deseasonalised monthly totals for District 71, together with the results of multiple simulations of this series using a two-state lognormal HSMM and an AR(2) model with lognormal residuals, showing medians and 90% confidence intervals .....	242
Table 11.3 Statistics for the time series of deseasonalised monthly flows from the Burdekin River over various aggregations calculated with one-month moving windows, along with the median and 90% confidence intervals of statistics from 1000 simulations using a three-state NP HMM and an AR(3) model with lognormal residuals .....	244
Table 11.4 Run statistics for the time series of deseasonalised monthly flows from the Burdekin River, together with the results of multiple simulations of this series using a two-state lognormal HSMM and an AR(3) model with lognormal residuals, showing medians and 90% confidence intervals.....	246
Table 11.5 Statistics for monthly rainfall from the four rain gauges in the Warragamba catchment, together with ratios of catchment area represented by each rain gauge, and the catchment composite rainfall series.....	248
Table 11.6 Statistics for the time series of deseasonalised monthly totals from the Warragamba composite series over various aggregations calculated with one-month moving windows, along with the median and 90% confidence intervals of statistics from 1000 simulations using a three-state lognormal HMM and an AR(3) model with lognormal residuals .....	249
Table 11.7 Run statistics for the time series of deseasonalised monthly totals from the Warragamba composite series, together with the results of multiple simulations of this series using a three-state lognormal HMM and an AR(3) model with lognormal residuals, showing medians and 90% confidence intervals.....	251
Table 11.8 Restriction factors for monthly demand volumes from Warragamba reservoir corresponding to simulation storage volumes .....	253
Table 11.9 Average water balance statistics from 1000 simulations of monthly composite rainfall series for Warragamba .....	253

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# Abstract

Hydrological observations are characterised by wet and dry cycles, a characteristic that is termed hydrological persistence. Interactions between global climate phenomena and the hydrological cycle result in rainfall and streamflow data clustering into wetter and drier states. These states have implications for the management and planning of water resources. Statistical tests constructed from the theory of wet and dry spells indicate that evidence for persistence in monthly observations is more compelling than at an annual scale. This thesis demonstrates that examination of monthly data yields spatially-consistent patterns of persistence across a range of hydrological variables. It is imperative that time series models for rainfall and streamflow replicate the observed fluctuations between the climate regimes. Monthly time series are generally represented with linear models such as ARMA variants; however simulations from such models may underestimate the magnitude and frequency of persistence. A different approach to modelling these data is to incorporate shifting levels in the broader climate with a tendency to persist within these regimes. Hidden Markov models (HMMs) provide a strong conceptual basis for describing hydrological persistence, and are shown to provide accurate descriptions of fluctuating climate states. These models are calibrated here with a full Bayesian approach to quantify parameter uncertainty.

A range of novel variations to standard HMMs are introduced, in particular Autoregressive HMMs and hidden semi-Markov models which have rarely been used to model monthly rainfall totals. The former model combines temporal persistence within observations with fluctuations between persistent climate states, and is particularly appropriate for modelling streamflow time series. The latter model extends the modelling capability of HMMs by fitting explicit probability distributions for state durations. These models have received little attention for modelling persistence at monthly scale. A non-parametric (NP) HMM, which overcomes the major shortcomings of standard parametric HMMs, is also described. Through removing the requirement to assume parametric forms of conditional distributions prior to model calibration, the innovative NP HMM framework provides an improved estimation of persistence in discrete and continuous data that remains unaffected by incorrect parametric assumptions about the state distributions. Spatially-consistent persistence is identified across Australia with the NP HMM, showing a tendency toward stronger persistence in low-rainfall regions. Coherent signatures of persistence are also identified across time series of total monthly rainfall, numbers of rain-days each month, and the intensities of the most extreme rain events recorded each month over various short durations, illustrating that persistent climate states modulate both the numbers of rain events and the amount of moisture contained within these events. These results provide a new interpretation of the climatic interactions that underlie hydrological persistence.

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The value of HMMs to water resource management is illustrated with the accurate simulation of a range of hydrologic data, which in each case preserves statistics and spell properties over a range of aggregations. Catchment-scale rainfall for the Warragamba Reservoir is simulated accurately with HMMs, and rainfall-runoff transformations from these simulations provide reservoir inflows of lower drought risk than provided from ARMA models.

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## Statement of Originality

*I hereby certify that this work has not been submitted for the award of any other degree or diploma in any university of other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give my consent to this thesis, when deposited in the University Library, being available for photocopying and loan*

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Julian Peter Whiting

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Date

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# List of Symbols

## Climate terminology

ACW	Antarctic Circumpolar Wave
DMI	(Indian Ocean) Dipole Mode Index
ENSO	El-Niño Southern Oscillation
IOD	Indian Ocean Dipole
IPO	Interdecadal Pacific Oscillation
NAO	North Atlantic Oscillation
PDO	Pacific Decadal Oscillation
PC <sub>1</sub>	First Principal Component calculated from NINO3, IPO and DMI
SLP	Sea-level atmospheric pressures
SST	Sea-surface temperature
SO	Southern Oscillation
SOI	Southern Oscillation Index

## Time series notation

$y_t$	Observation at time $t$
$\{y_t\}$	Time series of observations
$Y_t$	Series of observations up to time $t$ , $\{y_1, y_2, \dots, y_t\}$
$\{u_t\}$	Time series of observations transformed into (0,1) interval
$\{x_t\}$	Time series of HMM states
$\{s_1, s_2, \dots, s_k\}$	Series of HMM states
$s_{i,t}$	HMM state series being in state $i$ at time $t$
$\{z_t\}$	Time series of white noise/ random variation
$T$	Time series length
$\phi_p$	Serial correlation coefficient at lag $p$ for a $AR(p)$ model
$\varphi_q$	Moving-average coefficient at lag $q$ for a $MA(q)$ model
5-mrm	Five-month running-means
$H$	Hurst exponent

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## Probability notation

$BF_{i,j}$	Bayes factor comparing model $i$ to model $j$
$N(\mu_j, \sigma_j^2)$	Gaussian distribution with mean $\mu_j$ and standard deviation $\sigma_j^2$ , with $j$ corresponding to model state where applicable
$\text{Gamma}(\alpha, \beta)$	Gamma distribution with shape $\alpha$ and scale $\beta$
$H$	Hurst exponent
$P_{ij}$	HMM transition probability from state $i$ to state $j$
$\Sigma TP$	Sum of HMM transition probabilities
$P(a b)$	Conditional probability of $a$ given $b$
$P(a,b)$	Joint probability of $a$ and $b$
$\theta$	Conditional vector of model parameters
$W$	HMM wet state
$D$	HMM dry state

## Stochastic modelling abbreviations

AD	Anderson-Darling goodness-of-fit statistic (Stephens, 1974)
AIC	Akaike's Information Criterion (Akaike, 1974)
AM	Adaptive Metropolis algorithm (Haario <i>et al.</i> , 2001)
LORT	Gold's length-of-runs test (Gold, 1929)
MCMC	Monte Carlo Markov Chain method
SCE	Shuffled Complex Evolution algorithm
SL	Shifting Level model