Aspects of Quantum Game Theory

by

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Contents

Abstract

Quantum game theory is an exciting new topic that combines the physical behaviour of information in quantum mechanical systems with game theory, the mathematical description of conflict and competition situations, to shed new light on the fields of quantum control and quantum information. This thesis presents quantizations of some classic game-theoretic problems, new results in existing quantization schemes for two player, two strategy non-zero sum games, and in quantum versions of Parrondo's games, where the combination of two losing games can result in a winning game. In addition, quantum cellular automata and quantum walks are discussed, with a history-dependent quantum walk being presented.

Statement of Originality

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of the thesis, when deposited in the University Library, being available for loan, photocopying, and dissemination through the digital thesis collection.

4th January, 2005

Signed

Date

Acknowledgments

A work of this magnitude could not possibly have been undertaken without scorning the advice of too many people to mention here. George W. Bush and Rupert Murdoch are just two of the people whose advice was not even sort, nor did they provide any funding. However, generous funding was provided by the GTECH Corporation with help from the SA Lotteries Commission. Travel funding was provided by The University of Adelaide postgraduate travel award, the D. R. Stranks postgraduate travel scholarship and the Department of Electrical and Electronic Engineering at The University of Adelaide. The support, direction and encouragement of my supervisor, A/Prof. Derek Abbott is gratefully acknowledged. I would also like to thank various colleagues and collaborators for useful discussions and help with various parts of this work: Prof. Jens Eisert of Potsdam University, Prof. Neil F. Johnson of Oxford University, Wanli Li of Princeton University, Prof. David Meyer of the University of California San Diego (UCSD), Joseph Ng of the University of Queensland, Dr. Arun Pati of the University of Bangor, and a number of others with whom I spoke at conferences and during interstate or overseas visits. I would like to thank my fellow students at the Centre for Biomedical Engineering and those I spent time with at the Physics Departments at Melbourne, Oxford and Potsdam Universities. Finally, I would like to thank my family and friends, without whom life would be impossible.

—Adrian Flitney

"Returning home I read a book on Physics. I don't understand it very well . . . Why isn't nature clearer and more directly comprehensible?"

—Shin'ichirō Tomonaga, Nobel prize winner in Physics, 1965

Thesis Conventions

Typesetting. This thesis is typeset using LaTeX2e software. Plots were generated by *Mathematica* 4.1. CorelDRAW 7.467 was used to generate some of the schematic diagrams, while the remainder were generated with standard LaTeX picture commands.

Spelling. Australian English spelling has been adopted throughout, as defined by the Macquarie English Dictionary (A. Delbridge (ed.) Macquarie Library, North Ryde, NSW, Australia, 2001). Where more than one spelling variant is permitted such as biassing or biasing and infra-red or infrared the option with the fewest characters has been chosen.

Mathematics. The International Standards Organization has established the recognized conventions for typesetting mathematics. The most important points are given below.

1. Equations are treated as part of the text and include the appropriate punctuation.
2. Simple variables are represented by italic letters, e.g., x, y or z.
3. Vectors are written in bold face italic, e.g., $\boldsymbol{B}$ or $\boldsymbol{\pi}$.
4. Superscripts or subscripts that are descriptions and not variables are in upright font, e.g., k_{A} where A stands for Alice as opposed to k_i where $i = 1, \ldots, n$.

Referencing. The Harvard style is used for referencing and citation.

Publications

Flitney-A. P and Abbott-D (2005). Quantum games with decoherence, *J. Phys. A*, **38**, 449–59.

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Corrigenda

Typographic errors in names: "Laundauer" should be "Landauer" (page 2), "Morgenstein" should be "Morgenstern" (pages 2 and 177), "Wohl" should be "Wolf" (pages 2 and 170), and "Wilkins" should be "Wilkens" (pages 3, 17, 50, 55 and 170).

Page 15: Replace the sentence above Eq. (2.7) with:
"A particular choice of maximally entangling operator $\hat{J}$, for a general N player two strategy game, may be written (Benjamin and Hayden 2001b) as"

Page 37: In the paragraph below Eq. (4.7) replace the phrase "... invert a general $|\psi\rangle$" with "... perform a universal not on a general $|\psi\rangle$" and drop the word "unitarily" from the end of the following sentence (that begins with "A general complementing operation ...").

Page 40: In the caption to Figure 4.6 the last sentence should end with " ... with the subscript A referring to Alice and B to Bob."

Page 66: In the first paragraph, change the sentence beginning "These techniques work by encoding ..." to "The former technique works by encoding ..."

Page 67: Equation (6.4) is more clearly written as

$$\rho \rightarrow \sum_{j_1,\ldots,j_N=0}^{2} \mathcal{E}_{j_1} \otimes \ldots \otimes \mathcal{E}_{j_N}\, \rho\, (\mathcal{E}_{j_1} \otimes \ldots \otimes \mathcal{E}_{j_N})^\dagger,$$

Page 69: In the fourth line of Eq. (6.6), $\hat{M}_k$ should be $\hat{U}_k$ in both instances.

Page 138: The command `Unprotect[Play]` should be inserted before `Play[...]`.

Page 142: In the code for `ResultsRandom` the `NextStep[c, p]` command should be `NextStep[c, p, p]`.

Page 144: In the usage commands, `inithist` has been omitted from the descriptions of `Results`, `Results2` and `ResultsRandom`. For these commands the usage should read `Results::usage = "Results[cap, inithist, p, ... ]"` etc.

Page 150: In the second line of the code for `Results[ cap_List, p_List, n_Integer]` the braces inside the `Table` command have been omitted. This line should read
`Module[ { results = Table[0, {i,n}], nc = cap, j },`

Page 157: The command `PlotPeakPosn` is redundant and should be deleted. It relies on another command that has been removed from the package.

Page 159: Braces are missing in the `MakeEmpty` and `SetQubit` commands. These command should read:

```
MakeEmpty[n_Integer] := Table[ 0. I + 0., {i,1,2^n} ]
SetQubit[ Q_List, n_Integer, val_ ] :=
   Module[ {newQ = Q}, ... ]
```

Page 160: In the code for `ApplyRule[ Q_List, t_, a_, b_, p_, sh_, m_Integer ]`, the variable `newqca` should be `newQ`.

Sections A.2–4: The code works best if the `BeginPackage[ ... ], Begin["Private`"]` and the corresponding `End[]` and `EndPackage[]` commands are omitted.

Known bugs in the code: (a) In Section A.2.2, if `n` is not a multiple of `(na + nb)` then executing `Results[cap, hist, p, p1, p2, p3, p4, na, nb, n]` will produce a list of results with trailing zeros. That is, only `Floor[n/(na + nb)] * (na + nb)` actual data points are produced. (b) In Section A.4.1, the configuration must have an even number of qubits. That is, `n` should be even when setting up the configuration with `MakeEmpty[n]`.