SKULL OF THE AUSTRALIAN ABORIGINAL

A Multivariate Analysis of Craniofacial Associations

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PREFACE

A growth study of Central Australian Aboriginals who live under settlement conditions at Yuendumu, 185 miles north-west of Alice Springs, was begun in 1951 by the Department of Dental Science, University of Adelaide. During the first stage of the investigation the oral health status of the Yuendumu subjects was determined, serial dental casts were obtained and attention was directed towards the analysis of tooth morphology, occlusal relations and the patterns of mastication (CAMPBELL and BARRETT, '53).

In 1961 the scope of the study was extended and emphasis was placed on dental development and its relation to the patterns of craniofacial and general skeletal growth. On each annual visit to the settlement the subjects enrolled were examined and a wide range of records obtained. The material now available for analysis comprises dental casts, standardised roentgenograms of the head, roentgenograms of the hands, observations of selected body measurements, genealogies, and photographic records. The objectives of the dental study, the methodology developed and the progress to date were outlined by BARRETT, BROWN and FANNING ('65).

During the course of this long-term study, which is continuing, it became evident that useful information would accrue from a collateral investigation of skulls selected from the collection of Australian

Aboriginal skeletal material housed in the South Australian Museum. Although the craniology of the Australian is well documented, limited use has been made of roentgenographic techniques of measurement or multivariate methods of data analysis. The museum study was undertaken to clarify the patterns of craniofacial associations within this ethnic group by the application of multivariate procedures.

Apart from the main objective, considerable attention has been given to the analytic methods used. High-speed digital computers have provided the research worker with means to apply penetrating analytic techniques that would otherwise be impractical because of arithmetic labour. However, the application of multivariate analysis in craniometric research has not kept pace with mathematical and technological developments in computing science and, with few exceptions, little attempt has been made to appraise the usefulness of this class of analysis in treating anthropometric data.

The skull collection in Adelaide was examined and 100 specimens were selected for study; subsequently measurements were obtained directly from the skulls or indirectly from standardised roentgenograms. Computer programs were developed to handle the special techniques required and attention was given to some of the difficulties that accompany the use of factor analysis, the multivariate procedure chosen as most appropriate for the study.

This report is concerned with the findings of the Aboriginal skull study. Although the conclusions refer to this particular sample, the sections dealing with the analytic methods have a broader application. In the first section, a brief survey of previous craniological studies of the Australian Aboriginal is given, the material is described and the methods and use of factor analysis in craniometry are outlined. The remainder of the report presents the findings derived from standard statistical procedures and from the special multivariate techniques.

Numerical analyses were carried out partly at the University of Adelaide, and partly at the Royal Dental College, Copenhagen, Denmark, during a period of study leave.

ACKNOWLEDGMENTS

Apart from a short period spent in Denmark where some of the final analyses were made, the present investigation was carried out at the University of Adelaide, South Australia. Professor A.M. Horsnell, Department of Dental Science, and Professor A.A. Abbie, Department of Anatomy and Histology made the facilities of their departments available whenever required. This assistance is acknowledged with deep appreciation.

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THESIS SUBMITTED FOR THE DEGREE OF

DOCTOR OF DENTAL SCIENCE

I hereby certify that the text of this thesis is entirely my own composition, that the findings reported (except where due reference is made) are the result of my own personal investigations, and that no part of the work has been previously submitted for a degree in this or any other University.

Tasman Brown,

University of Adelaide

October, 1967

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CRANIOLOGY OF THE AUSTRALIAN ABORIGINAL

The morphological characters of the Australian Aboriginal skull are well known and have been reported in an extensive literature that spans a period of more than a century. While the majority of reports have stemmed from the examination of museum material, many have been concerned with the metric and non-metric characters of head-form in living subjects. However, little attention has been given to the analysis of variations in cranial form and furthermore the manner in which genetic, environmental and functional determinants of cranio-facial morphology interact is still largely conjectural. These topics are important in the general understanding of human growth patterns but they require the application of analytic procedures that are more searching than those used previously for the study of this ethnic group. In this regard several authors have stressed the need for new strategies in physical anthropology including the application of multivariate methods of data analysis and the collection of information suitable for genetic study (WASHBURN, '51, '62; GARN, '62; SCHULL, '62; BENNETT, '63).

The following brief review of studies concerned with Australian craniology is intended as a frame of reference for the present investigation. Reports cited were selected from many available in

order to indicate the gradual transition in objectives and methodology that has taken place over the years. Whereas most earlier studies were descriptive in nature, emphasis is now placed on biological problems of more fundamental interest. Comprehensive reviews are available in standard anthropological texts and therefore no attempt has been made to summarise the present state of knowledge in detail.

Studies of museum material

By the end of the 19th century the metric and non-metric characters of the Australian skull had been described by a number of European anthropologists. In the main, these reports were based on examinations of relatively few specimens from museums or private collections. TURNER (1884) referred to existing literature and described 35 skulls including three which had been collected during the Challenger expedition. In Turner's view, a description of two Australian skulls presented to Blumenbach in 1795 by Sir Joseph Banks had initiated interest in this ethnic group. Turner gave particular attention to the use of cranial indices in the description of skull shape illustrating his account with data derived from earlier writers. In view of the modern acceptance of roentgenographic cephalometry, it is interesting that Turner noted the value of sagittal sections in craniometry and provided measurements obtained from rubbings of three such sections.

Skulls in the Cambridge University Museum were described and individual measurements of the specimens tabulated by DUCKWORTH ('04). Although only 38 skulls were available for study, a valuable feature of Duckworth's report was a summary of previous findings derived by Davis, Flower, Quatrefages and Hamy, and Turner. By pooling the earlier observations, Duckworth provided mean values of cranial capacity and craniofacial indices for a sample of 214 skulls.

KLAATSCH ('08) reported the metric characters of 87 skulls from the Roth collection, Queensland, and provided detailed descriptions and contour drawings in three planes for 11 of these. This work was followed by the publication of dioptrographic tracings prepared in three normae from 90 Australian crania, by BERRY and ROBERTSON ('14).

To determine reliable statistical constants for Australian and Tasmanian skulls, MORANT ('27) pooled data from previous studies in which skulls had been carefully sexed. Eighteen reports, dating from Pruner-Bey in 1865 to Schultz in 1918 were used to provide parameters from a sample of 300 Australian skulls. In addition, two studies of unsexed material by Krause in 1897 and Robertson in 1911 were reviewed. Morant gave considerable attention to the variability of cranial components and to correlation among cranial variables. On the evidence obtained from coefficients of racial likeness, Morant concluded that skulls from different regions of Australia were sufficiently alike to be classified together as

Australian Type A, with the exception of those from the Northern coast which he found different in cranial vault size and shape but similar in general facial morphology.

A comprehensive metric survey of nearly 1,000 skulls from museums in Adelaide, Melbourne, Sydney and London was made by HRDLIČKA ('28). The skulls were categorised according to State of origin and sex, and the tabulations included individual observations and mean values for most standard craniometric variables and indices. Although Hrdlička found regional variations within his sample, he was not prepared to subdivide the skulls into specific morphological groups.

During this active period, a further contribution to Australian craniology was made by JONES ('29) who presented average type contours for 90 unsexed crania. Although little discussion was given, the contours were drawn in the three normae - lateralis, facialis and verticalis.

As part of a morphological study of Oceanic skulls, WAGNER ('37) analysed data obtained from 114 Australian specimens located in museums at Oslo and London. This series, Wagner pointed out, was quite distinct from that of Hrdlička. Besides providing a summary of the more important contributions to that time, an attempt was made to define areas of overlap between skull series described by previous workers. Wagner calculated parameters for most standard craniometric variables and used coefficients of correlation to analyse craniofacial associations. Although he supported Morant's specific

grouping of Northern Australian skulls, Wagner could not confirm that skulls from other regions were sufficiently alike to be placed in one common group and therefore proposed a comparatively even sequence of craniological types with the extremes from the Northern regions on the one hand and South Australia on the other.

A recent craniometric study of the Australian skull was conducted along traditional lines by MILICEROWA ('55) who examined 80 specimens from the Anthropology Department at Wrocław. Individual measurements and statistical parameters for 94 variables were listed together with photographs of each skull examined. Milicerowa explained that the Wrocław collection is part of a larger series of skulls presented by Klaatsch. However, although series numbers given by Klaatsch agreed with those in the Polish collection, sexing and individual measurements did not, so that the precise relation of Milicerowa's study to the earlier one by Klaatsch is uncertain.

The non-metric characters of the Australian skull were described by a number of earlier investigators but without benefit from the scheme of standardisation put forward by JONES ('31). Since then several studies have been made on large samples of Australian skulls. KROGMAN ('32) examined 113 male and 70 female specimens from the museum of the Royal College of Surgeons, London, and FENNER ('39) provided a definitive account of the non-metric characters by documenting observations made on over 1,000 adult skulls sexed and classified according to state of origin. More

recently LARNACH and FREEDMAN ('64) and LARNACH and MACINTOSH ('66) provided extensive non-metric observations on a group of 118 skulls from coastal New South Wales. The metric characters of the same skulls were described by FREEDMAN ('64). The last three studies originated from the Anatomy Department, University of Sydney, and are complemented by a review of published data and theories relating to the osteology and origins of the Tasmanian Aboriginal (MACINTOSH and BARKER, '65).

In addition to the accounts referred to above, several texts on physical anthropology list parameters for various craniofacial variables and outline the principal non-metric characters of the Australian skull. In particular, MARTIN-SALLER ('57), ASHLEY-MONTAGU ('60) and COON ('63) tabulate extensive data and provide guides to relevant literature.

With the exception of the works by MORANT ('27) and WAGNER ('37) previous metrical studies have been essentially descriptive in nature, little attention being given to the nature of intragroup variation. However, in more recent times, metric data have been obtained from Australian crania and studied for purposes other than description or ethnic group comparison.

ABBIE ('47) obtained measurements of 50 male and 50 female specimens for a study of human head form in relation to evolution, racial characters, heredity and environment. Through the use of coefficients of correlation, Abbie found little evidence of any

important associations between the form of the head and the size of the jaws. Since 1950 a number of reports have originated from the Department of Anatomy, University of Adelaide. ABBIE ('50) discussed the closure of cranial sutures in the Australian skull and HONE ('52) traced phylogenetic changes in the post-orbital structures. MURPHY, in a series of papers, has investigated the spheno-ethmoidal junction ('55), the pterion region ('56), the chin and mental foramen ('57a), and the post-natal changes in mandibular form ('57b). In addition, Murphy has described the patterns of tooth attrition and temporomandibular joint function in Australians ('65). Recent papers from this Department deal with the selection of reference points and lines for use in comparative craniometry (ABBIE, '63a, 63b).

Field studies

A review of the principal field studies of Australian Aboriginals was made by ABBIE ('63c) who listed over 90 references dating from the early observations of the English explorer Dampier in 1729. The following summary is restricted to investigations undertaken by the University of Adelaide. Those including observations of head form have been given precedence.

CAMPBELL and HACKETT ('27) obtained data, including many recordings of head dimensions from 57 Arunta tribesmen. Mean values were compared with those previously reported and particular attention was given to values for head indices. In two later papers (CAMPBELL, GRAY and HACKETT, '36a, 36b), findings from a study of 480 Aboriginals, grouped according to sex and dental age, were summarised and statistical parameters for most standard anthropometric variables were calculated. The authors could find no evidence to support the view that distinct physical types exist among Aboriginals from Central Australia.

Since 1945 Abbie has published many reports dealing with the physical anthropology of Australian Aboriginals using data derived during a series of field expeditions to several parts of the continent (ABBIE, '63c, '66). Craniofacial morphology was discussed in papers describing the metric and non-metric characters of the Wailbri tribe from Central Australia (ABBIE, '57; ABBIE and ADEY, '55). Preliminary attention has been given to the nature of physical changes in Aboriginals consequent upon contact with European environments (ABBIE, '60) and to the patterns of physical growth in three tribal groups (ABBIE, '61a). Included in the latter study were observations of growth changes in morphologic face height, bizygomatic diameter and cranial shape. The same author (ABBIE, '61b) listed selected mean values for anthropometric variables measured on Aboriginals living in the southern coastal regions, the central desert and northern Arnhem land. A remarkable degree of physical homogeneity was found in

subjects from quite different geographic and cultural environments.

Field work on the physical anthropology of Australian Aboriginals is continuing and to date Abbie and his colleagues have covered extensive areas of the continent to collect as many relevant data as possible before the traditional tribal structure deteriorates further. Apart from Abbie's survey, the dental study referred to in the Preface will provide information on patterns of craniofacial growth in Wailbri children from Central Australia.

Roentgenographic cephalometry

The technique by which measurements of the skull are obtained indirectly from standardised roentgenograms is well known and widely accepted in clinical orthodontics and facial growth research although applications in physical anthropology are limited in number. The method has been used to study craniofacial morphology in Australian Aboriginals.

CRAVEN ('58) used head roentgenograms obtained by HEATH ('47) to investigate a group of Aboriginals from the Haasts Bluff Settlement in Central Australia. Mean values were computed for selected angular variables and indices in males and females grouped according to age. This was the first application of roentgenographic cephalometry in Australian Aboriginal studies but it was limited in

scope by the mixed tribal origin of the subjects and by the number of films available for analysis.

Since 1961 roentgenographic cephalometry has been incorporated in a long-term growth study of Wailbri Aboriginals (BARRETT, BROWN and FANNING, '65). Facial prognathism in the Wailbri was investigated by BARRETT, BROWN and MACDONALD ('63) and attention was given to experimental errors involved in the use of roentgenograms to derive indirect skull measurements. The findings were compared with those of other ethnic groups to demonstrate that although prognathism of the alveolar regions of the face was marked in the Wailbri, the measures of basal jaw prognathism were remarkably uniform in the groups compared. More general aspects of facial morphology in the Wailbri were investigated by BROWN and BARRETT ('64) who reported sex differences in mean values and variances; a preliminary correlation analysis failed to demonstrate any craniofacial associations of marked biological interest. A more detailed account of craniofacial variations in young adult members of the Wailbri tribe was given by BROWN ('65a) who used correlation and regression analysis to study variations in facial prognathism in relation to cranial base morphology and the size and shape of other dentofacial structures.

Until now, the roentgenographic study of the Wailbri Aboriginals has been concerned with craniofacial morphology in adults but many records have been obtained from younger subjects. A preliminary comparison of facial characters in Australian Aboriginal children

and children from Melbourne, New Zealand and North America has been made by GRESHAM, BROWN and BARRETT ('65). In addition, multivariate techniques have been applied to craniometric data to illustrate the use of factor analysis (BROWN, BARRETT and DARROCH, '65a) and to make a factorial comparison between Aboriginals and Europeans (BROWN, BARRETT and DARROCH, '65b).

Summary

Early studies in Australian craniology were designed to describe and classify morphological characters of the skull but recently there has been a trend to probe for the underlying causes of observed variation both within and between groups. This approach, as WASHBURN ('62) pointed out, is concerned with the development of hypotheses that can be subsequently tested by further experimentation, in contrast to investigations where anthropometry is the principal method and the tabulation of observations a main objective.

The metric and non-metric characters of the Australian skull are now well documented and it would seem that little is to be gained from further descriptive studies unless they are designed to throw light on the question of regional variations. However, the underlying nature of associations between craniofacial components is less certain even though the topic has received preliminary attention.

If substantial progress is to be made in these research areas, there is little doubt that modern computer facilities should be effectively used for data recording and analysis (BARRETT, BROWN and SIMMONS, '66; BARRETT, BROWN and McNULTY, '67).

Recent developments in computer technology have provided the biometrician with refined analytic methods that would otherwise be unmanageable on account of the mathematical labour involved. For example, multivariate procedures offer new avenues for craniometric research particularly in the study of associations between measurable characters and in the discrimination between groups. Unfortunately, relatively few examples are available with which to assess the general usefulness of these procedures in craniometric data analysis.

Multivariate methods have not been applied to the study of cranio-facial variations in Australian Aboriginals in any systematic way; it would seem that an investigation incorporating the technique of roentgenographic cephalometry and designed to take advantage of recent developments in multivariate analysis would contribute to knowledge in this field and supplement the information obtainable by more conventional means.

2

MATERIALS AND METHODS

General considerations

Facial growth and morphology have been studied by the quantitative analysis of measurements obtained directly from subjects or indirectly from roentgenograms of the head. Cross-sectional or serial research may be indicated in particular situations but information on individual growth patterns is provided only by carefully obtained serial observations (GARN and SHAMIR, '58). Metallic implants used in conjunction with serial head roentgenograms constitute the best method for accurate interpretation of the direction and amount of facial growth (BJÖRK, '55a, '63, '64a).

Cross-sectional studies of the human skull are usually limited to the presentation of statistical parameters for size and shape; only when carried out over several age groups do they provide information on growth trends. It is possible, however, to gain an insight into craniofacial growth patterns by means of correlation analyses carried out on cross-sectional data. This approach assumes that associations between variables provide evidence of the presence but not necessarily the nature of coordinating mechanisms that have operated during the growth of the components correlated. The information obtained in

this way is limited because the use of correlation analysis in craniometry is complicated by statistical problems and difficulty in the interpretation of observed values. Another disadvantage in correlation studies lies in the necessity of handling variables in pairs or adopting a partial or multiple correlation approach. In either case a thorough examination of all possible combinations of variables is difficult and time-consuming. Notwithstanding these limitations, correlation studies can provide useful information provided they are carefully interpreted.

Studies of associations may be clarified by applying a multivariate approach to craniometric data analysis. The main advantage of this technique over the more usual uni- and bivariate methods stems from the manner in which the variables are treated collectively thus avoiding the necessity of making prior assumptions of dependence or independence among them. Some of these aspects have been discussed by HOWELLS ('51) and more recently by SOLOW ('66, p75) who carried out correlation and multivariate analyses on data obtained from young adult Danish males.

Apart from the numerical analysis of craniometric data many techniques of experimental biology have been developed to examine the sites and patterns of bone growth in the mammalian skull. Among several methods commonly used have been the histological examination of growth sites (HOYTE, '60; BAUME, '61; CLEALL, PERKINS and GILDA, '64), the experimental manipulation of sutures, synchondroses and

growth cartilages (DuBRUL and LASKIN, '61; KOSKI and MASON, '64; SARNAT, '63; DAS, MEYER and SICHER, '65) and the artificial deformation of growing crania (MOSS, '59).

In addition to the study of sites and mechanisms of facial growth, attention has been given to genetic aspects of facial growth (KRAUS, WISE and FREI, '59; GARN, LEWIS and VICINUS, '63), the relation between facial growth and general body growth (LINDEGÅRD, '53; BJÖRK, '55b; NANDA, '55; BAMBHA and VAN NATTA, '63; JOHNSTON, HUFRAM, MORESCHI and TERRY, '65) and the coordination of facial growth (MOSS and YOUNG, '60; MEREDITH, '62).

As a result of numerous laboratory investigations as well as longitudinal and cross-sectional studies of human material a clearer understanding of the mechanisms and sites of facial growth is emerging; these topics have been adequately summarised by BAUME ('61), SCOTT ('62), MOSS ('64) and BJÖRK ('64b).

With the foregoing considerations in mind, the general objectives of the present study were developed and the methodology considered to be most appropriate was determined. The available material made it possible to investigate craniofacial associations in an ethnic group which is probably more homogeneous genetically than other living populations. Factor analysis, one particular form of multivariate technique, was selected to explore the patterns of covariation present in the pre-European Australian skull. An important facet of the investigation was to assess the general usefulness of factor

analysis in craniometric research.

Variables were selected to represent the size and shape of several anatomical regions of the skull and the data were obtained by measuring the skulls directly or by measuring standardised roentgenograms taken in the lateral and postero-anterior positions. The remainder of the present chapter deals with the material, the variables analysed and the general statistical methods. Attention is also given to the reliability of measurements derived from roentgenograms. The multivariate technique is relatively new so far as craniometric research is concerned and for this reason an outline of its main features is presented in a separate section (Chapter 3).

Material

The skulls examined in the present investigation form part of an extensive collection of Aboriginal skeletal material housed in the South Australian Museum, Adelaide. Only specimens assessed as male were included. Although no accurate datings were available for most of the specimens, it is fairly certain that many of those selected represent Aboriginal man in Australia at a time prior to European settlement.

About 1,000 skulls were examined to select those best suited for the requirements of the study. Only adult specimens were accepted,

that is those in which the third molars had erupted or, in a few instances, where agenesis of one or both lower third molars was evident, but the specimen was obviously adult. During this inspection, sexing was carried out and the dentitions charted but no attempt was made to place the skulls in specific age groups.

The criteria for selection of a skull for inclusion in the study were as follows; mandible available, no major fragmentation in the region of measuring landmarks, no noticeable postmortem distortion, a dentition sufficiently intact to allow placement of the mandible in the position of maximum tooth contact.

Sexing was carried out by the author using the features of discrimination listed by SICHER ('60, p88) and FENNER ('39). In the South Australian Museum the post-cranial specimens are stored apart from the skulls and for many specimens sex ratings had been made independently of the skulls by other workers (DAVIVONGS, '63; RAO, '66). In some instances the sex of specimens was known and available from museum records. Only for a few skulls were no independent sex ratings available and for these a colleague made separate assessments. Finally, by comparing the author's ratings with those derived from other sources, 106 skulls were classified as male and registered for inclusion in the study.

It is necessary to recognise two limitations that apply to craniometric studies such as the present one. Because sexing was attempted by inspection alone there was a considerable element of subjectivity

even though the secondary sex features are better differentiated in Australian crania than in other groups. The skulls selected were assessed as male but there is no certainty that this classification was correct in all instances. However, the risk of selecting skulls with erroneous sex ratings was considerably reduced by referring to records derived from the post-cranial skeletons. It is probable that the inclusion of a few skulls of the opposite sex would hardly affect the overall interpretation of the findings derived from a correlation analysis.

The second limitation was imposed by inevitable post-mortem dimensional changes in the specimens. It is difficult to assess the exact degree of shrinkage in museum material but BERGLAND ('63, p23) quoted sources showing reductions of linear dimensions of up to two per cent with an average shrinkage of about one per cent. Statistical parameters derived from museum material should be considered in the light of post-mortem shrinkage but, so far as correlation studies are concerned, this limitation should have little if any effect on the findings provided that the dimensional change is fairly uniform throughout the skull.

Standardised cephalograms were obtained for the 106 skulls by the method to be described and after inspection of these six skulls were excluded because fragmentation was found in the cranial base region. The final sample consisted of 100 adult skulls classified as male.

Table 1 lists the skulls according to the Australian State of origin. Apart from the South Australian specimens there were few skulls from any other State and it has been assumed that the variation between different groups of Australian skulls would be similar in magnitude to the variation within a single group. Although this assumption could not be verified because of small sample numbers in the sub-groups, an examination of recorded measurements gave no evidence to suggest the skulls could not be placed in a common group for the multivariate study. However, for a preliminary analysis of cranial vault characters the northern coastal skulls were separated from the others (Chapter 4).

Table 1. Distribution of Australian skulls according to state of origin

State	Number
South Australia	73
Victoria	1
New South Wales	4
Western Australia	5
Northern Territory	
Coastal	12
Central Desert	3
No data available	2
Total	100

Roentgenographic methods

Roentgenographic cephalometry is an established research method in craniofacial growth studies and several reviews of its applications in this field are available (BJÖRK, '54; KROGMAN and SOUSSINI, '57; GARN and SHAMIR, '58; SALZMAN, '61; SCOTT, '63). It is unfortunate, however, that the technique has not found the same acceptance in physical anthropology, particularly as a complement to traditional techniques for studying head form in different population groups.

Roentgenograms were taken with the skulls positioned in a cephalostat constructed for use at Yuendumu, Central Australia and described previously (BARRETT, BROWN and MACDONALD, '63; BROWN, '65a). The anode to median sagittal plane distance was fixed at 180 cm, while the median sagittal plane to film distance was a constant 15 cm. These distances, which are the same as those used at Yuendumu for living subjects, produced a calculated enlargement of 8.3 per cent on the roentgenograms for linear dimensions situated in the median sagittal plane or in the trans-porionic plane. Roentgenographic enlargement was checked against the image of a standard millimetre scale placed in the median sagittal plane at the time of exposure.

To facilitate skull positioning, the cephalostat was mounted on a stable base in an inverted position (Figure 1), the target-median plane and median plane-film distances were set and alignment of the central beam was checked by exposing intra-oral dental films on which

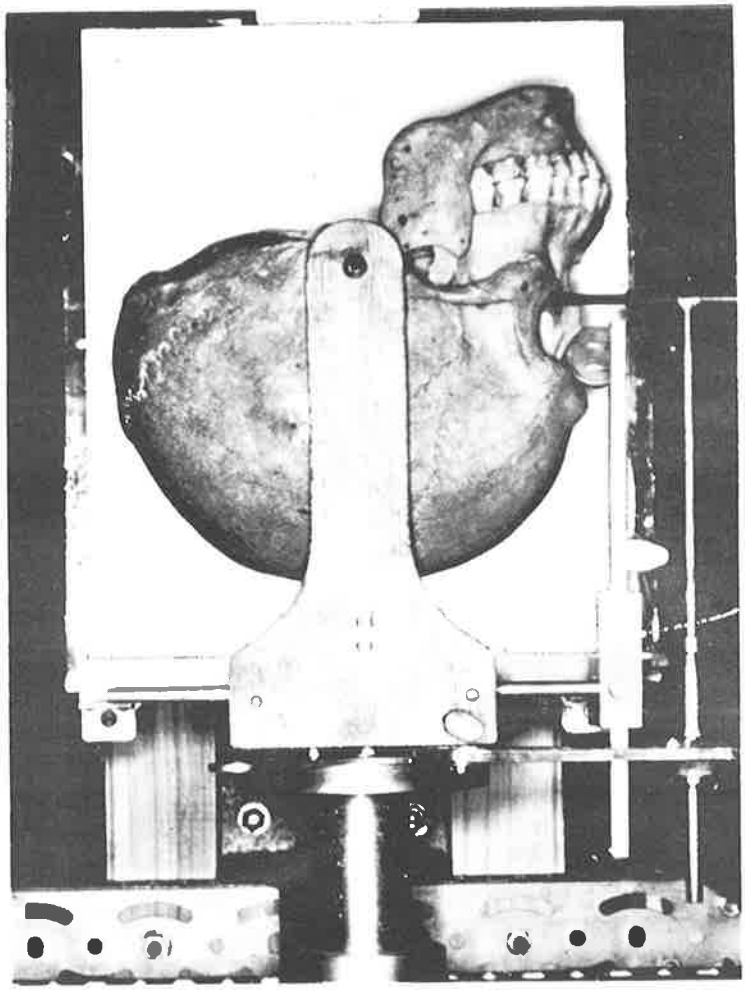

FIGURE 1. Method of skull positioning for roentgenography.

symmetrical superimposition of the images of right and left ear-rods indicated satisfactory alignment of central beam and cephalostat. The following procedure was used to obtain one lateral and one postero-anterior roentgenogram for each skull:

1. Small lead shot were fixed on each side of the skull over the craniometric points porion and euryon. These indicators were required for some of the measurements carried out;

2. The mandible was located in the position of maximum occlusal contact of the teeth and, when necessary, small pieces of soft wax were placed between condyles and mandibular fossae to stabilise the mandible;

3. The skull was carefully inverted with the mandible retained in the correct position and the specimen fixed in the cephalostat with the Frankfort plane horizontal and at right angles to the central beam;

4. For the postero-anterior film the cephalostat was rotated through 90 degrees and locked in this position without disturbing the skull fixation.

The roentgenographic apparatus, a General Electric hospital installation, was set at 80 Kvp and 10 Mas for most skulls but at times the exposure was varied according to the density of the specimen. Kodak Blue Brand film was used with Watson Victor Kontax cassettes, each fitted with two DuPont Stainless Fast Speed intensifying screens. As far as possible the technique paralleled

that used in Central Australia for living subjects. All films were processed in accord with the manufacturer's recommendations.

Measurement methods

Craniometric measurements were made directly on the skulls using standard techniques and instruments* (MARTIN-SALLER, '57). Dimensions were measured on the films with the aid of transparent ruled overlays similar to those suggested by BJÖRK and SOLOW ('62). By using this procedure landmarks were located separately for each dimension and the chance of perpetuating errors in landmark identification was avoided. The repeated determinations of landmarks reduced the possibility of artificially inflating the values of coefficients of correlation derived from the observations.

Linear measurements were recorded to the nearest 0.5 mm on both skulls and roentgenograms; angular measurements were recorded to the nearest 0.5 degrees. Linear measurements obtained on the lateral roentgenograms were corrected for enlargement by the use of a special rule, calibrated to compensate for the calculated enlargement value. For the postero-anterior films, all linear measurements were corrected for differential enlargement according to the formula:

$$x = \frac{(180 \pm d) \cdot y}{195}$$

*Manufactured by Siber Hegner and Co. Ltd., Zurich, Switzerland.

where x is the corrected value, y is the value measured on the postero-anterior film, and d is the corrected distance of the landmark from the trans-porionic plane measured on the lateral film. These adjustments were performed by digital computer.

Craniometric and roentgenographic reference points

Reference points used in the study are listed and defined below. Craniometric points are designated (C) and those located on roentgenograms are designated (R). Seven of the landmarks were defined for the present study; these were the points A, B, C, D, condylion, posterior nasal spine and scaphoid fossa point. The point tuberculum pharyngicum (pharyngeal tubercle) was located according to the definition of BERGLAND ('63, p16) and the remaining points were defined with minor modifications in some instances according to BJÖRK ('60) for roentgenographic locations and either WILDER ('20), MARTIN-SALLER ('57) or ASHLEY-MONTAGU ('60) for craniometric locations. Where a roentgenographic point was situated bilaterally and the two images did not coincide, the mid-point of left and right images was accepted as the point in question.

The points are illustrated in Figures 2, 3 and 4 for craniometric locations, in Figures 5 and 6 for roentgenographic locations and in Figure 8 for the endocranium.

POINT A (A): The posterior termination of the basal endocranial contour, determined irrespective of the anterior clinoid processes. Point A is approximately at the junction of the contours of the ethmoid horizontal plate and the orbital roof. (R)

POINT B (B): The anterior point of the line defining the greatest length diameter of the endocranial contour. (R)

POINT C (C): The superior point on the endocranial contour furthest from the line defining the greatest length diameter of the endocranial contour. (R)

POINT D (D): The posterior point of the line defining the greatest length diameter of the endocranial contour. (R)

ARTICULARE (ar): The intersection of the contour of the external cranial base and the dorsal contour of the mandibular neck or condyle. (R)

BASION (ba): The perpendicular projection of the anterior border of the foramen magnum on a tangent to the lower contour of the foramen (R), or the median point on the anterior margin of the foramen magnum. (C)

CONDYLION (cd): The most superior point on the crest of the mandibular condyle. (C and R)

ECTOMOLARE (ecm): The most lateral point on the outer surface of the alveolar ridge, opposite the centre of the maxillary second molar. (C and R)

ENDOMOLARE (enm): The most medial point on the inner surface of the alveolar ridge opposite the centre of the maxillary second molar. (C)

ETHMOIDALE (eth): The lowest median point on the contour of the anterior cranial fossa corresponding to the cribriform plate of the ethmoid bone. (R)

EURYON (eu): The two points opposite each other on the sides of the skull which form the termini of the line of greatest breadth. (C)

GLABELLA (g): The most prominent point in the median line between the two eyebrow ridges, a little above the fronto-nasal suture. (C)

GNATHION (gn): The lowest point on the symphysis of the mandible in the median sagittal plane. (C and R)

GONION (go): A point on the bony contour of the gonial angle located by the bisection of the angle formed by the mandibular line and the ramus line. (C and R)

GONIAL TANGENT POINT (tgo): The intersection of the mandibular line and the ramus line. (R)

HORMION (h): The median point in the suture between vomer and sphenoid where the former overlaps the latter. Located in norma basilaris. (C)

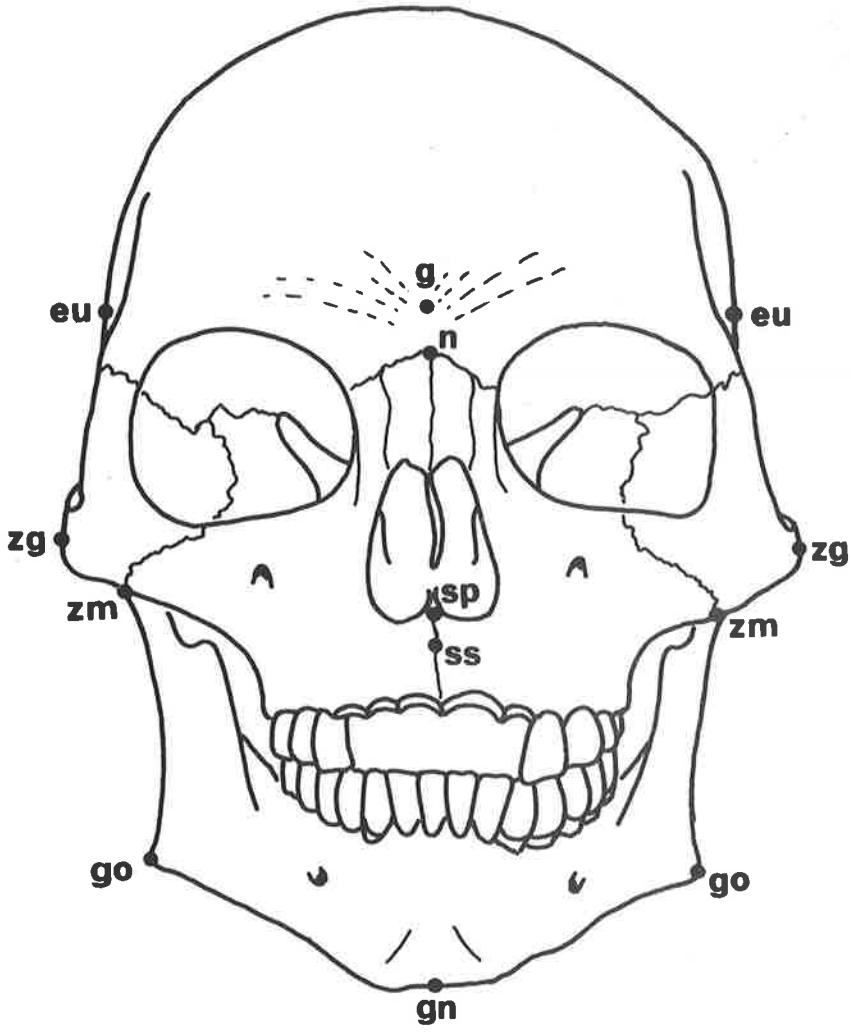

FIGURE 2. Craniometric points – frontal.

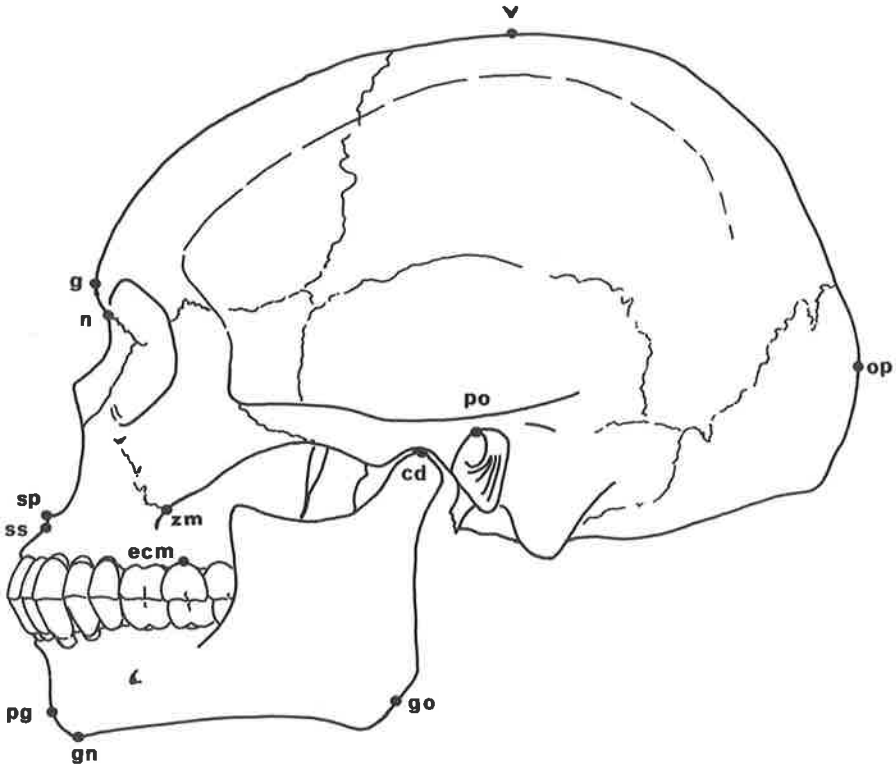

FIGURE 3. Craniometric points – lateral.

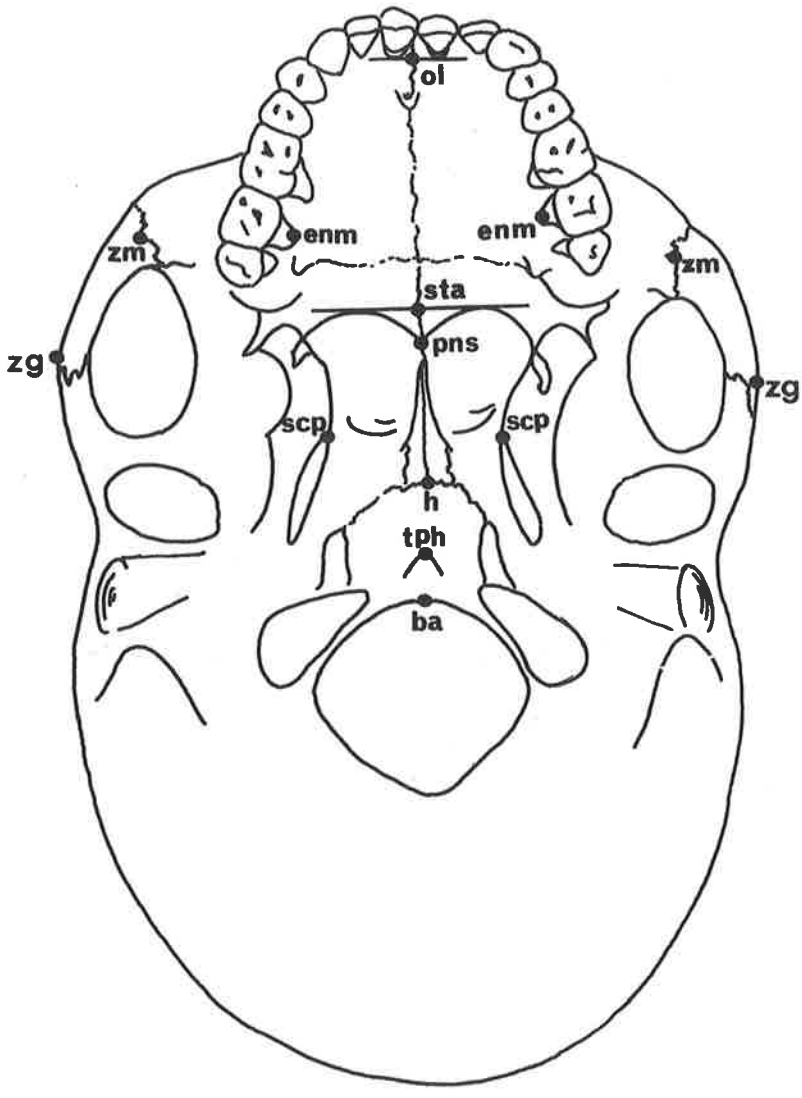

FIGURE 4. Craniometric points – basal.

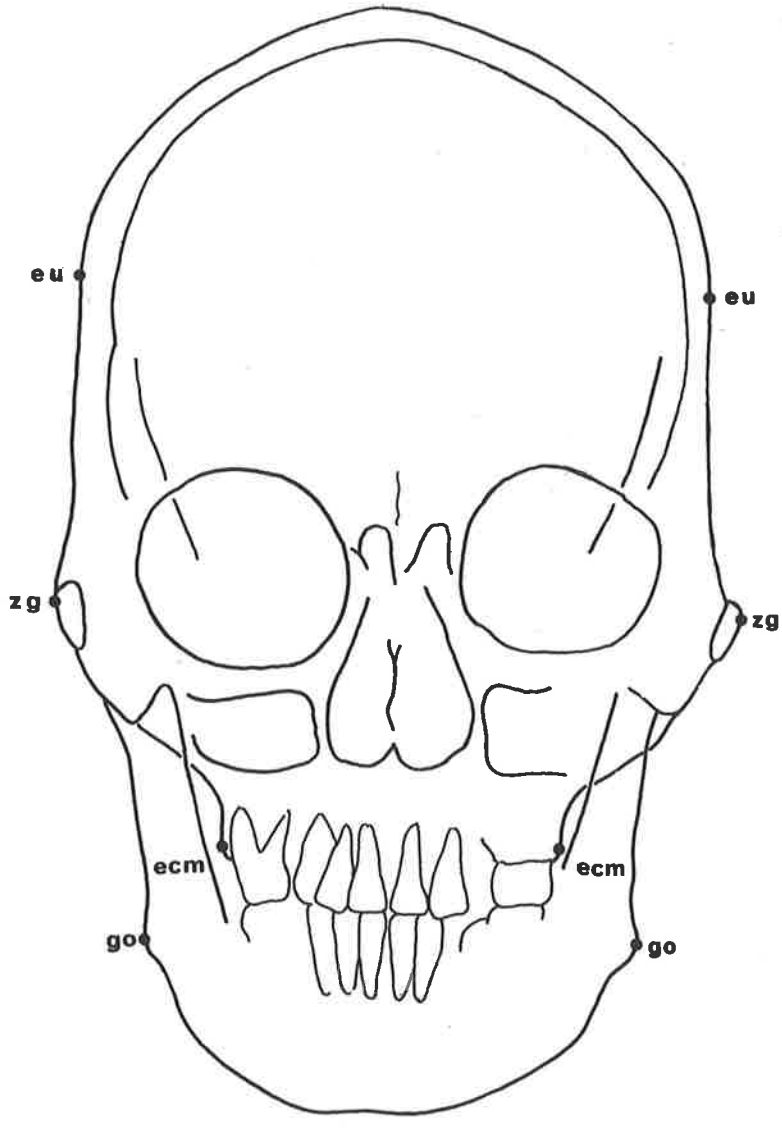

FIGURE 5. Roentgenographic points – frontal.

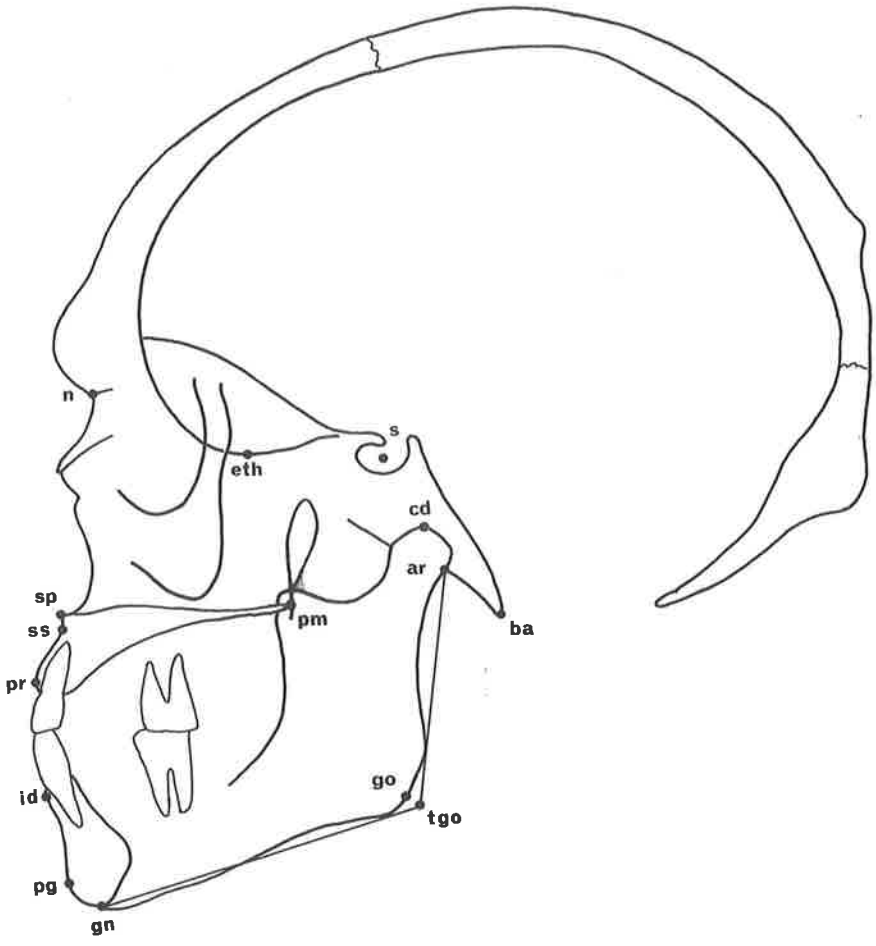

FIGURE 6. Roentgenographic points — lateral.

INFRADENTALE (id): The highest and most prominent point on the the lower alveolar process. (R)

NASION (n): The most anterior point of the fronto-nasal suture. (C and R)

OPISTHOCRANIUM (op): The most distal point on the skull from the glabella in the median sagittal plane, excluding the external occipital protuberance. (C)

ORALE (ol): The point on the bony palate where the median sagittal plane intersects a line drawn tangentially to the points of maximum convexity of the lingual margins of the alveoli of the two central incisor teeth. (C)

POGONION (pg): The most anterior point on the mandibular symphysis. (R)

PORION (po): The most superior point on the margin of the external acoustic meatus. (C)

POSTERIOR NASAL SPINE (pns): The apex of the posterior nasal spine. (C)

PROSTHION (pr): The lowest and most prominent point of the upper alveolar process. (R)

PTERYGOMAXILLARE (pm): The intersection of the superior contour of the nasal floor and the anterior contour of the pterygopalatine fossa. (R)

SELLA (s): The centre of the sella turcica determined as the mid-point of the maximum diameter of the fossa from the tuberculum sellae. (R)

SCAPHOID FOSSA POINT (scp): The most anterior extremity of the scaphoid fossa immediately adjacent to the medial pterygoid lamina. (C)

SPINAL POINT (sp): The apex of the anterior nasal spine. (C and R)

STAPHYLION (sta): The intersection of the median palatal suture and a line drawn tangentially to the curves of the posterior margin of the palate. (C)

SUBSPINALE (ss): The most posterior point on the anterior contour of the upper alveolar process in the median sagittal plane. (C and R)

TUBERCULUM PHARYNGICUM (tph): Point of intersection between the median sagittal plane and the line of attachment of the pharyngeal raphe. (C)

ZYGION (zg): The most lateral point on the zygomatic arch. (C and R)

ZYGOMAXILLARE (zm): The lowest point in the external suture between zygomatic and maxillary bones. (C and R)

VERTEX (v): The highest medial point on the skull when placed in the Frankfort Horizontal. (C)

Roentgenographic reference lines (Figure 7)

NASION-SELLA LINE (NSL): The line through points nasion and sella.

NASION-SELLA PERPENDICULAR (NSP): The line through sella and perpendicular to NSL.

ETHMOIDALE-SELLA LINE (ESL): The line through points ethmoidale and sella.

NASAL LINE (NL): The line through spinal point and pterygo-maxillare. In some instances where there was marked curvature of the palate this line was located by inspection to conform with the general inclination of the palate.

MANDIBULAR LINE (ML): The line through gnathion, tangent to the mandibular border at the angle region.

RAMUS LINE (RL): The line through articulare, tangent to the posterior border of the mandibular ramus.

Variables used

For the multivariate study, variables were selected to represent several anatomical regions of the skull. It was not practical to include more than a few of many available and the variables chosen were considered to provide the best indication of the sources of variation to be studied. It is stressed that the selection was made primarily with the requirements of factor analysis in mind, and

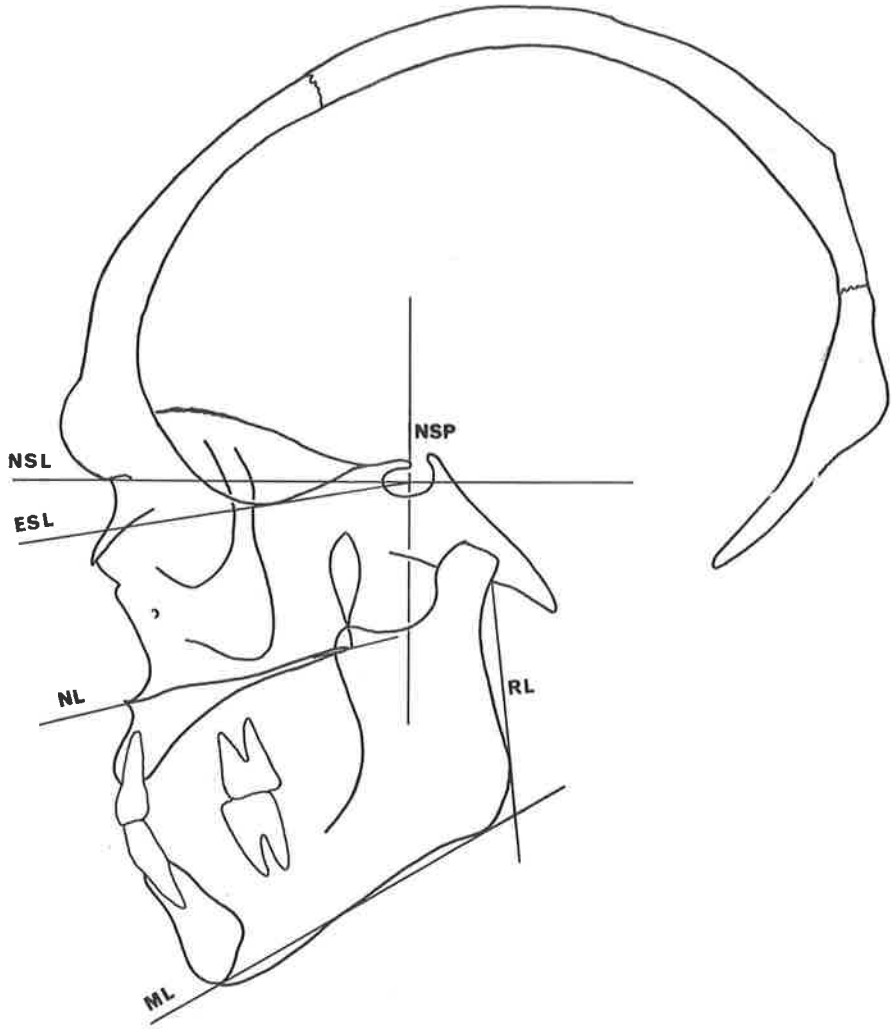

FIGURE 7. Roentgenographic reference lines.

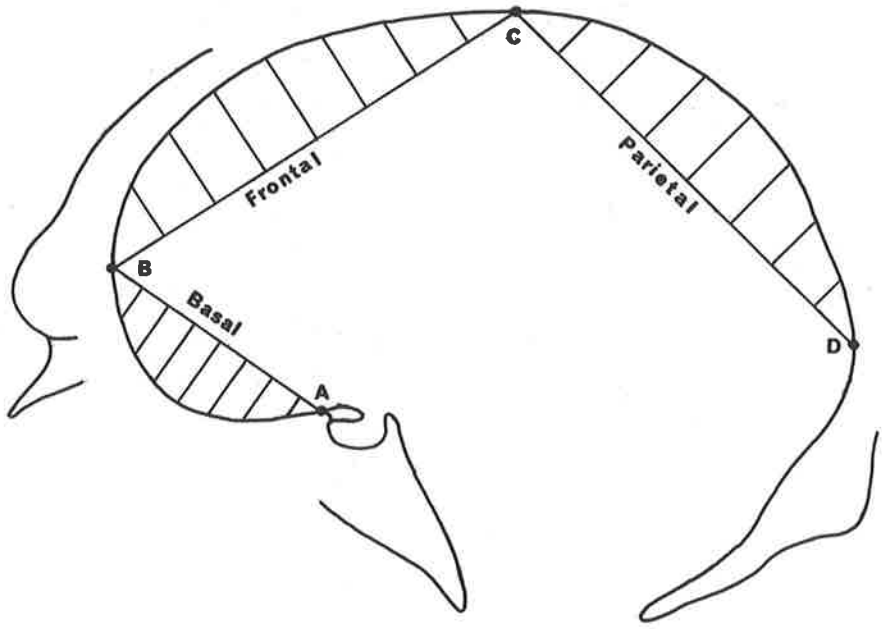

FIGURE 8. Measurement of endocranial contours.

therefore would be expected to differ from one made for the purpose of morphological description of the sample.

The neurocranium was described by two groups of variables, one including measurements of the ectocranial and endocranial contours, and the other consisting of size and shape indicators for the cranial base. The endocranial variables measured dimensions of the skull component referred to as the cerebral capsule by MOSS ('62). The method of YOUNG ('56) was modified to obtain variables that would adequately express the size and shape of the endocranium as chords, indices and angles relating to the basal, frontal and parietal segments of the endocranium (Figure 8). The endocranial surface was considered as a continuous curve rather than the separate curves of the sphenoid, ethmoid, frontal, parietal and occipital bones. Inadequate definition on the roentgenograms precluded a reliable analysis of the curvature in the sub-occipital region of the endocranium. Cranial base variables consisted of size and shape measurements of the base segments between nasion and basion, measurements of the frontal bone and frontal sinus and the inclination of the foramen magnum to the nasion-sella line.

The facial skeleton was represented by variables selected to indicate nasal and nasopharyngeal dimensions, upper and lower facial size and general facial shape. The nasal and nasopharyngeal group included depth, height and breadth measures of these regions and the orientation of the nasal floor to the cranial base. Facial size

was represented by breadth dimensions of the mid-facial regions, measurements of the palate, mandibular dimensions and depth of the infratemporal fossa, this variable being included to show the degree of muscular development. Finally, the shape of the facial skeleton was expressed by a series of angles chosen to indicate prognathic build and the inclination of the jaw bases to the cranial base.

The inclusion of a large number of variables in the initial stages was deliberate and had the effect of revealing the sources of covariation more readily by maximising the variance associated with groups of related variables. When more was known of the relationships between the variables and the sources of common variation they represented many were excluded from further analysis without loss of important information.

The variables are listed below; those measured directly on the skull are indicated by C (craniometric) and those obtained indirectly on lateral or postero-anterior roentgenograms are indicated by R (roentgenographic). A variable is defined only if the definition is not inferred by the notation used for the reference points determining the variable. For example, the variable nasion-ethmoidale distance is defined as the distance between points nasion and ethmoidale and was given the notation n-eth. Similarly, the median cranial base angle, defined as the angle between straight lines joining nasion-sella and sella-basion was given the notation n-s-ba.

Endocranium

BASAL CHORD: The distance between endocranial points A and B. (R)

BASAL INDICES 1 to 7: The perpendicular distances from seven equidistant points on the basal chord to the nearest parts of the basal endocranial contour, expressed as percentages of the basal chord length. (R)

BASAL-FRONTAL CHORD ANGLE (A-B-C) (R)

FRONTAL CHORD: The distance between endocranial points B and C. (R)

FRONTAL INDICES 1 to 7: The perpendicular distances from seven equidistant points on the frontal chord to the nearest parts of the frontal endocranial contour, expressed as percentages of the frontal chord length. (R)

FRONTO-PARIETAL CHORD ANGLE (B-C-D) (R)

PARIETAL CHORD: The distance between endocranial points C and D. (R)

PARIETAL INDICES 1 to 7: The perpendicular distances from seven equidistant points on the parietal chord to the nearest parts of the parietal endocranial contour, expressed as percentages of the parietal chord length. (R)

ENDOCRANIAL LENGTH (endo. l): The distance between endocranial points B and D. (R)

ENDOCRANIAL BREADTH (endo. b): The maximum distance between right and left endocranial contours measured on the postero-anterior roentgenogram. The measuring points correspond closely with the external points euryon. (R)

ENDOCRANIAL HEIGHT (endo. h): The sum of the separate perpendicular distances measured from basion and endocranial point C to a line joining endocranial points B and D. (R)

MAXIMUM CRANIAL LENGTH (g-op) (C)

MAXIMUM CRANIAL BREADTH (eu-eu) (C)

AURICULO-VERTEX HEIGHT (po-v): The perpendicular distance between the vertex and a line joining the bilateral points porion. (C)

Cranial base

NASION-ETHMOIDALE DISTANCE (n-eth) (R)

ETHMOIDALE-SELLA DISTANCE (eth-s) (R)

ANTERIOR CRANIAL BASE LENGTH (n-s) (R)

POSTERIOR CRANIAL BASE LENGTH (s-ba) (R)

TOTAL CRANIAL BASE LENGTH (n-ba) (R)

MEDIAN CRANIAL BASE ANGLE (n-s-ba) (R)

INTERNAL CRANIAL BASE ANGLE (eth-s-ba) (R)

FORAMEN ANGLE (for. angle): The angle between the NSL and a line drawn perpendicular to the tangent to the lower contour of the foramen magnum. (R)

MINIMUM FRONTAL THICKNESS (min. f): The shortest distance measured from nasion to the endocranial contour of the frontal bone. (R)

MAXIMUM FRONTAL THICKNESS (max. f): The greatest distance between ecto- and endocranial contours of the frontal bone measured in the median sagittal plane. This dimension usually spans the frontal sinus. (R)

FRONTAL SINUS HEIGHT (f. sinus h): The greatest distance between superior and inferior extremities of the frontal sinus as seen on the lateral roentgenogram. (R)

FRONTAL SINUS BREADTH (f. sinus b): The greatest distance between the lateral extremities of the frontal sinus as seen on the postero-anterior roentgenogram. (R)

Nasal and nasopharyngeal cavities

SPHENOID DIAMETER (sphen. d): The shortest distance between the floor of the pituitary fossa and the pharyngeal surface of the sphenoid bone. This dimension spans the sphenoidal air sinus. (R)

NASAL BREADTH (nasal b): The maximum distance between the lateral margins of the nasal aperture perpendicular to the median sagittal plane. (C)

NASAL DEPTH (ss-pns): The distance between points subspinale and posterior nasal spine. (C) or The distance between points subspinale and pterygomaxillare. (R)

ANTERIOR NASAL HEIGHT (n-sp) (C)

NASOPHARYNGEAL DEPTH (ba-pns): The distance between points basion and posterior nasal spine. (C) or The distance between points basion and pterygomaxillare. (R)

INTRAPHARYNGEAL DEPTH (tph-pns) (C)

NASOPHARYNGEAL BREADTH (scp-scp) (C)

NASOPHARYNGEAL HEIGHT (phar. h): The perpendicular distance from hormion to a line joining posterior nasal spine and basion, usually measured with a palatometer. (C)

MAXILLARY PROTRUSION (s-pm hor.): The projected distance of point pterygomaxillare to the NSP line. (R)

POSTERIOR UPPER FACE HEIGHT (s-pm vert.): The perpendicular distance from point pterygomaxillare to the NSL. (R)

Facial size

MORPHOLOGICAL FACE HEIGHT (n-gn) (C) and (R)

MAXILLARY BREADTH (zm-zm) (C)

BIZYGOMATIC BREADTH (zg-zg) (C)

MASSETERIC BREADTH (mass. b): Calculated as the difference between bizygomatic and bigonial breadths. (C)

MAXILLO-ALVEOLAR BREADTH (ecm-ecm) (C)

PALATAL BREADTH (palate b): The distance between bilateral points endomolare. (C)

PALATAL LENGTH (palate l): The distance between points orale and staphylion. (C)

PALATAL HEIGHT (palate h): The perpendicular distance from the highest point on the palatal vault to a line joining the bilateral points endomolare. Measured with a palatometer held at right angles to the transverse plane. (C)

BIGONIAL BREADTH (go-go) (C)

MANDIBULAR BODY LENGTH (gn-go): Measured on the skull as the perpendicular distance from gnathion to a line joining the bilateral points gonion. (C)

TOTAL MANDIBULAR LENGTH (gn-cd): Measured on the skull as the perpendicular distance between gnathion and a line joining the bilateral points condylion. (C)

MINIMUM RAMUS BREADTH (ramus b): The least distance between anterior and posterior borders of the ramus. In this study the right ramus was used except in a few instances where post-mortem fracture of this ramus had occurred. (C)

MINIMUM RAMUS HEIGHT (ramus h): The least distance between the mandibular and preangular notches, usually measured on the right ramus. (C)

INFRATEMPORAL FOSSA DEPTH (infra t.f.d.): The distance between the upper border of the right zygomatic arch and the deepest part of the infratemporal fossa. Measured with a calibrated probe directed horizontally inwards. (C)

Facial shape

MAXILLARY BASAL PROGNATHISM (s-n-ss) (R)

MANDIBULAR BASAL PROGNATHISM (s-n-pg) (R)

MAXILLARY ALVEOLAR PROGNATHISM (s-n-pr) (R)

MANDIBULAR ALVEOLAR PROGNATHISM (s-n-id) (R)

PROFILE ANGLE (n-ss-pg) (R)

GONIAL ANGLE (ar-tgo-gn) (R)

NASAL FLOOR INCLINATION (NL/NSL): The angle between NL and NSL. (R)

MANDIBULAR BASE INCLINATION (ML/NSL): The angle between ML and NSL. (R)

NASO-MANDIBULAR ANGLE (NL/ML): The angle between NL and ML. (R)

A summary of the variables used and the adopted notations is given in Table 2.

Table 2. The variables and their notations

	ENDOCRANIUM	
1.	Basal chord	
2.	Basal indices 1 to 7	
3.	Basal-frontal chord angle	A-B-C
4.	Frontal chord	
5.	Frontal indices 1 to 7	
6.	Fronto-parietal chord angle	B-C-D
7.	Parietal chord	
8.	Parietal indices 1 to 7	
9.	Endocranial length	endo. l
10.	Endocranial breadth	endo. b
11.	Endocranial height	endo. h
12.	Maximum cranial length	g-op
13.	Maximum cranial breadth	eu-eu
14.	Auriculo-vertex height	po-v
	CRANIAL BASE	
15.	Nasion-ethmoidale distance	n-eth
16.	Ethmoidale-sella distance	eth-s
17.	Anterior cranial base length	n-s
18.	Posterior cranial base length	s-ba
19.	Total cranial base length	n-ba
20.	Median cranial base angle	n-s-ba
21.	Internal cranial base angle	eth-s-ba
22.	Foramen angle	for. angle
23.	Minimum frontal thickness	min. f
24.	Maximum frontal thickness	max. f
25.	Frontal sinus height	f. sinus h
26.	Frontal sinus breadth	f. sinus b
	NASAL AND NASOPHARYNGEAL CAVITIES	
27.	Sphenoid diameter	sphen. d
28.	Nasal breadth	nasal b
29.	Nasal depth	ss-pns
30.	Anterior nasal height	n-sp
31.	Nasopharyngeal depth	ba-pns
32.	Intrapharyngeal depth	tph-pns
33.	Nasopharyngeal breadth	scp-scp
34.	Nasopharyngeal height	phar. h
35.	Maxillary protrusion	s-pm hor.
36.	Posterior upper face height	s-pm vert.

Table 2. (contd.) The variables and their notations

	FACIAL SIZE	
37.	Morphological face height	n-gn
38.	Maxillary breadth	zm-zm
39.	Bizygomatic breadth	zg-zg
40.	Masseteric breadth	mass. b
41.	Maxillo-alveolar breadth	ecm-ecm
42.	Palatal breadth	palate b
43.	Palatal length	palate l
44.	Palatal height	palate h
45.	Bigonial breadth	go-go
46.	Mandibular body length	gn-go
47.	Total mandibular length	gn-cd
48.	Minimum ramus breadth	ramus b
49.	Minimum ramus height	ramus h
50.	Infratemporal fossa depth	infra t.f.d.
	FACIAL SHAPE	
51.	Maxillary basal prognathism	s-n-ss
52.	Mandibular basal prognathism	s-n-pg
53.	Maxillary alveolar prognathism	s-n-pr
54.	Mandibular alveolar prognathism	s-n-id
55.	Profile angle	n-ss-pg
56.	Gonial angle	ar-tgo-gn
57.	Nasal floor inclination	NL/NSL
58.	Mandibular base inclination	ML/NSL
59.	Naso-mandibular angle	NL/ML

Statistical methods

Estimates of the descriptive parameters - mean, standard deviation, standard error of the mean and the linear correlation coefficient between two variables - were computed according to the usual procedures:

$\bar{x}$	Mean	$\frac{\Sigma x}{N}$
s	Standard deviation	$\sqrt{\frac{\Sigma (x - \bar{x})^2}{N - 1}}$
$\epsilon(\bar{x})$	Standard error of the mean	$\frac{s}{\sqrt{N}}$
r_{xy}	Correlation coefficient	$\frac{\Sigma (x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma (x - \bar{x})^2 \Sigma (y - \bar{y})^2}}$

where x and y are observed scores and N is the number of observations.

In addition, the coefficients of variation were computed but are not listed in the tables; the ranges of variation are indicated by inclusion of the minima and maxima observations for variables. To assess the significance of differences in variances or means of two groups, the F-ratio test of Snedecor and the t-test of Student were used.

The value of assessing the forms of distributions was stressed by SOLOW ('66, p53-55) who drew attention to the use of the statistics $\sqrt{b_1}$ and b_2 as described by PEARSON ('31) and RAO ('52). Although kurtosis and skewness of biometric data were studied by FAWCETT ('02) and MACDONELL ('04), relatively few attempts to analyse the

distribution forms of anthropometric data have been made since.

Skewness is indicated by $\sqrt{b_1}$ which is calculated from the second and third moments of the deviations from the mean according to:

$$\sqrt{b_1} = \sqrt{\frac{N.(\Sigma(x - \bar{x})^3)^2}{(\Sigma(x - \bar{x})^2)^3}}$$

where the sign of $\sqrt{b_1}$ is the same as that of the third moment. Kurtosis is indicated by b_2 which is computed from the second and fourth moments of the deviations from the mean according to:

$$b_2 = \frac{N.\Sigma(x - \bar{x})^4}{(\Sigma(x - \bar{x})^2)^2}$$

The significance limits for $\sqrt{b_1}$ and b_2, which are shown in Table 3 for a sample of 100, were taken from PEARSON and HARTLEY ('54).

Table 3. Significance limits of $\sqrt{b_1}$ and b_2 for N = 100*

	p = .01	p = .05	Mean	p = .05	p = .01
$\sqrt{b_1}$	Negative skewness			Positive skewness	
	-.567	-.389	0	.389	.567
b_2	Platykurtosis			Leptokurtosis	
	2.18	2.35	3	3.77	4.39

*From PEARSON and HARTLEY ('54)

The special statistical procedures used for the factor analyses are discussed separately in subsequent sections. All computations were carried out on computers in Australia and Denmark.*

Reliability of roentgenographic measuring methods

Some variables included for study could be measured with equal convenience on the skulls or roentgenograms and to compare these direct and indirect methods a series of double determinations was made on 50 specimens selected at random from the sample. Ten dimensions were obtained directly from the skulls and the same dimensions were then measured on the lateral or postero-anterior roentgenograms and corrected for differential enlargement. In most instances dimensions were chosen to assess landmarks known to be difficult to locate precisely on roentgenograms.

The reliability of the roentgenographic measuring technique, which included location of landmarks, was assessed by calculating the mean of the differences between direct and indirect determinations of a variable, M_{diff}, the standard error of the mean difference,

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* IBM 1620 University of Adelaide, Adelaide.
IBM 7090 Northern Europe University Computing Center, Copenhagen.
CDC 3600 C.S.I.R.O., Canberra.
CDC 6400 University of Adelaide, Adelaide.
GIER Data Regnecentralen, Copenhagen.

$\varepsilon(M_{diff})$, the standard deviation of a single determination, s, and the coefficient of correlation, r, between the two determinations. The statistics M_{diff}, $\varepsilon(M_{diff})$ and r were calculated by treating the differences between two determinations as normally distributed variables and applying the usual methods but the procedure of DAHLBERG ('40) was followed to compute s according to:

$$s = \sqrt{\frac{\Sigma diff^2}{2N}}$$

where diff is the difference between direct and indirect determinations. The findings for the series of ten sets of double determinations are summarised in Table 4.

Of the ten mean differences, all but three, which differed significantly from zero at the one per cent level, were less than one millimetre. A further four mean differences were significantly different from zero, two at the one per cent level and two at the five per cent level. These findings indicate that for most of the ten variables analysed the determinations differed between the two methods of measurement even though the differences were numerically small in relation to the means of the variables concerned. Except for the variable maxillary breadth, the variances of the single determinations computed as s^2 were small compared with the relevant sample variances.

The correlations between skull and roentgenographic measures were high for morphological face height, bigonial breadth, bizygomatic

breadth, maxillo-alveolar breadth and maximum cranial breadth, ranging from 0.92 to 0.99. However, the correlation for maxillary breadth was almost zero and for the remaining four variables ranged from 0.72 to 0.84.

These findings can be taken as a reflection of the relative accuracy of landmark locations on roentgenograms but no generalisations can be made because few variables were included and landmarks difficult to locate precisely on roentgenograms were deliberately chosen. Only for the bilateral points euryon were metallic indicators used as an aid in identification of maximum cranial convexity. The almost perfect correlation for cranial breadth ($r = 0.99$) justified the use of metallic indicators and produced fairly consistent determinations of this variable from roentgenograms. It may be added that the region of maximum convexity of a skull can only be guessed on a lateral roentgenogram if the points euryon are not identified by markers.

High mean differences and low correlation coefficients for the variables nasal depth, nasopharyngeal depth and maxillary breadth are explained by the difficulties in accurate location of points pterygomaxillare and zygomaxillare on roentgenograms. Pterygomaxillare as defined did not correspond with the craniometric point posterior nasal spine, even though many previous studies have indicated such a correspondence. Moreover, zygomaxillare, which is the lowermost point on the zygomaxillary suture, was impossible to

Table 4. Analysis of the differences in mm between measurements obtained directly on skulls and indirectly on roentgenograms for 50 specimens. The standard deviation of a single determination, s, and the correlation coefficient between the two determinations, r, are shown

Variable	M_{diff}	$\varepsilon(M_{diff})$	s	r
	measurements on lateral films			
ss-pns	-1.54**	0.24	1.65	.80
ba-pns	2.16**	0.27	2.02	.72
gn-go	0.09	0.27	1.88	.78
gn-cd	-0.36	0.34	1.66	.84
n-gn	0.59**	0.15	0.89	.99
	measurements on postero-anterior films			
go-go	-0.65	0.41	2.07	.92
zg-zg	0.39*	0.18	0.92	.95
zm-zm	-5.44**	1.05	6.45	.07
ecm-ecm	-0.51**	0.13	0.74	.97
eu-eu	-0.33*	0.14	0.77	.99

** M_{diff} differs from zero at the 1 per cent probability level.

* M_{diff} differs from zero at the 5 per cent probability level.

All correlation coefficients except that for zm-zm differ from zero at the 0.1 per cent probability level.

identify consistently on the roentgenograms, without the aid of metallic markers.

In view of the findings, craniometric determinations of variables were used in preference to roentgenographic when this was possible. However, provided landmarks are clearly discernible on roentgenograms and when correct compensation for differential enlargement of the image has been carried out, there is no strong indication that roentgenographic measurements will not provide statistical parameters free of gross errors and closely approximating those obtainable from direct skull measurements. Moreover, it would seem that a correlation analysis would result in very similar findings whether it was based on reliable roentgenographic measures or on direct craniometric evaluations. Nevertheless, it should be borne in mind that landmarks difficult to locate on roentgenograms will almost certainly lead to inflated variance estimates even though mean values may not reflect significant discrepancies.

It was not considered necessary to carry out double determination evaluations to test the reproducability of measurements taken from roentgenograms or skulls. Many previous investigators have shown that measurements can be reproduced within limits that do not markedly affect true values provided that the measuring techniques are carefully standardised. An analysis and discussion of the sources of error involved in cephalometric roentgenography and the subsequent tracing and measuring of head films has been reported previously (BROWN, '65a).

3

FACTOR ANALYSIS

Principles and Applications in Anthropometry

Factor analysis is one of a group of multivariate procedures that permit variables to be analysed collectively rather than individually. The method requires no prior assumptions of dependence or independence between the variables but groups them according to what they measure in common. In this way it differs from the statistical techniques of correlation and regression analysis.

One essential feature of factor analysis lies in the postulation of a set of unknown variables termed factors upon which the observed variables depend. It is the object of the analysis to locate and define these factors and to study the dependence of the variables on them. This is somewhat analogous to a regression analysis in which each variable is treated as dependent and where the independent variables are unknown. When the factors have been found, an attempt is usually made to identify them with the influences, biological or otherwise, that determine the observed covariation among the experimental variables. However, factor interpretation is not essential and the analysis may be used as a means of disclosing sources of common variation among the variables, or alternatively as

a sorting technique whereby the variables are placed in groups determined by the shared variation.

Because of these features, factor analysis is a useful tool in fields where little is known of the relationships between variables. In particular, the metric data obtained in studies of craniofacial morphology and growth are well suited to this type of analysis. It can be said, then, that factor analysis treats the experimental variables as effects and probes beneath the surface to look for evidence of common variation and its causes; according to CATTELL ('52, p17) it "has its functions, therefore, in basic research to provide measurement foundations for later special problems in pure and applied research".

Mathematical procedures

A distinction should be made between factor analysis and principal components analysis which it superficially resembles and with which it is sometimes confused. Although the two methods employ similar computations and can be made to provide similar results, the underlying assumptions are quite different. Principal components analysis is a relatively straightforward procedure for rescaling variables into components which are equal in number to the variables. The correlations among the variables are explainable by

and can be reproduced exactly from the component coefficients. Thus there is no reduction in the number of variables required to represent the original data even though in practice only the major components are given distinction.

On the other hand, factor analysis seeks to explain the sources of variation as common factors less in number than the variables. Apart from the common factors influencing more than one variable there is assumed to be a set of unique factors, equal in number to the variables, with squared coefficients that are more correctly termed residual variances. The residual variances are required to explain the variance component not accounted for by the common factors. Thus, the factor model assumes that the total number of factors, common and unique, is greater than the number of variables, but the common factors are given most importance.

According to the above assumptions, the basic factor model can be represented in regression form as:

$$z = a_1F_1 + a_2F_2 + \ldots\ldots + a_mF_m + bU$$

where z is a variable in standard form, F_1, F_2 ..... F_m are the scores on the m common factors influencing z, a_1, a_2 ..... a_m are the factor coefficients and b is the coefficient for U the unique factor belonging to z.

It is convenient to consider z to be in standard form in which case it has unit variance. If the factors are also standardised and

defined as being uncorrelated it is possible to show that:

$$\text{variance } (z) = 1 = a_1^2 + a_2^2 + \ldots + a_m^2 + b^2$$

also if:

$$a_1^2 + a_2^2 + \ldots\ldots + a_m^2 = h^2$$

then:

$$\text{variance } (z) = 1 = h^2 + b^2$$

The term h^2 is called the communality of the variable and is part of the variance of z accounted for by all of the common factors while b^2 is the contribution of the uniqueness or residual variance.

Extending the notation to cater for n variables and m factors:

$$z_1 = a_{11}F_1 + a_{12}F_2 + \ldots\ldots + a_{1m}F_m + b_1U_1$$

$$z_2 = a_{21}F_1 + a_{22}F_2 + \ldots\ldots + a_{2m}F_m + b_2U_2$$

. . . . . .

$$z_n = a_{n1}F_1 + a_{n2}F_2 + \ldots\ldots + a_{nm}F_m + b_nU_n$$

The above set of linear equations can be resolved in terms of the observed correlations and expressed in a form suitable for solution as shown in the matrix equation:

$$\mathbf{R} = \mathbf{A}\mathbf{A}' + \mathbf{B}$$

where $\mathbf{R}$ is the n x n matrix of correlations (or covariances) among the observed variables;

A is the n x m matrix of common factor coefficients;

A' is the transpose of **A**;

B is the diagonal matrix containing n residual variances.

The mathematical procedures involved in the solution of the matrix equation are complex and their description is beyond the scope of the present text. However, details of various computing methods, geometric representations and underlying theory are given in standard texts on the subject (CATTELL, '52; THURSTONE, '47; HOLZINGER and HARMAN, '41; HARMAN, '60a, 60b; AYRES, '62; JÖRESKOG, '63; SEAL, '64; LAWLEY and MAXWELL, '63). Appendix C outlines computing algorithms for the factoring methods applied in this investigation.

In recent years a great deal of attention has been given to the mathematical and statistical requirements of factor analysis with the result that several factoring procedures, each different in approach have been proposed. The biologist, however, is concerned with the derivation of solutions that are capable of meaningful interpretation as well as being mathematically precise. One of the problems associated with the biometric use of factor analysis lies in the selection of appropriate factor models and computing techniques that will satisfy both the mathematical and biological requirements. It is perhaps unfortunate that developments in computing methods have not been matched by a clearer understanding of the properties of factor analysis applied in biological situations.

To compare the suitability of different factoring procedures, six methods were used to analyse correlation matrices derived from biometric data (BROWN, '67a). The six methods used were: principal components preserving only the major components, principal factor analysis, image-covariance analysis, Jöreskog factor analysis, iterative principal factor analysis and maximum likelihood factor analysis. The precision of the methods was assessed by the accuracy with which the original correlations could be reproduced from the factor coefficients. It was shown that the maximum likelihood method of Lawley (LAWLEY and MAXWELL, '63, p10-27) was the most precise mathematically but the technique proposed by JÖRESKOG ('63) and an iterative procedure based on the well known principal factor method (SEAL, '64, p187) led to solutions that could be accepted as adequate. Although only these three solutions showed a high degree of mathematical precision, all techniques resulted in similar patterns of factor coefficients and so far as factor recognition is concerned, the choice of factoring method appeared to be relatively unimportant. Nevertheless, the more efficient procedures should be used providing computing facilities are available, particularly if a clearer interpretation of factors is desired.

In the present study, three different factoring methods were used in the manner and combination described in Chapter 5. The methods are briefly outlined below.

Principal factor analysis (HARMAN, '60a, p154-191)*

Principal factor analysis, which has largely replaced the once popular but less precise centroid method, is probably the most widely applied of all current factoring methods. Before commencing the analysis, the unit diagonal elements of the correlation matrix are replaced with estimates of the communalities of the variables. It is usual to accept for these estimates the squared multiple correlations of each variable with the remainder in the set, these values being determined from the inverse of the correlation matrix.

The factoring procedure obtains the eigenvalues and eigenvectors of the modified correlation matrix and by a simple normalisation the factor coefficients are obtained. The communalities of the variables can be re-estimated from the factor coefficients and, if a more precise solution is warranted, the procedure is repeated a number of times with successively closer approximations to the communalities until these converge to stable values.

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* Detailed descriptions, mathematical derivations and computing procedures are given in the texts referred to above. In addition, the computer programmes coded by the author in FORTRAN IV together with computing algorithms and instructions for usage have been placed in the libraries of the Computing Section, C.S.I.R.O., Canberra and the Department of Computing Science, University of Adelaide (BROWN, '65b, 67b). The principal factor analysis programme used by SOLOW ('66) was made available for the stages of the investigation carried out in Denmark.

Jöreskog factor analysis (JÖRESKOG, '63)

In the Jöreskog analysis specifications are added to the basic factor model to make it more determinate; the residual variances are assumed to be proportional to the diagonal elements of the population correlation matrix. The computing procedure entails a rescaling of the correlation matrix prior to eigenvalue extraction. Unlike principal factor analysis, all eigenvalues of the rescaled Jöreskog matrix are positive and any number of factors judged to be significant can be retained. The method has a great advantage over principal factor analysis in that it is non-iterative and therefore can be rapidly carried out on a computer and, in addition, the controversial question of selecting initial communality estimates is avoided. Moreover, the final factor coefficients are very close in value to those obtainable by a maximum likelihood estimation.

Maximum likelihood factor analysis (LAWLEY and MAXWELL, '63, p10)

Lawley's method has not been widely applied because of the complexity and magnitude of the calculations involved. However, with the increasing availability of digital computers it can be expected to enjoy more frequent use.

Maximum likelihood estimation uses the sample correlation matrix to derive a set of consistent and efficient estimates of the unknown population factor coefficients and residual variances.

It does this by first accepting a set of trial values for the unknown coefficients and proceeding through a number of iterations until convergence of successive solutions within a desirable level of tolerance is obtained.

In practice it is desirable to select an initial set of factor coefficients which are close in value to the ones expected and, although theoretically almost any set of trial values will suffice, experience has shown that those derived by the Jöreskog method are close approximations and lead to a more rapid and complete convergence of the likelihood coefficients.

While it is possible to interpret the initial solutions derived by any of the above methods, the interpretation is simplified and appears to have more biological meaning when a transformation of the factor matrix is carried out in which the factor coefficients are redistributed over a set of new rotated factors. The geometric principles and underlying assumptions involved in factor rotation are discussed by HARMAN ('60a, p233-288).

Transformations of a factor matrix may take the form of an orthogonal rotation where the factors remain uncorrelated or an oblique rotation where factors are correlated. The choice between orthogonal and oblique final solutions is made by the analyst, but insufficient evidence is available to allow any general conclusions about the relative merits of these transformations under varying biological conditions. CATTELL ('65a) has discussed the problems of factor

rotation and SOLOW ('66) referred to them in relation to the vector configuration of overlapping or independent variables. In the present study orthogonal transformations of the initial solutions were carried out in every instance by the varimax method of KAISER ('58).

The "correct" number of factors operating in a biological situation is not easily determined and the choice of how many factors to retain will be influenced by the object of the analysis and the mathematical model chosen. Usually it has been the practice to adopt a minimum rank model in which the correlations between variables can be satisfactorily reproduced from a minimum number of factor coefficients. The number of common factors is then made equal to the minimum rank of the factored matrix. A solution based on this model is best achieved by repeated iterative procedures wherein the number of factors is gradually increased until a mathematically acceptable solution is obtained. In this regard, statistical tests are available to assess the "significance" of successive factors but it is desirable to carry out preliminary analyses that allow an estimate of the probable number of factors to be made.

It is, however, by no means certain that the minimum rank model is the most suitable for anthropometric data even though it is used frequently in other fields. As CATTELL ('65a) has pointed out, it may be erroneous to minimise the number of factors in a complex biological situation and as SOLOW ('66) has shown, the analyst may

find it more useful to extract as many factors as the mathematical situation allows, followed by a process of transformation, sorting and subsequent discard of the factors judged to have little biological importance.

Biometric applications of factor analysis

Since the foundations of factor analysis were laid early in the century by PEARSON ('01) and SPEARMAN ('04), the method has enjoyed wide acceptance in psychology as a method for determining patterns of human behavior. Although factor analysis has been employed in other fields, its application in the treatment of anthropometric data is still limited except for research into human constitution.

Reference to factorial studies of human body-build has been made by TANNER ('47), HOWELLS ('51; '52), HUNT ('52) and HAMMOND ('57a; 57b). Other biologically orientated factor studies have been carried out by ROBINOW ('42) who investigated the time of appearance of human ossification centres in the extremeties, by ROBINOW, RICHARDS and ANDERSON ('42) who grouped deciduous teeth according to their times of eruption and related tooth eruption to general skeletal maturation, and by KRAUS and CHOI ('58) who used factor analysis to determine whether growth of the foetal skeleton was influenced by single or multiple regulatory fields. Recently principal components

and other multivariate analyses were applied to dental problems by HARRIS ('65) to distinguish craniofacial patterns associated with Class II malocclusions of the teeth.

Apart from the studies referred to above, factor analysis has been used in anthropometry to investigate associations between components of the human craniofacial skeleton. A review of research along these lines is included to illustrate the method and the information available from its use. Interpretation of solutions is probably the most controversial aspect of factor analysis and, as more experience in its use is gained, it can be expected that current practices in factor interpretation will change. In the following outline the interpretations made by the various authors are presented with little comment.

HOWELLS ('51) analysed a set of 20 variables, including general body and head dimensions, that were measured on 76 brother pairs from Wisconsin University. The seven centroid factors obtained by analysis of the correlations among the variables were transformed into oblique positions before interpretation. The factors were taken to represent the following: general body size, long bone length, cranial size, brain size, lateral craniofacial development, facial length and ear size. The author discussed the complexity of the experimental variables in terms of the factor solution, pointing out that head length was associated with general head size and not with brain size, whereas head breadth was strongly associated with brain

size. He also computed three second order factors from the inter-correlations among the oblique primary factors.

In a later study HOWELLS ('53) continued the factorial study of the Wisconsin students and used regression formulae to calculate factor scores for each of the 152 students. Correlations of brothers in the 20 variables were compared with the correlations of brothers in the factor scores on seven factors. After finding that the correlations of brothers were increased when measured on factor scores, he proposed that these scores might form a more useful basis in genetic studies than direct anthropometric measurements.

HOWELLS ('57) applied factor analysis to a study of cranial vault morphology. He obtained 54 measurements, selected to represent the size and shape of the cranial vault, from contour tracings of 100 crania. As an initial step the matrix of partial correlation coefficients with the effects of cranial length, breadth and height removed was computed. Thus, length, breadth and height of the cranium were regarded as factors of general size and the subsequent analysis revealed seven secondary factors independent of the first three.

The secondary factors were taken to indicate variations in the following regions: supraorbital ridges, forehead breadth, frontal height, parietal fullness, obelionic height, fullness of the lower occiput and breadth at the base of the auricular meatus. Howells

interpreted his findings in the light of other experimental growth studies. He suggested that cranial morphology was determined by two distinct influences, skull growth which was mainly in the antero-posterior direction and brain growth in the transverse direction. These two influences determined the general cranial form and the degree of brachycephaly or dolichocephaly. The secondary factors were associated with variations in local regions and were believed to represent regional growth patterns resulting from influences such as muscle attachments or remodelling adjustments.

SCHWIDETZKY ('59) used factor analysis to investigate morphological associations in a large group of skulls from the Canary Islands. The experimental variables, 39 in number, included 24 direct measurements and 15 derived indices. Ten common factors were retained from the analysis. A factor of facial robustness contributed 22.2 per cent to the total variance with significant loadings on measures of frontal bone slope, supraorbital ridge development, mandibular robustness and eversion of the gonial angle. The second factor, termed one of cranial breadth, contributed 14.1 per cent to the common variance. The remaining factors accounted for progressively less of the common variance and were concerned with local regions of the skull; they indicated variations in facial breadth, zygomatic breadth, nasal breadth, nasal prominence, nasal shape, mandibular robustness, frontal bone shape and mandibular prognathism. After applying an oblique rotation, Schwidetzky was

unable to demonstrate any high correlations between the factors and this was taken as evidence of morphological independence between regions of the skull.

LANDAUER ('62) undertook a centroid analysis of data collected from 70 Egyptian skulls. The analysis, based on 23 variables, showed factors of general size, skeletal mass and robustness, zygomatic breadth, frontal fullness and lower facial breadth. A significant finding was that three of the factors seemed to operate in the zygomatic area but in different ways. Landauer applied oblique rotation to the factor matrix but was unable to demonstrate any correlations of great interest between the factors.

Factor analysis was applied by BROWN, BARRETT and DARROCH ('65a) in a study of associations between eight variables representing head and general body dimensions, obtained from a group of 58 Central Australian Aboriginals. Principal factor analysis followed by varimax rotation revealed three common factors that were interpreted as follows: a head length factor with loadings also for head circumference, bizygomatic diameter, weight and femoral condyle diameter; a factor of general skeletal length determined by stature, radius length, femoral condyle diameter and weight; a factor of head breadth with loadings also on head circumference, bizygomatic diameter and weight.

In a second factor study (BROWN, BARRETT and DARROCH, '65b) two ethnic groups, Swedes and Australian Aboriginals, were compared by

the analysis of 11 cephalometric variables measured on roentgenograms. Matrices of correlation coefficients among the variables were obtained from previous investigations of the two groups (LINDEGÅRD, '53; BROWN, '65a) and the analyses were carried out by the principal factor method followed by varimax rotation.

Five common factors were retained in each group to account for the major sources of covariation and a comparison of the two varimax solutions, by the use of coefficients of congruence, showed a general similarity in the patterns of four of the five factors. This was taken to add some validity to the biological interpretation of the factors. The common factors were interpreted as follows: a mandibular length factor, an anterior nasal factor, a posterior nasal factor, a ramus height factor and a cranial base factor. The study, however, was limited by the number of variables common to each ethnic group and could not provide any detailed information. It was, nevertheless, the first attempt made to compare craniofacial morphology in two ethnic groups by the factor analysis of data obtained from cephalometric roentgenograms.

Current status and future trends

The factor investigations referred to above represent early applications of a controversial method to analyse sources of covariation

among components of the human skull. There is a general similarity in many of the factors disclosed by different studies and although this adds some validity to the method, it is premature to propose any general principles governing morphological coordination in the skull. Many aspects of factor analysis require further study. In particular, the precise manner in which the selection of variables can influence a factor solution, the relative merits of orthogonal and oblique transformations, the degree of mathematical precision required, and methods for the objective recognition and interpretation of valid factors are topics in need of clarification. TANNER ('64) and CATTELL ('65a; 65b) have referred to some of these problems and recently SOLOW ('66) studied several biological aspects of correlation and factor analysis. He developed systematic techniques that depart from usually accepted procedures and provide the biometrician with a more penetrating method than was available previously.

The main departure of Solow's work was in the method developed for handling overlapping variables, that is those which represent the same general source of variability and become located within a narrow vector bundle in the factor space. This author pointed out the effect on a factor solution of adding or omitting overlapping variables and, furthermore, he indicated the effectiveness of oblique and orthogonal transformations in representing the relationships between sources of variation. In order to avoid inconsistencies Solow used a method based on the omission of overlapping variables. Initially 88 variables were chosen to represent sources of variation in general

skeletal dimensions, cephalometric dimensions and dimensions of the teeth and dental arches. A preliminary factor analysis of all variables was supported by a series of analyses of the correlation matrix partitioned into smaller groups of related variables. Overlapping variables, recognised in each analysis by similarity in their patterns of factor coefficients, were subsequently discarded when this was deemed advisable. Finally 38 variables were retained to represent the sources of variation considered worthwhile in biological interest. Many overlapping variables and, in some instances, groups of overlapping variables were discarded while at other times overlapping variables were deliberately retained to establish a known source of variation.

Principal factor analysis of the 38 retained variables was followed by varimax rotation. Nineteen common factors were interpreted as follows: the postcranial skeleton was represented by a factor for length and one for breadth; eight factors were determined by variations in cranial and facial dimensions and of these three were concerned with the cranial base region; nine factors were located by variables describing dimensions of the teeth and dental arches. The final interpretation of the patterns of craniofacial associations was based on the correlations among the variables and the findings of the factor analyses.

Clear comparisons between the available factor studies of craniofacial morphology are complicated by the use of dissimilar variables, different mathematical approaches and different objectives.

Although many disclosed factors appear to represent biological coordinating mechanisms, it is not possible to positively identify them with genetical, hormonal or environmental influences. Apart from the work of HOWELLS ('53), little interest has been shown in quantifying factors by the estimation of factor scores. It seems reasonable to suggest that further development along these lines is required to clarify the scope and limitations of factor analysis in anthropometric research.

There is little doubt that future applications of factor analysis will depart in many ways from those in current use. When more is known of the method itself and indications for its use, it may be possible to provide connecting links between various approaches to the study of craniofacial growth and morphology.

In order to summarise the progress made to date, a brief survey of the factor studies referred to is given in Table 5.

Table 5. Summary of previous studies of craniofacial morphology showing factor designations and interpretations

HOWELLS ('51) Sample : 152 Male Wisconsin students
Variables : 7 General body, 13 head

V-2	General size	V-14	Lateral craniofacial development
V-5	Long bone factor	V-17	Facial length (upper facial height)
V-9	General cranial size	V-20	Ear size
V-10	Head breadth (brain size)		

HOWELLS ('57) Sample : 100 Male crania
Variables : 54 Cranial contours

V-34	Dolichocephaly	V-13	Breadth across frontal angles
V-19	Mid-parietal breadth	V-15	Forehead breadth
V-50	Brain size (unspecific)	V-22	Occipital fullness
V-9	Upper parietal fullness	V-25	Frontal fullness
V-11	Breadth at base of skull	V-33	Obelionic fullness

SCHWIDETSKY ('59) Sample : 1290 Male Canary Island skulls
Variables : 24 direct measurements, 15 indices

A	Facial robustness Cranial breadth	F	Nasal factor 1
		G	Nasal factor 2
C	Facial breadth (upper)	H	Mandibular robustness
D	Facial Breadth (zygomatic)	I	Frontal bone slope
E	Nasal breadth	K	Prognathism

LANDAUER ('62) Sample : 70 Male and female crania
Variables : 23 Craniometric

V-19	Brain factor (general volume)	V-17	Frontal fullness
V-21	Cranial ruggedness (general size)	V-9	Lower facial breadth (naso-maxillary)
V-3	Facial breadth		

BROWN, BARRETT and DARROCH ('65a) Sample : 58 Male and female Australian Aboriginals
Variables : 4 General body, 4 head

I	Head length	III	Head breadth
II	General skeletal length		

BROWN, BARRETT and DARROCH ('65b) Sample : 243 Swedish males
Variables : 11 Roentgenographic dimensions*

I-S	Mandibular length	IV-S	Ramus height
II-S	Anterior nasal factor	V-S	Anterior cranial base
III-S	Posterior nasal factor		

* Matrix of correlation coefficients obtained from LINDEGÅRD ('53)

BROWN, BARRETT and DARROCH ('65b) Sample : 58 Male and female Australian Aboriginals
Variables : 11 Roentgenographic dimensions

I-A	Mandibular size	IV-A	Anterior nasal factor
II-A	Posterior nasal factor	V-A	Anterior cranial base
III-A	Cranial base angulation		

SOLOW ('66) Sample : 102 Young adult Danish males
Variables : 88 Dimensions obtained directly, from dental casts or from radiographs

1	Extremity length	11	Upper incisor inclination
2	Extremity width	12	Lower incisor inclination
3	Anterior cranial base	13	Tooth size
4	Facial width	14	Dental arch width
5	Clivus length	15	Maxillary arch inclination (buccal)
6	Mandibular length	16	Mandibular arch inclination (buccal)
7	Cranial base flexion	17	Dental occlusion
8	Maxillary prognathism	18	Spacing of teeth
9	Anterior maxillary height	19	Mesial molar occlusion - tooth crowding
10	Mandibular inclination		

4

STATISTICAL DESCRIPTION OF VARIABLES

The present investigation was designed for multivariate analysis and detailed descriptions of the metric characters would be superfluous, particularly as they have been dealt with at great length in previous studies of the Australian skull. Therefore the computed parameters for the variables are presented as statistical summaries and illustrated by diagrams constructed from the mean values for the dimensions analysed.

Although only 12 skulls from the northern coastal region of Australia were included in the sample, a comparison between these and the remainder has been made to throw light on the question of regional variations in Australian crania and to detect any indication that the two groups should not be pooled for the multivariate procedures. The data also permit a comparison to be made between the present findings, those derived from previous craniometric studies of the Australian and those obtained from a roentgenographic investigation of young adult male Aboriginals living at Yuendumu in Central Australia.

In the past little attention has been given to the comparative morphology of the bony nasopharynx and it was considered worthwhile

to compare the present results with those obtained by BERGLAND ('63) for Norwegian and Lapp skulls, particularly as evidence points to a close relation between this region and adjacent bony structures during growth.

Results

Statistical estimates of parameters describing the 77 variables included in this phase of the investigation are shown for the complete skull sample in Table 6. The parameters for linear variables are presented in millimetres, those for angular variables in degrees, and those for indices as percentages. Figure 9, which illustrates the craniofacial characters of the Australian Aboriginal skull, was constructed from the average values of the dimensions studied.

Discussion

A. Regional variations in Australian skulls

For this aspect of the investigation, the sample of skulls was divided into two sub-groups; Group A consisted of 12 skulls from Melville Island and the north coast, while the specimens from all other parts of the continent formed Group B. Statistical para-

meters for the 77 variables were calculated for each group separately and the mean values compared. Table 7 summarises the findings and presents the parameters for variables with mean values different in the two groups at a probability level of five per cent or less. Variables with mean values that did not differ significantly between groups have been omitted.

Of the 77 variables compared, 10 had mean values differing between groups at the one per cent level of probability and 16 had mean values differing at the five per cent level. By far the greatest number of significant differences was found in the group of variables representing size and shape of the cranial vault and cranial base. The basal chord length and most of the basal indices were greater in Group A skulls than in Group B indicating that the basal endocranium was more convex in the median sagittal plane in the northern skulls.

There are no differences of interest between groups in the measures of the frontal and parietal endocranium contours, but the angle B-C-D, which indirectly indicates general curvature of the cranial vault, was smaller in the northern group pointing to a tendency towards greater cranial vault curvature. This tendency was also demonstrated by greater mean values in Group A skulls for endocranial height (128.5 mm compared with 125.7 mm) and auriculo-vertex height (111.5 mm compared with 109.8 mm). The differences in these means, however, were non-significant. Endocranial and cranial lengths and breadths were smaller in the northern skulls.

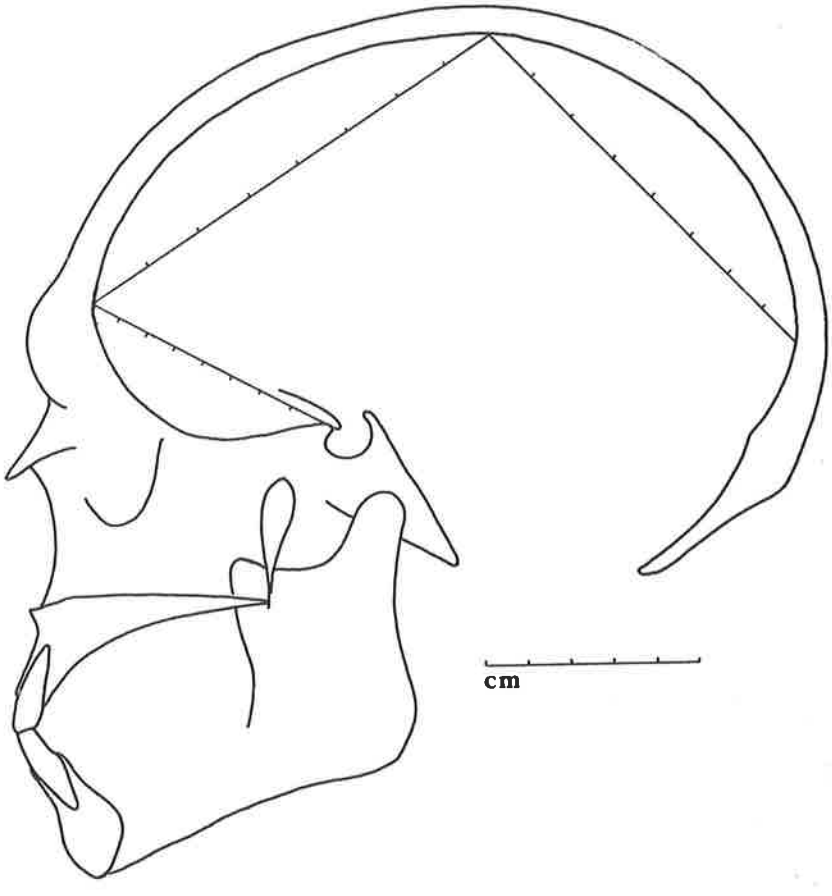


FIGURE 9. Craniofacial pattern constructed from mean values for 100 Australian Aboriginal skulls.

Table 6. (Continued)

	Variable	Mean	ϵ(M)	s	Range Min.	Max.	Skewness $\sqrt{b_1}$	Kurtosis b_2
25	f. sinus h	26.0	.87	8.7	10.0	51.0	.27	2.49
26	f. sinus b	37.7	1.24	12.4	18.9	72.6	.77**	2.98
27	sphen. d	15.0	.18	1.8	10.5	19.5	.05	3.14
28	nasal b	26.8	.18	1.8	22.5	31.5	-.12	3.06
29	ss-pns	55.3	.29	2.9	48.0	63.0	.01	3.14
30	n-sp	49.2	.28	2.8	41.5	56.0	-.21	2.97
31	ba-pns	42.4	.28	2.8	37.0	51.5	.49*	3.27
32	tph-pns	35.2	.26	2.6	29.5	44.5	.54*	3.76
33	scp-scp	27.0	.20	2.0	23.0	31.5	.02	2.51
34	phar. h	16.6	.17	1.7	13.0	20.5	.08	2.71
35	s-pm hor.	15.6	.26	2.6	5.5	22.0	-.44*	4.36*
36	s-pm vert.	40.6	.24	2.4	35.5	47.5	.08	2.93
37	n-gn	112.4	.65	6.5	94.0	132.0	.19	3.44
38	zm-zm	94.1	.45	4.5	82.5	103.5	-.18	2.87
39	zg-zg	135.0	.41	4.1	126.5	147.0	.11	2.89
40	mass. b	33.6	.62	6.2	19.5	51.0	.18	3.25
41	ecm-ecm	66.7	.31	3.1	60.5	76.0	.36	2.84
42	palate b	40.6	.26	2.6	35.0	47.5	.30	2.56
43	palate l	52.3	.30	3.0	46.0	60.5	.18	2.88
44	palate h	12.6	.23	2.3	7.0	18.0	.22	2.90
45	go-go	101.5	.66	6.6	85.5	116.5	-.17	2.74
46	gn-go	78.5	.46	4.6	64.5	93.5	.11	3.75
47	gn-cd	114.6	.45	4.5	101.0	125.5	-.38	3.20
48	ramus b	35.0	.32	3.2	26.5	44.0	.07	3.04
49	ramus h	51.5	.37	3.7	43.0	62.5	.55*	3.27
50	infra t.f.d.	27.0	.24	2.4	22.0	34.0	.28	2.87
51	s-n-ss	86.8	.43	4.3	78.0	96.0	.07	2.25*
52	s-n-pg	83.2	.38	3.8	72.5	95.0	.17	3.62
53	s-n-pr	92.0	.36	3.6	83.0	99.5	.02	2.80
54	s-n-id	87.4	.37	3.7	77.5	97.5	.08	3.39
55	n-ss-pg	170.7	.56	5.6	156.0	184.5	-.03	2.64
56	ar-tgo-gn	114.5	.61	6.1	100.0	133.0	.16	3.05
57	NL/NSL	7.8	.31	3.1	1.0	15.0	-.01	2.74
58	ML/NSL	26.6	.62	6.2	12.5	47.0	.39*	3.61
59	NL/ML	18.5	.56	5.6	5.5	33.0	.14	2.62

** Skewness or kurtosis significant at $p < .01$

* Skewness or kurtosis significant at $p < .05$

Table 7. Regional variations in craniofacial dimensions of Australian skulls. Group A = 12 skulls from northern coastal region, Group B = 88 skulls from rest of continent

Variable		Mean	$\epsilon(\bar{x})$	s	p
1 Basal chord	A	61.5	1.00	3.5	*
	B	58.9	0.39	3.7	
2 Basal index 3	A	29.1	1.09	3.8	*
	B	26.6	0.40	3.8	
Basal index 4	A	31.8	0.75	2.6	**
	B	29.0	0.35	3.3	
Basal index 5	A	31.0	0.79	2.7	**
	B	28.3	0.31	2.9	
Basal index 6	A	26.9	0.88	3.0	*
	B	24.7	0.30	2.8	
Basal index 7	A	18.1	0.61	2.1	*
	B	16.6	0.23	2.1	
5 Frontal index 2	A	16.4	0.38	1.3	*
	B	15.6	0.13	1.2	
6 B-C-D	A	99.3	1.04	3.6	**
	B	101.7	0.27	2.6	
9 endo. 1	A	163.2	2.03	7.0	*
	B	167.8	0.59	5.5	
10 endo. b	A	116.0	1.60	5.6	**
	B	122.2	0.63	5.9	
12 g-op	A	182.9	1.75	6.1	**
	B	188.0	0.54	5.1	
13 eu-eu	A	126.0	1.38	4.8	**
	B	132.2	0.52	4.9	
15 n-eth	A	35.1	0.57	2.0	**
	B	37.9	0.29	2.7	
16 eth-s	A	34.2	0.58	2.0	**
	B	32.3	0.25	2.3	

* Difference in mean value significant at $p < .05$
** Difference in mean value significant at $p < .01$

Table 7. Continued

Variable		Mean	$\epsilon(\bar{x})$	s	p
18 s-ba	A	42.5	0.88	3.0	*
	B	40.9	0.27	2.5	
20 n-s-ba	A	131.6	1.84	6.4	**
	B	136.0	0.53	5.0	
26 f. sinus b	A	45.0	4.8	16.6	*
	B	36.7	1.2	11.4	
28 nasal b	A	27.8	0.40	1.4	*
	B	26.6	0.19	1.8	
29 ss-pns	A	53.5	0.70	2.4	*
	B	55.5	0.30	2.8	
36 s-pm vert.	A	41.9	0.44	1.5	*
	B	40.4	0.26	2.4	
37 n-gn	A	108.5	2.00	6.9	*
	B	113.0	0.67	6.3	
43 palate l	A	50.3	0.64	2.2	*
	B	52.6	0.42	3.0	
44 palate h	A	14.2	0.77	2.7	*
	B	12.4	0.23	2.2	
49 ramus h	A	53.6	1.23	4.3	*
	B	51.2	0.38	3.5	
57 NL/ NSL	A	5.5	1.14	4.0	**
	B	8.1	0.30	2.8	
58 ML/NSL	A	23.3	1.78	6.2	*
	B	27.1	0.65	6.1	

* Difference in mean value significant at $p < .05$
** Difference in mean value significant at $p < .01$

Although the anterior cranial base length (n-s) did not differ between groups, its component lengths n-eth and eth-s did, probably as a result of positional variation in ethmoidale brought about by the greater convexity of the anterior cranial fossa in the northern skulls. No explanation can be offered for the significant differences in mean values of the median cranial base angle and the frontal sinus breadth.

There were relatively few significant differences between Groups A and B in the size and shape dimensions of the facial skeleton, and only one of these differences was significant at the one per cent level of probability. Nasal breadth was slightly greater and nasal depth shorter in Group A skulls, and the palate was shorter and its vault higher than in Group B. The finding that morphological face height was shorter in Group A was associated with group differences in nasal floor and mandibular base inclinations. The only group difference found in the lower face was for the variable ramus height which was slightly greater in the northern skulls.

The comparisons taken as a whole indicate a general similarity in the dimensions of the facial skeleton but some differences in the cranial vault regions of the two skull groups. These differences were not marked, being most apparent in the variables cranial length and breadth which were smaller in the northern skulls, and in the sagittal shape of the basal endocranial segment which was more convex in the northern specimens. Although obtained from a small sample,

the results reveal regional variations in Australian crania similar to those described by earlier investigators (Chapter 1). However, the differences in most mean values were numerically small and there appeared to be no strong objection to pooling the entire data for the correlation and multivariate phases of the study, particularly in view of the similarity in facial dimensions of the two groups.

B. Comparison with previous craniometric studies

The comparison between mean values for craniofacial variables derived from the present and previous studies of Australian material (Table 8) shows close agreement between the two sets of values. Of the 15 comparisons available only four variables had mean values that differed by more than one mm, and only one, auriculo-vertex height, differed by more than two mm.

Unfortunately, standard deviations were not available for many variables included in the earlier studies and statistical analysis of the mean differences could not be carried out. However, it can be assumed that the small differences found could reasonably stem from dissimilar measuring techniques or disparities in sample sizes rather than morphological differences between the skull groups studied. The values reported in the present text can therefore be taken as reliable estimates for statistical parameters of the pre-European Australian skull.

Table 8. Comparison of craniofacial dimensions in present and previous studies of Australian skulls

Variable	Previous			Present	
	Mean	s	N	Mean	s
Exocranium					
g-op	187.8	6.7	82	187.4	5.4
eu-eu	132.2	5.0	162	131.5	5.3
po-v	115.0	-	13	110.0	5.2
Cranial base					
n-ba	102.1	4.3	137	101.4	3.8
Nasal cavity					
nasal b	26.9	2.0	120	26.8	1.8
n-sp	49.5	3.1	118	49.2	2.8
Upper face					
zm-zm	93.9	-	37	94.1	4.5
zg-zg	133.6	6.1	139	135.0	4.1
ecm-ecm	65.8	3.3	55	66.7	3.1
palate b	41.1	-	28	40.6	2.6
palate l	51.5	-	106*	52.3	3.0
palate h	11.0	-	90*	12.6	2.3
Lower face					
ramus b	35.4	-	107**	35.0	3.2
ramus h	52.5	-	107**	51.5	3.7
Total face					
n-gn	113.7	-	292†	112.4	6.5

*CAMPBELL ('25); **MURPHY ('57b); †HRDLIČKA ('28)

All other values obtained from MORANT ('27)

Table 9. Comparison of craniofacial dimensions in museum material and young adult male Aboriginals from Yuendumu, Central Australia

Variable	Yuendumu N = 31		Museum N = 100		Probability	
	Mean	s	Mean	s	s^2	Mean
Cranial base						
n-s	70.5	3.2	68.8	2.7		**
s-ba	45.5	3.3	41.1	2.6	*	**
n-ba	105.4	4.2	101.4	3.8		**
n-s-ba	129.6	4.2	135.5	5.3		**
for. angle	91.7	5.2	92.6	4.2		
Nasal cavity						
ss-pns	52.1	2.9	55.3	2.9		**
n-sp	49.4	4.0	49.2	2.8	**	
s-pm hor.	17.1	2.5	15.6	2.6		**
s-pm vert.	42.9	2.5	40.6	2.4		**
Total face						
n-gn	119.1	7.1	112.4	6.5		**
Facial shape						
s-n-ss	87.1	3.8	86.8	4.3		
s-n-pg	81.3	4.0	83.2	3.8		*
s-n-pr	91.8	3.7	92.0	3.6		
s-n-id	86.5	3.9	87.4	3.7		
n-ss-pg	169.1	5.1	170.7	5.6		
ar-tgo-gn	120.7	5.9	114.5	6.1		**
NL/NSL	6.9	3.5	7.8	3.1		
ML/NSL	32.0	4.8	26.6	6.2		**
NL/ML	25.1	3.9	18.5	5.6	*	**

* Difference in means or variances significant at $p < 0.05$

** Difference in means or variances significant at $p < 0.01$

C. Comparison with Aboriginals from Yuendumu

Data relating to adult male members of an Aboriginal tribe were presented by BROWN ('65a). Of the 19 dimensions available for comparison (Table 9), three showed statistically significant differences in variances between the pre-European and the Yuendumu Aboriginals. Of these, the variances for n-sp differed at the one per cent level while for s-ba and NL/ML the variances differed at the five per cent level.

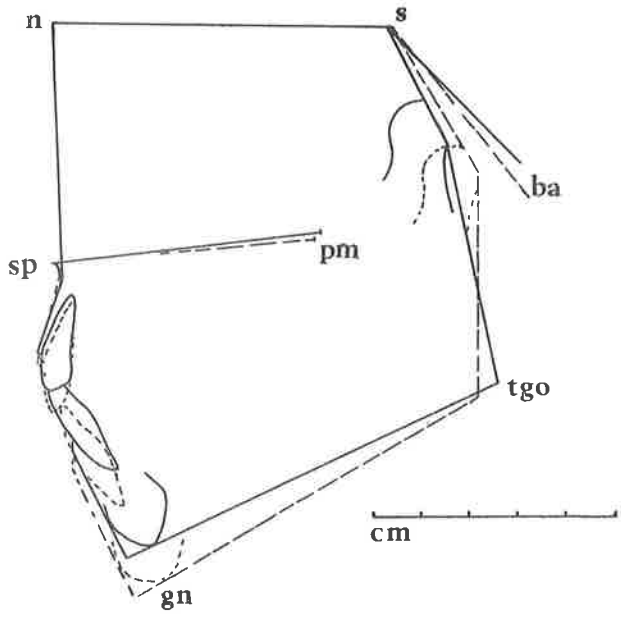
Mean values for all linear dimensions except n-sp differed significantly between the two groups at the one per cent level; the variable ss-pns had a mean value that was smaller in the Yuendumu group, whereas the other means were greater than in the museum material. The dimension ss-pns, however, is not strictly comparable in the two groups; it was measured directly on the skulls between points subspinale and posterior nasal spine, whereas in the Yuendumu subjects profile roentgenograms were used and the point pterygo-maxillare determined the posterior limit of the dimension. For the angular variables, the mean values for s-n-pg differed between groups at the five per cent level, and four others, n-s-ba, ar-tgo-gn, ML/NSL and NL/ML had mean values differing at the one per cent level.

In general, the differences in skull form between the museum material and the Yuendumu Aboriginals were in overall size, which was greater in the Yuendumu subjects, and in mandibular prognathism which was greater in the museum specimens.

Mandibular form, expressed by variables ar-tgo-gn, ML/NSL and NL/ML, differed in the two groups; the mandibular base was more acutely inclined to the cranial base and the gonial angle was smaller in the museum skulls. These differences are illustrated in Figure 10 which was constructed from the group mean values.

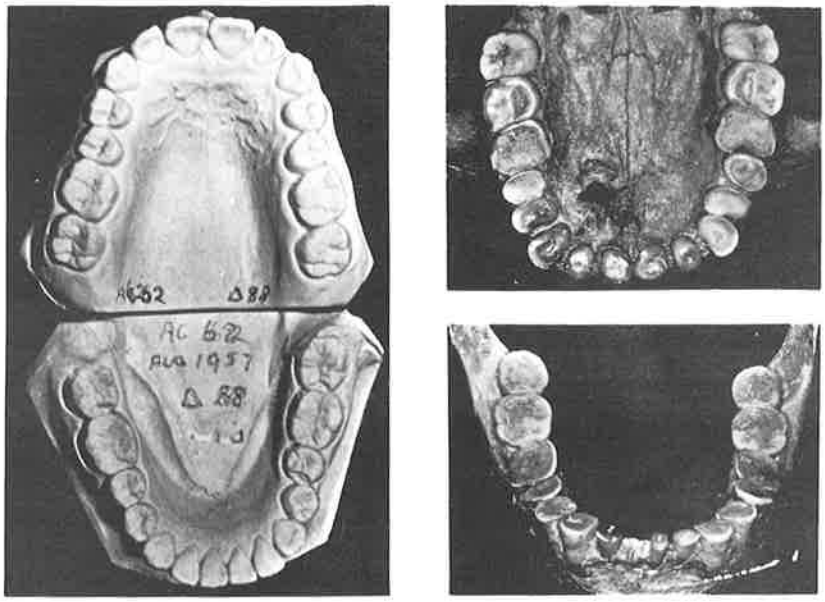
Many of the differences found could reasonably be ascribed to post-mortem shrinkage in the museum material. However, the possibility of distinct morphological differences should not be overlooked even though additional information is required before light can be shed on the nature of the changes, if any, that have occurred in recent generations of Aboriginals.

On the one hand, the Yuendumu group was small in number, represented a single Central Australian tribe and probably displayed greater genetic homogeneity than the skull sample drawn from several regions of Australia. The differences may therefore be due to distinct regional variations. On the other hand, the differences may indicate changing patterns of facial growth, particularly if it is borne in mind that the Yuendumu group of young adult males had received the benefit of improved nutrition during most of their growing period. It is unfortunate that reliable information on the nutritive content of food taken by Aboriginals living under nomadic conditions is scarce and that opportunities for studying growth in these people no longer exist.



—— 100 Aboriginal skulls
----- 31 Young adult Aboriginal males

FIGURE 10. Comparison of mean craniofacial patterns.



A B

A Dental casts of a Central Australian Aborigine
B Dentition of a specimen from the S.A.Museum

FIGURE 11. Tooth attrition.

The effects of tooth attrition on facial morphology may also contribute to the group differences, particularly the reduction in facial height and the development of an edge-to-edge incisor bite which is characteristic of many mature museum specimens. In many instances the skulls showed severe occlusal and interproximal tooth attrition but in contrast tooth attrition was slight in the Yuendumu subjects (Figure 11). Although the two groups had almost identical values for upper face height there was a mean difference of 6.7 mm in the values for morphological face height. The facial height difference was therefore confined to the subnasal region and could reasonably be explained by marked occlusal tooth attrition in the museum group. The findings are similar to those of MURPHY ('59) who described in some detail the changes in several facial height dimensions consequent upon marked loss of tooth substance through attrition.

The group difference of two degrees in mandibular prognathism could also be associated with severe tooth attrition in the museum skulls. It has been observed that Australian skulls exhibiting gross attrition usually show morphological changes in the temporo-mandibular joints together with an anterior repositioning of the mandible. These changes, which have been briefly mentioned by BROWN ('65c), could partly account for the greater angle of mandibular prognathism in the museum material.

D. Comparison with Norwegian and Lapp cranial material

The findings reported by BERGLAND ('63) included observations of the bony nasopharynx in a group of Norwegian and Lapp crania from the 17th to 19th centuries. Because this region is seldom included in craniometric studies, a comparison was made with the findings for Australian Aboriginal skulls. However, Bergland's mean values were not corrected for roentgenographic enlargement and before comparisons were made, the means and variances were adjusted to compensate for the stated enlargement of 6.25 per cent. The values listed in Table 10 have been corrected in this manner and should be reasonably close to true dimensions. Because of this adjustment differences in group means and variances were not assessed statistically.

In general, the nasopharyngeal dimensions in the Australian skulls were similar to those for the Norwegian and Lapp groups. Allowing for differences in the definition of measuring points, it would seem that the Australian skulls were slightly greater in pharyngeal length, slightly smaller in pharyngeal height, but about the same in breadth and capacity of the bony nasopharynx. In this regard it is interesting to recall the findings of Bergland, who on the basis of similar values for nasopharyngeal volume in his two cranial groups, suggested a functional adjustment to respiratory requirements involving compensations in the height and depth of the nasopharynx. SOLOW ('66, p125) also found indications of a compensatory mechanism which kept nasopharyngeal volume independent of cranial base flexion.

Table 10. Comparison of craniofacial dimensions in Australian Aboriginal, Norwegian and Lapp skulls. Values recorded in mm or degrees and corrected for roentgenographic enlargement.

Variable	Norwegian N = 60*		Lapp N = 30*		Australian N = 100	
	Mean	s	Mean	s	Mean	s
Nasopharynx						
ba-pns	40.0	4.0	41.8	2.8	42.4	2.8
tph-pns	32.5	3.2	34.9	2.4	35.2	2.6
phar. h	18.0	2.1	17.6	1.9	16.6	1.7
choanal width	28.0	1.9	28.0	2.2	-	-
scp-scp	-	-	-	-	27.0	2.0
phar. capacity	9.8	-	10.3	-	9.5†	-
Cranial base						
n-s	66.2	2.7	65.0	3.0	68.8	2.7
s-ba	43.0	2.4	39.7	2.5	41.1	2.6
n-ba	99.8	4.0	98.2	3.6	101.4	3.8
n-s-ba	131.6	5.1	138.3	4.8	135.5	5.3
Upper face						
n-sp	52.1	3.6	50.5	2.4	49.2	2.8
s-n-ss	82.4	4.5	83.1	3.9	86.8	4.3

* Derived from findings presented by BERGLAND ('63)
Norwegian skulls - adult male from early 19th century
Lapp skulls - adult male from 17th and 18th centuries

† Calculated according to Bergland's formula:
pharyngeal capacity = (ba-pns).(phar.h).(choanal width). $\frac{1}{2}$

The other craniofacial variables available for comparison had similar mean values in the three populations with the exception of the angle s-n-ss which was larger in the Australian skulls. This angle, usually taken as a measure of basal maxillary prognathism, is influenced by the degree of alveolar development and was probably greater in the Australian group as a result of larger tooth dimensions or greater procumbency of the incisors.

E. Distributions of the variables

For the multivariate analyses the largest in each set of seven segmental indices was selected to represent the shape of the endocranial segments. For the basal endocranium, index 4 was retained to represent general shape; for the frontal endocranium, index 3; for the parietal endocranium, index 4. These three variables were designated basal index, frontal index and parietal index. Thus 18 indices were eliminated from future analyses.

The statistics $\sqrt{b_1}$ and b_2 were computed for each of the remaining 59 variables to indicate consistency with or departure from a normal distribution, $\sqrt{b_1}$ being the measure of skewness and b_2 the measure of leptokurtosis or platykurtosis. Eleven variables showed departures from normality statistically significant at the one or five per cent levels of probability. These findings are summarised in Table 11.

One variable, s-pm hor., showed leptokurtosis significant at the five per cent level while two variables, A-B-C and s-n-ss, showed platykurtosis significant at the five per cent level. Skewness to the left, significant at the one per cent level was displayed by the parietal index and at the five per cent level by s-pm hor. Skewness to the right, significant at the one per cent level was shown by f. sinus b, and at the five per cent level by endo. h, ramus h, ba-pns, tph-pns, n-eth and ML/NSL. Examination of the data did not provide any obvious reason for the disclosed departures from normality and it appeared that no particular cranium or group of crania was consistently responsible for the deviations. Furthermore, the departures from normality followed no set pattern although skewness to the right was the most common finding.

As SOLOW ('66, p55) has pointed out, insight into the causes of departures from normality in anthropometric data must await the presentation of distribution statistics for many other groups. No explanation can be made for the observed departures in the present material, but in view of the relatively small number of variables displaying significant skewness or kurtosis it was considered justified to include all 59 variables in the initial stages of the multivariate analysis.

Table 11. Variables showing statistically significant departures from normality

	Variable	$\sqrt{b_1}$	b_2	Type of departure
3	A-B-C	.244	2.267*	Platykurtosis
8	Parietal index	-.628**	3.436	Left skewness
11	endo. h	.464*	3.522	Right skewness
15	n-eth	.416*	3.231	Right skewness
26	f. sinus b	.769**	2.982	Right skewness
31	ba-pns	.494*	3.267	Right skewness
32	tph-pns	.536*	3.761	Right skewness
35	s-pm hor.	-.440*	4.358*	Leptokurtosis, left skewness
49	ramus h	.548*	3.272	Right skewness
51	s-n-ss	.067	2.252*	Platykurtosis
58	ML/NSL	.394*	3.607	Right skewness

* Departure from normality significant at $p < .05$

** Departure from normality significant at $p < .01$

5

MULTIVARIATE ANALYSIS OF THE AUSTRALIAN SKULL

Preliminary Factor Analyses

A brief account of the computing procedures commonly used in factor analysis was given in Chapter 3 and reference was made to the relative merits of different methods so far as mathematical precision is concerned. On the basis of previous trials with empirical data, the maximum likelihood method of factor estimation was shown to be the most precise mathematically, even though several methods compared led to similar solutions and would be equally suitable for preliminary factor interpretation. However, when factor analysis is used to examine complex biological phenomena, considerations of a more specific nature become involved. Some of the special biological aspects are discussed in the first section of this chapter.

The presence of a significant correlation coefficient between two anthropometric variables has usually been accepted as evidence of a biological coordination, but SOLOW ('66, p75) has shown that recognition of non-biological causes for associations between variables is important. Solow's work follows that of PEARSON and DAVIN ('24), WALLIS ('34) and others who discussed "spurious" correlations between indices that shared common components or

between linear variables that spanned the same anatomical region and therefore "covered" each other.

A "spurious" correlation exists in the association between bizygomatic breadth and cranial breadth (reported by Pearson and Davin as 0.54 for 72 male skulls; 0.42 in the present sample). The two dimensions span the same anatomical region and bizygomatic breadth "covers" cranial breadth so that a significant correlation between them could be expected. The correlation expresses the rather obvious morphological association between breadths of the zygomatic arches and cranial vault brought about by their anatomical proximity. This type of correlation may be less informative than one of lower magnitude existing between variables not covering each other.

Variables measured on cephalometric roentgenograms are also subject to "spurious" coordination and, moreover, a further source of non-biological correlation may arise if the variables share common reference points, reference lines or reference structures. This concept was advanced by SOLOW ('66, p77) who termed associations of this type "topographical" to distinguish them from "non-topographical" correlations between variables that did not share common points or lines.

To explain the nature of topographical associations it is reasonable to assume, first, that the variability of a linear dimension is determined by the joint variability of the two reference points used to define the variable. From this it follows that when two linear

dimensions share a reference point the variability of the common point will be included in both variables resulting in their correlation even if all three points vary independently. Angular variables sharing a common reference line will be topographically associated for similar reasons. Correlations between topographically related variables can be expected to be as high as those between "spuriously" related variables. Furthermore, the sign and approximate magnitude of topographical associations can be predicted from a knowledge of the topography involved (SOLOW, '66, p83).

An example of a topographical association from the present study is found in the observed correlation ($r = 0.66$) between the two linear variables n-s and n-ba which shared the reference point nasion. The angular variables n-s-ba and s-n-pg shared the reference line NSL and were topographically related with an observed correlation of $r = -0.52$.

The correlation between two cephalometric variables determined by common references may be conditioned by a true biological coordination as well as the topographical effect described. In these instances it is difficult to interpret the observed correlation values because no satisfactory method is available to separate the topographical and non-topographical components of an observed correlation coefficient.

Analysis of the Australian data was designed with the above considerations in mind and differed from the usual factor techniques. The procedures adopted, although more complex than many previous ones,

were flexible and allowed greater control over the experimental variables. Special attention was given to the use of correlation and factor analysis in the selection of variables to represent meaningful sources of covariation. Factoring methods have seldom been used to analyse cephalometric data and the procedures applied for variable selection and for mathematical resolution of the various correlation matrices are discussed in some detail.

Methods of analysis

Fifty-nine variables, defined in Chapter 2, were chosen as reasonable indicators for the size and shape of several anatomical regions of the skull. For convenient tabulation and reference the variables were placed in groups shown in Table 12 although the manner of grouping had no bearing on the analyses carried out.

The 1,711 correlation coefficients between these variables were computed for the first stage of the multivariate analysis. The correlation matrix was then inspected, the statistically significant coefficients identified, and a search made for correlations that could be explained, at least in part, by one of the types of non-biological coordination. The aim of the preliminary matrix inspection was to gain a clearer insight into the associations present before commencing the multivariate procedures.

Table 12. Variables included in the first stage of the multivariate analysis

CRANIAL VAULT

1. Basal chord
2. Basal index
3. A-B-C
4. Frontal chord
5. Frontal index
6. B-C-D
7. Parietal chord
8. Parietal index
9. endo. l
10. endo. b
11. endo. h
12. g-op
13. eu-eu
14. po-v

CRANIAL BASE

15. n-eth
16. eth-s
17. n-s
18. s-ba
19. n-ba
20. n-s-ba
21. eth-s-ba
22. for. angle
23. min. f
24. max. f
25. f. sinus h
26. f. sinus b

FACIAL SIZE

37. n-gn
38. zm-zm
39. zg-zg
40. mass. b
41. ecm-ecm
42. palate b
43. palate l
44. palate h
45. go-go
46. gn-gn
47. gn-cd
48. ramus b
49. ramus h
50. infra t.f.d.

FACIAL SHAPE

51. s-n-ss
52. s-n-pg
53. s-n-pr
54. s-n-id
55. n-ss-pg
56. ar-tgo-gn
57. NL/NSL
58. ML/NSL
59. NL/ML

NASAL AND NASOPHARYNGEAL DIMENSIONS

27. sphen. d
28. nasal b
29. ss-pns
30. n-sp
31. ba-pns
32. tph-pns
33. scp-scp
34. phar. h
35. s-pm hor.
36. s-pm vert.

Distinction was made between four types of correlations, depending on the anatomical relationships between the variables concerned: spurious correlations between variables that included a common component, for example basal chord and basal index; spurious correlations between variables that spanned the same or adjacent anatomical regions, for example cranial breadth and bizygomatic breadth; topographical correlations between variables sharing common reference points or lines, for example n-s-ba and s-n-pg; correlations not falling into these groups. The first three types of correlations, which can be regarded as being conditioned by topographical or non-biological situations, are termed "specious" in the present text.

In many instances it was difficult to assign a correlation to a specific group and associations between variables spanning adjacent anatomical regions were regarded as specious only if they were measured from reference points situated in fairly close proximity. To explain further, bizygomatic breadth and cranial breadth were considered to be speciously related on account of the proximity of the reference points euryon and zygion and because of the anatomical connection between zygomatic arches and cranial vault. However, bizygomatic and bigonial breadths were not considered to be speciously related even though they both measured facial breadths and could be expected to show a coefficient of correlation ($r = 0.38$ in the present study) consistent with a coordination between anatomical parts jointly influenced by the development of the masticatory musculature. Until more is known of the nature of correlations

between anthropometric variables, a grouping such as the one described must remain somewhat subjective.

Following the inspection of the correlation matrix, five factor analyses were carried out in the sequence and manner summarised in Table 13.

Analysis 1

The first analysis of all 59 variables was regarded as a preliminary exploration of the associations between the variables in which the main sources of covariation were disclosed as common factors. Trial factor loadings were derived by the method of JÖRESKOG ('63, p43), and the use of his "k-min" criterion and other tests* suggested that 23 factors would explain the major sources of variation present.

Subsequently a maximum likelihood analysis was carried out using the method of Lawley (LAWLEY and MAXWELL, '63, p10) with the trial loadings derived from the Jöreskog analysis. Finally, a varimax orthogonal transformation (also referred to as rotation) was

* It has been found useful to examine the relative magnitudes of the eigenvalues of the correlation matrix and the eigenvalues of the reduced correlation matrix, that is the matrix with estimates of the communalities inserted in the main diagonal elements, prior to factor extraction. The percentage contributions of the eigenvalues to the matrix trace provide a guide to the relative importance of the factors and the probable number of factors required to account for most of the variance present.

Table 13. Factor procedures used for the five analyses of cranio-facial associations[1]

Analysis	Number of Variables	Initial Solution	Factors Transformed	Factors Interpreted
1	59	Maximum Likelihood[2]	23	23
2	59	Principal Factor[3]	40	30
3	36	Principal Factor[3]	23	18
4	30	Principal Factor[3]	19	16
5	30	Maximum Likelihood[4]	16	16

[1] In all analyses the sample number was 100 and the transformation was carried out by the orthogonal varimax method.

[2] Approximations to the factor coefficients were derived from a preliminary analysis by the Jöreskog method. Forty iterations were made.

[3] The number of factors retained for transformation was determined by selecting positive eigenvalues of the correlation matrix reduced by inserting communality estimates in the main diagonal elements.

[4] Approximations to the factor coefficients were derived from Analysis 4. Twenty-five iterations were made.

performed on the likelihood solution by the method of KAISER ('58), and the contributions of the variables to the estimated factor variances were estimated by the methods outlined by HARMAN ('60a, p337-361). The maximum likelihood estimation is more precise mathematically than other factoring procedures, and although precision is not essential for gross factor interpretation, the method was used in the first stage of the multivariate analysis when an accurate representation of the sources of covariation was desirable.

The matrix of transformed factor coefficients was examined and rearranged for easier interpretation along the lines for factor revision described by SOLOW ('66, p101). Essentially, the rearrangement consisted of reversing the signs of the coefficients of a factor when the highest coefficients were negative, and changing the order of both factors and variables so that the variables with similar patterns of coefficients were grouped closer together than they were initially.

Interpretation of the main sources of variation was based on the examination of both the magnitude of the factor coefficients, that is the correlations between variables and factors, and the contributions of the variables to the estimated variances of the factors. It is normal practice to use only factor coefficients as a guide for factor interpretation. However, while it is true that an interpretation based on the contributions to the factor variances would,

in most instances, lead to similar conclusions, the interpretation is simplified by considering each set of values in relation to the other. It would seem that the contributions to the factor variances provide the clearer guide to the true nature of the factor. HARMAN ('60a, p347) has discussed the use of factor variance contributions in factor interpretation but as far as can be ascertained this procedure has not been applied previously in a biometric investigation.

Analysis 2

For the second analysis all 59 variables were included but in the order determined by the rearrangement carried out during the previous stage. The object of this analysis was to disclose factors of small magnitude that had not been included in the transformation procedure of Analysis 1. The number of variables included for analysis determines the total communality and, furthermore, the correlations among the variables determine the factors and their contributions to the communality. The inclusion of groups of speciously related variables would have the effect of producing factors which, although contributing significantly to the total communality, might not be as important biologically as factors determined by variables not so related. If the number of factors is minimised, according to the usually accepted minimum rank model, it becomes likely that some factors with small communality contributions would be excluded or at least overlooked. For these reasons the extraction of additional factors was deemed necessary to provide a more objective basis from

which to recognise variation sources, whether specious or otherwise.

Mathematical precision was not so essential at this stage and Analysis 2 was carried out by the principal factor method. Initial communality estimates, calculated as the squared multiple correlations of each variable with the remainder, were placed in the main diagonal elements of the correlation matrix prior to factoring. All eigenvalues greater than zero were retained to compute the coefficients for 40 common factors. Although all 40 factors were not required for final interpretation, they were rotated orthogonally by the varimax method. SOLOW ('66, p100) has shown that the pattern of varimax transformed factor coefficients is not markedly changed even when a number of factors greater than that finally retained is included in the transformation.

The transformation procedure led to a redistribution of the factor loadings and the contributions of the rotated factors to the common variance provided a clearer basis for a decision on which factors to retain as biologically significant. Twenty-nine transformed factors had contributions to the common variance diminishing from 11.2 per cent to 1.5 per cent; these factors could be readily interpreted in biological terms. The contributions of the remaining 11 transformed factors fell abruptly and only one, with a contribution of 0.8 per cent, was judged to be biologically meaningful. Accordingly 30 factors were retained for interpretation.

Interpretation of the factors in Analysis 2 was based on the magnitude of the factor coefficients but attention was given to specious associations between variables. Subsequently it was possible to select factors for inclusion in future analyses and to decide the combination of variables best suited for the factor requirements. Of the 59 original variables, 23 were eliminated either because of specious relationships or because they represented sources of minor variation. In a few instances, however, speciously related variables were deliberately retained to establish a given source of variation as a recognisable factor in future analyses. The use of a preliminary factor analysis provided a more objective basis for the retention and discard of variables than would have been possible had the selection been based solely on an examination of the correlation matrix.

Analysis 3

Principal factor analysis followed by varimax transformation was carried out on the 36 variables retained from the previous stage. After factor interpretation, the selection procedure was applied once more and with the additional knowledge now available it was feasible to eliminate a further six variables with minimal loss of information. Thirty remained and these were considered to represent the major sources of variation of interest in the skull group under examination.

Analysis 4

Analysis 4 was carried out on the 30 variables to obtain an initial set of factor coefficients that could be used as approximations for a more precise maximum likelihood estimation. Principal factor analysis followed by varimax transformation was carried out as before.

Nineteen factors were transformed and of these three were considered minor and not interpreted. Of the 16 remaining factors, one contributed only two per cent to the common variance and, although retained, its value as an indication of significant covariation was doubtful. No further reduction in the number of variables was necessary.

Analysis 5

The final analysis was carried out by Lawley's maximum likelihood method to obtain a set of factor coefficients from which the original set of correlation coefficients between the variables could be reproduced precisely. The maximum likelihood coefficients were used to compute factor scores for the 100 skulls in the sample. The procedure used and the interpretation for Analysis 5 is discussed separately in the next chapter.

Results

Correlation matrix

The matrix of correlation coefficients among the 59 variables is shown in Table 14. Specious associations are considered in Table 15 under the three main groups referred to previously; variables sharing common components, variables spanning adjacent anatomical regions, and variables sharing common reference points or lines. The list is not exhaustive including only the obvious sources of specious coordination.

The presence of a large number of specious associations was expected from the experimental design and Table 15 emphasises the complexities encountered during the interpretation of large correlation matrices computed from anthropometric data. It is stressed that the presence of a specious coordination between two variables does not preclude the possibility of additional biological coordination; it means, however, that until more is known of the nature of anthropometric associations, observed correlation values should be interpreted cautiously.

Analysis 1

Maximum likelihood estimation was used to obtain the initial set of factor coefficients. The computer program specified that the iterative procedure should cease when either all residual variances for the 59 variables had converged with a maximum difference between

successive residuals of 0.001, or when 40, the specified maximum number of iterations, had been completed. In this instance 40 iterations were performed after which the 59 residual variances had converged within 0.03, 58 had converged within 0.02 and 50 within the specified value of 0.001. The degree of convergence was accepted as adequate.

The varimax solution for Analysis 1, obtained by orthogonal transformation of the likelihood coefficients, is shown in Table 16 with loadings less than 0.15 omitted. The revised varimax solution is given in Table 17 in which the factors and variables have been rearranged according to the procedure outlined and, in addition, the factor pattern has been simplified by the exclusion of all coefficients judged to be non-significant*.

Table 18 lists the contributions of the variables to the estimated factor variances. Very small contributions, whose absolute values were less than 0.01, were omitted from the table on the assumption that these were obviously non-significant and most likely spurious, being generated during the sequence of complex arithmetic operations incorporating eigenvalue extraction and matrix inversion routines.

The magnitudes of the correlations between variables and the 23 rotated factors and the contributions of the variables to the

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* See note on the significance of factor coefficients at end of Chapter 5. (p105)

estimated factor variances were taken into account when the factors were interpreted. The interpretation summarised in Table 19 is preliminary only; because Analysis 1 was exploratory in nature no attempt has been made to assess the biological or topographical significance of the factors.

An assessment of the associations among the 59 variables revealed by Analysis 1 showed that most of the variance could be accounted for by 23 factors. However, the inclusion of speciously related variables led to the appearance of some factors that were of doubtful biological importance although they contributed significantly to the common variance.

The 23 common factors fell fairly readily into four main groups. Variations in endocranial size and shape were represented by factors 4, 7, 16, 14 and 18; cranial base dimensions by factors 12, 21, 8, 15, 17 and 11; the nasopharynx was represented in height by factor 6 and in depth by factor 3; facial size and shape variations were indicated by a large group of factors - 5, 20, 9, 10, 13, 22, 1, 19, 23 and 2.

Because the common factors extracted by any factoring method are determined by the correlations among the variables, it was not surprising that the major sources of covariation were revealed as four groups of factors each representing an anatomical region of the skull. This feature of factor analysis can provide useful informa-

tion when comparisons are made between successive analyses with modifications in the number and nature of the variables.

Analysis 2

The second analysis, carried out by the principal factor method followed by varimax transformation, resulted in 40 common factors. Examination of the varimax loadings and the percentage contributions of the factors to the common variance (shown in Table 20) suggested that 30 were capable of meaningful interpretation even though some of these were of little biological significance. The coefficients for the 30 retained factors are listed in Table 21 wherein the order of the factors is changed, some factors are shown with sign reversal and the coefficients judged as non-significant are omitted. For this analysis a value of 0.21 was accepted as the level of significance for a factor coefficient by applying the same criteria as before. Factor interpretation, based on the magnitude of the factor coefficients, is summarised in Table 22.

The 30 common factors were distributed as follows: the endocranium was represented by eight factors, 19, 16, 5, 21, 14, 30, 27 and 23; the cranial base by six factors, 24, 11, 1, 7, 17 and 3; the nasopharyngeal region by four factors, 10, 13, 18 and 15; facial size and shape by twelve factors, 28, 4, 6, 8, 9, 22, 2, 12, 31, 25, 29 and 20. These factors could be placed in the same four basic groups as in Analysis 1.

More factors were extracted in Analysis 2 and several sources of variation not disclosed in the first analysis were represented as common factors in the second analysis. For example, the endocranium was represented by three additional factors, frontal bone shape (30), endocranial length in the basal segment (23) and inclination of the foramen magnum (27). Two additional nasopharyngeal factors were present, one of breadth (15) and the other representing sphenoid or clivus thickness (18). The size and shape of the facial skeleton were indicated by two additional factors, one for nasal breadth (20) and the other for upper facial breadth (29). It was interesting that the 23 common factors of Analysis 1 were readily identified among the 30 common factors of Analysis 2.

An examination of the distribution and magnitude of the factor coefficients brought to light many instances of overlapping variables identified by similarity in their patterns of coefficients. The variable pair endo. l and g-op overlapped to measure variation in the general length of the cranium, and the variable pair min. f and max. f overlapped to determine variation in frontal bone dimensions. Moreover, in some instances sources of variation were expressed as two orthogonally related factors whereas in nature an oblique relationship would almost certainly exist. This situation, brought about when two or more variables were topographically associated, was present in the variable set, n-s, n-eth and eth-s representing lengths of the anterior cranial base. Two factors appeared, one for the eth-s segment of the base and the other for the n-eth segment.

The total anterior cranial base length n-s had significant loadings on both factors. CATTELL ('65a) has referred to this effect which results from mathematical restrictions inherent in an orthogonal factor model and the consequent preclusion of correlated factors no matter how they might occur in nature.

To reduce sources of minor covariation, the correlations between variables and factors were systematically examined and 23 of the 59 variables were omitted from the next stage. Referring to Table 22, variation in length of the endocranium was expressed by factors representing the individual basal, frontal and parietal segments. It would seem that the variable endo. 1 would satisfactorily locate this general source of variability and accordingly the variables basal chord, frontal chord and parietal chord, which showed no other factor coefficients of biological interest, were eliminated. The variable g-op overlapped endo. 1 by spanning the same anatomical region and on these grounds it was also eliminated.

Variations in height of the endocranium were expressed by two factors (5 and 14) determined by the variable set po-v, endo. h and B-C-D. The patterns of factor coefficients for po-v and endo. h differed so both were retained to determine this variation source but B-C-D was eliminated as it had no other significant loadings.

The indices, basal index, frontal index and parietal index had factor coefficients of little biological interest and were not retained. Variable eu-eu overlapped endo. b to determine the

factor of cranial breadth (16) and was therefore eliminated. Angle A-B-C showed only topographical relationships with the other variables and was also rejected.

Turning now to the cranial base group of factors, max. f was retained to express the frontal thickness factor (24) and min. f, which it overlapped, was eliminated. Anterior cranial base length (factors 1 and 7) could be effectively represented by the variable n-s and accordingly the variables n-eth and eth-s which were topographically related to each other and to n-s were eliminated. Variable f. sinus h was retained to determine a frontal sinus factor in preference to f. sinus b which had a similar loading pattern. Variables eth-s-ba, n-s-ba, s-ba and n-ba, although topographically related, were retained for the next stage to locate factors of cranial base inclination and clivus length. The variable for. angle was also included in the retained set.

Variations in the size of the nasopharyngeal region were expressed in Analysis 2 by the factors 10, 13, 18 and 15. The variables ba-pns, phar. h, scp-scp and sphen. d were retained to represent the variation of biological interest, and variables tph-pns and s-pm vert., which contained little additional information, were eliminated.

Among the large group of variables measuring size and shape of the facial skeleton, several sets of speciously related variables were encountered. Mass. b was topographically related to zg-zg and go-go; palate b was overlapped by ecm-ecm; s-n-id and s-n-pr expressed the

same variation source as s-n-pg and s-n-ss; NL/ML was topographically related to NL/NSL and ML/NSL. In addition, s-pm hor. and ar-tgo-gn had factor coefficients that were of little biological interest. Consequently the following variables were dispensed with in future analyses: mass. b, palate b, s-n-id, s-n-pr, NL/ML, s-pm hor. and ar-tgo-gn. The remaining facial variables were retained to locate the major sources of covariation in size and shape of the facial skeleton. A summary of the selection procedure is given in Table 23.

Analysis 3

The principal factor method followed by varimax transformation was used for the third analysis of the 36 retained variables. The percentage contributions of the factors to the common variance are shown in Table 24.

Of the resulting 23 common factors, 18 appeared capable of straightforward interpretation while the remainder had low variance contributions and were of little biological interest. The coefficients of the 18 retained factors are given in Table 25 which is simplified as in the previous analyses. For Analysis 3 a level of 0.21 was accepted as significant for a factor coefficient and factor interpretation, based on the magnitude of the factor coefficients, is summarised in Table 26.

The selection of variables carried out after the previous stage resulted in a more satisfactory factor pattern. Many sources of variation that were biologically unimportant had been eliminated and the distribution of the variable loadings over a new set of factors resulted in a varimax solution that was reduced in complexity and easier to interpret.

Variation in the endocranial dimensions was represented by three factors, factor 10 for breadth, factor 5 for height, and factor 8 for the foramen magnum inclination. Cranial base factors were reduced to four in number, factor 12 representing frontal bone size, factors 4 and 16 indicating the anterior and posterior cranial base segments and factor 11 representing the cranial base flexion angle.

Two factors, 13 and 14, were concerned with variations in the nasopharyngeal region and were identified as clivus thickness and nasopharyngeal breadth. Facial size and shape variations were determined by nine clearly defined factors, 15, 6, 3, 17, 18, 9, 1, 2 and 7 which together located the main sources of covariation in upper and lower facial depths, facial breadths and facial profile shape.

After an examination of the pattern of factor coefficients and an appraisal of the biological and specious associations remaining, it seemed desirable to exclude a further six variables that duplicated information contained in the others. The selection procedure, summarised in Table 27, was carried out as follows. Variables po-v and endo. h overlapped to locate the factor of endocranial height (16);

endo. h was retained to locate this variation source. The cranial base angles eth-s-ba and n-s-ba were topographically related and only n-s-ba was retained to express cranial base flexion in the median plane. Total cranial base length, n-ba, overlapped the variables n-s and s-ba but did not show other associations of interest and on these grounds it was eliminated. The variable ss-pns overlapped palate 1 to determine factor 15, and furthermore its factor coefficients were, in the main, topographically determined; ss-pns was therefore not retained. Mandibular length variation could be effectively represented by the variable gn-cd and the overlapped variable gn-go was eliminated from further analysis. Finally the angle of maxillary prognathism, s-n-ss, was eliminated as it did not contain much information that was not effectively included in the variables s-n-pg and n-ss-pg.

Analysis 4

The fourth analysis was entered with the 30 variables retained from Analysis 3 and carried out as before by the principal factor method followed by orthogonal transformation. The percentage contributions of the resulting 19 factors to the common variance are shown in Table 28.

For Analysis 4, a value of 0.21 was accepted as the criterion for significance of a factor coefficient. Of the 19 common factors, three had extremely low variance contributions and were not

interpreted. In addition, one factor (number 14) contributed only two per cent to the common variance and was of doubtful value even though it could be clearly identified. The coefficients for the 16 retained factors are shown in Table 29 which has been rearranged as before. Factor interpretation, summarised in Table 30, was based on the magnitude of the factor coefficients.

Compared with Analysis 3, there were few departures in the overall scheme of interpretation of the sources of variation. The endocranium was represented by three factors, 13, 7 and 8; the cranial base by two factors, 4 and 9; the nasopharynx by three factors, 16, 6 and 12; size and shape variations of the facial skeleton by the group of eight factors, 11, 3, 10, 1, 5, 15, 2 and 14.

The pattern of coefficients resulting from Analysis 4 was capable of meaningful interpretation and, furthermore, additional sources of specious association had been eliminated after the previous stage. The solution was accepted as a satisfactory representation of the major sources of variation present in the original set of craniofacial variables, so far as their biological interest was concerned. Therefore, without further revision, the factor loadings were retained as approximations with which to enter the more precise maximum likelihood estimation described in the next chapter.

The advantage in carrying out a series of factor analyses lies in the opportunity to examine relationships between variables and factors at several stages. The biological importance of each variable and

factor can be assessed and the subsequent analyses modified accordingly. Although it could be argued that specious associations can be recognised in a correlation matrix and eliminated at this stage, there is a distinct advantage in conducting this elimination over a series of factor analyses each of which increases the understanding of the associations present. This procedure permits a degree of experimental control not possible when a single analysis is made with little knowledge of the expected outcome. Moreover, serial analysis of this type preserves the maximum information content and leads to more efficient factor recognition.

Note on the significance of factor coefficients

At present, no universally acceptable test for the significance of a factor coefficient is available. The test suggested by HARMAN ('60a, p439) has been used for small matrices with meaningful results (BROWN, BARRETT and DARROCH, '65a; 65b), but in the present analyses the application of this test resulted in criteria for significance that were not in accord with the obvious biological relations between variables and factors. Accordingly a frequency count was made for all factor coefficients with absolute values falling between 0.15 and 0.26, the limits between which the significant level might reasonably be expected; Table 31 shows this count for Analysis 1. Inspection of this table shows that coefficients with values 0.15 to 0.21 had

frequencies ranging from 13 to 22 whereas the frequencies for coefficients with values 0.22 to 0.26 fell abruptly to 5 or 7. On the assumption that the smaller coefficient values were more likely to have arisen by chance alone, a value of 0.22 was accepted as the level of significance for a factor coefficient in Analysis 1.

This procedure, although it has no strict statistical foundation, led to a satisfactory interpretation in so far that most coefficients accepted as significant could be accounted for by the known biological or specious relationships among the variables. It is almost certain, however, that the procedure would exclude a few meaningful coefficients that did not quite reach the significance level. The method adopted finds some support from CATTELL ('65a) who selected a hyperplane band width (or estimated standard error of a zero factor loading) of ± 0.10 as a guide to determine which variables "belonged" to the hyperplane. Also, an empirical level of 0.20 was set by SOLOW ('66, p100) after consideration of sample size, the limits for a zero correlation, and the general applicability of this level for biological interpretation.

6

MULTIVARIATE ANALYSIS OF THE AUSTRALIAN SKULL

Final Factor Analysis

The factor representation of the main sources of variation present in the Australian Aboriginal skull was clarified during the previous analyses. After the elimination of variables of little biological interest or which duplicated sources of variation adequately represented by the remaining variables, a stage was reached when the important metric characters of the skull group under investigation were considered to be effectively described by the 30 remaining variables.

Subsequently a principal factor analysis of these 30 variables revealed 16 common factors that could be interpreted fairly readily in meaningful terms. However, the principal factor method, although suitable for factor recognition, leads to a set of factor coefficients that are mathematically less precise than those derived by more efficient procedures. The final analysis (number 5 in the series) was carried out by maximum likelihood estimation (LAWLEY and MAXWELL, '63, p10; HARMAN, '60a, p366).

Method of analysis

Trial factor loadings were derived by varimax transformation of the principal factor coefficients calculated for 30 variables and 16 common factors in Analysis 4. These values were accepted as approximations with which to enter the likelihood calculations which were solved by an iterative method (see Chapter 3). After each iteration, residual variances were computed for the variables and compared with those obtained during the previous iteration. The computer program specified that the iterations should cease when either all 30 residual variances had converged to stable values with differences between successive residuals less than 0.001, or when the maximum number of permitted iterations (in this instance an arbitrary value of 25) had been performed.

In the present analysis 25 iterations were performed after which the maximum difference between successive residual variances was 0.003; 17 residuals had converged to stable values within the specified limit of 0.001, 26 residuals had converged within 0.002 and all 30 residuals had converged within a value of 0.003. The degree of convergence achieved was considered satisfactory for the present analysis and the factor loadings so obtained were accepted as the maximum likelihood coefficients. Orthogonal transformation by the varimax method was carried out to obtain a new set of rotated loadings before factor identification was attempted.

The accuracy of the solution was assessed by reproducing the matrix of 435 correlations between the 30 variables from the factor coefficients and examining the magnitude of the residual coefficients, that is the differences between the observed correlations and those reproduced from the likelihood solution. In addition, the approximate χ^2 criterion (LAWLEY and MAXWELL, '63, p24 Equation 2.17) was used as a guide to the statistical significance of the residual coefficients. These findings are shown below:

Mean of residual coefficients	0.009
Standard error of mean	0.001
Standard deviation	0.011
Minimum residual coefficient	0.003
Maximum residual coefficient	0.088

$\chi^2 = 75.9$ for 75 d.o.f. $(.40 < p < .50$; non-significant)

The finding of a non-significant χ^2 criterion as well as low values for the residual correlations justified the conclusion that 16 common factors satisfactorily accounted for the correlations among the 30 craniofacial variables.

Finally, as a guide in factor interpretation, the contributions of the variables to the estimated factor variances were computed by the method outlined by HARMAN ('60a, p346) who considers that these contributions provide a better indication of the relative importance of the variables so far as factor prediction is concerned. Factor

coefficients represent the correlations between variables and factors and do not take into account the indirect contributions to a factor resulting from the intercorrelations among the variables in the set. The 16 factors were interpreted after an examination of the pattern of varimax loadings and consideration of the variable contributions to the estimated factor variances.

Results

The orthogonal varimax solution obtained by rotation of the initial maximum likelihood matrix of loadings is shown in Table 32. This solution was rearranged in the way described for previous analyses and is given in simpler form in Table 33 which includes factor coefficients with values greater than 0.20, the level of significance accepted for this solution being 0.21.

Contributions of the 16 factors to the total communality are given in Table 34. It is interesting to note that the three factors contributing least to the common variance (factor 16, 3.3 per cent; factor 8, 2.1 per cent; factor 13, 1.7 per cent) were the only ones that could not be readily identified among the 16 factors resulting from the principal factor analysis 4. It is quite probable that the elimination of these three factors followed by a repeat maximum likelihood estimation based on the loadings for 13 factors would lead

to an efficient solution with a non-significant criterion. However, it is unlikely that the overall interpretation would change and this step was omitted, particularly as there is insufficient evidence to support the view that minimising the number of common factors is desirable in biological situations as it might be at other times.

The total contributions of the variables to the estimated factor score variances are shown in Table 35 which has been simplified by omitting low values that were negligible and probably spurious. The interpretation of the 16 common factors is summarised in Table 36.

Factor interpretation

The common factors were interpreted by taking into account the magnitudes of the correlations between variables and factors (Table 33), the relative importance of the variables for factor score prediction (Table 35) and the recognition of non-biological or specious associations. Factors resulting from a factor analysis indicate sources of shared variability among the variables and although it is often possible to identify them with biological influences that might bring about the common variability, the factors should not be taken as direct evidence of causation.

In the following interpretations each factor is considered to represent a source of variation common to a group of variables whose

intercorrelations and factor loadings are shown in the supporting tables. For convenience the identifying title given to a factor is, in most instances, similar to that of the variable with the highest correlation on the factor. This is not meant to infer that the factor represents a source of variation identical to the variable in question.

Factor 12 - Endocranial breadth

Variable	Correlation coefficients[1] 10	43	39	45	a[2]	b[3]
10. endo. b	1				.79	.46
43. palate 1	.32	1			.28	.05
39. zg-zg	.36	.22	1		.42	.14
45. go-go	.34	-.04	.38	1	.43	.08

[1]The minimum value of a correlation coefficient differing from zero at the p = .05 level is 0.20
p = .01 level is 0.26

[2]The column headed a contains the correlations between the variables and the factor, that is the factor coefficients derived by varimax transformation of the maximum likelihood solution.

[3]The column headed b contains the contributions of the variables to the estimated variance of the likelihood factor.

These foot-notes and headings apply to each of the tables accompanying the factor descriptions.

The variable endocranial breadth had the highest loading on this factor. Upper and lower facial breadths, indicated by bizygomatic and bigonial diameters showed moderate loadings indicating a general coordination in breadth between the cranial vault and the facial skeleton. However, these three variables could be considered overlapping to some extent by spanning adjacent structures in transverse planes so that the revealed common variation was not altogether unexpected. Only one other variable, palate length, had a significant loading on Factor 12. This loading was not high in value but gave evidence of an association between general skull breadths and the length of the upper jaw. The variables endocranial breadth and bizygomatic diameter contributed most to the estimated factor variance.

Factor 12 was interpreted as one of general breadth of the skull reflecting a coordination between the cranial vault, the upper face and the mandible so far as their variability in width was concerned. The absence of significant loadings for facial depth and facial shape variables, apart from the one for palatal length, indicated that the various facial breadths, although coordinated within the general framework of skull breadths, were largely independent of depth measures of the face and calvarium and shape of the facial profile. This finding is similar to the observations made by BJÖRK (64b, p35) when discussing symmetric development of the face. Factor 12 is also reminiscent of the general facial width factor reported by SOLOW ('66, p116).

Factor 5 - Endocranial height

Variable	Correlation coefficients 11	9	18	34	49	41	a	b
11. endo. h	1						.76	.38
9. endo. l	.27	1					.34	.07
18. s-ba	.39	.14	1				.54	.08
34. phar. h	.38	.09	.35	1			.65	.27
49. ramus h	.35	.19	.37	.30	1		.38	.02
41. ecm-ecm	.22	.19	.36	.10	.32	1	.21	.00

Endocranial height and pharyngeal height had the highest loadings on Factor 5 and were the only variables contributing to the factor variance to any extent. However, these variables were topographically related by sharing the reference point basion together with s-ba which had the third highest loading on the factor. Ramus height and endocranial length had moderate loadings on Factor 5 and maxillo-alveolar breadth had a coefficient slightly higher than the accepted significance level.

Factor 5 was interpreted as one of general cranial height expressing a source of variation common to the brain case, the adjacent clivus and nasopharynx and, to a lesser extent, the height of the mandibular ramus. However, no biological explanation can be offered for the revealed associations between the ramus height and the heights of the endocranium and pharynx.

Factor 14 - Frontal bone size

Variable	Correlation coefficients 25	24	a	b
25. f. sinus h	1		.71	.32
24. max. f	.46	1	.62	.22

Two variables, maximum frontal thickness and frontal sinus height, had significant loadings on this factor which appeared to indicate the coordination in sagittal and vertical dimensions of the frontal bone in the vicinity of the frontal air sinuses. Biologically, it is reasonable to take this factor as representing the general influence of sinus development on the morphology of adjacent parts of the frontal bone. Factor 14 was therefore interpreted as frontal bone size.

The lack of even a weak association between the frontal bone factor and other variables emphasised the morphological independence of this region and furthermore supports the contention of ABBIE ('52) and MOSS and YOUNG ('60) that a functional correlation between brow ridging and jaw size is unjustified. The finding, however, does not agree with the association between supra-orbital ridging and mandibular robustness displayed through factor analysis by SCHWIDETZKY ('59). The frontal bone dimensions in the present study were measured on sagittal roentgenograms and would provide only a crude indication of supra-orbital ridging.

Factor 3 - Cranial base flexion

Variable	Correlation coefficients 20	30	31	39	a	b
20. n-s-ba	1				.79	.85
30. n-sp	.41	1			.22	.00
31. ba-pns	.28	.23	1		.41	.06
39. zg-zg	.33	.14	.22	1	.30	.00

Factor 3 was associated most strongly with the angle of cranial base flexion, n-s-ba. The variables anterior nasal height and naso-pharyngeal depth also had significant loadings on this factor but the associations between the three variables could be anticipated on topographical grounds through the sharing of reference points nasion and basion. However, the significant loading on Factor 3 for bizygomatic breadth could be taken as evidence of weak biological coordination between the breadth of the upper face and the cranial base angulation. Skulls with flattening of the cranial base would tend to be broad in the upper facial region, and have a deeper nasopharynx.

Factor 3 was interpreted as one of cranial base flexion expressing a source of variability shared by the cranial base angulation, the breadth of the upper face and the depth of the nasopharynx. The association between cranial base angulation and anterior nasal height (n-sp) although partly expected on topographical grounds, confirms the observation of BJÖRK ('64b, p10) that cranial base flattening is

accompanied by marked overdevelopment of the upper face height at the expense of the posterior face height.

Factor 9 - Head balance

Variable	Correlation coefficients 22	9	18	31	a	b
22. for. angle	1				.83	.59
9. endo. 1	-.26	1			-.40	.10
18. s-ba	.26	.14	1		.26	.03
31. ba-pns	.23	.13	.25	1	.27	.04

Three variables, foramen angle, posterior cranial base length and nasopharyngeal depth had significant loadings on Factor 9 that could be explained by the sharing of point basion. The variable foramen angle contributed most to the estimated factor variance. However, the negative loading for endocranial length appeared to represent a true biological relationship between skull length and the inclination of the foramen magnum. Thus, long skulls would tend to be positioned upon the cervical column in such a way that the inclination of the foramen magnum to the cranial base would be more acute than in shorter skulls.

Factor 9 was therefore interpreted as one of head balance, expressing the relationship between head length and foramen magnum inclination.

Factor 4 - Anterior cranial base length

Variable	Correlation coefficients 17	9	24	31	a	b
17. n-s	1				.90	.81
9. endo. 1	.37	1			.34	.02
24. max. f	.41	.01	1		.28	.00
31. ba-pns	.37	.13	.22	1	.35	.03

Factor 4 appeared to indicate a source of variation common to the lengths of the cranial vault, cranial base and nasopharynx. The highest coefficient was for anterior cranial base length, and the other significant loadings were for endocranial length, maximum frontal thickness and nasopharyngeal depth. These associations would be expected as the variables spanned adjacent anatomical areas. The variance of Factor 4 was almost entirely accounted for by the contribution from the anterior cranial base length.

The factor was interpreted as one of anterior cranial base length which expressed coordination in the sagittal lengths of the adjacent areas, the endocranium, cranial base and nasopharynx. In contrast to the findings of SOLOW ('66, p116) so association between the anterior cranial base factor and jaw lengths was found in the Australian sample, even though these variables were significantly correlated with each other (Table 14).

Factor 7 - Clivus thickness

Variable	Correlation coefficients 27	18	30	31	49	58	a	b
27. sphen. d	1						.78	.33
18. s-ba	.28	1					.34	.04
30. n-sp	.24	.26	1				.40	.13
31. ba-pns	.21	.25	.23	1			.27	.05
49. ramus h	.33	.37	.18	-.03	1		.29	.00
58. ML/NSL	-.32	-.07	.25	-.07	-.23	1	-.28	.13

This factor had a high loading for sphenoid diameter and moderate loadings for the other variables. Of these, the association with posterior cranial base length (s-ba) could be expected as the variables sphen. d and s-ba were measured on a common reference structure, the clivus of the skull. Sphenoid diameter contributed most to the estimated factor variance.

The set of factor coefficients represented a true biological coordination between the thickness of the clivus, the height of the nasal cavity, depth of the nasopharynx and height of the mandibular ramus. The negative loading for mandibular base inclination probably resulted from the inverse relationship between this variable and ramus height.

Factor 7 was interpreted as one of clivus thickness demonstrating a coordination between the length and thickness of the clivus, anterior nasal height, nasopharyngeal depth and ramus height. This

factor bears some resemblance to the factor of clivus length reported by SOLOW ('66, p116) to be associated with maxillary height and breadth. It also confirms the view of BJÖRK ('64b, p34) that development of the upper face and nasopharynx is associated with that of the cranial base and mandible.

Factor 8 - Pharyngeal height

Variable	Correlation coefficients 34	57	a	b
34. phar. h	1		.37	.19
57. NL/NSL	-.23	1	-.28	.13

Factor 8 was of little biological interest. It was revealed with significant loadings on two topographically related variables, pharyngeal height and the nasal floor inclination. Its interpretation as a pharyngeal height factor indicates a morphological character of the skull group examined, namely the presence of a small angle of inclination between the nasal floor and the cranial base in association with a high bony nasopharynx.

Factor 15 - Palatal height

Variable	Correlation coefficients 44	37	49	45	a	b
44. palate h	1				.61	.19
37. n-gn	.38	1			.28	.09
49. ramus h	.32	.26	1		.35	.09
45. go-go	-.22	-.12	.08	1	-.31	.06

Factor 15 appeared to be one of general facial height with the highest loading and highest variance contribution for the variable palate height. None of the associations could be expected on purely topographical grounds and the revealed pattern of loadings therefore indicates a biological coordination between palatal height, morphological face height and ramus height. The negative loading for bigonial diameter suggested an inverse relationship between breadth of the lower face and the height of the anterior face, palate and mandibular ramus.

Factor 6 - Infratemporal fossa depth

Variable	Correlation coefficients 9	50	39	48	34	44	38	58	a	b
9. endo. 1	1								.41	.05
50. infra t.f.d.	.33	1							.81	.35
39. zg-zg	.28	.55	1						.62	.20
48. ramus b	.34	.53	.35	1					.67	.16
34. phar. h	.09	-.14	-.16	-.18	1				-.25	.04
44. palate h	-.04	-.24	-.05	-.27	.09	1			-.23	.02
38. zm-zm	.37	.37	.42	.30	-.03	.03	1		.40	.02
58. ML/NSL	-.17	-.26	-.21	-.24	.04	.27	.02	1	-.25	.05

Three variables infratemporal fossa depth, bizygomatic breadth and maxillary breadth spanned the same anatomical region and their high loadings on Factor 6 could be expected for this reason. The highest factor coefficient and the highest contribution to the factor variance was for infratemporal fossa depth. The other significant loadings could not be predicted on the grounds of anatomical proximity of the dimensions and therefore probably represent a true biological coordination.

The pattern of factor loadings indicated an association in size between anatomical components functionally related to the masticatory musculature. For example, powerfully developed masseter and medial pterygoid muscles would explain high values for upper facial breadth measured across the zygomatic arches, infratemporal fossa depth and breadth of the mandibular ramus. The significant loading on Factor 6 for the variable endo. 1 is interesting. This suggested a biological association between endocranial length and the mid-facial breadths so that skulls well developed in zygomatic breadth and ramus breadth tended to be long in the cranial vault. Other characters found in conjunction with a high score on Factor 6 included a shallow palate and nasopharynx and a mandibular base more parallel with the cranial base line NSL.

Factor 6 was interpreted as one of infratemporal fossa depth, indicative of an association between the development of the masticatory muscles and the morphology of adjacent bony structures.

Factor 13 - Nasal height

Variable	Correlation coefficients 30	43	41	a	b
30. n-sp	1			.34	.22
43. palate 1	.24	1		.22	.10
41. ecm-ecm	.20	.25	1	-.27	.08

Factor 13 had little biological interest, contributed least to the total common variance and expressed an association between three variables intercorrelated at the five per cent level of probability. It was identified as a nasal height factor positively associated with the height and depth of the nasal cavity and negatively with the maxillo-alveolar breadth.

Factor 10 - Nasal breadth

Variable	Correlation coefficients 28	33	11	31	39	38	41	a	b
28. nasal b	1							.62	.20
33. scp-scp	.17	1						.53	.10
11. endo. h	.24	.16	1					.21	.03
31. ba-pns	.09	.19	.07	1				.22	.04
39. zg-zg	.14	.24	.01	.22	1			.29	.05
38. zm-zm	.20	.24	.03	.15	.42	1		.38	.07
41. ecm-ecm	.25	.31	.22	.28	.24	.42	1	.51	.15

Factor 10 revealed a source of variation common to the breadth dimensions of the nasal, pharyngeal and upper facial regions. These associations could be predicted to some extent because the variables with the highest factor loadings spanned adjacent anatomical regions. The variables nasal breadth, nasopharyngeal breadth and maxillo-alveolar breadth contributed most to the estimated factor variance.

Moderate loadings for the variables endocranial height and naso-pharyngeal depth were present but no biological explanation for these associations can be given. Factor 10 was interpreted as a source of variation common to breadths of the nasal, nasopharyngeal and upper facial regions, but distinct from the other breadth factors 12 and 6.

Factor 2 - Mandibular length

	Correlation coefficients									
Variable	47	37	43	34	30	38	41	52	a	b
47. gn-cd	1								.83	.36
37. n-gn	.50	1							.52	.24
43. palate 1	.46	.43	1						.47	.02
34. phar. h	.29	.26	.11	1					.27	.00
30. n-sp	.20	.53	.24	.21	1				.21	.01
38. zm-zm	.31	.27	.37	-.03	.24	1			.26	.02
41. ecm-ecm	.26	.39	.25	.10	.20	.42	1		.24	.02
52. s-n-pg	.22	-.51	.06	.12	-.37	-.01	-.15	1	.24	.18

Eight variables shared a source of common variability indicated by Factor 2, although many of these associations could be expected either through the use of common reference points and reference structures or because adjacent anatomical areas were spanned. For example, the variables mandibular length, morphological face height, anterior nasal height and mandibular prognathism shared one of the reference points nasion or gnathion in common. In addition, bizygomatic breadth and maxillo-alveolar breadth were both measures of upper facial breadth. Only three variables, mandibular length, morphological face height and mandibular prognathism contributed to the estimated factor variance.

Essentially, the factor expressed a coordination in anterior facial heights, mandibular length, palatal and pharyngeal heights and upper facial breadths. Because Factor 2 affected variables measured in several anatomical regions, it was taken to indicate general coordination in size of the skeletal components of the face.

Factor 11 - Facial convexity

Variable	Correlation coefficients 55	43	37	45	a	b
55. n-ss-pg	1				.62	.21
43. palate 1	-.32	1			-.62	.41
37. n-gn	-.34	.43	1		-.22	-.03
45. go-go	.11	-.04	-.12	1	.23	.02

Factor 11 expressed variability in the convexity of the facial profile. The highest factor loadings and contributions to the factor variance were for the variables profile angle and palate length. This relationship could be expected from the anatomical proximity of the reference points subspinale and orale used in the determination of these variables. It was interesting that although the factor coefficients for profile angle and palate length were equal, the palate length had the greater variance contribution and was therefore more important so far as factor score prediction was concerned.

The negative loading on Factor 11 for variable n-gn could be partly accounted for by the topographical relationship with variable n-ss-pg. No biological explanation can be offered for the low positive factor loading for variable go-go.

Factor 11 was interpreted as one of facial convexity indicating, in the main, the relationship between palatal length and the shape of the facial profile.

Factor 1 - Mandibular prognathism

Variable	Correlation coefficients 52	57	58	20	30	37	55	a	b
52. s-n-pg	1							.84	.28
57. NL/NSL	-.64	1						-.77	.18
58. ML/NSL	-.71	.43	1					-.76	.21
20. n-s-ba	-.52	.56	.27	1				-.49	.01
30. n-sp	-.37	.51	.25	.41	1			-.59	.13
37. n-gn	-.51	.32	.64	.29	.53	1		-.66	.19
55. n-ss-pg	.42	-.26	-.44	-.23	-.20	-.34	1	.37	.01

Factor 1 had the greatest contribution to the common variance (15.0 per cent) and indicated a high degree of coordination between the seven variables concerned. In all instances, however, the significant correlations among the variables could be explained in part by topographical relationships. Reference line NSL was common to four variables, reference point nasion was common to all, reference point gnathion was common to two and pogonion was common to two. The variable s-n-pg contributed most to the estimated factor variance.

Even though the associations were conditioned to some extent by the topographical situation, the factor is not without interest. It provides further evidence of many craniofacial associations that have been described previously in other groups (BJÖRK, '47; LINDEGÅRD, '53; BROWN, '65a; WEI, '65). If the factor is taken as one of mandibular prognathism, the factor loadings provide an indication of the cranial characters likely to be found in conjunction with marked prognathism. These are: nasal and mandibular bases more acutely inclined to the cranial base, increased cranial base flexion, short anterior face heights and reduced facial convexity. It should be noted that because variable s-n-ss was eliminated in earlier analyses, the present Factor 1 indicates the associations found in conjunction with high or low mandibular prognathism. Had maxillary prognathism been retained as a variable, it would undoubtably have appeared with a strong loading on this factor. The omission of s-n-ss also explains the finding of a positive loading

for variable n-ss-pg which is a measure of relative prognathism of the mandible as well as an indication of convexity of the facial profile. It is interesting that a prognathism factor appeared in the factor analyses reported in the previous chapter but, in these instances, with loadings on additional variables that were later eliminated.

Although the revealed associations could be partly expected on the grounds of topographical relationships, this does not imply that there were no biological coordinating mechanisms present.

Factor 16 - Ramus height

Variable	Correlation coefficients 49	31	45	58	a	b
49. ramus h	1				.49	.21
31. ba-pns	-.03	1			-.34	.12
45. go-go	.08	-.14	1		.29	.05
58. ML/NSL	-.23	-.07	-.25	1	-.27	.15

Factor 16 had the third lowest contribution to the total common variance and was determined by four variables among which only two correlation coefficients reached significance at the five per cent probability level. The factor appeared to be one of mandibular ramus height but there was little of biological interest in the revealed loadings so that the factor could probably be safely disregarded

without loss of important information. To some extent, Factor 16 overlapped the clivus thickness factor number 7, and because of similarities in the pattern of factor coefficients, most likely represented a related source of variability.

Variable descriptions

Apart from the possibility of describing factors resulting from a factor analysis in terms of the variables examined, it is valid to reverse the procedure by considering the variables to consist of contributions from the common factors and the unique factors concerned with the variables in question. In other words the mathematical representation of a variable, stated in Chapter 3 is applied:

$$z = a_1F_1 + a_2F_2 + \ldots\ldots\ldots + a_mF_m + bU$$

where z is the variable concerned in standard form, $F_1, F_2, \ldots F_m$ are the scores on the m common factors affecting z, $a_1, a_2, \ldots a_m$ are the factor coefficients and b is the coefficient of U, the unique factor belonging to z. If the values for the common factors are known, it becomes possible to use the above model to predict a variable score for a particular subject. According to the assumptions stated in Chapter 3, the variance of variable z, can be partitioned into components whose numerical weights are determined by

the coefficients of the common factors:

$$\text{variance } z = 1 = a_1^2 + a_2^2 + \ldots\ldots a_m^2 + b^2$$

Variables are often used in anthropometric investigations with incomplete knowledge of the sources of variation they represent. It may also be difficult to select the most appropriate variables to indicate a source of variation that is to be analysed. For example, it might be desired to include a measure of general head breadth in a battery of variables. Breadth of the head could be measured in several regions; across the cranial vault, across the zygomatic arches, across the maxillae or across the mandible. Unless multivariate techniques are applied, it is difficult to determine if these variables represent related or independent sources of variation and therefore the most appropriate indicator for general head breadth is not easily selected. One application of factor analysis is to throw light on these problems by disclosing the relationships between variables and between variables and factors. A variable that is determined by several different sources of variation might be considered lower in biometric value than one whose variance is determined at a single source. However, it should be remembered that any factor solution will be decided by the variables included for analysis and, to a lesser degree, by the factor method applied. Therefore generalisations should be made with caution.

In the present section an attempt is made to reconsider the variables in the light of the factor findings. Figure 12 presents the variable

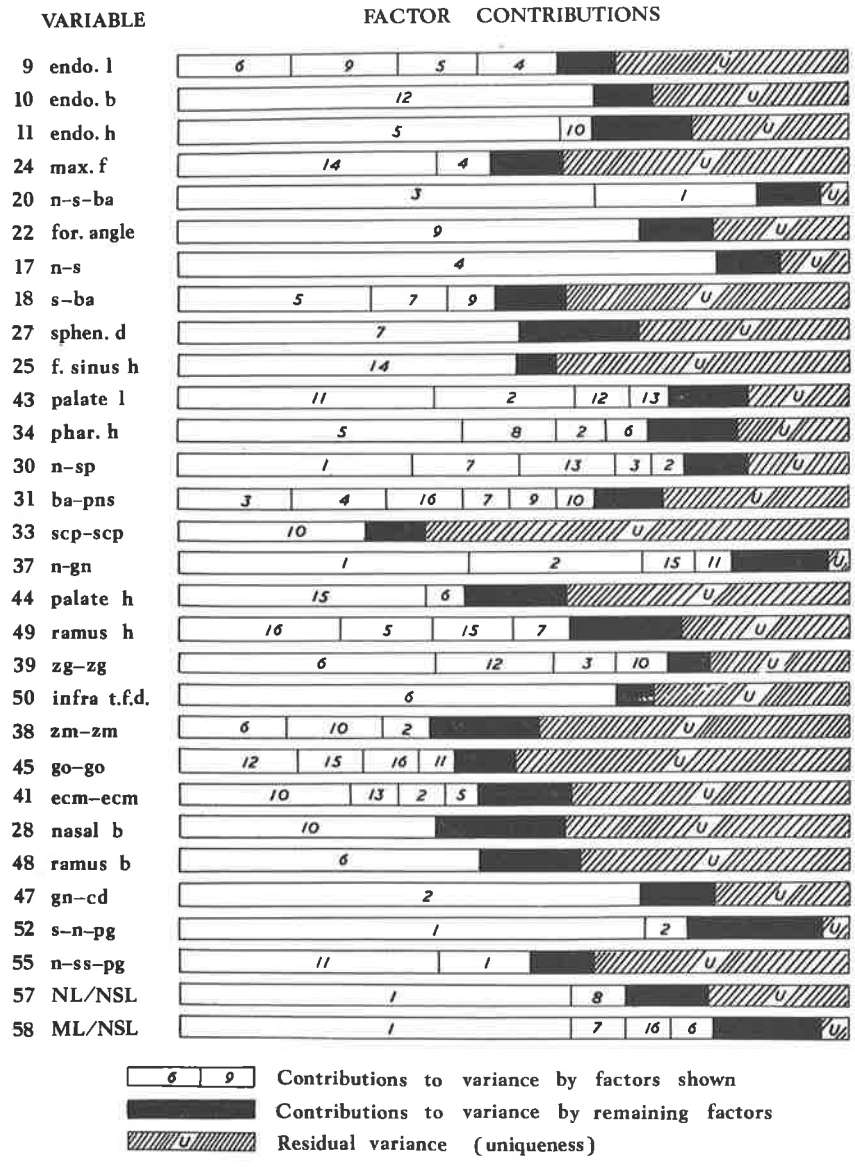

FIGURE 12. Contributions of factors to variables in Analysis No. 5.

low communalities indicating that in each instance there was a fairly high contribution to the variance from the unique factor specific for the variable concerned. The variables palate length, anterior nasal height and morphological face height had high communalities but their variances were determined by a group of common factors and, in a sense, these variables were more "complex" in their structure than those loaded on single common factors.

By examining the varimax pattern for Analysis 5 (Table 33) it is possible to assess the extent to which different variables measured the same source of variation. Variables with significant loadings on Factor 1 formed a major group indicating the common variation included in measures of prognathism, jaw base inclinations, facial heights and cranial base inclination. In this instance, however, the revealed common variability resulted in large measure from the topographical situation brought about by the sharing of reference points and reference lines.

Factor 5 variables together formed another large group expressing common variation in height dimensions of the endocranium, cranial base, nasopharynx and mandibular ramus. The variables concerned with Factor 6 together indicated a source of variation present in the breadth dimensions of the upper face and the breadth of the mandibular ramus. Factor 10 variables together measured variation in nasal, nasopharyngeal and upper facial breadths but this source of variation differed from Factor 12 which expressed skull breadths in a more

general way. It is not surprising that bizygomatic breadth would be concerned with the three breadth factors.

Apart from the variable groupings referred to, several other sets of variables, each concerned with regional sources of variation in the skull group examined, were revealed by the factor results.

Discussion

The concept that skull components are relatively independent of each other in their morphology has been recently restated by MOSS and YOUNG ('60) and MOSS ('62) who suggest that the size and shape of individual bones of the skull are determined to a great extent by local functional demands. This view is supported by experimental work which has demonstrated that interference with normal growth processes in one region of the skull does not necessarily result in abnormal growth elsewhere. However, it can be expected that a skull allowed to grow to normal maturity will show some morphological coordination between its components. This coordination could be expected as a result of genetic influences and environmental agencies affecting different skull regions in common. Coordinations of this type can be studied by a multivariate approach to metric data analysis where the variables are treated collectively rather than individually.

In the present investigation the 30 variables selected for final analysis represented most of the interesting features of craniofacial morphology in the skull group examined. Although the material was cross-sectional in nature, the disclosed factors highlighted the sources of covariation and indirectly pointed to coordinating mechanisms which in some instances reflected biological influences while in others were undoubtably topographical in nature.

The selected variables located factors that fell into several groups representing covariation in different regions of the skull. The factors significantly correlated with several variables from different skull regions can be considered "general" in nature in so far that they express covariation in two or more distinct skull regions. For example, the endocranial breadth factor was related to breadth variation in the calvarium, across the middle face and between the angles of the mandible. On the other hand, some factors had more restricted influences and were correlated with variables measured at the same or adjacent skull regions. The frontal bone factor provides an example of this type of coordination which was considered "local" in nature.

An indication of the overall patterns of covariation in the Australian Aboriginal skull can be gained by taking each factor group in turn.

Endocranial factors

The endocranium was represented by three factors designated endocranial breadth, endocranial height and head balance. Although these factors related to the endocranium in Analysis 5, the earlier factor analyses showed that they represented the same variation sources as expressed in the external skull dimensions.

The breadth factor (Factor 12) expressed a general coordination in skull breadths measured at different regions extending from the cranial vault to the lower border of the mandible. Thus in the group examined, the upper and lower jaws would tend to be broad or narrow according to the breadth of the calvarium. There was also a weak association between this factor and the length of the palate. The endocranial height factor (Factor 5) represented a general coordination in height of the cranial vault, the nasopharynx and the mandibular ramus. In addition, the endocranial length and the breadth of the upper jaw were weakly associated with this factor. The main association with the head balance factor (Factor 9) was for endocranial length which varied inversely with the inclination of the foramen magnum to the cranial base. In a skull with a high value for cranial length, the foramen magnum tended to be positioned more ventrally in relation to the cranial base than in one where cranial length was not so great.

The endocranial factors taken together revealed the general associations in breadths and heights between the endocranium, the

nasopharynx and the face and the coordination between skull length and inclination of the foramen magnum to the cranial base.

Cranial base factors

Of the three cranial base factors, one (Factor 14) was concerned with dimensions of the frontal bone in the anterior region of the median sagittal plane. This factor was independent of other craniofacial variables and was taken as an indication of coordination of frontal bone dimensions in the vicinity of the frontal air sinuses.

The second cranial base factor (Factor 4) disclosed a source of variation in length common to the anterior component of the median cranial base, the cranial vault and the adjacent nasopharynx. However, no facial variables were correlated with this factor suggesting that size of the facial skeleton and sagittal dimensions of the cranial base and nasopharynx were largely independent.

Cranial base flexion was concerned with the third factor of this group (Factor 3) which also showed correlations with nasopharyngeal depth and to a lesser extent with breadth and height of the upper face. The associations could be predicted from topographical relationships but gave some indication of a compensatory adjustment between cranial base flexion and nasopharyngeal depth - an acutely inclined cranial base being associated with a narrow nasopharynx.

The cranial base factors demonstrated a coordination between dimensions of the cranial base and the adjacent calvarium and nasopharynx. Apart from the rather low positive relationship between Factor 3 and upper facial height and breadth, variations in the cranial base were largely independent of variation in linear facial dimensions.

Nasopharyngeal factors

The nasopharyngeal region was represented by two factors, one for nasopharyngeal height and the other for clivus thickness. The height factor (Factor 8) showed a negative loading for the nasal line orientation but, because the posterior extremity of the hard palate was a common reference structure for variables pharyngeal height and nasal floor inclination, the relationship was not unexpected. The clivus thickness factor (Factor 7) was associated with nasal height, ramus height and to a lesser extent with the mandibular base inclination and depth of the nasopharynx. In the main, the nasopharyngeal factors expressed a source of variation common to the nasopharynx, the adjacent nasal cavity and to some extent the mandibular ramus.

Factors of upper facial size

Four factors were concerned with variations in upper facial dimensions and of these Factor 13 contributed least to the common variance and was of little biological interest in the present

analysis. One factor (Factor 10) expressed a source of variation in breadth dimensions common to several upper facial variables. This factor was interpreted as an indication of coordination in breadths of the nasal cavity, the nasopharynx, the alveolar arches and the zygomatic arches. Biologically, the factor loadings reveal the interdependence of nasal and nasopharyngeal dimensions and the effect of development in these regions on adjacent bony structures.

A height factor (Factor 15) was concerned with shared variation in facial heights, palatal height and ramus height and, in addition, was negatively correlated with the bigonial breadth. A second breadth factor (Factor 6) indicated a source of common variation in facial breadths that was different in nature to the variation revealed by Factor 10. Factor 6 represented coordination in dimensions of bony structures that are associated with the masticatory musculature. The highest correlations were with infratemporal fossa depth and mandibular ramus breadth, variables measuring the approximate thickness and breadth of the masseter and medial pterygoid muscles at their origins and insertions. Other significant loadings on this factor were for breadths of the zygomatic arches and maxillae and, to a lesser extent, for endocranial length. Nasopharyngeal height, palate height and the mandibular base inclination were negatively correlated with Factor 6. The factor indicated the features that would be expected in skulls showing a high degree of muscular development, namely, broad zygomatic arches, capacious infratemporal fossae, broad mandibular rami, a shallow palate and nasopharynx and a

mandibular base inclined acutely to the cranial base. Factor 6 of the present study bears a strong similarity to one of the facial breadth factors revealed by LANDAUER ('62) in her factor study of Egyptian crania.

In summary, the upper facial factors demonstrated coordination in breadth and height between components of the upper facial skeleton. The common variability in upper facial breadths appeared to stem from two distinct sources, one concerned with the development of the nasal and nasopharyngeal cavities and the other associated with the development of the masticatory musculature.

Factors of lower facial size

Two factors were associated with lower facial dimensions. Factor 2 was a general length factor with positive loadings for mandibular and palatal lengths, facial and nasopharyngeal heights, mid-facial breadths and the angle of mandibular prognathism. Many of the relationships could be predicted from the topographical associations between the variables but, nevertheless, the factor was taken as an indicator of general coordination in the length, height and breadth dimensions of the facial skeleton.

The second factor of this group (Factor 16) was concerned with height of the mandibular ramus, but the contribution of the factor to the total common variance was low and the loading pattern revealed

little of biological significance. The lower facial factors demonstrated a coordination in size of the facial components and a weak association between ramus height, ramus breadth and nasopharyngeal depth.

Facial profile factors

The pattern of associations concerned with variations in the shape of the facial profile was revealed by two factors. Factor 1 contributed most to the common variance and was termed mandibular prognathism. The variables correlated with Factor 1 shared common reference points or reference lines and for this reason their intercorrelations were partly expected. However, the pattern of factor loadings provides a mathematical summary of the morphological characters expected in skulls with varying degrees of mandibular prognathism.

The second profile factor (Factor 11) was one of profile shape or facial convexity. A high score on Factor 11 would be expected in skulls that had low values for palatal length and morphological face height and a high value for bigonial breadth. Skulls of this type would tend to present a flatter facial profile, with a larger angle n-ss-pg, compared with those showing opposite characters.

The two profile factors highlighted the associations between certain measurable characters of the facial skeleton which, when combined in various ways, resulted in the types of facial profile

indicated by the angles of mandibular prognathism and facial convexity.

It is difficult and, in many instances, misleading to make a comparison between findings of different factor studies. Because the number of variables and their nature will determine the common variance and its break-down into component factors, close resemblance between the results of differently designed studies can hardly be expected. Nevertheless, it was possible to match some of the factors of this study with those previously disclosed, particularly by HOWELLS ('57), LANDAUER ('62) and SOLOW ('66). Because of obvious difficulties referred to above, no precise analysis of factor congruence has been made; Table 5 summarises previous findings from factor studies of the skull.

In general, factor studies should be designed along similar lines and include comparable variables before precise factor comparisons are attempted. When dissimilar studies are compared, the factor matching should be carried out on the basis of overall factor patterns rather than comparison between isolated factors in separate studies. A factor that appears well defined in one study may, on close inspection, be recognised within the loading patterns of two or more factors in a second solution.

For these reasons it is premature to propose any general principles governing morphological coordination in the human skull. However, the findings from the several studies now reported add confidence to

the view that factor analysis, by providing a valuable and penetrating technique in craniometric research and related fields, can be expected to enjoy more frequent application when modern computer facilities are readily available and utilised.

7

MULTIVARIATE ANALYSIS OF THE AUSTRALIAN SKULL

Quantification of the Factors

The most common objective of factor analysis is to disclose sources of covariation existing among a group of variables. Analytic procedures are based on the assumption that the variance of each variable is determined in part by contributions from a number of common factors, so-called because they affect more than one variable of the set, and in part by a specific variance component not accounted for by the action of common factors. Demonstration of common factors is taken as evidence of covariation within smaller groups of variables forming sub-sets of the original set of variables.

In the past most biometric applications of factor analysis have been concerned with the estimation of correlations between factors and variables and, in the case of oblique models, the correlations between factors. In these instances the factors remain somewhat abstract even though they may be evaluated in qualitative terms on the basis of their correlations with the variables; at times the factors may be identified with biological processes. However, it is possible to extend the methods of factor analysis to include quantification of the factors by the calculation of scores on each

common factor for the members of the sample. HOWELLS ('53), who provided one of the few examples of factor quantification in biometry, considered that factor scores were more meaningful in genetic studies than scores on anthropometric variables.

The calculation of factor scores involves no new assumptions; it merely gives numerical values to the factors for each individual. Once computed, the factor scores can be regarded as values for a new set of variables each representing a composite of several related characters. A factor score consists of contributions from each of the original variables; variables that contribute very little to the factor variance will have negligible weight on the scores for that factor.

The calculation of factor scores provides further information to assist in the identification of factors operating within a group of variables. Comparison of specimens with different factor scores presents visual evidence of the factors at work and, moreover, assists in the assessment of their biological validity. This use of factor analysis in the present study is similar to the application of the method in somatotyping where subjects of different body-build are grouped according to their factorial make-up (TANNER, '64, p377; HUNT, '52; HAMMOND, '57a).

Method

Factor scores were computed by a short regression method outlined by HARMAN ('60a, p349) and LAWLEY and MAXWELL ('63, p88). The mathematical procedures are complex but were rapidly carried out by digital computer. In the present study the matrix of varimax transformed factor coefficients shown in Table 32 was used. The computing algorithm is outlined in the following section and given in more detail in Appendix C.

The basic factor equation (see Chapter 3):

$$z = a_1F_1 + a_2F_2 + \ldots\ldots\ldots a_mF_m + bU$$

can be restated in matrix notation as:

$$x = \bar{x} + Af + u$$

where x is the vector of N deviate scores for one subject,

$\bar{x}$ is the vector of mean values for N variables,

A is the matrix of loadings for N variables on K factors,

f is the vector of K factor scores for one subject,

u is the vector of N residual or unique components.

The objective is to estimate vector f from the known values of x, $\bar{x}$ and A. Normally the vector u is unknown and the calculation is not straightforward as it is in principal components analysis where vector u does not exist. The basic equation for the

estimation of a subject's score on a single factor F_1 is:

$$f_1 = b_{11}z_1 + b_{12}z_2 + \ldots\ldots b_{1N}z_N$$

where f_1 is the non-normalised score on Factor 1,

b_{1i} $(i = 1,2,\ldots\ldots N)$ is the vector of N beta coefficients of estimation for Factor 1,

z_i $(i = 1,2,\ldots\ldots N)$ is the vector of standard deviate scores on N variables.

This equation can be extended to cater for all factors and in matrix notation becomes:

$F = BZ$ where F is the matrix of factor scores,

B is the matrix of estimation coefficients,

Z is the matrix of standard deviate scores.

Estimation of the matrix of beta coefficients and computation of the scores for K factors is carried out in several stages, commencing with the matrix of factor coefficients.

The vector f_i contains the required scores on K factors for one subject and the calculations are repeated for each subsequent subject to complete the factor score matrix F. Factor scores computed in this way are non-normalised, have zero means and standard deviations approaching but not reaching one. In the present study all factor scores were normalised to a mean value of 50 and a standard deviation

of 10 according to:

$$f_i \text{ (normal)} = 50 + \frac{10.f_i}{s_i}$$

where s_i is the standard deviation of factor i.

This step eliminated negative scores and made comparisons between subjects more straightforward by taking into account the different factor variances. The procedure is similar to obtaining standard deviate scores on anthropometric variables.

Once obtained, the entire matrix of likelihood factor scores was written on magnetic tape and subsequently listed in both normalised and non-normalised forms together with subject identification. The computer was then programmed to list the scores in order of magnitude for each factor so that specimens with high or low scores could be quickly located.

The computations carried out during many stages of the multivariate analysis involved extensive matrix manipulations with the risk of lowered arithmetic precision even though eight significant figures were retained by the computer. As a check on the accuracy of the computations, correlations between the factors, as well as the means and variances of the factor scores were derived. The finding of zero correlations among the sixteen likelihood factors confirmed that the assumption of orthogonality in the factor model had not broken down. In addition, the calculated variances of the factor scores agreed very closely with the estimated variances derived earlier in

the analysis (see Chapter 6). The comparison between the two sets of factor variances is shown in Table 37 with the means, standard deviations, minima and maxima values of the factor scores. These checks carried out on the factor scores indicated that no significant loss of arithmetic accuracy had taken place during the computations.

Selected factor types from the skull sample

Morphological comparisons between skulls with high and low scores on a particular factor makes it possible to visualise the factor far more clearly than by an interpretation based on the magnitude of correlations with a set of variables. In the present section some of the sixteen likelihood factors are studied by comparing skulls with contrasting morphology, preference being given to those with either the highest contributions to the common variance or with interesting patterns of associations.

In effect, the comparisons match the computed factor solution with the sample material and provide a method for biological evaluation of the factors. The validity of factoring techniques may be judged more effectively on this basis than by the interpretation of a factor coefficient matrix.

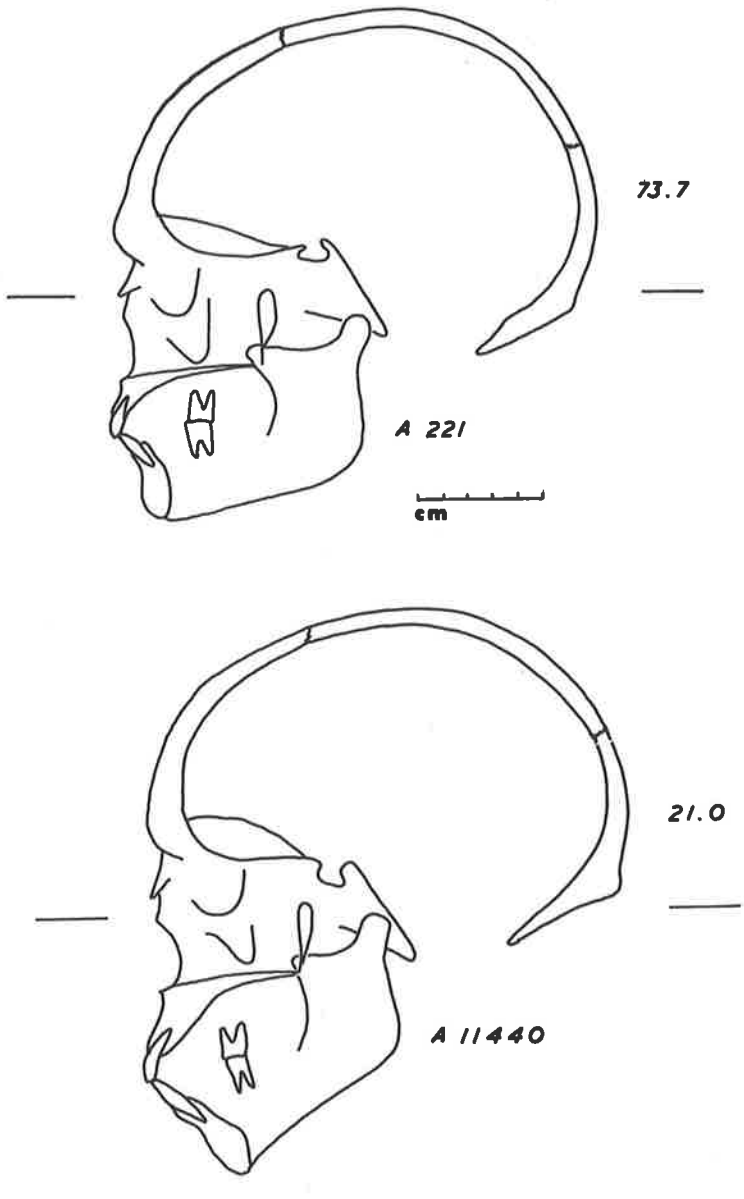

FIGURE 13. Factor 1.

Factor 1 : Mandibular prognathism

Figure 13

Factor 1 was determined by a group of seven variables sharing common reference points or reference lines and the associations expressed by the factor could be partly explained on topographical grounds. The lateral skull tracings shown in Figure 13 illustrate the contrasting craniofacial morphology associated with high and low scores for Factor 1.

Skull A221, with the group maximum score of 73.7, shows marked facial prognathism and low values for variables that were negatively correlated with the factor. The nasal floor and mandibular base are acutely inclined to the cranial base, cranial base flexion is more pronounced and the nasal and morphological face heights are low in value. On the other hand specimen A11440, with the group minimum score of 21.0, shows low mandibular prognathism, an obtuse cranial base angle, marked development of anterior face heights and greater inclination of the nasal and mandibular bases.

The cranial vault dimensions do not differ greatly in the two specimens but mandibular length and palatal length are greater in skull A11440. This suggests that the cranial vault morphology and absolute jaw size have little relation to the degree of facial prognathism in the group under study. The marked contrast in facial characters of the two specimens is indicative of quite different

mandibular growth patterns, skull A221 showing the result of anterior rotation of the mandible, and skull A11440 the result of posterior rotation. The term rotation is applied in the sense explained by BJÖRK ('55a; '64b, p18), who used metallic implants to investigate mandibular growth patterns and discussed the relation between anterior and posterior facial heights and the type of mandibular growth.

Factor 2 : Mandibular length

Figure 14

The morphology associated with different ratings of Factor 2, which was interpreted as one of mandibular length, is illustrated by the lateral skull tracings shown in Figure 14. Specimen A11528 had the group maximum score of 72.1 and specimen A25438 the group minimum of 24.8.

Morphological differences in the two skulls are most evident in the values for the two variables most strongly correlated with Factor 2, that is total mandibular length and morphological face height. Palatal length, nasopharyngeal height and the angle of mandibular prognathism are greater in skull A11528. However, the variable s-n-pg had a low loading of 0.24 on Factor 2 suggesting that no great association existed between mandibular prognathism and the mandibular length factor.

Although there are marked differences in facial size between the two specimens, the cranial vault and cranial base are similar both in

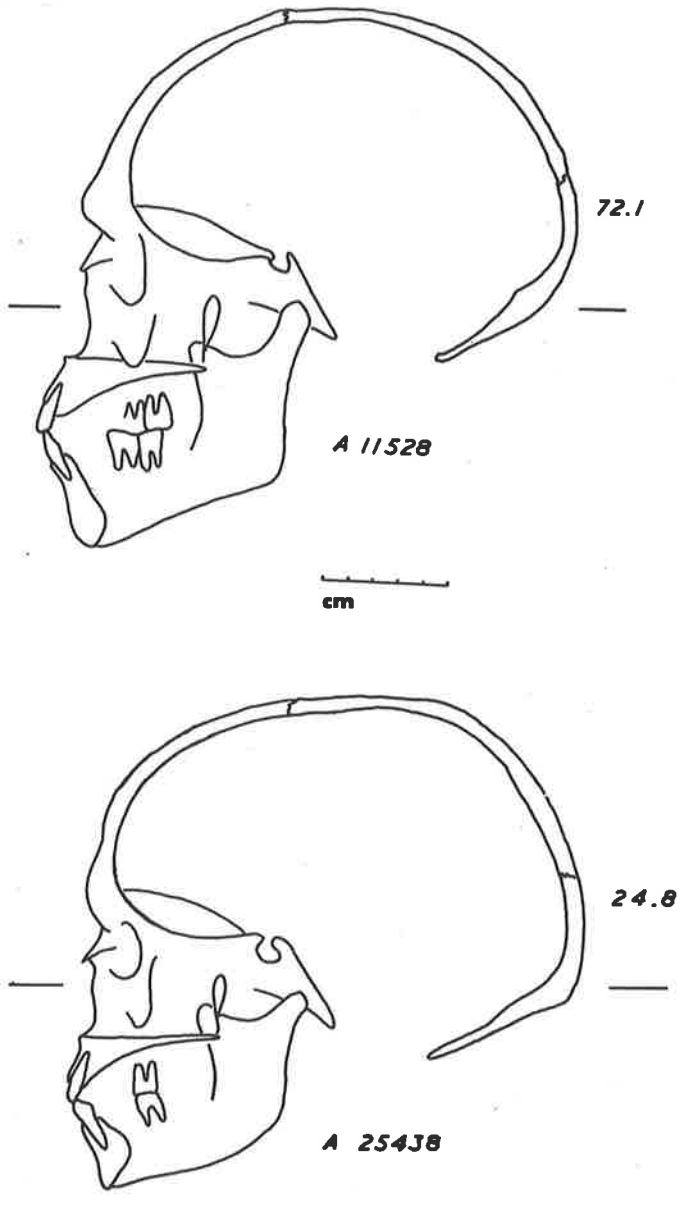

FIGURE 14. Factor 2.

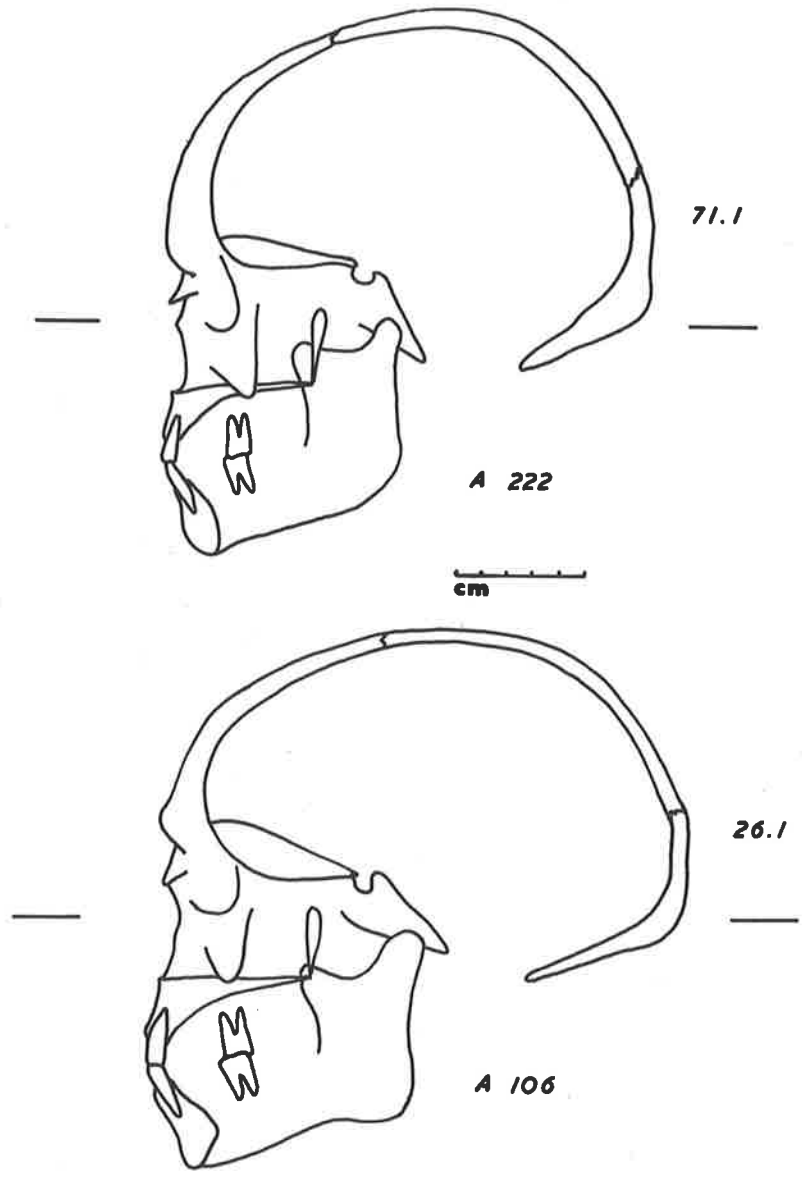

FIGURE 15. Factor 3.

Factor 4 : Anterior cranial base length

Figure 16

Factor 4 was associated with four variables, n-s, endo. 1, max. f and ba-pns, of which n-s contributed most to the factor variance. During interpretation the signs of the factor coefficients were reversed so that specimen A25422, with a factor score of 72.2, shows low values for the variables while specimen A99, with a factor score of 24.1, shows opposite trends.

Figure 16 illustrates the contrasting features of the two specimens. Skull A99 shows marked development in endocranial length, in depth of the nasopharynx and in facial depth; in skull A25422 these dimensions are much smaller. The comparison also shows that anterior cranial base lengths differ, but the posterior segment from sella to basion has almost the same length in both specimens.

The factor appears to operate in the region of the anterior cranial base and the adjacent nasopharynx. Although the illustrations reveal marked differences in the skeletal structures of the face, no relation between the facial variables and Factor 4 was indicated by the pattern of loadings for the factor. The differences can be explained by the specimens' scores on other factors associated with the facial skeleton.

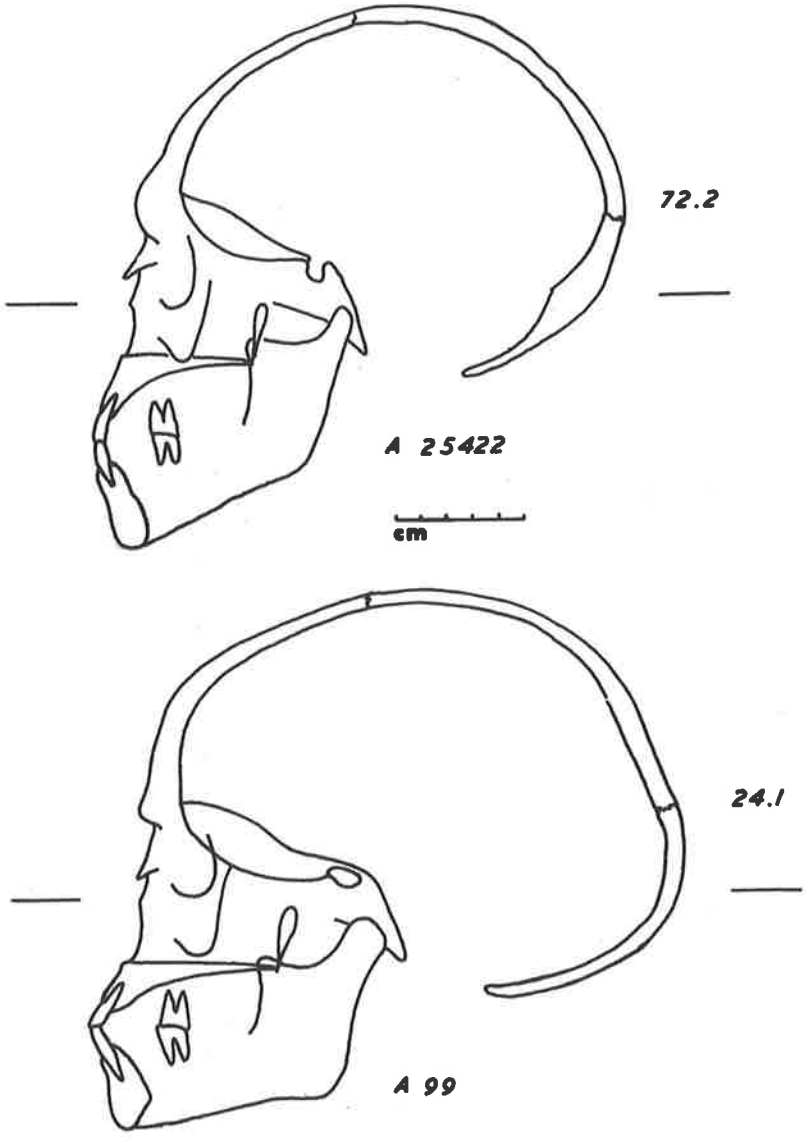

FIGURE 16. Factor 4.

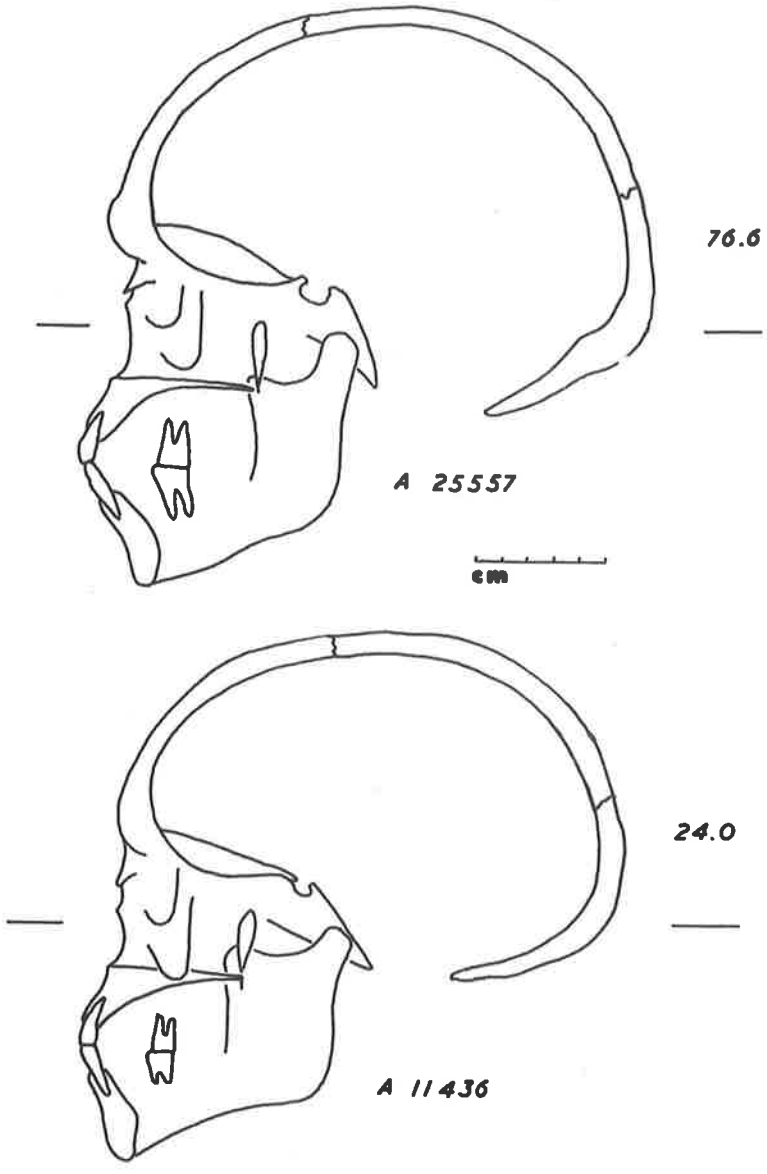

FIGURE 17. Factor 5.

Factor 6 : Infratemporal fossa depth

Figures 18-19

Skulls with the group maximum and minimum scores on Factor 6 are compared by lateral and frontal roentgenogram tracings shown in Figures 18 and 19. The differences are particularly striking in the variables most strongly correlated with the factor, that is infra t.f.d., zg-zg, ramus b and zm-zm.

Specimen A37, with a factor score of 83.4, is generally larger in facial breadth dimensions and in breadth of the mandibular ramus than specimen A25422 with a score of 29.9. Cranial heights and breadths of the two specimens differ very little but the endocranium is considerably shorter in skull A25422, demonstrating the association between the variable endo. 1 and Factor 6.

In skull A37 the zygomatic arches are well developed and extend laterally past the region of maximum cranial vault convexity. The mandibular ramus is broad, the palate low and the mandibular base is inclined acutely to the cranial base. These characters suggest that the masticatory musculature was well developed in this skull. In contrast, skull A25422 shows a type of morphology consistent with reduced muscularity. Factor 6, therefore, represented covariation in structures anatomically associated with the jaw musculature.

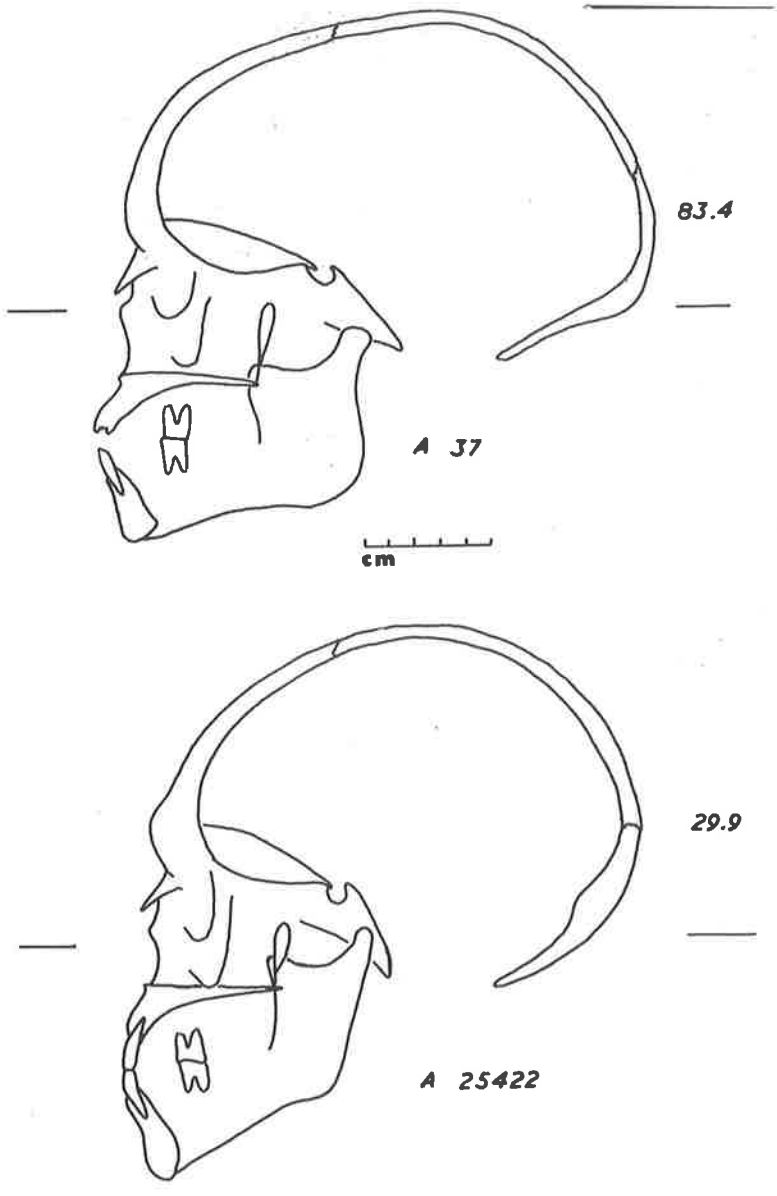

FIGURE 18. Factor 6.

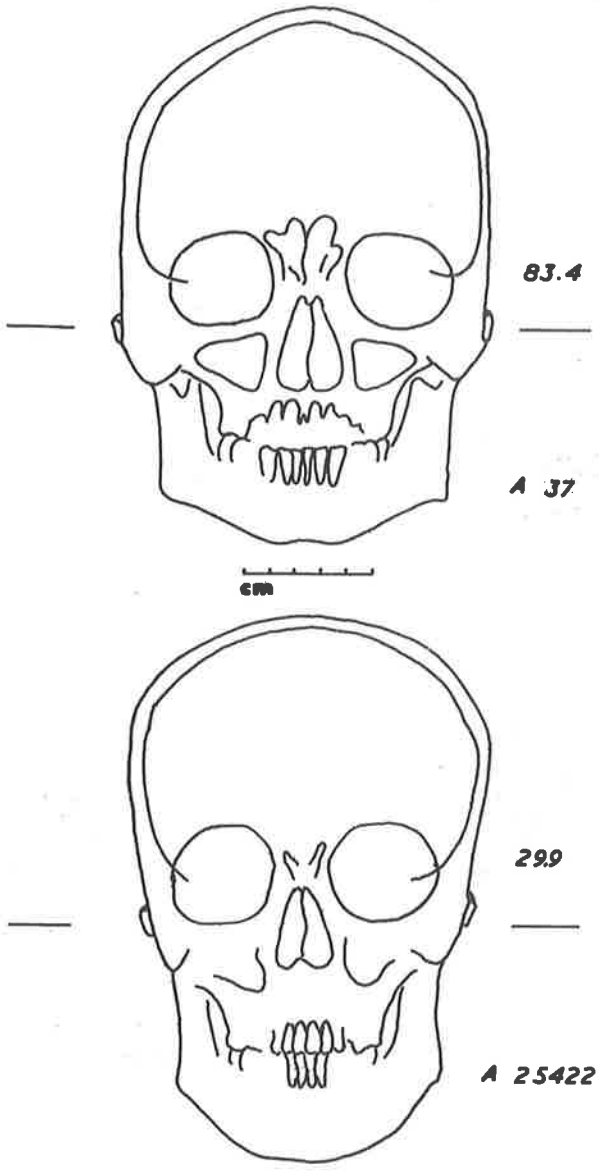

FIGURE 19. Factor 6.

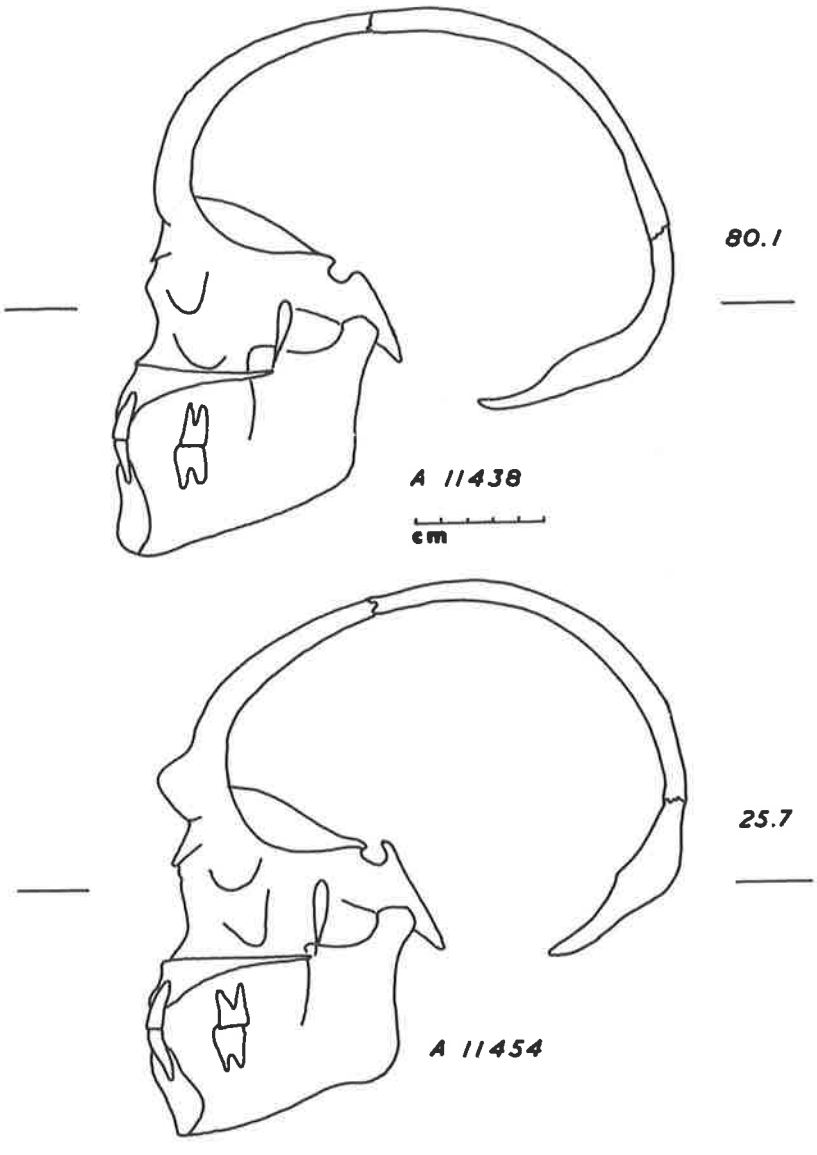

FIGURE 20. Factor 9.

Factor 9 : Head balance

Figure 20

Tracings of specimens A11438 and A11454, which had the group maximum and minimum scores on Factor 9 of 80.1 and 25.7, are shown in Figure 20. Because sign reversal of the factor coefficients was carried out during interpretation, the characters associated with the factor are more pronounced in the specimen with the minimum score.

The principal characters found in conjunction with a low computed factor score were a short cranial vault, an obtuse foramen angle, a long posterior cranial base segment and a deep nasopharynx. The variables for. angle and endo. 1 differ greatly in the two skulls compared. The more acute foramen angle in specimen A11438 is accompanied by greater development in the occipital segment of the cranial vault bringing the posterior margin of the foramen magnum into a more inferior and ventral position. This development is at the expense of nasopharyngeal depth and the posterior cranial base which is shorter than it is in specimen A11454.

Factor 10 : Nasal breadth

Figure 21

Factor 10 was interpreted as representing coordination in several mid-facial breadth dimensions. Variables with the highest loadings for the factor were nasal b, scp-scp, ecm-ecm, zm-zm and zg-zg; those contributing most to the variance of the factor were nasal b, ecm-ecm and scp-scp.

Figure 21 shows postero-anterior tracings for skulls A851 and A25449 which had factor scores of 72.4 and 25.2 respectively. The marked development in breadth of the mid-facial structures is clearly demonstrated in the tracing of A851. In this specimen the nasal cavity, nasopharynx, zygomatic arches and upper dental arcade are much broader than they are in A25449.

It is interesting, however, that cranial breadth was less in skull A851 demonstrating independence of the head breadth Factor 12 and the facial breadth Factor 10. Moreover, specimen A25449 with the group minimum score for Factor 10, had an above-average score of 57.3 for Factor 6, the factor interpreted as infratemporal fossa depth. These observations lend weight to the view that the three breadth factors 6, 10 and 12 influence breadth dimensions of the skull but in different ways and independently of each other.

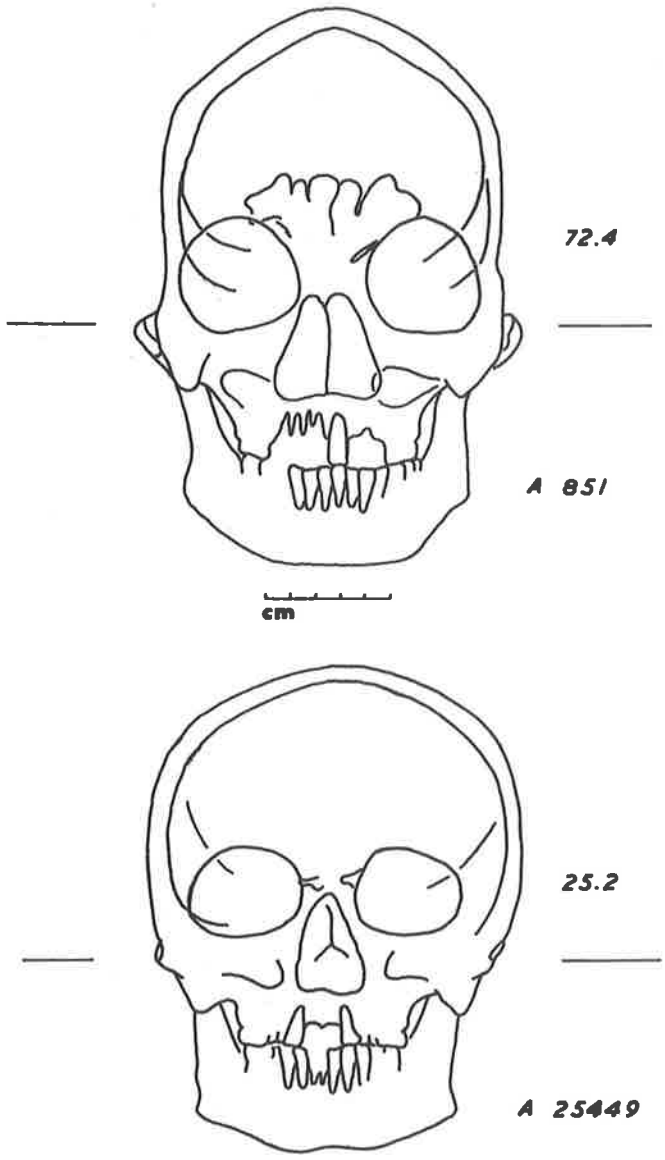

FIGURE 21. Factor 10.

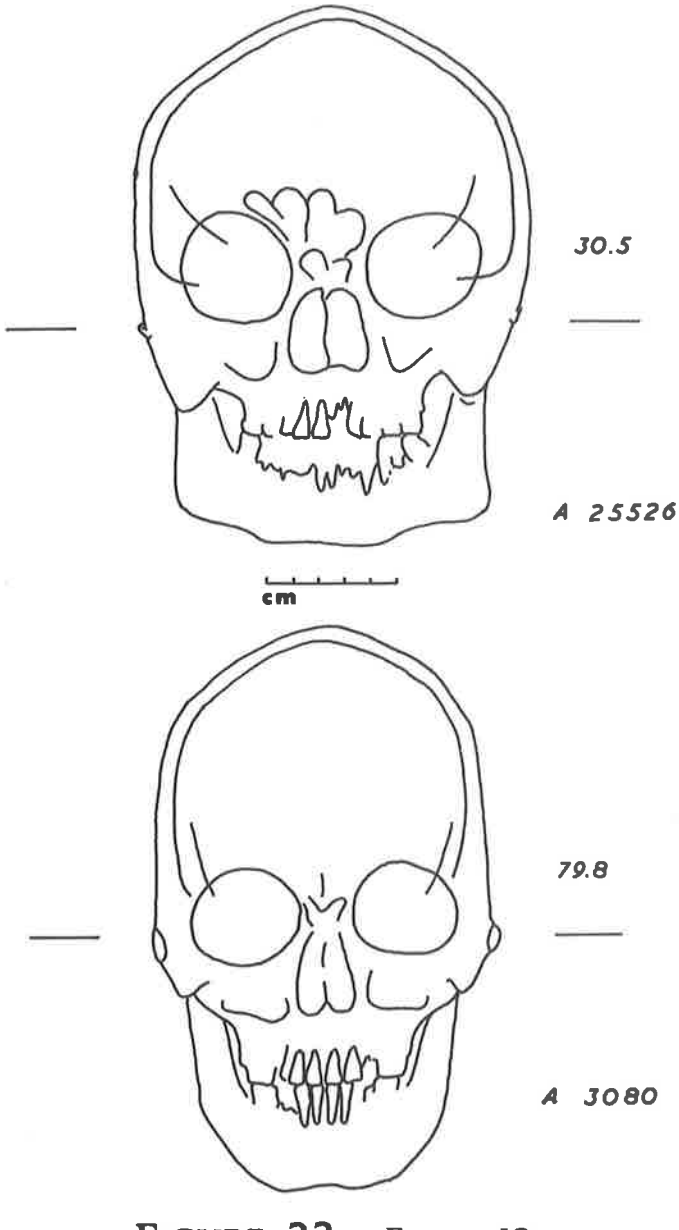

FIGURE 22. Factor 12.

Factor 12 : Endocranial breadth

Figure 22

Postero-anterior tracings of the skulls with maximum and minimum scores on Factor 12 are shown in Figure 22. The three variables most strongly correlated with Factor 12 were endo. b, zg-zg and go-go, the first of which contributed most to the factor variance. Because the signs of the factor coefficients were reversed during interpretation the characters associated with Factor 12 are most pronounced in skull A25526 with the minimum score of 30.5. In contrast skull A3080 had a factor score of 79.8, three standard deviations above the adjusted mean of 50.0.

Morphologically, the two skulls show contrasting features. Specimen A25526 is broad in the cranial vault and has high values for the variables bizygomatic breadth and bigonial breadth while the opposite characters are displayed by specimen A3080. It is interesting that the zygomatic arches in A25526 do not project laterally past the cranial vault as they do in A3080. The variable bizygomatic breadth appears to be determined in part by the action of Factor 12 and in part by agencies acting locally on the arches. Factor 12 reveals a general coordination in breadths of the cranial vault, upper face and mandible.

Factor 14 : Frontal bone size

Figure 23

Sagittal tracings of the frontal regions of skulls with high and low scores for Factor 14 are shown in Figure 23. The highest scores for the group were found in skulls A905, A115 and A11419 with values of 74.9, 73.4 and 71.0 respectively. Lowest factor scores for the group were found in skulls A3080, A97 and A1036 with scores of 31.9, 32.5 and 33.4 respectively.

Two variables contributed to the variance of Factor 14, f. sinus h and max. f. The scores on the factor effectively discriminated between two skull types on the basis of the magnitudes of the variables associated with the factor. It is obvious from the illustrations that the roentgenographic morphology of the frontal bone in the median plane is determined to a great extent by the dimensions of the frontal sinus. However, the outer surface of the frontal bone does not always reflect the magnitude of the underlying cavity. Skull A115 had the second highest score for Factor 14 and yet the outer surface of the frontal bone was flat compared with the other specimens regardless of their factor scores.

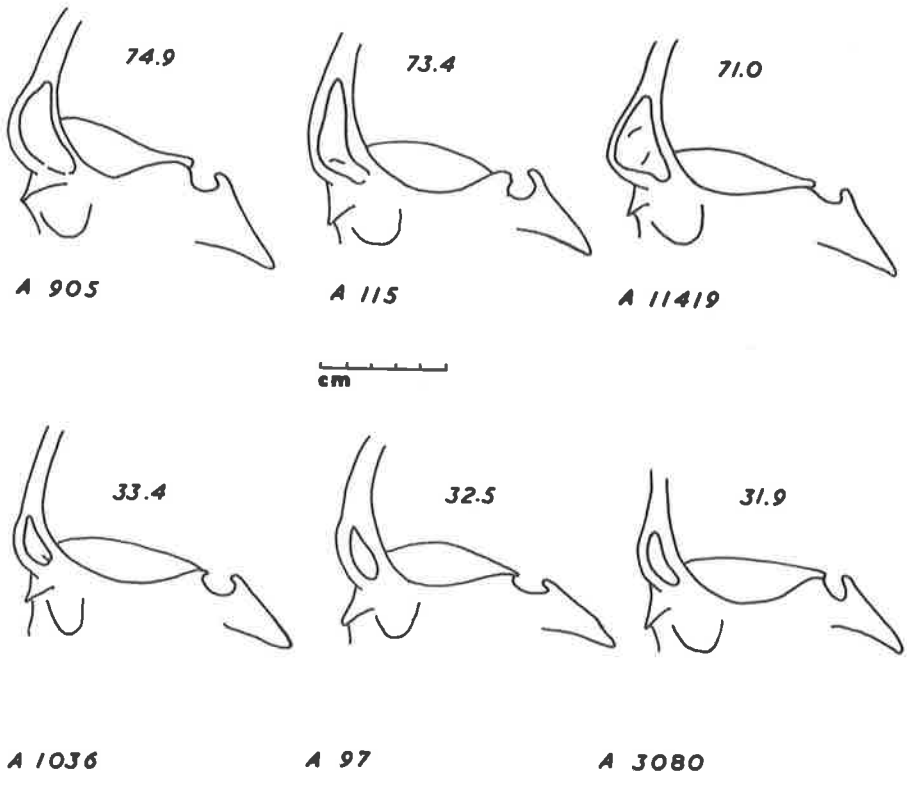

FIGURE 23. Factor 14.

Summary

For the final stage of the multivariate analysis of craniofacial associations in the Australian Aboriginal skull, factor scores were calculated for each of the 100 specimens included in the study. Although the numerical values of the scores had little meaning when considered alone, they provided a useful basis for the comparison of skulls with contrasting morphology. In the present study the associations between craniofacial variables were explained by 16 common factors computed by the maximum likelihood method of Lawley. A restriction of orthogonality was placed on the factors which therefore remained completely independent of each other throughout the various analytic stages. Comparison of skulls with different scores for a factor allowed the factor to be visualised, its interpretation validated and its biological significance assessed. Ten of the factors were treated in this way.

Although the estimation of factor scores has seldom been carried out in anthropometric studies, it would seem that this extension of factor analysis is worthwhile. The nature of factors can be readily understood when contrasting morphological types are compared on the basis of their factor scores. Moreover, the scores can be treated as values of a new set of variables and subjected to further analysis. It has been suggested that factor scores may be more useful than direct anthropometric observations in genetic studies. In this regard

it would be interesting to apply the methods outlined in this section to a group of Aboriginal subjects for whom genealogies had been collected; the Wailbri Aboriginals of Central Australia constitute such a group.

GENERAL DISCUSSION AND SUMMARY

The investigation was undertaken to provide information on the patterns of covariation in measurable characters of the Australian Aboriginal skull. Factor analysis, one of several multivariate techniques, was used as the central method of data reduction to disclose sources of coordination within the craniofacial components. The findings supplement the information gained from previous descriptive studies of this ethnic group and contribute to the general understanding of variations in human head-form.

Standardised lateral and postero-anterior roentgenograms were obtained from 100 Australian Aboriginal skulls selected from the collection housed in the South Australian Museum, Adelaide. The skulls were adult and sexed as male, the sex ratings being checked by comparison with those derived by previous workers who had examined the post-cranial skeletons of the same specimens. Age records were not available but it is reasonably certain that most specimens date from the period prior to the European settlement of Australia. The skulls originated from several different regions of the continent but were grouped together for the multivariate analyses.

Initially 77 linear and angular variables were selected to represent the craniofacial characters that would contribute most to a study of morphological coordination. The variables included measures of the sagittal endocranial curvatures, general cranial form, the size and shape of the cranial base, nasopharyngeal size and size and shape of the facial skeleton. Measurements were obtained either directly from the skulls or indirectly from the roentgenograms and in the latter case compensation was made for differential enlargement of the image.

A comparison of direct and indirect determinations of selected variables showed that the discrepancies between determinations were small if landmarks could be easily recognised on the roentgenograms. However, the use of landmarks difficult to locate precisely on roentgenograms gave rise to errors of significant magnitude. The direct measurement technique was used for preference whenever measurements could be obtained in either way.

Before commencing the multivariate analysis a descriptive statistical survey was carried out on the observations. Mean values of the linear variables were very similar to those reported in previous craniometric studies of the Aboriginal skull but were generally smaller than the corresponding values derived from a roentgenographic study of Wailbri Aboriginals living under settlement conditions in Central Australia. Additional research is required to ascertain if the differences arose from post-mortem shrinkage of the

skulls alone or whether they indicated differences between tribal groups or changes in head-form that have taken place since the time of European contact and the adoption of changing food patterns.

In order to throw light on the nature of regional variations in head-form, a comparison was made between 12 skulls from Melville Island and the north coast and the remainder in the sample, most of which originated from the southern regions of the continent. The main differences between the groups were confined to the cranial vault which was smaller in length and breadth but greater in height in the northern specimens which therefore presented a more globular cranial form. The endocranial contour in the basal region was more convex in the northern skulls but there were few significant differences between the two groups in facial variables.

The distributions of the 59 variables selected for the multivariate analysis were examined by computing estimates for the parameters of skewness and kurtosis. Eleven variables showed significant departures from normality at the one or five per cent probability levels and although skewness to the right was the most common finding, there appeared to be no set pattern in the disclosed departures. An understanding of the distribution forms of craniometric variables must await the presentation of statistics for many other groups.

The first stage of the multivariate analysis was concerned with the identification of sources of specious associations among the 59

variables. The term specious was used to describe a correlation between two variables that could be partly explained and to a certain extent predicted on a non-biological basis. Specious associations were presented when two variables shared common reference points, reference lines or anatomical structures, when variables shared common components, for example indices, and when variables overlapped each other by spanning the same or adjacent anatomical regions. Recognition of these sources of association is important in roentgenographic analyses where it is usual to employ linear and angular variables that are defined from a number of common reference points. Associations of this type have been given special attention by SOLOW ('66).

Correlations between the variables were examined by using the technique of factor analysis which seeks to explain the shared variance present in a set of variables in terms of common factors affecting more than one variable and unique factors, affecting single variables. Five sequential factor analyses were carried out and the revealed patterns of association were examined at each stage. This procedure permitted some experimental control over the variables and provided a guide to the elimination of variables duplicating the information contained in others.

The most precise factoring method is the Lawley maximum likelihood estimation which is an iterative procedure leading to convergence of the factor loadings to stable values. This method was used for the first and last analyses when a high level of mathematical precision

was desirable but the simpler principal factor technique was applied for the intermediate analyses when factor interpretation was the prime objective. To assist in factor interpretation all initial solutions were subjected to orthogonal transformation by the Kaiser varimax method.

In order to give substance to the factors disclosed in the final analysis, factor scores were computed for all skulls in the sample. Specimens with high and low scores for a factor were then compared. This extension of the factor technique allowed the factors to be visualised by highlighting the contrasting morphological characters displayed by skulls placed at the extremes of the range of scores for a factor.

For the final analysis, 30 variables were retained from the original 59 to represent most of the interesting features of craniofacial morphology in the Australian Aboriginal. The derived factors were taken as indications of sources of variability common to groups of variables. Although 16 common factors were retained, not all of these were of major importance and this number could probably have been reduced by two or three without destroying the mathematical precision of the solution.

It appeared that coordination between the craniofacial components could be either general or local in character. General factors represented sources of variability shared by two or more distinct regions of the skull, for example between cranial vault variables

and facial variables. Local factors, however, were more restricted in their effect and were determined by two or more variables from the same or, in some instances, adjacent anatomical regions.

The general factors were concerned with the various breadth and height dimensions of the skull and with the size of the facial skeleton. Of these, the factors indicating breadth coordination were particularly interesting. One breadth factor (Factor 12) was concerned with breadths of the endocranium, zygomatic arches and the mandible measured at the gonial angles. The dimensions concerned were all measured in approximately the same plane and although some correlation between them could be expected for this reason, the factor indicated coordination in breadths of the cranial vault, upper face and mandible in the coronal plane.

Breadths of the facial skeleton were determined by the operation of two additional factors, one expressing coordination in nasal, nasopharyngeal and upper facial breadths and the other demonstrating the coordination in size of facial components that are conjointly influenced in morphology by the development of the masticatory musculature. The nasal breadth factor (Factor 10) was correlated with nasal and nasopharyngeal breadths and with upper alveolar arch and facial breadths. These dimensions were measured in adjacent planes and the factor can be taken as evidence of coordination in facial and nasopharyngeal breadths in the transverse plane. There was no association with either cranial or mandibular breadths.

The third breadth factor (Factor 6) was determined by the depth of the infratemporal fossa, breadths of the upper face measured in the zygomatic region and mandibular ramus breadth. The factor appeared to indicate quite definitely the effect of different degrees of muscular development on adjacent bony morphology. Endocranial length was positively associated with this factor.

There was some indication of coordination in heights of the endocranium, nasopharynx and mandibular ramus as expressed by the endocranial height factor which was also correlated positively with endocranial length. A second height factor revealed an association between height dimensions of the sphenoid bone, nasal cavity and mandibular ramus. The factor of mandibular length indirectly indicated the conjoint variation in lengths of the mandible and palate, breadth of the upper face and heights of the face and nasopharynx. This factor could be taken as evidence of a general coordination in the size of several facial components.

From the patterns of loadings on the general factors it is reasonable to conclude that the craniofacial characters in the skull group studied were determined to a large extent by the interaction of factors indicating the developmental state of the endocranium (brain), nasopharynx, nasal cavity and masticatory musculature. There was strong evidence of coordination in various skull breadth dimensions. A moderate degree of coordination existed in height dimensions but there was little indication of close associations between the depth

dimensions of the cranial vault and facial skeleton although within the facial skeleton the linear variables showed some concomitant variation. These findings offer an explanation for the clinical observation that dental malocclusions arising from a disparity in jaw relationships are more prevalent in the sagittal plane than in the coronal.

Apart from the general factors referred to, several factors indicated covariation in local regions of the skull. There was a fairly close association between the length of the skull and the position of the foramen magnum in relation to the cranial base, the foramen being situated more ventrally with an increase in cranial length. A local factor accounted for the concomitance in thickness of the frontal bone and height of the frontal air sinus in the anterior sagittal plane. Within the cranial base region there were two coordinating mechanisms, one concerned with lengths of the base, the endocranium and the nasopharynx, and the other expressing a close relationship between nasopharyngeal depth and the angle of cranial base flexion. The thickness of the sphenoid bone also varied conjointly with the height of the upper face and mandibular ramus.

Within the facial skeleton one local factor indicated a relationship between heights of the palate and mandibular ramus and two were concerned with the facial profile. The profile factors disclosed the morphological characters associated with different profile shapes

as measured by the angles of mandibular prognathism and facial convexity.

There is no doubt that many significant factor loadings arose from the presence of a number of specious correlations between the variables. In the present analysis almost half of the variables were eliminated in the earlier stages with a consequent reduction in the number of speciously related variables. However, specious associations are informative in so far as they disclose the extent and direction of intracranial relationships even though this information can sometimes be predicted without resort to mathematical analysis. In many instances it was difficult to interpret specious correlations because they could have arisen from a combination of biological and non-biological coordination.

Evidence of biological coordination was disclosed by the factor analysis but it was thought unjustified to identify the factors with specific biological processes. Although components of the skull were not independent in their morphology, many associations revealed in the study were confined to anatomically or functionally related structures; these associations could sometimes be considered as indications of compensatory adjustments that had taken place during growth of the components concerned. There was, however, evidence of more general coordinating mechanisms, particularly in breadth dimensions measured in the coronal and transverse planes. It would be reasonable to conclude that the development of the skull as a

whole is a highly integrated process and that while each component has a certain measure of independence, associations between functionally related regions exist. The identification of causative agents underlying any revealed coordinations is beyond the scope of factor analysis and must be left to other techniques, for example, experimental biology and genetics.

The present use of factor analysis is seen as an extension of previous factor studies of the human skull and supports the view that the method is a useful tool in craniometric research. By treating variables collectively the method explores the sources of covariation between them more objectively than is possible with conventional statistical techniques. In this way factor analysis brings an entirely new approach to the study of craniofacial variation. In particular, the estimation of factor scores appears to be a worthwhile extension of the method in so far that it provides a basis for more penetrating studies aimed to separate the genetic, environmental and functional determinants of cranial morphology. Research along these lines will be continued in a group of Central Australian Aboriginals for whom longitudinal growth data and genealogies are available.

APPENDIXES

APPENDIX A

Skulls included in the study with South Australian Museum Catalogue numbers shown

SOUTH AUSTRALIA

A	37	Swanport
	42	Swanport
	66	Swanport
	77	Swanport
	97	Swanport
	98	Swanport
	99	Swanport
	101	Swanport
	102	Swanport
	106	Swanport
	114	Swanport
	115	Swanport
	117	Swanport
	125	Swanport
	126	Swanport
	129	Swanport
	242	Mingbool
	306	Mypolonga
	450	Coorong
	480	Swanport
	481	Swanport
	569	Coorong
	719	Ardrossan
	777	Robe
	799	Meningie
	847	Cournamony
	989	Morgan
	1036	Ardrossan
	1081	Wallaroo
	3075	Glenelg
	3077	Glenelg
	11418	Swanport
	11423	Innamincka
	11436	Tailem Bend
	11438	Salt Creek
	11440	Oodnadatta
	11515	Myponga
A	11518	Port Lincoln
	11526	Cape Jervis
	11528	Torrens Island
	11529	Coorong
	11536	Coffin Bay
	13167	Meningie
	13171	Coorong
	14474	Adelaide
	15550	Hardwicke Bay
	15553	Fulham
	15554	Fulham
	15555	Fulham
	15557	Fulham
	16505	Alawoona
	16518	Port Adelaide
	16521	Milang
	16524	Wallaroo
	20583	Corny Point
	20584	Lake Albert
	20587	Fulham
	20589	Fulham
	20591	Wallaroo
	20596	Allandale
	20606	Renmark
	20615	Ardrossan
	20619	Moorook
	20629	Coorong
	20654	Locality unknown
	25437	Streaky Bay
	25438	Coorong
	25449	Coorong
	25455	Umberatana
	25501	Coorong
	25526	Lake Albert
	25553	Milang
	25557	Morgan

WESTERN AUSTRALIA

A	838	Fitzroy River
	13196	Derby
	25422	Cygnet Bay
A	25425	Cygnet Bay
	25564	Eucla

NEW SOUTH WALES

A	905	Lake Victoria
	994	Lake Victoria
A	11454	Moorna
	25456	Silverton

VICTORIA

A	16868	Echuca

NORTHERN TERRITORY

A	220	Melville Island
	221	Melville Island
	222	Melville Island
	851	Melville Island
	853	Melville Island
	3080	Melville Island
	11419	Melville Island
	11420	Melville Island
A	11434	Melville Island
	11455	Boroloola
	13144	Tennant Creek
	20104	Hermannsburg
	20105	Hermannsburg
	25426	Charlotte Waters
	25429	Adelaide River

LOCALITY NOT KNOWN

A	132	
A	11416	

APPENDIX B

Tables relating to the five factor analyses

CONTENTS

TABLE 14. CORRELATION COEFFICIENTS

	VARIABLE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	NASAL CHORD	1.00	.27	-.04	.27	.10	.11	.17	-.25	.36	-.07	.35	.21	-.08	.22	.15	.28	.41	.04	.18	-.20	1
2	NASAL INDEX	.27	1.00	-.01	.03	.02	-.06	-.05	-.11	-.04	-.19	.13	-.08	-.19	.04	-.23	.32	.00	.07	-.05	-.17	2
3	A-N-C	-.04	-.01	1.00	-.20	-.38	-.20	.26	.13	-.07	-.01	.41	-.00	-.00	.28	.10	-.28	-.15	.08	.05	.19	3
4	FRONTAL CHORD	.27	.03	-.20	1.00	.17	-.33	.12	-.38	.52	.03	.36	.43	.05	.33	.08	.23	.26	-.09	.12	.03	4
5	FRONTAL INDEX	.10	.02	-.38	.17	1.00	-.11	.01	.05	.05	-.00	.03	-.02	.04	.09	-.05	.06	.01	.04	-.12	-.26	5
6	N-C-O	.11	-.06	-.20	-.33	-.11	1.00	-.04	-.11	.33	.07	-.45	.32	.09	-.19	.11	-.02	.13	.00	.19	.20	6
7	PARIETAL CHORD	.17	-.05	.26	.12	.01	-.04	1.00	-.11	.71	.18	.44	.60	.24	.33	.21	.01	.19	.30	.30	.05	7
8	PARIETAL INDEX	-.25	-.11	.13	-.38	.05	-.11	-.11	1.00	-.36	-.05	.06	-.22	-.04	.01	-.08	-.02	-.10	.20	-.02	-.17	8
9	ENDO. L	.36	-.04	-.07	.52	.05	.33	.71	-.36	1.00	.15	.27	.87	.21	.33	.26	.15	.37	.14	.39	.17	9
10	ENDO. R	-.07	-.19	-.01	.03	-.00	.07	.18	-.05	.15	1.00	.07	.08	.92	.12	-.08	-.05	-.16	.11	.00	.05	10
11	ENDO. M	.35	.13	.41	.36	.03	-.45	.44	.06	.27	.07	1.00	.25	.08	.57	.06	.20	.21	.39	.24	-.28	11
12	G-OP	.21	-.08	-.00	.43	-.02	.32	.60	-.22	.87	.08	.25	1.00	.14	.33	.44	.15	.54	.12	.50	.18	12
13	EU-EU	-.08	-.19	-.00	.05	.04	.09	.24	-.04	.21	.92	.08	.14	1.00	.20	-.04	-.10	-.17	.10	-.00	.09	13
14	PO-V	.22	.04	.28	.33	.09	-.19	.33	.01	.33	.12	.57	.33	.20	1.00	.12	.07	.14	.16	.16	-.07	14
15	N-ETH	.15	-.23	.10	.08	-.05	.11	.21	-.08	.26	-.08	.06	.44	-.04	.12	1.00	-.35	.64	.02	.52	.14	15
16	ETH-S	.25	.32	-.26	.23	.06	-.02	.01	-.02	.15	-.05	.20	.15	-.10	.07	-.35	1.00	.42	.02	.19	-.20	16
17	N-S	.41	.00	-.15	.26	.01	.13	.19	-.10	.37	-.16	.21	.54	-.17	.14	.64	.42	1.00	.04	.66	-.08	17
18	S-BA	.04	.07	.08	-.09	.04	.00	.30	.20	.14	.11	.39	.12	.10	.16	.02	.02	.04	1.00	.59	-.07	18
19	N-BA	.18	-.05	.05	.12	-.12	.19	.30	-.02	.39	.00	.24	.50	-.00	.16	.52	.19	.66	.59	1.00	.36	19
20	N-S-BA	-.20	-.17	.19	.03	-.26	.20	.05	-.17	.17	.05	-.28	.18	.09	-.07	.14	-.20	-.08	-.07	.36	1.00	20
21	ETH-S-BA	-.25	-.37	.24	.03	-.34	.11	.00	-.13	.08	.02	-.16	.15	.07	.01	.08	-.16	-.06	-.11	.28	.79	21
22	FOR. ANGLE	-.10	-.04	.22	-.23	-.10	.03	-.23	.20	-.26	-.12	.13	-.10	-.16	-.04	.01	-.02	.01	.26	.23	.17	22
23	MIN. F	-.05	.02	.21	.05	-.25	-.09	.07	.01	.03	-.29	.22	.26	-.28	.11	.27	.07	.38	.11	.34	-.12	23
24	MAX. F	-.04	-.01	.32	-.02	-.23	-.04	.07	-.04	.01	-.24	.23	.30	-.24	.05	.37	.01	.41	.13	.34	-.15	24
25	F. SINUS M	-.04	.12	.15	.01	-.23	.01	.06	-.06	.04	-.14	.11	.17	-.15	.10	.10	.06	.17	.19	.34	.02	25
26	F. SINUS M	.06	.15	.21	-.04	-.21	.00	.05	-.03	.00	-.17	.18	.07	-.23	.08	.00	.13	.13	.27	.30	-.07	26
27	SPHEN. O	.03	-.04	-.13	-.12	.13	.15	.14	-.09	.08	.16	-.07	.14	.14	.06	.08	.13	.15	.28	.20	-.19	27
28	NASAL. M	.21	.20	.03	.03	-.07	-.14	.05	.06	-.03	-.10	.24	-.01	-.12	.08	-.02	.16	.13	.18	.14	-.14	28
29	SS-PNS	-.00	-.08	-.15	.06	.07	.20	.16	-.23	.25	.15	-.10	.35	.15	-.02	.25	.09	.30	.08	.33	.17	29
30	N-SP	-.12	-.06	.03	-.05	-.12	.15	.11	-.12	.10	.04	-.04	.15	.02	.08	.10	.07	.17	.26	.48	.41	30

VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .05 IS 0.197
VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .01 IS 0.257

TABLE 14. CORRELATION COEFFICIENTS

	VARIABLE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
31	BA-PNS	.04	-.19	.12	-.07	-.15	.10	.15	.03	.13	-.00	.07	.23	-.05	.08	.30	.15	.37	.25	.52	.28	31
32	IPM-PNS	.03	-.26	-.02	-.09	-.12	.27	.23	-.03	.26	.03	-.08	.34	.00	-.00	.30	.14	.39	.18	.47	.24	32
33	SCP-SCP	.06	.10	.07	-.07	.05	.11	.07	.18	.05	.12	.16	.09	.12	.20	.02	.02	.03	.14	.12	-.02	33
34	PHAR. H	.06	.02	.05	.10	.18	-.13	.19	-.03	.09	.14	.38	.04	.15	.21	-.05	.08	.02	.35	.09	-.27	34
35	S-PM HOR.	.25	.04	-.31	.12	.23	-.09	-.03	.05	.02	-.21	.08	.06	-.18	.12	.13	.31	.38	-.09	-.10	-.56	35
36	S-PM VERT.	-.06	.00	.09	-.24	.14	.04	.21	.07	-.01	.01	.10	.08	.01	.07	.06	.01	.07	.36	.16	-.21	36
37	N-GN	-.13	-.15	.10	.02	-.18	.07	.15	.02	.14	.08	.07	.22	.10	.15	.14	-.03	.03	.31	.37	.29	37
38	ZM-ZM	.11	-.08	-.19	.20	.07	.16	.21	-.14	.37	.22	.03	.35	.18	.07	.11	.11	.19	.13	.25	.16	38
39	ZG-ZG	-.04	-.08	-.01	.11	-.04	.20	.19	-.02	.28	.36	.01	.29	.42	.10	.03	.11	.09	.19	.30	.33	39
40	MASS. H	.06	.00	.05	.03	-.09	.03	-.07	.29	.03	-.12	.08	.11	-.14	.20	.02	.13	.11	.08	.19	.19	40
41	ECM-ECM	.09	-.30	-.00	.10	-.07	.05	.15	-.00	.19	.15	.22	.20	.14	.17	.11	.00	.14	.36	.37	.11	41
42	PALATE H	.08	-.06	-.01	-.00	-.04	-.08	.01	.06	-.04	.03	.17	-.04	.05	.07	.03	.03	.11	.20	.17	-.07	42
43	PALATE L	-.03	-.07	.03	-.07	-.13	.25	.19	-.07	.22	.32	-.06	.32	.30	.00	.14	.10	.11	.17	.28	.19	43
44	PALATE H	.09	.14	.11	-.05	-.11	-.12	.06	-.00	-.04	-.10	.15	-.03	-.10	.13	-.03	.09	-.02	.25	.10	-.05	44
45	GO-GO	-.08	-.06	-.06	.04	.07	.09	.18	-.29	.15	.34	-.06	.07	.39	-.13	.00	-.06	-.05	.04	.00	.02	45
46	GN-GO	-.09	-.05	.11	-.02	-.10	.02	.24	.12	.16	.19	.17	.30	.17	.18	.23	.15	.29	.16	.26	-.04	46
47	GN-CO	-.07	-.16	.13	.08	-.09	-.03	.19	-.04	.16	.04	.19	.26	.08	.17	.26	.04	.22	.22	.26	-.00	47
48	RAMUS H	.01	-.09	-.04	.20	-.01	.13	.18	-.03	.34	.03	-.03	.38	.02	-.04	.15	.09	.24	-.03	.22	.17	48
49	RAMUS H	.14	-.01	-.02	.27	.04	-.09	.09	-.12	.19	-.01	.35	.20	.04	.31	.05	.16	.20	.37	.23	-.30	49
50	INFRA T.F.O.	-.05	-.13	-.11	.16	.07	.15	.23	-.13	.33	.10	-.05	.30	.20	.08	-.06	.15	.07	.09	.15	.13	50
51	S-N-SS	-.11	-.11	-.14	-.16	.18	.07	.14	-.05	.04	.25	-.06	.04	.27	-.04	-.05	-.10	-.11	.04	-.25	-.37	51
52	S-N-PG	.01	.04	-.10	-.04	.15	-.07	.14	-.01	.01	.15	.06	-.01	.17	.02	-.06	.08	.02	.01	-.25	-.52	52
53	S-N-PR	-.11	-.07	-.11	-.09	.20	-.01	.06	.03	-.02	.18	-.07	-.02	.17	-.02	-.16	-.03	-.22	.03	-.34	-.38	53
54	S-N-IO	-.09	-.02	-.09	-.13	.15	-.04	.10	.04	-.04	.16	-.06	-.06	.14	-.10	-.12	-.01	-.12	-.02	-.33	-.46	54
55	N-SS-PG	.13	.20	.05	.14	-.13	-.21	.04	.06	-.02	-.12	.20	-.03	-.10	-.01	.05	.16	.21	-.02	.03	-.23	55
56	AR-GO-GN	.10	.13	-.05	-.14	.02	-.03	-.13	.09	-.21	-.09	-.14	-.28	-.08	-.13	-.08	-.07	-.19	-.18	-.29	-.07	56
57	NL/NSL	-.08	-.11	.05	.06	-.12	.08	-.02	-.12	.07	.09	-.12	.00	.05	-.04	.04	-.02	-.01	-.02	.27	.56	57
58	ML/NSL	-.08	-.04	.14	-.10	-.12	-.03	-.12	.19	-.17	-.02	-.06	-.15	-.03	-.02	-.04	-.12	-.20	-.07	-.04	.27	58
59	NL/ML	-.04	.02	.12	-.14	-.03	-.09	-.13	.27	-.24	-.08	.02	-.17	-.08	.01	-.07	-.12	-.22	-.06	-.20	-.03	59

VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .05 IS 0.197
VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .01 IS 0.257

TABLE 14. CORRELATION COEFFICIENTS

	VARIABLE	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
1	BASAL CHORD	-.25	-.10	-.05	-.04	-.04	.06	.03	.21	-.00	-.12	.04	.03	.06	.06	.25	-.06	-.13	.13	-.04	.06	1
2	BASAL INDEX	-.37	-.04	.02	-.01	.12	.15	-.04	.20	-.08	-.06	-.19	-.26	.10	.02	.04	.00	-.15	-.08	-.08	.00	2
3	A-B-C	.24	.22	.21	.32	.15	.21	-.13	.03	-.15	.03	.12	-.02	.07	.05	-.31	.09	.10	-.19	-.01	.05	3
4	FRONTAL CHORD	.01	-.23	.05	-.02	.01	-.04	-.12	.03	.06	-.05	-.07	-.09	-.07	.10	.12	-.24	.02	.20	.11	.03	4
5	FRONTAL INDEX	-.34	-.10	-.25	-.23	-.23	-.21	.13	-.07	.07	-.12	-.15	-.12	.05	.16	.23	.14	-.18	.07	-.04	-.09	5
6	A-C-D	.11	.03	-.09	-.04	.01	.00	.15	-.14	.20	.15	.10	.27	.11	-.13	-.09	.04	.07	.18	.20	.03	6
7	PARIETAL CHORD	.00	-.23	.07	.07	.06	.05	.14	.05	.16	.11	.15	.23	.07	.19	-.03	.21	.15	.21	.19	-.07	7
8	PARIETAL INDEX	-.1.	.20	.01	-.04	-.06	-.03	-.09	.06	-.23	-.12	.03	-.03	.18	-.03	.05	.07	.02	-.14	-.02	.29	8
9	ENDO. L	.04	-.26	.03	.01	.04	.00	.08	-.03	.25	.10	.13	.26	.05	.09	.02	-.01	.14	.37	.28	.03	9
10	ENDO. R	.02	-.12	-.29	-.24	-.14	-.17	.16	-.10	.15	.04	-.00	.03	.12	.14	-.21	.01	.08	.22	.36	-.12	10
11	ENDO. R	-.1.	.13	.22	.23	.11	.18	-.07	.24	-.10	-.04	.07	-.08	.16	.38	.08	.10	.07	.03	.01	.08	11
12	G-OP	.15	-.10	.26	.30	.17	.07	.14	-.01	.35	.15	.23	.34	.09	.04	.06	.08	.22	.35	.29	.11	12
13	EU-EU	.07	-.16	-.28	-.24	-.15	-.23	.14	-.12	.15	.02	-.05	.00	.12	.15	-.18	.01	.10	.18	.42	-.14	13
14	PO-V	.01	-.04	.11	.05	.10	.08	.06	.08	-.02	.08	.08	-.00	.20	.21	.12	.07	.15	.07	.10	.20	14
15	N-ETH	.08	.01	.27	.37	.10	.00	.08	-.02	.25	.10	.30	.30	.02	-.05	.13	.06	.14	.11	.03	.02	15
16	ETH-S	-.16	-.02	.07	.01	.06	.13	.13	.16	.09	.07	.15	.14	.02	.08	.31	.01	-.03	.11	.11	.13	16
17	N-S	-.05	.01	.38	.41	.17	.13	.15	.13	.30	.17	.37	.39	.03	.02	.38	.07	.03	.19	.09	.11	17
18	S-BA	-.11	.26	.11	.13	.19	.27	.28	.18	.08	.26	.25	.18	.14	.35	-.09	.36	.31	.13	.19	.08	18
19	N-BA	.22	.23	.34	.34	.34	.30	.20	.1-	.33	.48	.52	.47	.12	.09	-.10	.16	.37	.25	.30	.19	19
20	N-S-BA	.19	.17	-.1.	-.15	.02	-.07	-.19	-.14	.17	.41	.28	.24	-.02	-.27	-.56	-.21	.29	.16	.33	.19	20
21	ETH-S-BA	1.00	.15	.18	.09	.19	.14	-.16	-.07	.07	.36	.26	.20	-.09	-.15	-.49	-.17	.32	.09	.21	.21	21
22	FOR. ANGLE	.15	1.00	.11	.20	.12	.10	-.25	-.09	-.08	.16	.23	.06	-.02	-.06	-.19	-.01	.16	-.12	-.08	.21	22
23	MIN. F	.18	.11	1.00	.79	.51	.46	.08	.12	.02	.14	.18	.10	.07	.06	.10	.15	.19	-.02	-.11	.01	23
24	MAX. F	.09	.20	.79	1.00	.46	.40	.14	.17	.05	.05	.22	.16	-.02	.10	.07	.21	.10	-.11	.00	-.02	24
25	F. SINUS R	.19	.12	.51	.46	1.00	.80	.13	.23	-.04	.20	.11	.09	.18	.05	-.17	.07	.29	.04	.06	.08	25
26	F. SINUS R	.14	.10	.46	.40	.80	1.00	.24	.30	-.10	.18	.10	.04	.24	.13	-.20	.19	.25	.03	-.01	.08	26
27	SPHEN. D	-.16	-.25	.08	.14	.13	.24	1.00	.11	.17	.24	.21	.26	.13	.07	.16	.43	.10	.08	.15	-.04	27
28	NASAL. R	-.07	-.09	.12	.17	.23	.30	.11	1.00	.06	.05	.09	.07	.17	-.01	-.02	.09	-.03	.20	.14	-.02	28
29	SS-PNS	.07	-.08	.02	.05	-.04	-.10	.17	.06	1.00	.31	-.01	.13	-.02	-.02	.04	.18	.08	.35	.24	.03	29
30	N-SP	.36	.16	.14	.05	.20	.18	.24	.05	.31	1.00	.23	.24	.09	.21	-.38	.35	.53	.24	.14	.03	30

VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .05 IS 0.197
VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .01 IS 0.257

TABLE 14. CORRELATION COEFFICIENTS

	VARIABLE	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	
31	PA-PNS	.20	.23	.18	.22	.11	.10	.21	.09	-.01	.23	1.00	.88	.19	-.01	.22	.26	.21	.15	.22	.29	31
32	IPH-PNS	.20	.06	.10	.16	.09	.04	.26	.07	.13	.24	.88	1.00	.09	.08	.24	.34	.19	.23	.25	.18	32
33	SCP-SCP	-.09	-.02	.07	-.02	.18	.24	.13	.17	-.02	.09	.19	.09	1.00	-.15	-.06	.12	.15	.24	.24	.05	33
34	PHAR. H	-.15	-.06	.06	.10	.05	.13	.07	-.01	-.02	.21	-.01	.08	-.15	1.00	.07	.51	.26	-.03	-.16	-.14	34
35	S-PM HOR.	-.44	-.19	.10	.07	-.17	-.20	.16	-.02	.04	-.38	.22	.24	-.06	.07	1.00	.13	-.27	-.12	-.21	.02	35
36	S-PM VERT.	-.17	-.01	.15	.21	.07	.19	.43	.09	.18	.35	.26	.34	.12	.51	.13	1.00	.19	.06	.02	-.08	36
37	N-GN	.32	.16	.19	.10	.29	.25	.10	-.03	.08	.53	.21	.19	.15	.26	-.27	.19	1.00	.27	.09	.18	37
38	ZM-ZM	.09	-.12	-.02	-.11	.04	.03	.06	.20	.35	.24	.15	.23	.24	-.03	-.12	.06	.27	1.00	.42	.08	38
39	ZG-ZG	.21	-.08	-.11	.00	.06	-.01	.15	.14	.24	.14	.22	.25	.24	-.16	-.21	.02	.09	.42	1.00	.25	39
40	MASS. H	.21	.21	.01	-.02	.08	.08	-.04	-.02	.03	.03	.29	.18	.05	-.14	.02	-.08	.18	.08	.25	1.00	40
41	ECM-ECM	.14	.06	.11	.10	.09	.13	.17	.25	.14	.20	.28	.19	.31	.10	-.11	.06	.39	.42	.24	.11	41
42	PALATE H	.01	-.05	.08	.12	.04	.10	.03	.33	.01	.02	.07	.01	.17	.05	-.03	.07	-.05	.18	.10	-.03	42
43	PALATE L	.12	-.02	-.00	.01	.06	-.01	.15	-.04	.57	.24	.23	.32	.11	.11	-.02	.25	.43	.37	.22	.18	43
44	PALATE H	.03	.05	.12	.11	.15	.18	.06	.19	-.17	.01	.01	-.06	-.01	.09	-.04	-.02	.38	.03	-.05	.20	44
45	GO-GO	-.07	-.25	-.08	.02	-.04	-.08	.13	.11	.13	.06	-.14	-.02	.10	.04	-.15	.09	-.12	.19	.38	-.80	45
46	GN-GO	-.03	.02	.15	.21	.12	.18	.21	.06	.22	.13	.27	.30	.09	.25	.07	.40	.29	.24	.22	.23	46
47	GN-CO	-.02	.05	.25	.22	.12	.07	.12	-.07	.28	.20	.26	.30	.07	.29	.17	.43	.50	.31	.16	.06	47
48	RAMUS H	.05	.06	.07	.06	.02	-.05	-.02	-.14	.30	.01	.29	.34	.03	-.18	.07	.04	.03	.30	.35	.15	48
49	RAMUS H	-.10	-.12	.31	.27	.22	.31	.33	.22	.10	.18	-.03	-.04	.04	.30	.12	.23	.26	.20	.15	.02	49
50	INFRA T.F.O.	.13	-.06	.01	.00	-.03	-.06	.14	-.07	.28	.13	.16	.22	.04	-.14	-.01	.10	.08	.37	.55	.08	50
51	S-N-SS	-.35	-.26	-.11	-.05	-.22	-.23	.22	.09	.41	-.24	.04	.21	-.03	.11	.44	.28	-.26	.14	.08	-.18	51
52	S-N-PG	-.45	-.35	-.02	.04	-.20	-.19	.29	-.03	.16	-.37	.00	.10	-.06	.12	.53	.34	-.51	-.01	.08	-.24	52
53	S-N-PR	-.34	-.20	-.14	-.08	-.22	-.20	.22	-.05	.20	-.37	.13	.23	.02	.16	.52	.32	-.16	.07	.02	.02	53
54	S-N-LD	-.44	-.29	-.07	-.05	-.21	-.23	.25	-.07	.18	-.38	.05	.18	-.02	.09	.54	.32	-.35	.03	.01	-.15	54
55	N-SS-PG	-.18	-.12	.20	.21	.11	.12	.07	.02	-.30	-.20	-.08	-.14	-.09	-.05	.09	.03	-.34	-.15	.02	-.10	55
56	AR-GO-GN	-.17	-.07	-.16	-.34	-.23	-.28	-.28	-.00	-.10	-.10	-.20	-.22	.02	-.06	.13	-.17	.12	-.03	-.33	-.12	56
57	ML/NSL	.44	.20	-.05	-.16	.10	.01	-.12	-.13	.06	.51	-.03	-.12	.08	-.23	-.52	-.43	.32	.13	.05	-.02	57
58	ML/NSL	.24	.20	-.00	-.17	.06	.03	-.32	-.01	-.17	.25	-.07	-.15	.13	.04	-.32	-.18	.64	.02	-.21	.13	58
59	ML/ML	.02	.11	.03	-.09	.01	.04	-.27	.08	-.20	-.00	-.07	-.10	.11	.19	-.05	.05	.53	-.02	-.26	.16	59

VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .05 IS 0.197
VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .01 IS 0.257

TABLE 14. CORRELATION COEFFICIENTS

	VARIABLE	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	
1	NASAL CHORD	.09	.08	-.03	.09	-.08	-.09	-.07	.01	.14	-.05	-.11	.01	-.11	-.09	.13	.10	-.08	-.08	-.04	1
2	NASAL INDEX	-.30	-.06	-.07	.14	-.06	-.05	-.16	-.09	-.01	-.13	-.11	.04	-.07	-.02	.20	.13	-.11	-.04	.02	2
3	A-B-C	-.06	-.01	.03	.11	-.06	.11	.13	-.04	-.02	-.11	-.14	-.10	-.11	-.09	.05	-.05	.05	.14	.12	3
4	FRONTAL CHORD	.10	-.00	-.07	-.05	.04	-.02	.08	.20	.27	.16	-.16	-.04	-.09	-.13	.14	-.14	.06	-.10	-.14	4
5	FRONTAL INDEX	-.07	-.04	-.13	-.11	.07	-.10	-.09	-.01	.04	.07	.18	.15	.20	.15	-.13	.02	-.12	-.12	-.03	5
6	R-C-D	.05	-.08	.25	-.12	.09	.02	-.03	.13	-.09	.15	.07	-.07	-.01	-.04	-.21	-.03	.08	-.03	-.09	6
7	PARIETAL CHORD	.15	.01	.19	.06	.18	.24	.19	.18	.09	.23	.14	.14	.06	.10	.04	-.13	-.02	-.12	-.13	7
8	PARIETAL INDEX	-.00	.06	-.07	-.00	-.29	.12	-.04	-.03	-.12	-.13	-.05	-.01	.03	.04	.06	.09	-.12	.19	.27	8
9	ENDO. L	.19	-.04	.22	-.04	.15	.16	.16	.34	.19	.33	.04	.01	-.02	-.04	-.02	-.21	.07	-.17	-.24	9
10	ENDO. R	.15	.03	.32	-.10	.34	.19	.04	.03	-.01	.10	.25	.15	.18	.16	-.12	-.09	.09	-.02	-.08	10
11	ENDO. M	.22	.17	-.06	.15	-.06	.17	.19	-.03	.35	-.05	-.06	.06	-.07	-.06	.20	-.14	-.12	-.06	.02	11
12	G-OP	.20	-.04	.32	-.03	.07	.30	.26	.38	.20	.30	.04	-.01	-.02	-.06	-.03	-.28	.00	-.15	-.17	12
13	EU-EU	.14	.05	.30	-.10	.39	.17	.08	.02	.04	.20	.27	.17	.17	.14	-.10	-.08	.05	-.03	-.08	13
14	PO-V	.17	.07	.00	.13	-.13	.18	.17	-.04	.31	.08	-.04	.02	-.02	-.10	-.01	-.13	-.04	-.02	.01	14
15	N-ETH	.11	.03	.14	-.03	.00	.23	.26	.15	.05	-.06	-.05	-.06	-.16	-.12	.05	-.08	.04	-.04	-.07	15
16	ETH-S	.09	.03	.10	.09	-.06	.15	.04	.09	.16	.15	-.10	.08	-.03	-.01	.16	-.07	-.02	-.12	-.12	16
17	N-S	.14	.11	.11	-.02	-.05	.29	.22	.24	.20	.07	-.11	.02	-.22	-.12	.21	-.19	-.01	-.20	-.22	17
18	S-BA	.36	.20	.17	.25	.04	.16	.22	-.03	.37	.09	.04	.01	.03	-.02	-.02	-.18	-.02	-.07	-.06	18
19	N-BA	.37	.17	.28	.10	.00	.26	.26	.22	.23	.15	-.25	-.25	-.34	-.33	.03	-.29	.27	-.04	-.20	19
20	N-S-BA	.11	-.07	.19	-.05	.02	-.04	-.00	.17	-.30	.13	-.37	-.52	-.38	-.46	-.23	-.07	.56	.27	-.03	20
21	ETH-S-BA	.14	.01	.12	.03	-.07	-.03	-.02	.06	-.10	.10	-.35	-.46	-.34	-.44	-.18	-.17	.44	.24	.02	21
22	FOR. ANGLE	.05	-.05	-.02	.05	-.25	.02	.05	.06	-.12	-.06	-.26	-.35	-.20	-.29	-.12	-.07	.20	.20	.11	22
23	MIN. F	.11	.08	-.00	.12	-.08	.15	.25	.07	.31	.01	-.11	-.02	-.14	-.07	.20	-.16	-.05	-.00	.03	23
24	MAX. F	.10	.12	.01	.11	.02	.21	.22	.06	.27	.00	-.05	.04	-.08	-.05	.21	-.34	-.16	-.17	-.09	24
25	F. SINUS M	.09	.04	.06	.15	-.04	.12	.12	.02	.22	-.03	-.22	-.20	-.22	-.21	.11	-.23	.10	.06	.01	25
26	F. SINUS M	.13	.10	-.01	.18	-.08	.18	.07	-.05	.31	-.06	-.23	-.19	-.20	-.23	.12	-.28	.01	.03	.04	26
27	SPHEN. D	.17	.03	.15	.08	.13	.21	.12	-.02	.33	.14	.22	.29	.22	.25	.07	-.28	-.12	-.32	-.27	27
28	NASAL. M	.25	.33	-.04	.19	.11	.06	-.07	-.14	.22	-.07	.09	-.03	-.05	-.07	.02	-.00	-.13	-.01	.08	28
29	SS-PNS	.14	.01	.57	-.17	.13	.22	.28	.30	.10	.28	.41	.16	.20	.18	-.30	-.10	.06	-.17	-.20	29
30	N-SP	.20	.02	.24	.01	.06	.13	.20	.01	.18	.13	-.24	-.37	-.37	-.38	-.20	-.10	.51	.25	-.00	30

VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .05 IS 0.197
VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .01 IS 0.257

TABLE 14. CORRELATION COEFFICIENTS

	VARIABLE	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	
31	MA-PNS	.28	.07	.23	.01	-.14	.27	.26	.29	-.03	.16	.04	-.00	.13	.05	-.08	-.20	-.03	-.07	-.07	31
32	IPM-PNS	.19	.01	.32	-.06	-.02	.30	.30	.34	-.04	.22	.20	.10	.23	.18	-.14	-.22	-.12	-.15	-.10	32
33	SCP-SCP	.31	.17	.11	-.01	.10	.09	.07	.03	.04	.04	-.03	-.06	.02	-.02	-.09	.02	.08	.13	.11	33
34	PHAR. H	.10	.05	.11	.09	.04	.25	.29	-.18	.30	-.14	.11	.12	.16	.09	-.05	-.06	-.23	.04	.19	34
35	S-PM HOR.	-.11	-.03	-.02	-.04	-.15	.07	.17	.07	.12	-.01	.44	.53	.52	.54	.09	.13	-.52	-.32	-.05	35
36	S-PM VERT.	.06	.07	.25	-.02	.09	.40	.43	.04	.23	.10	.28	.34	.32	.32	.03	-.17	-.43	-.18	.05	36
37	N-GN	.39	-.05	.43	.38	-.12	.29	.50	.03	.26	.08	-.26	-.51	-.16	-.35	-.34	.12	.32	.64	.53	37
38	ZM-ZM	.42	.18	.37	.03	.19	.24	.31	.30	.20	.37	.14	-.01	.07	.03	-.15	-.03	.13	.02	-.02	38
39	ZG-ZG	.24	.10	.22	-.05	.38	.22	.16	.35	.15	.55	.08	.08	.02	.01	.02	-.33	.05	-.21	-.26	39
40	MASS. H	.11	-.03	.18	.20	-.60	.23	.06	.15	.02	.08	-.18	-.24	.02	-.15	-.10	-.12	-.02	.13	.16	40
41	ECM-ECM	1.00	.69	.25	.21	.04	.25	.26	.03	.32	.05	-.02	-.15	.03	-.15	-.23	-.10	.13	.15	.12	41
42	PALATE H	.69	1.00	-.00	.17	.09	.18	.04	-.18	.19	-.16	.01	.06	-.01	-.08	.09	-.11	-.05	-.08	-.03	42
43	PALATE L	.25	-.00	1.00	.12	-.04	.35	.46	.23	.14	.23	.30	.06	.40	.24	-.32	.01	-.01	.11	.13	43
44	PALATE H	.21	.17	.12	1.00	-.22	.18	.18	-.27	.32	-.24	-.23	-.20	-.03	-.16	-.03	.14	.00	.27	.31	44
45	GO-GO	.04	.09	-.04	-.22	1.00	-.08	.04	.07	.08	.26	.22	.28	-.00	.15	.11	-.09	.06	-.25	-.32	45
46	GN-GO	.25	.18	.35	.18	-.08	1.00	.57	.27	.17	.05	.15	.21	.18	.14	.09	-.29	-.22	-.02	.11	46
47	GN-CO	.26	.04	.46	.18	.04	.57	1.00	.20	.35	.21	.17	.22	.29	.29	.06	-.00	-.17	.03	.14	47
48	RAMUS H	.03	-.18	.23	-.27	.07	.27	.20	1.00	-.04	.53	.14	.11	.14	.14	-.04	-.43	-.03	-.24	-.26	48
49	RAMUS H	.32	.19	.14	.32	.08	.17	.35	-.04	1.00	.17	.04	.11	.06	.02	.09	-.32	-.17	-.23	-.13	49
50	INFRA T.F.O.	.05	-.16	.23	-.24	.26	.05	.21	.53	.17	1.00	.21	.19	.11	.14	-.00	-.32	.05	-.26	-.32	50
51	S-N-SS	-.02	.01	.30	-.23	.22	.15	.17	.14	.04	.21	1.00	.65	.74	.73	-.29	-.05	-.43	-.42	-.19	51
52	S-N-PG	-.15	.06	.06	-.20	.28	.21	.22	.11	.11	.19	.65	1.00	.68	.88	.42	-.16	-.64	-.71	-.42	52
53	S-N-PR	.03	-.01	.40	-.03	-.00	.18	.29	.14	.06	.11	.74	.68	1.00	.85	-.16	-.01	-.61	-.34	-.01	53
54	S-N-ID	-.15	-.08	.24	-.16	.15	.14	.29	.14	.02	.14	.73	.88	.85	1.00	.16	-.01	-.61	-.51	-.21	54
55	N-SS-PG	-.23	.09	-.32	-.03	.11	.09	.06	-.04	.09	-.00	-.29	.42	-.16	.16	1.00	-.18	-.26	-.44	-.37	55
56	AR-TGO-GN	-.10	-.11	.01	.14	-.09	-.29	-.00	-.43	-.32	-.32	-.05	-.16	-.01	-.01	-.18	1.00	.10	.61	.61	56
57	NL/NSL	.13	-.05	-.01	.00	.06	-.22	-.17	-.03	-.17	.05	-.43	-.64	-.61	-.61	-.26	.10	1.00	.43	-.08	57
58	ML/NSL	.15	-.08	.11	.27	-.25	-.02	.03	-.24	-.23	-.26	-.42	-.71	-.34	-.51	-.44	.61	.43	1.00	.86	58
59	NL/ML	.12	-.03	.13	.31	-.32	.11	.14	-.26	-.13	-.32	-.19	-.42	-.01	-.21	-.37	.61	-.08	.86	1.00	59

VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .05 IS 0.197
VALUE OF A CORRELATION COEFFICIENT DIFFERING FROM ZERO AT P = .01 IS 0.257

Table 15. Main sources of specious coordination

Group 1 - Variables sharing common components

Basal chord Basal index	Frontal chord Frontal index	Parietal chord Parietal index	zg-zg mass. b	go-go mass. b

Group 2 - Variables spanning related anatomical regions

General cranial length	General cranial breadth	Clivus length
g-op	endo. b	s-ba
endo. 1	eu-eu	sphen. d
Basal chord	zg-zg	
Parietal chord		
Frontal chord		
n-eth		
eth-s		
n-s		
n-ba		
min. f		
max. f		

Upper facial breadth	Facial length
zg-zg	palate 1
zm-zm	ss-pns
ecm-ecm	
palate b	
infra t.f.d.	

General cranial height

po-v
endo. h

Group 3 - Variables sharing common reference points or lines

NSL or points n or s in common

s-n-ss	n-sp
s-n-pg	n-gn
s-n-pr	n-s
s-n-id	n-eth
for. angle	eth-s
NL/NSL	n-ba
ML/NSL	n-s-ba
min. f	eth-s-ba
n-ss-pg	s-pm hor.
	s-pm vert.

ML or point gn in common

ML/NSL	n-gn
NL/ML	gn-go
ar-tgo-gn	gn-cd

Points pns or pm in common

ba-pns	s-pm hor.
tph-pns	s-pm vert.
phar. h	ss-pns

Points ba in common

n-ba	phar. h
n-s-ba	endo. h
eth-s-ba	ba-pns
	for. angle

Points A, B C or D in common

Basal chord	endo. 1
Frontal chord	endo. h
Parietal chord	A-B-C
	B-C-D

Table 16. Analysis 1 - Varimax solution derived by transformation of initial maximum likelihood loadings - coefficients > .14 shown

	VARIABLE	COMMON FACTORS																					Σa^{2*}	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23		
1	Basal chord				21				36				-17	16	23	28	20		18			26			56	1
2	Basal index			-16										-18		21						66			61	2
3	A-B-C												16		-17	-29	68								70	3
4	Frontal chord				20										90	15									99	4
5	Frontal index	21											-25							-23					32	5
6	B-C-D																-17		93						98	6
7	Parietal chord				93												16		-15						99	7
8	Parietal index		23		-15	-17					30				-46			28							56	8
9	endo. l				81					15					43				27						99	9
10	endo. b							94																	95	10
11	endo. h				31	-17	18							17	20	21	60	28	-34						88	11
12	g-op				67	21			27	16			26		35				25				16		90	12
13	eu-eu							89		18															93	13
14	po-v				26		19				20				28		45								53	14
15	n-eth			20		14			82				20			-37									96	15
16	eth-s															93						20			97	16
17	n-s			22	18				77				25			38									98	17
18	s-ba						29					15		18				86			18				98	18
19	n-ba	-37		38	20	30			43			17	18	15				50							98	19
20	n-s-ba	-67		37		40	-19		-16	19						-26									99	20
21	eth-s-ba	-58		30		25			-23				18			-16		-17				-38	-17	-20	90	21
22	for. angle	-30		15	28						15		18		-19		30	31					26		57	22
23	min. f							-19				34	77												83	23
24	max. f								20			21	85												91	24
25	f. sinus h	-17										78	31												76	25
26	f. sinus b	-15										89	18												92	26
27	sphen. d	21	-28	17			34					22					-29				24			20	56	27
28	nasal b											26		37								33	-17		44	28
29	ss-pns					77			21	15										-19					81	29
30	n-sp	-53		19		32	57																	30	89	30

* Σa^2 The communalities of the variables computed as the sum of the squares of the common factor coefficients

Table 16. Analysis 1 - Varimax solution (continued)

	VARIABLE	COMMON FACTORS																							Σa^{2*}	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23		
31	ba-pns			94																					98	31
32	tph-pns			86																					89	32
33	scp-scp									20		25		29			17					15		21	36	33
34	phar. h						59			-17								21							56	34
35	s-pm hor.	69		18				-20	31							33									85	35
36	s-pm vert.	29		21			76								-16										80	36
37	n-gn	-34	51			21	33					18									43	-23	19		94	37
38	zm-zm				17	29				34				34	17										48	38
39	zg-zg		-16					28		90															99	39
40	mass. b									24	94														99	40
41	ecm-ecm													85				16			18	-19			92	41
42	palate b													77											62	42
43	palate l	17	16	18		65		27											15		20		17		73	43
44	palate h		22								15										62				53	44
45	go-go							21		33	-89														99	45
46	gn-go	17				24	31		18		18			15						20			35		59	46
47	gn-cd	26	19	16		38	33			18							23			25	30	-22	24		79	47
48	ramus b		-18	21	17	19				35											-24		52		66	48
49	ramus h		-20	-18			39			16		21	16	20	30						41				73	49
50	infra t.f.d.		-20		19					59												-19			57	50
51	s-n-ss	75	-15			25		15												-34					86	51
52	s-n-pg	83	-33								-16									30					96	52
53	s-n-pr	87		15		20			-22																92	53
54	s-n-id	90				17														15					94	54
55	n-ss-pg		-30			-29														83					96	55
56	ar-tgo-gn		72							-20		-20	-18										-32	17	84	56
57	NL/NSL	-76				20	-23																	51	98	57
58	ML/NSL	-44	85																						99	58
59	NL/ML		92							-12														-16	99	59
	Factor Contribution (Per Cent)	12.5	6.7	5.7	5.6	4.9	4.8	4.8	4.7	4.5	4.5	4.4	4.3	4.2	4.0	3.7	3.3	3.1	3.0	2.8	2.6	2.5	2.1	1.5	47.8 = Total Communality	

* Σa^2 The communalities of the variables computed as the sum of the squares of the common factor coefficients

Table 17. Analysis 1 - Varimax solution of Table 16 rearranged to bring high loadings near the main diagonal - coefficients > .21 shown

	VARIABLE	COMMON FACTORS																						Σa^{2*}	
		4	7	16	14	18	12	21	8	15	17	11	5	6	3	20	9	10	13	22	1	19	23	2	
7	Parietal chord	93																							99	7
9	endo. l	81			43	27																			99	9
12	g-op	67			35	25	26		27																90	12
10	endo. b		94																						95	10
13	eu-eu		89																						93	13
3	A-B-C			68						-29															70	3
5	Frontal index						-25															-23			32	5
11	endo. h	31		60		-34					28														88	11
14	po-v	26		45	28																				53	14
4	Frontal chord				90																				99	4
8	Parietal index				-46						28							30						23	56	8
6	B-C-D					93																			98	6
23	min. f						77					34													83	23
24	max. f						85																		91	24
2	Basal index							66																	61	2
21	eth-s-ba							-38	-23				25		30						-58				90	21
20	n-s-ba									-26			40		37						-67				99	20
22	for. angle	28		30							31									26	-30				57	22
15	n-eth								82	-37															96	15
17	n-s						25		77	38					22										98	17
1	Basal chord				23			26	36	28															56	1
16	eth-s									93															97	16
18	s-ba										86			29											98	18
19	n-ba								43		50		30		38						-37				98	19
27	sphen. d			-29								22		34		24								-28	56	27
25	f. sinus h						31					78													76	25
26	f. sinus b											89													92	26
29	ss-pns												77												81	29
43	palate l		27										65												73	43
34	phar. h													59											56	34

* Σa^2 The communalities of the variables computed as the sum of the squares of the common factor coefficients

Table 17. Analysis 1 - Varimax solution rearranged (continued)

	VARIABLE	COMMON FACTORS																						Σa^{2*}	
		4	7	16	14	18	12	21	8	15	17	11	5	6	3	20	9	10	13	22	1	19	23	2	
3	s-pm vert.													76							29				80	36
30	n-sp												32	57							-53		30		89	30
31	ba-pns														94										98	31
32	tph-pns														86										89	32
33	scp-scp											25							29						36	33
35	s-pm hor.								31	33											69				85	35
37	n-gn							-23						33		43					-34			51	94	37
44	palate h															62								22	53	44
49	ramus h				30									39		41									73	49
39	zg-zg		28														90								99	39
50	infra t.f.d.																59								57	50
38	zm-zm												29				34		34						48	38
45	go-go																33	-89							99	45
40	mass. b																24	94							99	40
41	ecm-ecm																		85						92	41
42	palate b																		77						62	42
28	nasal b							33				26							37						44	28
48	ramus b															-24	35			52					66	48
46	gn-go												24	31						35					59	46
47	gn-cd			23				-22					38	33		30				24	26	25			79	47
56	ar-tgo-gn																			-32				72	84	56
52	s-n-pg																				83	30		-33	96	52
54	s-n-id																				90				94	54
51	s-n-ss												25								75	-34			86	51
53	s-n-pr								-22												87				92	53
55	n-ss-pg												-29									83		-30	96	55
57	NL/NSL													-23							-76		51		98	57
58	ML/NSL																				-44			85	99	58
59	NL/ML																							92	99	59
	Factor Contribution Per Cent	5.6	4.	3.3	4.0	3.0	4.3	2.5	4.7	3.7	3.1	4.4	4.9	4.8	5.7	2.6	4.5	4.5	4.2	2.1	12.5	2.8	1.5	6.7	47.8 = Total Communality	

* Σa^2 The communalities of the variables computed as the sum of the squares of the common factor coefficients

Table 18. Analysis 1 - contributions of the variables to estimated factor score variances. Factor score variances shown in brackets after the factor number

Factor	Variable and contribution		Factor	Variable and contribution	
4 (.99)	Parietal chord	66	6 (.91)	n-sp	34
	endo. 1	44		s-pm vert.	22
7 (.96)	endo. b	62	3 (.97)	ba-pns	79
	eu-eu	37		tph-pns	14
16 (.83)	endo. h	41	20 (.82)	n-gn	32
	A-B-C	21		palate h	13
14 (.98)	frontal chord	68	9 (.99)	zg-zg	74
	endo. 1	24		go-go	20
				mass. b	13
18 (.98)	B-C-D	67			
	endo. 1	20	10 (.99)	go-go	62
				mass. b	43
12 (.90)	max. f	63			
	min. f	25	13 (.91)	ecm-ecm	70
				palate b	14
21 (.80)	eth-s-ba	22			
	basal index	19	22 (.80)	ramus b	16
				ar-tgo-gn	12
8 (.97)	n-s	49			
	n-eth	38	1 (.99)	s-n-pg	21
				s-n-id	21
15 (.96)	eth-s	67		n-s-ba	17
	n-s	14		s-n-pr	14
	n-eth	12			
			19 (.95)	n-ss-pg	73
17 (.96)	s-ba	69		s-n-pg	13
	n-ba	33			
			23 (.94)	NL/NSL	27
11 (.91)	f. sinus b	72		NL/ML	25
	f. sinus h	18		ML/NSL	18
5 (.94)	n-s-ba	30	2 (.99)	ML/NSL	74
	ss-pns	17		NL/ML	43
	n-ba	13			

Table 19. Analysis 1 - interpretation of common factors

Factor number (Table 17)	Highest factor coefficients		Highest contributions to factor variances		Interpretation
4	Parietal chord	93	Parietal chord	66	Endocranial length
	endo. l	81	endo. l	44	(parietal segment)
	g-op	67			
7	endo. b	94	endo. b	62	Endocranial breadth
	eu-eu	89	eu-eu	37	
16	A-B-C	68	endo. h	41	Endocranial height
	endo. h	60	A-B-C	21	
14	Frontal chord	90	Frontal chord	68	Endocranial length
	Parietal index	-46	endo. l	24	(frontal segment)
	endo. l	43			
18	B-C-D	93	B-C-D	67	Endocranial curvature
	endo. h	-34	endo. l	20	(fronto-parietal)
12	max. f	85	max. f	63	Frontal thickness
	min. f	77	min. f	25	
21	Basal index	66	eth-s-ba	22	Endocranial curvature
	eth-s-ba	-38	Basal index	19	(anterior fossa)
8	n-eth	82	n-s	49	Anterior cranial base
	n-s	77	n-eth	38	length
15	eth-s	93	eth-s	67	Anterior cranial base
	n-s	38	n-s	14	length (eth-s segment)
	n-eth	-37	n-eth	12	
17	s-ba	86	s-ba	69	Posterior cranial base
	n-ba	50	n-ba	33	length
11	f. sinus b	89	f. sinus b	72	Frontal sinus capacity
	f. sinus h	78	f. sinus h	18	
5	ss-pns	77	n-s-ba	30	Nasal depth
	palate l	65	ss-pns	17	
	n-s-ba	40	n-ba	13	

Table 19. Analysis 1 - interpretation of common factors (contd.)

Factor number (table 17)	Highest factor coefficients		Highest contributions to factor variances		Interpretation
6	s-pm vert.	76	n-sp	34	Upper facial height
	phar. h	59	s-pm vert.	22	(nasopharyngeal)
	n-sp	57			
3	ba-pns	94	ba-pns	79	Nasopharyngeal depth
	tph-pns	86	tph-pns	14	
20	palate h	62	n-gn	32	Facial height
	n-gn	43	palate h	13	(palate segment)
9	zg-zg	90	zg-zg	74	Upper facial breadth
	infra t.f.d.	59	go-go	20	
10	mass. b	94	go-go	62	Lower facial breadth
	go-go	-89	mass. b	43	
13	ecm-ecm	85	ecm-ecm	70	Maxillary breadth
	palate b	77	palate b	14	
22	ramus b	52	ramus b	16	Ramus breadth
	go-go	35	ar-tgo-gn	12	
1	s-n-id	90	s-n-pg	21	Facial prognathism
	s-n-pr	87	s-n-id	21	
	s-n-pg	83	n-s-ba	17	
19	n-ss-pg	83	n-ss-pg	73	Facial convexity
	s-n-ss	-34	s-n-pg	13	
	s-n-pg	30			
23	NL/NSL	51	NL/NSL	27	Nasal floor
	n-sp	30	NL/ML	25	inclination
			ML/NSL	18	
2	NL/ML	92	ML/NSL	74	Mandibular base
	ML/NSL	85	NL/ML	43	inclination
	ar-tgo-gn	72			

Table 20. Analysis 2 - percentage contributions of varimax factors to the common variance

Factor	Contribution	Factor	Contribution
1	3.8	21	3.1
2	11.2	22	2.0
3	3.7	23	2.0
4	2.4	24	3.7
5	2.3	25	5.8
6	3.4	26*	0.7
7	3.0	27	1.5
8	4.0	28	4.1
9	3.4	29	1.6
10	3.7	30	1.8
11	1.9	31	0.8
12	2.3	32*	0.5
13	5.1	33*	0.2
14	2.9	34*	0.6
15	1.8	35*	0.5
16	4.4	36*	0.3
17	2.7	37*	0.2
18	2.0	38*	0.1
19	4.8	39*	0.1
20	1.8	40*	0.0

* Factor not interpreted because of low factor contribution

Table 21. Analysis 2 - Varimax solution simplified with coefficients > .21 shown

VARIABLE	COMMON FACTORS																										Σa^{2*}	
	19	16	5	21	14	24	11	1	7	17	3	28	10	13	4	6	8	9	22	2	12	31	25	30	27	23	18	15	29	20		
7 Parietal chord	92																														99	7
9 endo. l	78			40	28																										99	9
12 g-op	67			35	28	21		31																							95	12
10 endo. b		93																													95	10
13 eu-eu		91																													95	13
3 A-B-C			33	-26	-24	22			-29															-41							84	3
5 Frontal index																								72							67	5
11 endo. h	31		43		-41					26			21													30					92	11
14 po-v	23		74																												73	14
4 Frontal chord				87	-22																										98	4
8 Parietal index				-38						28		-22					31									-30		22			77	8
6 B-C-D					93																										98	6
23 min. f						82					31																				90	23
24 max. f						79		24			25																				91	24
2 Basal index							79																								79	2
21 eth-s-ba							-29							30						-56											93	21
20 n-s-ba									-23			31		38						-64											98	20
22 for. angle																				-25				69							74	22
15 n-eth								82	-35																						96	15
17 n-s						25		73	39					23												22					98	17
1 Basal chord								21																		71					73	1
16 eth-s									92																						95	16
18 s-ba										85			25																		97	18
19 n-ba								41		50		29		39						-37											99	19
27 sphen. d																											75				80	27
25 f. sinus h						26					84																				85	25
26 f. sinus b											83																				91	26
29 ss-pns												82																			87	29
43 palate l		28										63			24																84	43
34 phar. h													75																		81	34

* Σa^2 The communalities of the variables computed as the sum of the squares of the common factor coefficients

Table 21. Analysis 2 - Varimax solution simplified (continued)

VARIABLE	COMMON FACTORS																												Σa^{2*}	
	19	16	5	21	14	24	11	1	7	17	3	28	10	13	4	6	8	9	22	2	12	31	25	30	27	23	18	15	29	20		
36 s-pm vert.													69																		87	36
30 n-sp												36	43							-53		21					27				89	30
31 ba-pns														91													28				96	31
32 tph-pns														87																	94	32
33 scp-scp																															71	33
35 s-pm hor.								28	31											69								76			91	35
37 n-gn													28		38					-36			53								96	37
44 palate h															65								23								65	44
49 ramus h			23	21									28		54																83	49
39 zg-zg		37														42													74		99	39
50 infra t.f.d.																79															81	50
38 zm-zm												29				35		24													69	38
45 go-go		24															-89												26		99	45
40 mass. b.																	93														99	40
41 ecm-ecm							-21											80													91	41
42 palate b																		86													82	42
28 nasal b																		23												76	74	28
48 ramus b																59			27				-21						22		77	48
46 gn-go													22						75												88	46
47 gn-cd												32	30		31				35	24			74								88	47
56 ar-tgo-gn											-21				-26				-22			28	-30								92	56
52 s-n-pg																				84											99	52
52 s-n-id																				91											97	54
51 s-n-ss												23								75		-37									95	51
53 s-n-pr																				87											96	53
55 n-ss-pg													-26									81	-30								93	55
57 NL/NSL														-23						-75			51								99	57
58 NL/NSL																				-46				84							99	58
59 NL/ML																								91							99	59
Factor Contribution (per cent)	4.8	4.4	2.3	3.1	2.9	3.7	1.9	3.8	3.0	2.7	3.7	4.1	3.7	5.1	2.4	3.4	4.0	3.4	2.0	11.2	2.3	0.8	5.8	1.8	1.5	2.0	2.0	1.8	1.6	1.8		

* Σa^2 The communalities of the variables computed as the sum of the squares of the common factor coefficients

Table 22. Analysis 2 - interpretation of common factors

Factor number (Table 21	Highest factor coefficients		Interpretation	Factor number (Table 21)	Highest factor coefficients		Interpretation
19	Parietal chord	92	Endocranial length (parietal segment)	6	infra t.f.d.	79	Mandibular robustness
	endo. l	78			ramus b	59	
16	endo. b	93	Endocranial breadth	8	mass. b	93	Lower facial breadth
	eu-eu	91			go-go	-89	
5	po-v	74	Cranial height	9	palate b	86	Palatal breadth
	endo. h	43			ecm-ecm	80	
21	Frontal chord	87	Endocranial length (frontal segment)	22	gn-go	75	Mandibular length
	endo. l	40			gn-cd	35	
14	B-C-D	93	Endocranial curvature (fronto-parietal)	2	s-n-id	91	Facial prognathism
	endo. h	-41			s-n-pr	87	
24	min. f	82	Frontal thickness	12	n-ss-pg	81	Facial convexity
	max. f	79			s-n-ss	-37	
11	Basal index	79	Endocranial curvature (anterior fossa)	31	NL/NSL	51	Nasal floor inclination
	eth-s-ba	-29			n-sp	21	
1	n-eth	82	Anterior cranial base (n-eth segment)	25	NL/ML	91	Mandibular base inclination
	n-s	73			ML/NSL	84	
7	eth-s	92	Anterior cranial base (eth-s segment)	30	Frontal index	72	Endocranial curvature (frontal segment)
	n-s	39			A-B-C	-41	
17	s-ba	85	Posterior cranial base length	27	for. angle	69	Inclination of foramen magnum
	n-ba	50					
3	f. sinus h	84	Frontal sinus capacity	23	Basal chord	71	Endocranial length (basal segment)
	f. sinus b	83					
28	ss-pns	82	Nasal depth	18	sphen. d	75	Sphenoid thickness
	palate l	63					
10	phar. h	75	Nasopharyngeal height	15	scp-scp	76	Nasopharyngeal breadth
	s-pm vert.	69					
13	ba-pns	91	Nasopharyngeal depth	29	zg-zg	74	Upper facial breadth
	tph-pns	87					
4	palate h	65	Palatal height	20	nasal b	76	Nasal breadth
	ramus h	54					

Table 23. Analysis 2 - summary of selection of variables

Variable included in Analysis 2	Eliminated	Variable retained for Analysis 3
Parietal chord	Parietal chord	
endo. 1		endo. 1
g-op	g-op	
endo. b		endo. b
eu-eu	eu-eu	
A-B-C	A-B-C	
Frontal index	Frontal index	
endo. h		endo. h
po-v		po-v
Frontal chord	Frontal chord	
Parietal index	Parietal index	
B-C-D	B-C-D	
min-f	min. f	
max. f		max. f
Basal index	Basal index	
eth-s-ba		eth-s-ba
n-s-ba		n-s-ba
for. angle		for. angle
n-eth	n-eth	
n-s		n-s
Basal chord	Basal chord	
eth-s	eth-s	
s-ba		s-ba
n-ba		n-ba
sphen. d		sphen. d
f. sinus h		f. sinus h
f. sinus b	f. sinus b	
ss-pns		ss-pns
palate 1		palate 1
phar. h		phar. h
s-pm vert.	s-pm vert.	
n-sp		n-sp
ba-pns		ba-pns
tph-pns	tph-pns	
scp-scp		scp-scp
s-pm hor.	s-pm hor.	

Table 23. Analyses 2 - summary of selection of variables (contd.)

Variable included in Analysis 2	Eliminated	Variable retained for Analysis 3
n-gn		n-gn
palate h		palate h
ramus h		ramus h
zg-zg		zg-zg
infra t.f.d.		infra t.f.d.
zm-zm		zm-zm
go-go		go-go
mass. b	mass. b	
ecm-ecm		ecm-ecm
palate b	palate b	
nasal b		nasal b
ramus b		ramus b
gn-go		gn-go
gn-cd		gn-cd
ar-tgo-gn	ar-tgo-gn	
s-n-pg		s-n-pg
s-n-id	s-n-id	
s-n-ss		s-n-ss
s-n-pr	s-n-pr	
n-ss-pg		n-ss-pg
NL/NSL		NL/NSL
ML/NSL		ML/NSL
NL/ML	NL/ML	

Table 24. Analysis 3 - percentage contributions of varimax factors to the common variance

Factor	Contribution	Factor	Contribution
1	8.3	13	4.2
2	12.4	14	3.3
3	8.2	15	4.9
4	6.9	16	5.6
5	6.3	17	3.4
6	3.2	18	2.2
7	4.8	19*	1.0
8	3.5	20*	0.8
9	3.2	21*	0.7
10	3.5	22*	0.6
11	8.4	23*	0.1
12	4.7		

* Factor not interpreted because of low variance contribution

Table 25. Analysis 3 - Varimax solution simplified with coefficients > .20 shown

VARIABLE	COMMON FACTORS																	Σa^2 *	
	10	5	12	11	8	4	16	13	15	14	6	3	17	18	9	1	2	7	
9 endo. l		35			-43	32						37							64	9
10 endo. b	73																		67	10
11 endo. h		74					21												76	11
14 po-v		74																	60	14
24 max. f			63			27													64	24
21 eth-s-ba				82													-26		83	21
20 n-s-ba				87													-34		97	20
22 for. angle					68														62	22
17 n-s						91													96	17
18 s-ba							89												96	18
19 n-ba			23	30		65	53												99	19
27 sphen. d								70											63	27
25 f. sinus h			68																57	25
29 ss-pns									80			22							84	29
43 palate l	29								48							42		-24	72	43
34 phar. h		33					28			-38		-24				34			68	34
30 n-sp				27				41	24								-54		77	30
31 ba-pns				35	22	42		22	-25	21						28			81	31
33 scp-scp										63									48	33
37 n-gn											-21					47	-67		93	37
44 palate h											62								58	44
49 ramus h		35	23					29			38			30					72	49
39 zg-zg	29			27						22		59	26						73	39
50 infra t.f.d.												78							70	50
38 sm-sm												41		29	23	23			58	38
45 go-go	22												69						64	45
41 ecm-ecm										25				52					59	41
28 nasal b															72				62	28
48 ramus b												67							65	48
46 gn-go																69			65	46
47 gn-cd																80			81	47
52 s-n-pg				-28												23	81		96	52
51 s-n-ss				-24					25								59	-49	92	51
55 n-ss-pg																	29	84	89	55
57 ML/NSL				33												-25	-65		78	57
58 ML/NSL								-24				-25					-78		92	58
Factor Contribution (per cent)	3.5	6.3	4.7	8.4	3.5	6.9	5.6	4.2	4.9	3.3	3.2	8.2	3.4	2.2	3.2	8.3	12.4	4.8		

* Σa^2 The communalities of the variables computed as the sum of squares of the common factor coefficients

Table 26. Analysis 3 - interpretation of common factors

Factor number (Table 25)	Highest factor coefficients		Interpretation
10	endo. b	73	Endocranial breadth
5	endo. h	74	Endocranial height
	po-v	74	
12	f. sinus h	68	Frontal bone size
	max. f	63	
11	n-s-ba	87	Cranial base inclination
	eth-s-ba	82	
8	for. angle	68	Inclination of foramen magnum
4	n-s	91	Anterior cranial base length
	n-ba	65	
16	s-ba	89	Posterior cranial base length
	n-ba	53	
13	sphen. d	70	Clivus thickness
15	ss-pns	80	Upper facial length
	palate l	48	
14	scp-scp	63	Pharyngeal breadth
6	palate h	62	Palatal height
3	infra t.f.d.	78	Mandibular robustness
	ramus b	67	
17	go-go	69	Lower facial breadth
18	ecm-ecm	52	Upper facial breadth
9	nasal b	72	Nasal breadth
1	gn-cd	80	Mandibular length
	gn-go	69	
2	s-n-pg	81	Facial prognathism
	ML/NSL	-78	
7	n-ss-pg	84	Facial convexity
	s-n-ss	-49	

Table 27. Analysis 3 - summary of selection of variables

Variable included in Analysis 3	Eliminated	Variable retained for Analysis 4
endo. l		endo. l
endo. b		endo. b
endo. h		endo. h
po-v	po-v	
max. f		max. f
eth-s-ba	eth-s-ba	
n-s-ba		n-s-ba
for. angle		for. angle
n-s		n-s
s-ba		s-ba
n-ba	n-ba	
sphen. d		sphen. d
f. sinus h		f. sinus h
ss-pns	ss-pns	
palate l		palate l
phar. h		phar. h
n-sp		n-sp
ba-pns		ba-pns
scp-scp		scp-scp
n-gn		n-gn
palate h		palate h
ramus h		ramus h
zg-zg		zg-zg
infra t.f.d.		infra t.f.d.
zm-zm		zm-zm
go-go		go-go
ecm-ecm		ecm-ecm
nasal b		nasal b
ramus b		ramus b
gn-go	gn-go	
gn-cd		gn-cd
s-n-pg		s-n-pg
s-n-ss	s-n-ss	
n-ss-pg		n-ss-pg
NL/NSL		NL/NSL
ML/NSL		ML/NSL

Table 28. Analysis 4 - percentage contributions of varimax factors to the common variance

Factor	Contribution
1	3.9
2	14.8
3	11.5
4	5.8
5	9.0
6	6.5
7	9.0
8	4.7
9	5.6
10	3.1
11	4.8
12	4.7
13	4.5
14	2.0
15	5.2
16	4.0
17*	0.3
18*	0.7
19*	0.0

* Factor not interpreted because of low variance contribution

Table 29. Analysis 4 - Varimax solution simplified with coefficients > .20 shown

VARIABLE	COMMON FACTORS																Σa^{2*}	
	13	7	4	16	8	9	6	12	11	3	10	1	5	15	2	14		
9 endo. l		28			-42	35				42							53	9
10 endo. b	68																60	10
11 endo. h		69				21											67	11
24 max. f			63			29											59	24
20 n-s-ba		-25		50						22					-65		85	20
22 for. angle					69												61	22
17 n-s			21			74											66	17
18 s-ba		57			25		31										61	18
27 sphen. d							73										61	27
25 f. sinus h			63														48	25
43 palate l	35									22			55	-32			64	43
34 phar. h		64						-22		-23			28				66	34
30 n-sp							34						27		-67		73	30
31 ba-pns				56	23	31											63	31
33 scp-scp								63									44	33
37 n-gn			22						31				53	-29	-50	30	94	37
44 palate h									61	-21							52	44
49 ramus h		40		-27			31		35				22				70	49
39 zg-zg	32									63	22						71	39
50 infra t.f.d.										79							67	50
38 zm-zm								25		42		28	32				57	38
45 go-go	26									22	56						54	45
41 ecm-ecm		23						38	27			21	21	-25			53	41
28 nasal b												64					52	28
48 ramus b										64							56	48
47 gn-cd													78				75	47
52 s-n-pg							27							31	75		93	52
55 n-ss-pg														66	26		57	55
57 NL/NSL															-83		77	57
58 ML/NSL							-34			-31				-32	-52	42	91	58
Factor Contribution (per cent)	4.5	9.0	5.8	4.0	4.7	5.6	6.5	4.7	4.8	11.5	3.1	3.9	9.0	5.2	14.8	2.0		

* Σa^2 The communalities of the variables computed as the sum of the squares of the common factor coefficients

Table 30. Analysis 4 - interpretation of common factors

Factor number (Table 29)	Highest factor coefficient		Interpretation
13	endo. b	68	Endocranial breadth
7	endo. h	69	Endocranial height
	phar. h	64	
4	max. f	63	Frontal bone size
	f. sinus h	63	
16	ba-pns	56	Pharyngeal depth
	n-s-ba	50	(Cranial base flexion)
8	for. angle	69	Foramen magnum inclination
9	n-s	74	Anterior cranial base length
6	sphen. d	73	Clivus thickness
12	scp-scp	63	Pharyngeal breadth
11	palate h	61	Palatal height
3	infra t.f.d.	79	Mandibular robustness
	ramus b	64	
10	go-go	56	Lower facial breadth
1	nasal b	64	Nasal breadth
5	gn-cd	78	Mandibular length
15	n-ss-pg	66	Facial convexity
2	NL/NSL	-83	Facial prognathism
	s-n-pg	75	
14	ML/NSL	42	Mandibular base inclination

Table 31. Analysis 1 - frequencies of factor coefficients in varimax solution falling between 0.15 and 0.26

Absolute value of coefficient	Frequency
0.15	16
0.16	15
0.17	22
0.18	17
0.19	12
0.20	21
0.21	13
0.22	5
0.23	7
0.24	5
0.25	7
0.26	5

Table 32. Analysis 5 - Varimax solution complete

VARIABLE	COMMON FACTORS																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Σa^2 *	
9 endo. l	-06	03	-16	-34	34	41	-02	-19	40	00	-08	-10	13	00	-01	01	66	9
10 endo. b	-02	03	-01	08	12	07	09	-04	06	05	-13	-79	02	-15	-05	-05	72	10
11 endo. h	09	04	10	-15	76	-02	-16	-13	-06	21	18	-03	01	10	11	-03	77	11
24 max. f	14	13	02	-28	09	-02	05	08	-17	03	09	11	05	62	03	06	57	24
20 n-s-ba	-49	03	-79	04	-20	19	-10	-06	-04	-07	-06	-06	-02	-03	-03	-04	96	20
22 for. angle	-20	01	-07	-01	11	-01	-13	-05	-83	-06	-05	09	02	12	00	-05	80	22
17 n-s	02	08	02	-90	07	12	07	-01	03	05	06	10	-02	19	-01	01	90	17
18 s-ba	-02	11	-06	04	54	06	34	03	-26	18	-06	-09	00	10	15	06	58	18
27 sphen. d	14	05	04	-08	00	03	78	01	13	10	-02	-11	00	10	06	03	69	27
25 f. sinus h	-14	03	-01	-01	07	00	01	-04	-02	14	01	06	-03	71	04	01	56	25
43 palate l	-01	47	-11	-10	-02	19	07	-16	04	04	-62	-28	-22	05	16	-07	85	43
34 phar. h	03	27	11	02	65	-25	07	37	12	-19	-17	-09	-06	04	-12	08	82	34
30 n-sp	-59	21	-22	-12	14	05	40	06	-07	03	-09	03	-34	08	-20	18	85	30
31 ba-pns	05	20	-41	-35	07	16	27	11	-27	22	-12	09	12	05	-05	-34	72	31
33 scp-scp	-11	03	07	01	02	09	09	-10	-02	53	00	-13	05	07	-02	-14	37	33
37 n-gn	-66	52	00	05	17	08	16	13	01	05	-22	-02	09	20	28	-09	97	37
44 palate h	-12	18	01	04	11	-23	06	00	-01	13	-01	07	02	09	61	06	53	44
49 ramus h	08	19	17	-08	38	14	29	03	09	11	00	05	02	20	35	49	75	49
39 zg-zg	02	00	-30	-03	-03	62	06	18	01	29	10	-42	-09	03	11	11	82	39
50 infra t.f.d.	03	11	00	03	-01	81	12	01	02	01	01	-05	-09	-04	-09	06	71	50
38 zm-zm	-13	26	-06	-15	03	40	02	-06	15	38	-19	-14	-01	-10	-01	19	54	38
45 go-go	07	05	-02	09	-05	20	06	11	20	13	23	-43	00	05	-31	29	55	45
41 ecm-ecm	-16	24	-12	-11	21	04	13	-01	-06	51	-17	-10	27	-03	12	17	59	41
28 nasal b	10	-14	-02	-06	11	-09	00	15	08	62	00	10	-19	20	11	13	58	28
48 ramus b	09	09	-07	-17	-04	67	-04	-08	-06	-05	-14	04	12	06	-17	-11	60	48
47 gn-cd	05	83	-02	-11	16	18	04	03	-04	01	-02	-02	02	10	11	05	80	47
52 s-n-pg	84	24	15	02	-01	09	20	-04	13	-04	16	-15	-11	-13	-18	00	96	52
55 n-ss-pg	37	03	00	-13	05	-02	03	-10	06	-13	62	01	-11	18	03	01	62	55
57 NL/NSL	-77	-12	-19	-04	-10	03	01	-28	-08	01	05	-11	-04	-04	-13	12	79	57
58 ML/NSL	-76	18	13	15	-07	-25	-28	14	-03	14	-16	04	00	-05	16	-27	96	58
Factor Contribution (Per cent)	15.0	8.0	5.2	6.2	8.4	10.4	6.2	2.1	5.5	6.7	5.3	5.9	1.7	5.6	4.5	3.3		

* Σa^2 The communalities of the variables computed as the sum of squares of the common factor coefficients

Table 33. Analysis 5 - Varimax solution simplified with coefficients > 0.20 shown

VARIABLE	COMMON FACTORS																Σa^2 *	
	12	5	14	3	9	4	7	8	15	6	13	10	2	11	1	16		
9 endo. l		34			-40	34				41							66	9
10 endo. b	79																72	10
11 endo. h		76										21					77	11
24 max. f			62			28											57	24
20 n-s-ba				79											-49		96	20
22 for. angle					83												80	22
17 n-s						90											90	17
18 s-ba		54			26		34										58	18
27 sphen. d							78										69	27
25 f. sinus h			71														56	25
43 palate l	28										22		47	-62			85	43
34 phar. h		65						37		-25			27				82	34
30 n-sp				22			40				34		21		-59		85	30
31 ba-pns				41	27	35	27					22				-34	72	31
33 scp-scp												53					37	33
37 n-gn									28				52	-22	-66		97	37
44 palate h									61	-23							53	44
49 ramus h		38					29		35							49	75	49
39 zg-zg	42			30						62		29					82	39
50 infra t.f.d.										81							71	50
38 zm-zm										40		38	26				54	38
45 go-go	43								-31					23		29	55	45
41 ecm-ecm		21									-27	51	24				59	41
28 nasal b												62					58	28
48 ramus b										67							60	48
47 gn-cd													83				80	47
52 s-n-pg													24		84		96	52
55 n-ss-pg														62	37		62	55
57 NL/NSL								-28							-77		79	57
58 ML/NSL							-28			-25					-76	-27	96	58
Factor Contribution (Per cent)	5.9	8.4	5.6	5.2	5.5	6.2	6.2	2.1	4.5	10.4	1.7	6.7	8.0	5.3	15.0	3.3		

* Σa^2 The communalities of the variables computed as the sum of squares of the common factor coefficients

Table 34. Analysis 5 - percentage contributions of varimax factors to the common variance

Factor	Contribution
1	15.0
2	8.0
3	5.2
4	6.2
5	8.4
6	10.4
7	6.2
8	2.1
9	5.5
10	6.7
11	5.3
12	5.9
13	1.7
14	5.6
15	4.5
16	3.3
Total	100.0

Table 35. Analysis 5 - contributions of the variables to the estimated factor variances - zero contributions are omitted from table *

VARIABLE	COMMON FACTORS															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
9 endo. l				02	07	05		03	10				02			
10 endo. b				01	01	01						46				
11 endo. h			01		38		03	04		03	03				01	01
24 max. f	01								01					22		
20 n-s-ba	01	01	85		01	03	02		01	01	01					
22 for. angle	01						01		59			01				
17 n-s				81	01	01			01		01	01		01		
18 s-ba		01			08		04		03	01					01	
27 sphen. d		01					33		01			01				
25 f. sinus h										01				32	01	
43 palate l		02		01		01	01	06			41	05	10	01	03	
34 phar. h			01		27	04	01	19	01	05	03	01			03	01
30 n-sp	13	01			01		13						22	01	07	03
31 ba-pns			06	03			05	02	04	04	01	01	02			12
33 scp-scp								01		10						01
37 n-gn	19	24		01	01	02	08	01		02	03		05	08	09	03
44 palate h						02								01	19	
49 ramus h					02									01	09	21
39 sg-sg						20		09		05	02	14	02		03	
50 infra t.f.d.						35						01	01			
38 sm-sm		02				02				07	01	01		01		03
45 go-go								01	01	01	02	08		01	06	05
41 ecm-ecm		02								15	01		08		01	03
28 nasal b	01	01						01		20		01	03	02		01
48 ramus b			01			16					01		01		03	01
47 gn-cd		36			01		01									01
52 s-n-ps	28	18				01	03	01	01	01	07	02	04	03	06	
55 n-ss-pg	01							01		01	21		02	02		
57 ML/NSL	18		02					13			01	02			03	02
58 ML/NSL	21	05	04	01	01	05	13	04		07	03			02	01	15
Estimated Factor Variance	.95	.87	.90	.87	.84	.84	.82	.66	.81	.76	.78	.76	.62	.69	.71	.70

* The leading decimal is omitted from table entries. Low values are shown in the table but many of these are probably spurious, being generated during the chain of arithmetic computations required to calculate the estimated factor variances and the contributions of the variables to these variances. Variable contributions with values exceeding 0.09 are underlined to indicate the major components of the estimated factor variances.

Table 36. Analysis 5 - interpretation of common factors

Factor number (Table 33)	Highest factor coefficients		Highest contributions to estimated factor variances		Interpretation
12	endo. b	79	endo. b	46	Endocranial breadth
			zg-zg	14	
5	endo. h	76	endo. h	38	Endocranial height
	phar. h	65	phar. h	27	
14	f. sinus h	71	f. sinus h	32	Frontal bone size
	max. f	62	max. f	22	
3	n-s-ba	79	n-s-ba	85	Cranial base flexion
9	for. angle	83	for. angle	59	Head balance angle
4	n-s	90	n-s	81	Anterior cranial base length
7	sphen. d	78	sphen. d	33	Clivus thickness
			n-sp	13	
8	phar. h	37	phar. h	19	Pharyngeal height
			NL/NSL	13	
15	palate h	61	palate h	19	Palatal height
6	infra t.f.d.	81	infra t.f.d.	35	Mandibular robustness (Infra-temporal fossa depth)
	ramus b	67	zg-zg	20	
			ramus b	16	
13	n-sp	34	n-sp	22	Nasal height
			palate l	10	
10	nasal b	62	nasal b	20	Nasal breadth
	scp-scp	53	ecm-ecm	15	
			scp-scp	10	
2	gn-cd	83	gn-cd	36	Mandibular length
			n-gn	24	
11	n-ss-pg	62	palate l	41	Facial convexity
	palate l	-62	n-ss-pg	21	
1	s-n-pg	84	s-n-pg	28	Facial prognathism
	NL/NSL	-77	ML/NSL	21	
	ML/NSL	-76	n-gn	19	
16	ramus h	49	ramus h	21	Ramus height
			ML/NSL	15	

Table 37. Descriptive statistics for the computed non-normalised scores on 16 maximum likelihood factors for 100 Australian skulls

	Estimated* St. Dev.	Computed** St. Dev.	Mean	Error of Mean	Min.	Max.	Range
1.	.9750	.9756	.0002	.0976	-2.83	2.31	5.14
2.	.9350	.9353	-.0006	.0936	-2.36	2.07	5.42
3.	.9479	.9490	-.0013	.0949	-2.61	2.00	4.61
4.	.9350	.9354	.0003	.0936	-2.42	2.07	4.49
5.	.9150	.9152	.0010	.0916	-2.38	2.44	4.81
6.	.9156	.9157	.0011	.0916	-1.84	3.05	4.90
7.	.9028	.9038	.0011	.0904	-2.35	2.16	4.51
8.	.8105	.8112	.0005	.0812	-1.93	1.69	3.61
9.	.8981	.8993	.0001	.0900	-2.18	2.71	4.89
10.	.8737	.8739	.0015	.0874	-2.17	1.96	4.13
11.	.8802	.8812	.0007	.0882	-2.29	2.20	4.50
12.	.8712	.8710	.0001	.0871	-1.70	2.59	4.29
13.	.7853	.7869	-.0006	.0787	-1.76	1.95	3.71
14.	.8326	.8325	-.0001	.0833	-1.51	2.07	3.58
15.	.8394	.8398	.0013	.0840	-1.87	2.39	4.27
16.	.8347	.8351	-.0008	.0836	-2.15	2.19	4.34

* Derived from the equation St. Dev. $= \sqrt{\text{diag}\left[I + (I + J)^{-1}\right]}$ see pp 223-224

** Derived from the computed factor scores by treating them as normally distributed variables.

APPENDIX C*

Computing Algorithms

* Appendix C is abstracted from a more detailed description of several computer programs, coded in FORTRAN IV and designed for execution on a CDC 6400 computer (BROWN, '67b). In addition to the computing algorithms, this publication outlines the usage of the programs, machine restrictions, subroutines required, data deck structures, program timing and materials available. References are also listed.

IDENTIFICATION

Factor Analysis Package. Included in this package are five computer programs with their subroutines which are designed to compute orthogonal solutions according to the basic mathematical model of factor analysis. A method for principal components analysis is included although this technique is different in mathematical foundation and computing procedure from those of true factor analysis. Several different approaches to the factor model are available; the final program estimates beta coefficients for the computation of factor scores and goes on to calculate these scores for any number of subjects if required.

Titles: PROGRAM PRELIM
PROGRAM FACTIT
PROGRAM JORIMA
PROGRAM MAXLIK
PROGRAM FACSCO

Category: Multivariate analysis
Language: 6400 FORTRAN
Author: T. Brown, Department of Dental Science, September 1965
Modified by J. Parfitt, January 1967
Installation: CDC 6400, University of Adelaide, Adelaide.

PURPOSE

PROGRAM PRELIM carries out preliminary analyses of a data matrix prior to factor analysis, giving information on the latent roots and their significance. The output allows a decision to be made regarding the number of statistically significant, or scientifically meaningful, factors to be extracted during further analysis of the data.

PROGRAMS FACTIT, JORIMA and MAXLIK compute orthogonal factor loadings by any of the following six methods:-

A. Non-iterative methods

PRINCIPAL FACTOR ANALYSIS - requiring initial communality estimates (PROGRAM FACTIT).

MODIFIED PRINCIPAL COMPONENTS ANALYSIS - no initial communality estimates required and a predetermined number of factors is retained (PROGRAM FACTIT).

JORESKOG FACTOR ANALYSIS - no initial communality estimates required; matrix is rescaled prior to factor extraction (PROGRAM JORIMA).

IMAGE-COVARIANCE FACTOR ANALYSIS - no initial communality estimates required; matrix is rescaled prior to factor extraction (PROGRAM JORIMA).

B. Iterative methods

ITERATIVE PRINCIPAL FACTOR ANALYSIS - requires initial estimates of communalities; the procedure entails cycling until successive solutions converge to a specified value or until the maximum permitted number of iterations is reached (PROGRAM FACTIT).

MAXIMUM LIKELIHOOD FACTOR ANALYSIS - a very precise method requiring initial approximations to the factor loadings obtained by some preliminary analysis. The program cycles until convergence of successive solutions to a predetermined level is obtained or until the maximum permitted number of iterations is reached (PROGRAM MAXLIK).

The above methods operate on data which may be in the form of raw scores, a matrix of correlation coefficients or a variance/covariance matrix. When raw scores are entered the appropriate matrix is computed prior to factor extraction. Options are provided to carry out varimax transformation of the initial solution by the method of Kaiser. Various tests and checks, including a chi-square test on the significance of residual coefficients, are included to serve as a guide to the mathematical fit of the solution.

PROGRAM FACSCO computes the factor loadings on orthogonal factors from the subjects' raw scores or standard deviate scores. Beta coefficients for the factor score estimation are first entered or computed from the factor loadings which have been obtained by any of the methods referred to above. The factor scores may be computed as non-normalised values or they may be normalised according to the estimated standard deviation of factor.

ALGORITHM

Detailed mathematical procedures are given in the references listed. The basic model of factor analysis assumes that a subject's deviate scores on N variables can be explained in terms of contributions from K common factors and N unique factors as follows:

$x = \bar{x} + Gf + u$ where

x (N x 1) is the vector of N deviate scores for one subject,
$\bar{x}$ (N x 1) is the vector of mean values for N variables,
G (N x K) is the matrix of loadings for N variables on K factors,
f (K x 1) is the vector of K factor scores, and
u (N x 1) is the vector of N residual or unique components.

The equation contains the unknown quantities G, f and u and the first stage of factor analysis is to estimate G and u according to the basic factor analysis equation:

$R = GG' + U$ where

R (N x N) is the sample matrix of correlation coefficients, or the variance/covariance matrix - this is an estimate of the true population matrix,
G (N x K) is the matrix of factor loadings and G' (K x N) is its transpose,
U (N x N) is the diagonal matrix of residual variances, or uniquenesses with diagonal elements u, and off-diagoanl elements 0.

The computing procedures carried out in the five programs included in this package are briefly outlined for each program in turn.

ALGORITHM FOR PROGRAM PRELIM

The following steps are carried out:-

1. The raw scores for M subjects on N variables are entered and the correlation or variance/covariance matrix R, with elements r_{ij}, $i = j = 1,2,\ldots,N$, is computed. Alternatively R may be entered direct from cards.

2. Trace R is computed.

3. Program proceeds with either or all of Stages I, II and III as specified on parameter card.

Stage I

4. The eigenvalues of R are computed by the Jacobi diagonalisation method to solve

$$|R - \lambda I| = 0$$

where λ_i, $i = 1,2,\ldots,N$ are eigenvalues and I is the identity matrix.

5. The elements, λ_i, of the eigenvalue vector L are arranged in descending order of magnitude and listed together with their percentage contributions to Trace R.

Stage II

6. Initial estimates of the residual variance vector U are entered, or alternatively are computed from the inverse of R according to:

$$U = \frac{1}{\text{diag } R^{-1}}$$

The Gauss-Jorgan method is used to compute R^{-1} and the determinant $|R|$.

7. The eigenvalues of the reduced data matrix (R - U), that is with communality estimates in the main diagonals, are computed to solve:

$$|(R - U) - \lambda I| = 0$$

where, as before, λ_i are eigenvalue elements of L.

8. The elements of L are arranged in descending order of magnitude and listed together with their percentage contributions to Trace (R - U).

Stage III

9. Vector D is set = diag R^{-1}, that is, $D = U^{-1}$

10. Matrix R is rescaled according to Jöreskog's method as follows:

$$R^* = D^{1/2} R D^{1/2}$$ where the elements of rescaled matrix R^* are given by

$$r_{ij}^* = d_i^{1/2} r_{ij} d_j^{1/2} \qquad i = j = 1,2,\ldots,N.$$

11. The eigenvalues of the rescaled matrix R^* are computed to solve:

$$|R^* - \lambda I| = 0$$

where, as before, λ_i are eigenvalue elements of L.

12. The elements of L are arranged in descending order of magnitude and listed together with their percentage contributions to Trace R^*.

13. The criterion value C, indicating the significance of N - K smallest roots of R^*, is computed for each value of K from 1 - N, according to the Jöreskog method:

$$T_K = \frac{1}{N - K} \sum \lambda_i \qquad i = K + 1,\ K + 2,\ldots,\ N$$

$$C_K = \frac{1}{2T_K^{\,2}} \sum (\lambda_i - T_K)^2 \qquad i = K + 1,\ K + 2,\ldots,\ N$$

14. Each value of C_K, which is distributed approximately as chi-square, is listed together with the degrees of freedom given by $(N - K - 1)(N - K + 2)/2$.

15. New problem is commenced or end of file card is read and job terminates.

Note

The decision on the number of factors to extract from any data matrix is always difficult to make and factor analysis should not be undertaken without clearly defined objectives and a detailed knowledge of the variables to be analysed. Depending on the nature of the proposed investigation, PROGRAM PRELIM provides information that will enable a reasonably objective approach to the problem of when to stop factoring, or how many factors to extract. If the objective is to explain the interactions among the variables with mathematical precision, the number of factors will generally be higher than if the objective is simply to explain the major sources of variation with less reliance on "statistical fit".

The following suggested criteria for the number of factors may be found useful but the final choice must depend on the nature of the problem:-

K = a) Number of eigenvalues of R greater than 1,
b) Number of eigenvalues of $(R - U) > 0$,
c) Number of eigenvalues of R contributing a given percentage to Trace R, say 50, 60 or 70 per cent depending on the nature of the problem,
d) Number of eigenvalues of (R - U) contributing a given percentage to Trace (R - U), say 90, 95 or 100 per cent,
e) Number of eigenvalues of R^* beyond which the chi-square criterion is non-significant - probably the most reliable assessment.

ALGORITHM FOR PROGRAM FACTIT

The following steps are carried out:-

1. The raw scores for M subjects on N variables are entered and the correlation matrix or the variance/covariance matrix R, with elements r_{ij}, $i = j = 1,2,\ldots,n$, is computed. Alternatively R may be entered direct from cards. The means, and standard deviations of the variables are computed and listed.

2. Trace R, inverse R, and the determinant $|R|$ are computed.

3. If specified, the initial variance vector U_1, with elements u_{1i}, $i = 1,2,\ldots,n$, is entered by cards or U may be computed from the inverse of R according to:

$$U_1 = \frac{1}{\text{diag } R^{-1}}$$

4. The elements of U_1 are scanned to ensure that their values lie between 0 and r_{ii}. If this condition is not fulfilled for each value, then the maximum absolute row values of R are substituted for U. This check may disclose errors in the original R matrix or an insufficient number of significant figures in the elements of R when these have been punched on cards.

5. The eigenvalues of the reduced data matrix $(R - U_1)$, that is with communality estimates in the main diagonal elements, are computed to solve:

$$|(R - U_1) - \lambda I| = 0 \quad \text{where } \lambda_i \quad i = 1,2,\ldots,N$$

are elements of eigenvalue vector L and I is the identity matrix.

6. The elements of L are arranged in descending order of magnitude and the associated eigenvectors, forming matrix E_{NN} with elements e_{ij}, $i = j = 1,2,\ldots,N$, are placed in the corresponding order.

7. The number of factors K to be retained is either specified in advance or computed according to the criteria entered on the parameter card by scanning the elements of L.

8. The retained eigenvalues are placed in vector L with elements λ_i, $i = 1,2,\ldots,K$, and the associated eigenvectors form matrix E with elements e_{ij}, $i = 1,2,\ldots,N$; $j = 1,2,\ldots,K$.

9. New estimates of the residual variance vector U_2 are computed according to:

$$U_2 = \text{diag } (R - ELE')$$

10. The elements of U_2 are compared with those of U_1 for convergence within the specified value:
Is absolute value $(u_{2i} - u_{1i}) >$ convergence value?

If convergence has not occurred program proceeds from step 11.
If convergence has occurred program proceeds from step 12.

11. The number of iterations is compared with the maximum number permitted and if this has not been reached the residual variance vector U_2 replaces vector U_1 and a new iteration commences at step 5. If the maximum number of iterations has been reached the program proceeds from step 12.

12. The matrix of orthogonal factor loadings G is computed:

$$G = EL^{1/2}$$

with elements, that is factor loadings, g_{ij} $\begin{matrix} i = 1,2,\ldots,N \\ j = 1,2,\ldots,K. \end{matrix}$

13. The orthogonality of the solution is checked according to:

$$ORTH = \sum_{j=1}^{N} g_{jp} g_{jq} \qquad p \neq q = 1,2,\ldots,K.$$

ORTH should = 0 for true orthogonality.

14. The original data matrix is restored according to:

$R_{restored} = GG' + U_2$ and the determinant $|R_{restored}|$ is found.

An approximate chi-square test is carried out according to:

$$\chi^2 = n' \log_e \left(|R_{restored}| \; / \; |R| \right)$$

where $n' = (N - 1) - (2N + 5)/6 - 2K/3$
with $(N - K)(N - K - 1)/2$ degrees of freedom.

15. A varimax orthogonal solution is obtained by the method of Kaiser if specified. The final transformed matrix of factor loadings V is computed by :

$V = GT$ where T is the orthogonal matrix which transforms G to V under conditions maximising the Kaiser varimax criterion.

16. The final communalities and residual variances of each variable are computed and the data matrix restored as in step 14 according to:

$$R_{restored} = VV' + U_2$$

17. The matrix of residual coefficients is computed with elements $\bar{r}_{ij}$, $i = j = 1,2,\ldots,N$

$$\bar{R} = R_{restored} - R$$

18. The mean, error of mean, standard deviation, minimum and maximum or $\bar{r}_{ij}$ are computed.

19. An approximate chi-square test on the elements of $\bar{R}$ is carried out according to:

$$\chi^2 = n' \sum \bar{r}_{ij}^{\,2} / u_{2i}u_{2j} \qquad i \neq j = 1,2,\ldots,N$$

where $n' = N - 1$,
with $(N - K)(N - K - 1)/2$ degrees of freedom.

20. Program commences new problem or terminates on reading an end of file card.

Note

PROGRAM FACTIT, although designed to carry out iterative principal factor analysis, can be made to perform non-iterative analysis by limiting the number of cycles specified on the parameter card to 1. Principal components analysis can be performed by limiting the number of iterations to 1 and reading in zero values as the first residual variances, that is values of 1 will be retained in the diagonal elements of a correlation matrix or variances will be retained in the diagonals of a variance/covariance matrix. For true principal components analysis all components are retained and K is set = N and no varimax rotation is specified. For modified principal components analysis, K is set to any desired value less than N and varimax rotation can be performed if required. Modified principal components is sometimes confused with factor analysis which it resembles, but its objectives are fundamentally different.

ALGORITHM FOR PROGRAM JORIMA

PROGRAM JORIMA enables a matrix to be factored by either of two methods.

Method I - Jöreskog Factor Analysis: the steps are as follows:-

1. As for PROGRAM FACTIT without calculation of means and standard deviations.

2. As for PROGRAM FACTIT.

3. Vector D with elements d_i, $i = 1,2,\ldots,N$ is set = $\text{diag}R^{-1}$, or alternatively the residual variance vector U is entered by cards as in FACTIT and
$$D = U^{-1}$$

4. The data matrix is rescaled according to Jöreskog's method according to:
$$R^* = D^{1/2} R D^{1/2} \quad \text{with elements}$$
r_{ij}^* $i = j = 1,2,\ldots,N$ where $r_{ij}^* = d_i^{1/2} r_{ij} d_j^{1/2}$

5. The eigenvalues and eigenvectors of the reduced data matrix R^* are computed to solve the equation:
$$|R^* - \lambda I| = 0 \quad \text{where } \lambda_i \text{ are the elements of L.}$$
The eigenvectors form matrix E with elements e_{ij}, $i = j = 1,2,\ldots,N$.

6. The elements of L are arranged in descending order of magnitude and the columns of E are placed in corresponding order.

7. The K largest eigenvalues are listed together with their percentage contributions to the Trace R^* - K is the number of common factors specified on the parameter card.

8. Jöreskog's T value is computed according to:
$$T = \frac{1}{N - K} \sum \lambda_i \qquad i = K + 1, K + 2, \ldots, N$$
or alternatively, if initial residual variances were entered at step 3, T is set = 1.

9. The Jöreskog matrix of factor loadings G is computed according to:
$$G = E(L - T)^{1/2} D^{1/2} \quad \text{with elements } g_{ij} \text{ given by:}$$
$$g_{ij} = e_{ij}(\lambda_j - T)^{1/2} d_i^{1/2} \qquad \begin{matrix} i = 1,2,..,N; \\ j = 1,2,..,K. \end{matrix}$$

10. Program proceeds as for steps 13 - 20 of PROGRAM FACTIT.

Method II - Image-Covariance Factor Analysis: the steps are as follows:-

1. As for PROGRAM FACTIT without calculation of means and standard deviations.

2. As for PROGRAM FACTIT.

3. The vector of residual variances U, with elements u_i, $i = 1,2,\ldots,N$, is entered by cards or alternatively, is computed from the inverse of R according to:

$$U = \frac{1}{\text{diag } R^{-1}}$$ (U forms a diagonal matrix with zero off-diagonal elements)

4. The data matrix is rescaled according to:

$$R^* = R + UR^{-1}U - 2U$$

For the diagonal elements of R^*

$$r_{ii}^* = r_{ii} + u_i - 2u_i \qquad i = 1,2,\ldots,N$$

$$= r_{ii} - u_i$$ This value is analogous to a communality estimate

For the off-diagonal elements of R^*

$$r_{ij}^* = r_{ij} + r^{ij}u_iu_j - 2(0) \qquad i \neq j = 1,2,\ldots,N$$

where r^{ij} are the elements of R^{-1}.

5. As for Method I - Jöreskog Factor Analysis (step 5).

6. As for Method I - Jöreskog Factor Analysis (step 6).

7. As for Method I - Jöreskog Factor Analysis (step 7).

8. The image-covariance matrix of factor loadings G is computed according to:

$$G = EL^{1/2}$$ with elements g_{ij} given by:

$$g_{ij} = e_{ij}\lambda_j^{1/2} \qquad i = 1,2,\ldots,N;\ j = 1,2,\ldots,K.$$

9. The program proceeds as for steps 13 - 20 of PROGRAM FACTIT.

ALGORITHM FOR PROGRAM MAXLIK

The following steps are carried out:-

1. The matrix of correlation coefficients or the variance/covariance matrix R is entered by cards. R has elements r_{ij} $i = j = 1,2,..N$. N is the number of variables and M is the number of subjects.

2. The Trace R is computed and the determinant $|R|$ is entered or computed. Vector D with elements d_i is set = diag R, that is $d_i = r_{ii}$, $i = 1,2,...,N$.

3. The matrix of initial approximations to the factor loadings G_1' is entered.

   Note: Matrix G_1' is the transpose of factor matrix G and has elements:

   $$g_{1\ ij} \qquad i = 1,2,...,K;\ j = 1,2,...,N$$

   where K is the number of factors entered on the parameter card.

4. Initial estimates of the residual variances are computed to form matrix V_1 with elements v_{1j} according to:

   $$v_{1j} = d_j - \sum g_{ij}^2 \qquad i = 1,2,...,K;\ j = 1,2,...,N$$

5. Compute vector W_1 according to: $W_1 = G_1^{(1)}\ V_1^{-1}$

   where $G_1^{(1)}$ is the first row of matrix G' and the transpose sign is dropped for simplicity. $G_1^{(1)}$ contains the variable loadings for the first factor.

6. Compute vector U_1 according to $U_1 = W_1\ R - G_1^{(1)}$.

7. Compute positive scalar $S_1 = U_1 W_1$.

8. Compute new estimates of the loadings for Factor 1 according to:

   $$G_2^{(1)} = U_1 / S_1^{1/2}\ .$$

9. Repeat steps 5 - 8 for each successive factor; for example for Factor 2:-

   $$W_2 = G_1^{(2)}\ V_1^{-1}$$

   $$U_2 = W_2\ R - G_1^{(2)} - W_2 G_2^{(1)}\ G_2^{(1)}$$

   $$S_2 = U_2\ W_2$$

   $$G_2^{(2)} = U_2 / S_2^{1/2}\ .$$

These steps result in the formation of the second estimate of the factor matrix G.

10. Successive estimates of the residual variance vector V_2 are obtained as in step 4:-

$$V_{2j} = d_j - \sum g_{ij}^2 \qquad i = 1,2,\ldots,K;\ j = 1,2,..,N$$

where the elements g_{ij} are from matrix G_2.

11. The new residual variances are compared with the initial set for convergence within the value specified on the parameter card.

Is absolute $(V_2 - V_1)$ > convergence value?

If convergence has not taken place program proceeds from step 12. If convergence of each element of V_2 has taken place, program proceeds from step 13.

12. If the maximum number of permitted iterations has been reached program proceeds from step 13. If the permitted number of iterations has not been reached the program commences a new cycle from step 5, substituting vector V_2 for V_1 and matrix G_2 for G_1.

13. The program proceeds as for steps 13 - 20 of PROGRAM FACTIT.

ALGORITHM FOR PROGRAM FACSCO

This program estimates the beta coefficients for the computation of factor scores for M subjects on K orthogonal factors using the scores for the M subjects on N variables and the orthogonal matrix of factor loadings G. Matrix G can be estimated by any of the methods outlined above and can be the initial non-transformed solution or the varimax transformed solution.

Options set on the data parameter card permit entry to the computing procedures at any of several points. If all options are used, PROGRAM FACSCO first estimates the beta coefficients from the matrix of factor loadings and then the estimated variances of the factor scores with the contributions of each variable to the estimated variances. The program then reads in the subjects' original observed scores together with the means and standard deviations of the variables. The subjects' standard deviate scores are computed, followed by the non-standardised factor scores for each subject on the K factors. Finally, the factor scores are normalised by dividing each by its standard deviation and adjusting to a mean of 50 and a standard deviation of 10.

Alternative procedures allow the program to:-

(1) Halt after calculation of beta coefficients;
(2) Read in beta coefficients direct from cards;
(3) Enter the standard deviate scores for the subjects direct from cards;
(4) Omission of the factor score normalisation.

The basic equation for the estimation of a subject's factor score can be written:

$$f_1 = b_{11}z_1 + b_{12}z_2 + b_{13}z_3 + \ldots\ldots + b_{1N}z_N$$

where f_1 is the non-normalised score on Factor 1,

b_{1i} $(i = 1,2,\ldots,N)$ is the vector of beta coefficients for Factor 1,

z_i $(i = 1,2,\ldots,N)$ is the vector of the subject's standard deviate scores on N variables.

If all options are specified PROGRAM FACSCO proceeds as follows:

1. The transpose of the matrix of orthogonal factor loadings is entered by cards. Matrix G' has elements:
g_{ij} $\quad i = 1,2,\ldots,K; \quad j = 1,2,\ldots,N.$

2. The residual variance vector V, with elements v_j, $j = 1,2,\ldots,N$, is computed according to:

$$v_j = 1.0 - \sum_i g_{ij}^2 \qquad i = 1,2,\ldots,K; \; j = 1,2,\ldots,N.$$

3. Matrix $S = G'V^{-1}$ is computed (S is of order K x N).

4. Matrix $J = G'V^{-1}G$ is computed (J is square of order K x K).

5. Matrix $(I + J)^{-1}$ is computed (I is the K-order identity matrix).

6. Matrix $\left[I - (I + J)^{-1}\right]$ is computed.

7. Variances of the factor scores are given by:

$$\text{variances} = \text{diag}\left[I - (I + J)^{-1}\right].$$

8. Standard deviations of the factor scores are computed:

$$\text{s.d.}_{\text{Factor } i} = \sqrt{\text{diag}\left[I - (I + J)^{-1}\right]} \qquad i = 1,2,\ldots,K.$$

9. The matrix of beta coefficients B with elements b_{ij} is computed:

$$B = (I + J)^{-1}G'V^{-1}$$

10. The total contributions of the variables to the factor score variances are computed:

$$\text{contribution}_{ij} = \sum b_{ij} g_{ij} \qquad \begin{matrix} i = 1,2,\ldots,K; \\ j = 1,2,\ldots,N. \end{matrix}$$

11. The means $\bar{x}_i$ and the standard deviations s_i of the N variables are entered and the standard scores for each subject on N variables are computed according to:

$$z_{ij} = (x_{ij} - \bar{x}_j) / s_j \qquad \begin{matrix} i = 1,2,\ldots,M; \\ j = 1,2,\ldots,N. \end{matrix}$$

where z_{ij} are the standard deviate scores and x_{ij} are observed scores.

12. Each subject's scores on K factors are computed; thus for one subject:

$$f_i = \sum b_{ij} z_j \qquad \begin{matrix} i = 1,2,\ldots,K; \\ j = 1,2,\ldots,N. \end{matrix}$$

13. The factor scores are normalised to a mean of 50 and a standard deviation of 10:

$$f_i \text{ (normal)} = (10 f_i / \text{s.d.}_{\text{Factor } i}) + 50.$$

14. The program commences a new problem after all factor scores are listed, or terminates on reading an end of file card.

Note: It may be found useful to list the computed factor scores on magnetic tape in the event that subsequent printing in order of magnitude is required with subject identification, or if additional analysis of the factor scores is anticipated.

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