Ant Colony Optimization for the Design of Water Distribution Systems

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Abstract

During the last decade, evolutionary methods such as genetic algorithms have been developed for the optimal design and operation of water distribution systems. More recently, ant colony optimization algorithms (ACOAs), which are evolutionary methods based on the foraging behavior of ants, have been successfully applied to a number of benchmark combinatorial optimization problems. For example, when applied to the traveling salesman problem, ACOAs have been shown to outperform genetic algorithms. In this paper, a formulation is developed which enables ACOAs to be used for the optimal design of water distribution systems. This formulation is applied to a benchmark water distribution system optimization problem and the results are compared with those obtained using genetic algorithms. The findings of this study indicate that the performance of ACOAs is comparable with that of GAs for the case study considered. The GA performed slightly better in terms of finding the optimal solution from different starting positions in the search space, whereas the ACOA performed better in terms of the number of evaluations needed to reach the optimum.

Introduction

Genetic algorithms (GAs) are an evolutionary optimization method based on the concept of survival of the fittest that have been used extensively for the optimal design and operation of water distribution systems (WDS) (e.g. Simpson et al. 1994). More recently, Dorigo et al. (1996) developed an evolutionary optimization algorithm based on the foraging behavior exhibited by ant colonies in their search for food. Ant colony optimization algorithms (ACOAs) have been successfully applied to a number of benchmark combinatorial optimization problems, such as the traveling salesman and quadratic assignment problems (Dorigo et al. 2000), and have been shown to outperform other evolutionary optimization algorithms, including GAs (e.g. Dorigo and Gambardella 1997). In the late 1990s, Dorigo and Di Caro (1999) introduced a general framework for developing ACOAs; the ant colony meta-heuristic. This enables ACOAs to be applied to a range of combinatorial optimization problems, provided problem specific formulations can be developed (Stützle and Hoos 2000). In this paper, the utility of ACOAs for the optimal design of WDS is explored.

Ant Colony Optimization

ACOAs are inspired by the fact that ants are able to find the shortest route between their nest and a food source, even though they are almost blind. This is accomplished by using pheromone (chemical) trails as a form of indirect communication. Ants deposit pheromone trails whenever they travel. The path taken by individual ants from the nest in search for a food source is essentially random. However, when many ants are searching for a food source simultaneously, the paths taken are affected by the pheromone trails laid by other ants. When ants encounter pheromone trails, there is a higher probability that trails with higher pheromone intensities will be chosen. As more ants travel on paths with higher pheromone intensities, the pheromone on these paths builds up further, making it more likely to be chosen by other ants. The way this form of positive reinforcement can be used to find the shortest path between the nest and a food source can best be illustrated with an example.

Let us consider the scenario shown in Fig. 1a, where an obstacle has been placed between the ants' nest (A) and the food source (B) so that one route from the nest to the food source (AEDFB) is shorter than the other (AECFB). In the example considered, the length of the shorter route is 4 unit distances (UD), whereas the length of the longer route is 6 unit distances (Fig. 1a). Let us assume that 16 ants leave the nest at time t=0, that the initial pheromone concentration (τ) on each path segment is zero (Fig. 1b), that each ant moves one unit distance per unit time (t), and that each ant deposits one unit of pheromone (τ) after reaching the next node. The numbers of ants (shown in brackets) and the pheromone concentrations on each path segment at times t = 0, 1, 2, 5, 7, 9 and 10 are shown in Figs. 1b to 1h.

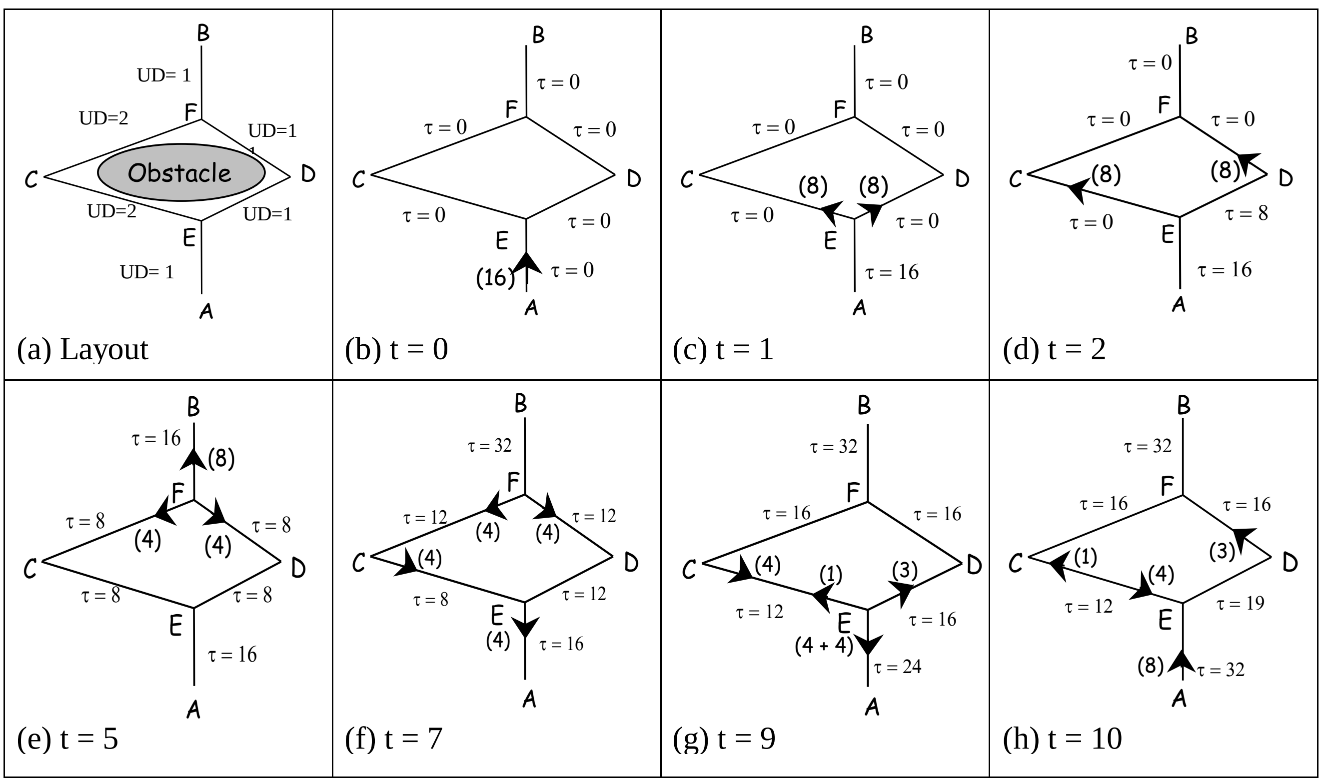


Figure 1. Example of reinforcement of shorter routes as ants travel from their nest (A) to a food source (B) and back.

At t = 1 (Fig. 1c), 16 ants arrive at E and have deposited 16 units of pheromone on AE. As there is no pheromone on EC and ED, there is an equal probability that the ants will choose either path. Consequently, it is assumed that 8 ants choose EC and 8 ants choose ED. At t = 2 (Fig. 1d), the 8 ants following path ED have reached D and have deposited 8 units of pheromone on ED. Since path EC is twice as long as path ED, the ants following path EC have not yet reached C, and thus pheromone has not yet been deposited on EC (assuming that the pheromone is only deposited once EC has been traversed completely). At t = 5 (Fig. 1e), the 8 ants travelling on the longer route are at F on their way to the food source (B) and have deposited 8 units of pheromone on EC and CF. At the same time, the 8 ants travelling on the shorter route are also at F on their way back to the nest (A), having already reached the food source (B) at t = 4. Consequently, they have deposited 8 units of pheromone on DF and 16 units of pheromone on FB (8 units going from F to B and 8 units going from B to F). At this stage, the pheromone intensities on FD and FC are 8 units each, and thus, by equal probability, it is assumed that 4 returning ants choose path FC and the other 4 choose path FD.

At t = 7 (Fig. 1f), the 4 returning ants that have chosen the shorter route to the nest (FDEA) have reached E and have deposited an additional 4 units of pheromone on FD and DE. In contrast, the 4 returning ants that have chosen the longer route to the nest (FCEA) have reached C and have deposited an additional 4 units of pheromone on FC. At the same time, the 8 ants that chose the longer route from the nest to the food source initially have returned to F, after reaching the food source (B) at t = 6. Consequently, they have deposited an additional 16 units of pheromone on BF (8 units going from F to B and 8 units going from B to F). At F, there is an equal probability that the 8 ants will choose paths FC and FD, as each has a pheromone intensity of 12 units. Consequently, it is assumed that 4 ants travel on each of these paths on their way back to the nest (A).

At t = 9 (Fig. 1g), 8 ants are at E on their way from the food source (B) back to the nest (A); 4 ants that took the long route to and the short route from the food source, and 4 ants that took the short route to and the long route from the food source. The former have deposited an additional 4 units of pheromone on FD and DE, while the latter have deposited an additional 4 units of pheromone on CE. At the same time, the 4 ants that took the long route to and from the nest have reached C on their way back to the nest (A), and have deposited an additional 4 units of pheromone on FC. In addition, the first 4 returning ants (i.e. the ants that chose the shorter route to and from the food source) are at E, having reached the nest (A) at t = 8, and have deposited an additional 8 units of pheromone on EA (4 units going from E to A and 4 units going from A to E). At E, there is now a greater probability that the ants will choose the shorter path (ED), as the pheromone concentration on ED is 16 units, compared with 12 units on EC. Consequently, it is assumed that 3 ants choose ED, while only 1 ant chooses EC. This further reinforces the shorter route, as shown in Fig. 1h. At t = 10, the gap between the pheromone intensity on EC and ED has widened, increasing the probability that ED will be chosen by the 8 ants leaving the nest (A) at that time. In this way, the probability that the shorter route is chosen increases with time.

Ant Colony Optimization Algorithms

The basic principle of positive reinforcement via the use of pheromone trails discussed above also underlies ACOAs. In addition, ACOAs make use of a colony of cooperating individuals and adopt a stochastic decision-making policy using local information. However, as the main purpose of artificial ant systems is to find optimal solutions to combinatorial optimization problems, they also incorporate features that are not found in their natural counterparts. For example, artificial ants generally have some memory, are not completely blind and live in an environment where time is discrete (Dorigo et al. 1996).

In order to implement ACOAs, the combinatorial optimization problem under consideration has to be able to be mapped onto a graph G = (**D**, **L**, **C**), where **D** = $\{d_1, d_2, \ldots, d_n\}$ is a set of points at which decisions have to be made, **L** = $\{l_{i(j)}\}$ is the set of options (j) available at each decision point (i), and **C** = {$c_{i(j)}$} is the set of costs associated with options **L** = {$l_{i(j)}$}. A set of finite constraints Ω (**D**, **L**) may be assigned over the elements of **D** and **L**. A feasible path over G is called a solution (φ) and a minimum cost path is an optimal solution (φ^*). The cost of a solution is denoted by f(φ) and the cost of the optimal solution by f(φ^*) (Dorigo and Di Caro 1999). For example, in the problem in Fig. 1, there are two decision points (E & F), at each of which two options exist (paths ECF and EDF). The costs associated with these two choices are the lengths of the respective paths (i.e. 4 unit distances for ECF and 2 unit distances for EDF). In this example, both paths constitute feasible solutions, as there are no constraints. However, path EDF is the optimal solution, as it is shorter than path ECF.

Once the problem has been defined in the terms set out above, the ACOA can be applied. The discussion in this paper will focus on the Ant System algorithm developed by Dorigo et al. (1996), although alternatives have been proposed since (e.g. Stützle and Hoos 2000). The main steps in the algorithm are shown in Fig. 2. Each cycle consists of three main steps: generation of a trial solution φ(k), calculation of the cost of the trial solution (f(φ(k)), and updating of the concentrations of the pheromone trail(s) associated with each of the choices. Each trial solution is built incrementally as an artificial ant moves from one decision point to the next. Each of these moves is referred to as an iteration (t). In contrast, a cycle (k) consists of n iterations, where n is the number of decision points. At each decision point, the component to be added to the trial solution is chosen stochastically in accordance with the following equation (Dorigo et al. 1996):

$$p_{i(j)}(t) = \frac{[\tau_{i(j)}(t)]^{\alpha} \cdot [\eta_{i(j)}]^{\beta}}{\sum_{l_{i(j)}} [\tau_{i(j)}(t)]^{\alpha} \cdot [\eta_{i(j)}]^{\beta}} \qquad (1)$$

where $p_{i(j)}$ (t) is the probability that option $l_{i(j)}$ is chosen at iteration t, $\tau_{i(j)}$ (t) is the concentration of pheromone associated with option $l_{i(j)}$ at iteration t, $\eta_{i(j)} = 1/c_{i(j)}$ is a heuristic factor favoring options that have smaller "local" costs, and α and β are parameters that control the relative importance of pheromone and the local heuristic factor, respectively. Artificial ants can also be provided with memory to ensure that each decision point is only visited once. In general, trial solutions are generated asynchronously by a number of ants (m), after which the costs of the trial solutions can be calculated (Fig. 2).

Once a cycle has been completed, the pheromone trails have to be updated in a way that reinforces good solutions (Fig. 2). The general form of the pheromone update equation is as follows (Dorigo et al. 1996):

$$\tau_{i(j)}(k+1) = \rho\, \tau_{i(j)}(k) + \Delta\tau_{i(j)} \qquad (2)$$

where $\tau_{i(j)}$ (k+1) is the concentration of pheromone associated with option $l_{i(j)}$ at cycle k+1, $\tau_{i(j)}$ (k) is the concentration of pheromone associated with option $l_{i(j)}$ at cycle k, ρ is a coefficient representing pheromone persistence, and the total change in pheromone $\Delta\tau_{i(j)}$ is given by (Dorigo et al. 1996):

$$\Delta\tau_{i(j)} = \sum_{h=1}^{m} \Delta\tau_{i(j)}^{h} \qquad (3)$$

where $\Delta\tau_{i(j)}{}^{h}$ is the change in the concentration of pheromone associated with option $l_{i(j)}$ made by ant h between cycles k and k+1. The pheromone persistence coefficient (ρ) must be less than one and therefore simulates pheromone evaporation. This enables greater exploration of the search space and avoids premature convergence to sub-optimal solutions as it reduces the difference in pheromone concentration between options at each decision point. In addition, evaporation reduces the likelihood that high cost solutions will be selected in future cycles. Dorigo et al. (1996) investigated three different ways of calculating $\Delta\tau_{i(j)}{}^{h}$. The discussion in this paper is restricted to the ant-cycle algorithm, as it performs significantly better than the other two algorithms (Dorigo et al. 1996). When the ant-cycle algorithm is used, $\Delta\tau_{i(j)}{}^{h}$ is given by (Dorigo et al. 1996):

$$\Delta\tau_{i(j)}^{h} = \begin{cases} \dfrac{R}{f(\varphi)^{h}} & \text{if the } h^{th} \text{ ant chooses option } l_{i(j)} \text{ at cycle } k \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

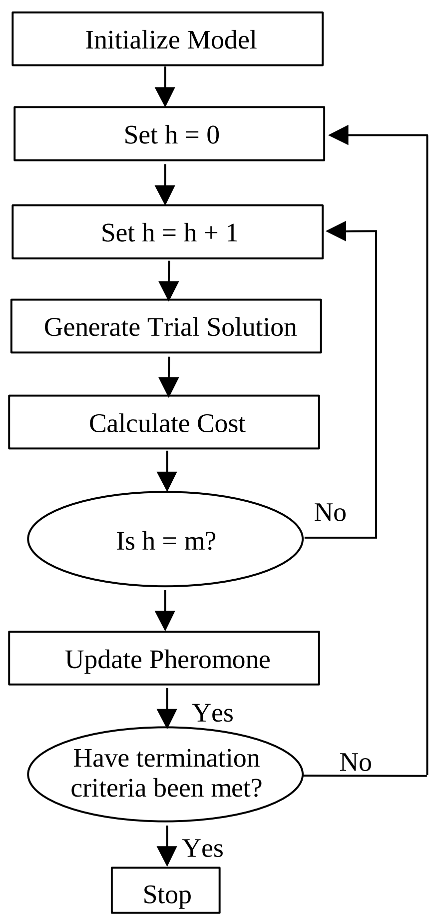


Figure 2. Steps in ACOA.

where R is the pheromone reward factor, and $f(\varphi)^h$ is the cost of the trial solution generated by ant h. It should be noted that in accordance with Eq. 4, the pheromone is updated only once a complete trial solution has been constructed. In addition, the amount of pheromone added to each of the options chosen as part of a particular trial solution is a function of the cost of the trial solution. The better the trial solution, and hence the lower the cost, the larger the amount of pheromone added. Consequently, solution components (options) that are used by many ants and form part of lower cost solutions receive more pheromone and are more likely to be chosen in future cycles (Stützle and Hoos 2000). The cycle of generating trial solutions, calculating the costs of the chosen solutions and updating the pheromone concentrations is repeated until certain stopping criteria are met (Fig. 2). The stopping criterion generally used is the completion of a certain number of cycles.

Application to Water Distribution Systems

In order to apply ACOAs to WDS optimization problems, the graph G = (**D**, **L**, **C**) needs to take the form shown in Fig. 3. One decision point is associated with each pipe. In the example in Fig. 3, there are 5 pipes and hence 5 decisions points (d_1, d_2, …, d_5). At each decision point, there are a number of options, corresponding to the available pipe diameters (ϕ_j). In the example shown in Fig. 3, there are 8 possible pipe diameters (ϕ_1, ϕ_2, …, ϕ_8), corresponding to 8 choices at each of the 5 decision points ($l_{i(1)}$, $l_{i(2)}$, …, $l_{i(8)}$, i = 1, 2, …, 5). The costs corresponding to each of these choices ($c_{i(1)}$, $c_{i(2)}$, …, $c_{i(8)}$, i = 1, 2, …, 5) are the product of the unit cost per meter length of each of the pipe diameters ($UC_{\phi j}$) and the length of the pipe segment under consideration (LE_i). The total pipe cost associated with a particular trial solution ($f_{PI}(\varphi)$) is then given by:

$$f_{PI}(\varphi) = \sum_{i=1}^{n} UC_{\phi\ j} * LE_i \qquad (5)$$

Another major difference between the pipe optimization problem as formulated here and other combinatorial optimization problems to which ACOAs have been applied is the way in which they are constrained. For example, in the traveling salesman problem, the only constraint is that each city must be visited once only and that the finishing point must be the same as the starting point. In this situation, tabu lists are used to ensure that only feasible solutions are generated (see Dorigo et al. 1996). However, the constraints that need to be satisfied in the optimal design of WDS are of a different nature. The feasibility of a particular trial solution (e.g. whether minimum pressure constraints have been satisfied) can only be assessed after it has been constructed in its entirety, and consequently, the constraints cannot be taken into account explicitly during the construction of trial solutions. The approach taken in this research to deal with this problem is to modify Eq. (4) in order to give negative reinforcement to pipe diameter options that result in solutions that violate the pressure constraints:

$$\Delta\tau_{i(j)}^{h} = \begin{cases} \dfrac{R}{f_{PI}(\varphi)^h} - P_{pher} * \Delta H_{max} & \text{if the } h^{th} \text{ ant chooses option } l_{i(j)} \text{ at cycle } k \\ 0 \ \text{otherwise} \end{cases} \qquad (6)$$

where P_{pher} is a pheromone penalty factor and ΔH_{max} is the maximum pressure deficit in the WDS, which is obtained using a hydraulic solver for each trial solution (i.e. combination of pipes) generated.

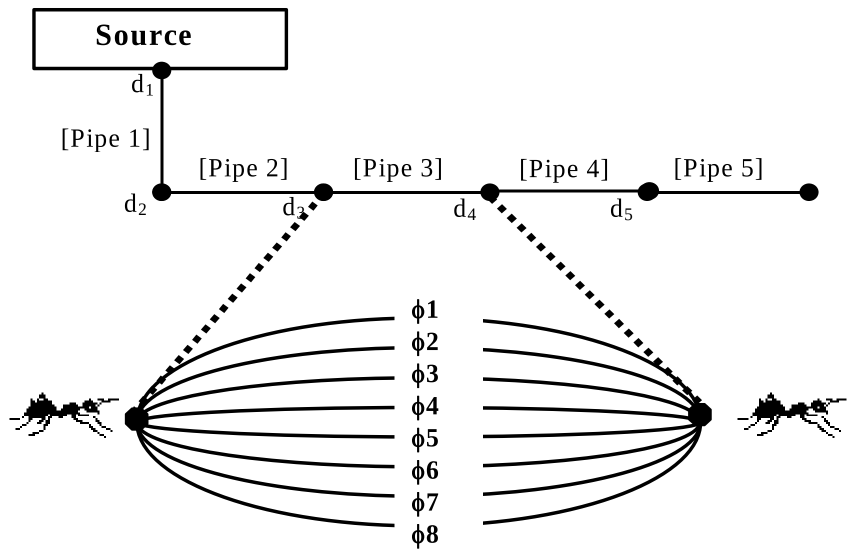
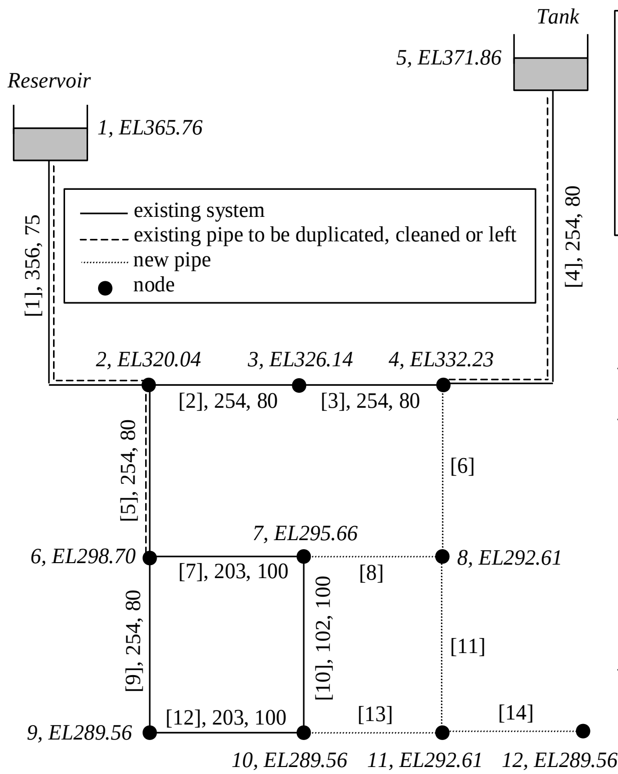


Figure 3. Representation of WDS optimization problems in terms of a graph.

Case Study

The case study used to assess the applicability of ACOAs to the optimal design of WDS is the 14-pipe network expansion problem to which Simpson et al. (1994) applied GAs. This provides a direct comparison between the performance of ACOAs and GAs. The case study network has a total of 14 pipes supplied by two water sources – a tank and a reservoir (Fig. 4). Both water supplies are assumed to be at a constant elevation. Eight pipes are existing while there are five new pipes to be sized (with at least the minimum size pipe). Three of the existing pipes may be duplicated with a new pipe in parallel (not necessarily of the same diameter as the existing pipe). Three water demand loading cases need to be considered – a peak hour case and two fire loading cases. A full description of the case study can be found in Simpson et al. (1994).

The software code required to implement the ACOA was developed in Fortran 77. This code was linked with the code for the hydraulic solver WADISO (Gessler and Walski 1985) in order to calculate the maximum pressure deficit for each trial solution generated, and hence the total cost of each trial solution in accordance with Eq. 7 and the required pheromone updates in accordance with Eq. 8. Some preliminary trials on the sensitivity of the results obtained to the parameters that control the ACOA were conducted. As a result of these trials, the following model parameters were adopted: $\alpha = 5$, $\beta = 3.5$, $\rho = 0.8$, m = 100, R = 200,000, and $P_{pher} = 0.005$. To be consistent with Simpson et al. (1994), the number of trial solutions generated was taken as 50,000. Consequently, the number of cycles used was 500, as m = 100. The analysis was repeated for 10 different random number seeds, which is in agreement with Simpson et al. (1994).



[1], 356, 75 [pipe number], diameter (mm), present day Hazen-Williams roughness coefficient (*C*)

2, EL320.04 node number, node elevation (m)

Note: 1. All pipe lengths are 1609m, except pipe [1] and pipe [4] which are 4828m and 6437m, respectively.
2. *C* is 120 for new and cleaned pipes.

	Demand Pattern 1		**Demand Pattern 2**		**Demand Pattern 3**	
Node	Q (L/s)	H_{min} (m)	Q (L/s)	H_{min} (m)	Q (L/s)	H_{min} (m)
2	12.62	28.18	12.62	14.09	12.62	14.09
3	12.62	17.61	12.62	14.09	12.62	14.09
4	0	17.61	0	14.09	0	14.09
6	18.93	35.22	18.93	14.09	18.93	14.09
7	18.93	35.22	82.03	10.57	18.93	14.09
8	18.93	35.22	18.93	14.09	18.93	14.09
9	12.62	35.22	12.62	14.09	12.62	14.09
10	18.93	35.22	18.93	14.09	18.93	14.09
11	18.93	35.22	18.93	14.09	18.93	14.09
12	12.62	35.22	12.62	14.09	50.48	10.57

Figure 4. Case study – 14-pipe network expansion problem (after Simpson et al. 1994).

Results and Discussion

A comparison of the results obtained using GAs and ACOAs is given in Table 1. It can be seen that the GA was able to find the global optimum solution for 8 of the 10 random number seeds tried, compared with 7 out of 10 when the ACOA was used. It should be noted that the global optimum solution of $1.750 million was obtained by Simpson et al. (1994) using complete enumeration. On average, the least cost solution found by the GA was $1.756 million (+ 0.34%), which is slightly less than the optimal solution of $1.769 million (+ 1.09%) obtained using the ACOA. However, the ACOA was able to reach the global optimum more quickly than the GA. On average, the ACOA found the optimum solution within 12,455 function evaluations (ranging from 2,014 to 32,791), whereas the GA required 20,790 function calls to the hydraulic solver (ranging from 10,080 to 43,740). These results suggest that ACOAs are a suitable technique for the optimal design of WDS.

The results in Table 1 represent a fair comparison between GAs and ACOAs, as relatively basic forms of the respective algorithms were used in both studies. However, better results have already been obtained when an improved GA was applied to the same case study. By using tournament instead of proportionate selection, Simpson and Goldberg (1994) were able to find the global optimum solution regardless of the starting position in search space. In addition, the average number of evaluations required to reach the global optimum ranged from 2,900 to 8,700, depending on the model parameters used. Significant improvements in performance have also been observed for several benchmark optimization problems by incorporating algorithmic changes into ACOAs (see Stützle and Hoos 2000). The effect of such improvements on the performance of ACOAs in relation to their ability to find the least cost design of WDS should be the focus of future research efforts.

Table 1: Results obtained using GAs and ACOAs

	GA (Simpson et al. 1994)			ACOA		
Run Number	Least Cost ($ million)	Difference from Optimum (%)	Evaluation No. First Achieved	Least Cost ($ million)	Difference from Optimum (%)	Evaluation No. First Achieved
1	1.773	1.3	29,070	1.750	0	2,014
2	1.750	0	10,350	1.750	0	6,538
3	1.750	0	43,740	1.813	3.6	29,978
4	1.812	3.5	40,860	1.750	0	32,791
5	1.750	0	17,190	1.750	0	11,371
6	1.750	0	11,070	1.813	3.6	7,188
7	1.750	0	10,080	1.750	0	5,016
8	1.750	0	41,490	1.750	0	18,312
9	1.750	0	12,510	1.750	0	11,146
10	1.750	0	19,890	1.812	3.5	21,042
		AVERAGE*	20,790		AVERAGE*	12,455

*Excludes non-global optimal solutions

Conclusions and Recommendations

A formulation for applying ACOAs to the optimal design of WDS has been presented in this paper. In order to apply ACOAs to the optimal design of WDS, a decision point was placed on each potential pipe in the system. At each decision point, the available choices corresponded to the available pipe diameters (or pipe rehabilitation options). Pheromone intensities and heuristic values were associated with each of these choices. The heuristic value was taken as the inverse of the cost of each choice. Pheromone intensities were modified in a way that favors choices that result in smaller total network costs. Penalty costs were applied to solutions that violate the required pressure constraints. In addition, the pheromone levels associated with choices that result in systems that violate the required pressure constraints were decreased.

Based on the results obtained in this research, in which ACOAs were applied to the 14-pipe network expansion problem used by Simpson et al. (1994), ACOAs appear to be a suitable method for the optimal design of WDS. The performance of the basic ACOA used in this research was comparable with that of the basic GA used by Simpson et al. (1994) for the same problem. The GA performed slightly better in terms of finding the optimal solution from different starting positions in the search space, whereas the ACOA performed better in terms of the number of evaluations needed to reach the optimum.

Future research efforts in this field should focus on (i) the application of ACOAs to more complex WDS optimization problems, (ii) investigations into of the sensitivity of ACOAs to the parameters that control their operation (e.g. α, β), with the aim of developing guidelines for users, (iii) the development of formulations that enable ACOAs to be applied to other water resources optimization problems, (iv) the determination of which problems can be efficiently solved by ACOAs

(Stützle and Hoos 2000) and under what circumstances ACOAs should be used in preference to GAs and vice versa, and (v) the evaluation of the effectiveness of algorithmic improvements to ACOAs (e.g. Stützle and Hoos 2000) for water resources problems.

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