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EXPERIMENTAL OBSERVATION AND ANALYSIS OF INVERSE TRANSIENTS FOR PIPELINE LEAK DETECTION

John P. Vítkovský[1], Martin F. Lambert[2], Angus R. Simpson[3], M. A.S.C.E., and James A. Liggett[4]

1 Graduate Hydrologist, Water Assessment Group, Department of Natural Resources and Water, Queensland, Indooroopilly QLD 4068, Australia. Email: john.vitkovsky@nrm.qld.gov.au

2 Associate Professor, School of Civil and Environmental Engineering, University of Adelaide, Adelaide SA 5005, Australia. (Corresponding author). Tel: +61 8 8303 5414; Fax: +61 8 8303 4359; Email: mlambert@civeng.adelaide.edu.au

3 Professor, School of Civil and Environmental Engineering, University of Adelaide, Adelaide SA 5005, Australia. Email: asimpson@civeng.adelaide.edu.au

4 Professor Emeritus, School of Civil and Environmental Engineering, Cornell University, Ithaca, NY 14853-3501, USA. Email: jal8@cornell.edu

ABSTRACT

Fluid transients result in a substantial amount of data as pressure waves propagate throughout pipes. A new generation of leak detection and pipe roughness calibration techniques has arisen to exploit those data. Using the interactions of transient waves with leaks, the detection, location and quantification of leakage using a combination of transient analysis and inverse mathematics is possible using inverse transient analysis (ITA). This paper presents further development of ITA and experimental observations for leak detection in a laboratory pipeline. The effects of data and model error on ITA results have been explored including strategies to minimize their effects using model error compensation techniques and ITA implementation approaches. The shape of the transient is important for successful application of ITA. A rapid input transient (which may be of small magnitude) contains maximum system response information, thus improving the uniqueness and quality of the ITA solution. The effect of using head measurements as boundary conditions for ITA has been shown to significantly reduce sensitivity, making both detection and quantification problematic. Model parsimony is used to limit the number of unknown leak candidates in ITA, thus reducing the minimization problem complexity. Experimental observations in a laboratory pipeline confirm the analysis and illustrate successful detection and quantification of both single and multiple leaks.

CE DATABASE KEYWORDS

Pipes; Transients; Unsteady Flow; Error Analysis, Leaks, Inverse Analysis

INTRODUCTION

Throughout the world, pipelines efficiently transport fluids. When a leak occurs in such a pipeline, there is an associated loss of product and increased pumping and treatments costs arising from the additional fluid needed to fill the loss. For fluids such as oil and gas, detrimental environmental impacts are potential consequences of any loss. Under certain transient conditions leaks may allow passage of contaminants into the pipeline causing concerns about purity and, in some cases, health. For these reasons, increasing both the accuracy and efficiency of detection and location of leaks is essential.

Wang *et al*. (2001) presented a review of many alternative leak detection techniques; however, the focus of this paper is on inverse transient analysis (ITA) for leak detection in pipelines. This paper examines ITA in greater detail including the general analysis of inverse problems, types of error and their effect (in particular model error), and strategies to deal with some error types. Additionally, the importance of transient shape for successful ITA application, use of measured boundary conditions for simulation, and a model parsimony approach are investigated. Finally, experimental observations in a laboratory pipeline illustrate successful single and multiple leak detection.

INVERSE TRANSIENT ANALYSIS

Liggett and Chen (1994) proposed calibration and leak detection in pipe networks using fluid transients. Their methodology used pressure head measurements made during a transient event. Using inverse mathematics the pipe parameters—leak areas, friction coefficients, wave speeds—were adjusted to match observed pressures in the numerical model. The solution parameter set was determined by minimizing an objective function that represents the match between the numerically modeled heads and measured heads. The objective function is derived from maximum likelihood estimators (Press *et al*. 1992) giving rise to the least-squares criterion,

$$E = \sum_{i=1}^{M} \left(H_i^m - H_i \right)^2 \tag{1}$$

where E = objective function, H_i^m = measured head, H_i = numerically modeled head and M = total number of measurements. The most important assumption is that the model is a good representation of the system behavior. The residuals of the fit should also be normally distributed, of zero mean, of stationary variance, and not correlated. Violation of any of these assumptions will result in ITA fits that are not of maximum likelihood.

Numerical aspects of ITA that have been studied are algorithmic efficiency (Nash and Karney 1999, Vítkovský 2001, and Vítkovský *et al*. 2002), minimization algorithms (Vítkovský *et al*. 2000, Kapelan *et al*. 2003), optimal sampling designs (Vítkovský *et al*. 2003), and prior information use (Kapelan *et al*. 2001). Tang *et al*. (2000), Vítkovský *et al*. (2001), Covas and Ramos (2001), Covas *et al*. (2003) performed experimental validations of ITA for leak detection in pipelines. Experimental validations of ITA in pipe networks have been undertaken by Covas and Ramos (2001) and Wang (2002). Field testing of ITA for leak detection has been performed in a trunk main by Stephens *et al*. (2002), Stephens *et al*. (2004), Covas *et al*. (2004) and Stephens *et al*. (2005).

APPLICATION OF INVERSE ANALYSIS

For any inverse calculation the basic properties of the particular inverse problem should be considered. A set of questions should be asked of any inverse analysis result. The inverse problem should be well-posed, meaning that a solution (a) exists, (b) is unique and (c) depends continuously on its data (Tarantola 1987). Ill-posed cases are not considered herein, although in pipe and network flows, as in other fields, useful information can be obtained from such cases (through singular value decomposition; see Press *et al*. 1992) even though a complete solution is not possible.

A common problem in ITA occurs when the transient event contains little information. In this situation the effect of a leak in one position can resemble the effect of a leak in a different position along the pipeline. In the presence of noisy data, the result of ITA is typically a non-unique solution that can be identified by decomposing the curvature matrix using singular value decomposition (SVD) to determine singular, or near-singular, values. The case of near-singular values represents an ill-conditioned problem and is associated with a large difference in the sensitivities between parameters, and it results in a set of simultaneous equations with a high condition number (e.g., 10^6 for single precision calculations). Kapelan *et al*. (2001) better conditioned the inverse problem for ITA resulting in a more clearly defined minimum in the objective function. For a number of parameters, a poorly conditioned problem or a non-unique solution causes the inverse problem to be unsolvable unless techniques like regularization are employed (Press *et al*. 1992), whereby supplementary data or relationships can render the inverse problem solvable. In many inverse applications, larger amounts of information can be used to reduce the effect of measurement noise, and limiting the parameter search space can make the problem solvable, techniques that are used in magnetic resonance imaging (MRI) (Kak and Slaney 1988). Another approach is to better locate the measurement stations or improve the transient event characteristics (the sampling design problem—Vítkovský *et al*. 2003).

In certain situations pressure waves—for example those that have not reached the suspected leak location and returned to a measurement site—do not carry useful information. In this situation the measured behavior has no dependence on leak parameters, a situation that can be identified by zero-valued columns (and corresponding rows) in the curvature matrix. In some cases, a portion of the parameter set can be determinable while the remainder is indeterminate (a mixed-determinate problem). Where some parameters are determinable while others are not, SVD can be used to identify those parameters that are determinable by eliminating the parameters with singular and near-singular values from the inverse analysis, thus making the problem solvable. Alternatively, Kuczera (1983a) used an approach whereby initial information is incorporated in the parameters' prior distributions, then a Bayesian technique is used to calculate the parameters' posterior distributions given the measured data. The result is that the solutions for those parameters that are determinate are improved.

A final question, which is approached by an analysis of the confidence of each solution parameter, is the plausibility of the inverse solution. If a normal error distribution is assumed in the solution parameters and the search space is approximately linear in the neighborhood of the solution, then computationally efficient methods, such as the first-order second-moment (FOSM) method (Press *et al*. 1992), can be used. In general the true error distributions of the solution parameters are difficult to parameterize easily and other techniques, such as the efficient Monte Carlo Markov Chain sampling methods (like the Metropolis algorithm) must be employed (Kavetski *et al*. 2002). If the parameter error is similar in magnitude to the value of a parameter, then that parameter has been poorly determined and the ITA result is likely to have a large error.

ERROR IN INVERSE TRANSIENT ANALYSIS

Sometimes, even though an apparently good fit may have been found using the least-squares objective function, the solution parameters are poorly determined. This situation can be due to a number of sources, such as violation of assumptions made in the derivation of the objective function and the presence of different types of error in the data or model. A classification has been developed of three main and eight sub-types of error in inverse analysis, which are:

I. Data Error:

- Ia. Random Data Error (e.g., random noise in measured data).
- Ib. Systematic Data Error (e.g., poorly calibrated measuring instruments).

II. Model Input Error:

- IIa. Parameter Input Error (e.g., poorly defined model parameters).
- IIb. Random Input Error (e.g., random noise in boundary condition data).
- IIc. Systematic Input Error (e.g., poorly calibrated/measured boundary condition data).

III. Model Structure Error:

- IIIa. Inconsistent Model Error (e.g., incorrectly modeled process).
- IIIb. Incomplete Model Error (e.g., unaccounted for model process).
- IIIc. Numerical Model Error (e.g., numerical algorithm error).

Any inverse solution should be tested for the presence of these errors. Typically the residuals of the fit or unrealistic solution parameters can be used to diagnose different types of error. The following sections detail the different error types.

Data Error

There has been considerable research into the diagnosis and solution to Type Ia errors, most of which are based on the violation of assumptions made when deriving the maximum likelihood objective function. Type Ia errors are identified by: (i) residuals that are not normally distributed, (ii) residuals with non-zero mean, (iii) non-stationary residuals, and (iv) correlated residuals. The diagnosis of Type Ia errors and their solution is relatively straightforward and outlined in Kuczera (1983a).

Type Ib errors can be difficult to identify and commonly involve incorrectly calibrated measurement devices resulting in a common mode error or systematic under- or over-estimation of data values. Additionally, interference from electrical and mechanical sources, such as from AC current and pump vibration, may cause error. Solutions to this type of error are the use of reliable measurement equipment, careful calibration of measurement devices and filtering undesirable signal content. The rest of this paper assumes that the measured data contain no systematic error (Type Ib error).

Model Input and Model Structure Errors

Model input and model structure errors introduce auto-correlated errors into the residuals, thus violating the assumption that the numerical model reasonably approximates reality and those assumptions made in the derivation of the standard maximum likelihood function (Eq. 1). Kavetski *et al*. (2002) presented the effect of, and a solution to, a systematic error in input data (Type IIc error) for rainfall-runoff model calibration. An effect of model error on the inverse problem solution is to produce data-inconsistent parameter estimates (different parameter estimates are determined for different data sets) and unrealistic parameter values and parameter confidence estimates. Kuczera (1982, 1983b) investigated the effect of different data sets yielding different parameter estimates in rainfall-runoff model calibration. However, the problem is complex in inverse transient analysis in that the parameter estimates may change given different data length and location. The symptoms of model input and structure errors can be used as a diagnostic for their identification. These symptoms are (a) parameter estimates outside of a reasonable range, (b) structure in the residuals that relates to model processes, and (c) parameter estimates that vary with data length and data location (data-inconsistent results). In the case of unsteady pipe flow, significant peaks or structure in the partial auto-correlation function (PACF) of the residuals, especially with lags equal to multiples of L/a (where L is pipe length, a is wave speed and where L/a is a characteristic time period relating to the wave travel time in the pipe), are symptomatic of model error.

Tang *et al*. (2000), Covas and Ramos (2001) and Covas *et al*. (2001) provided three examples of Type IIIb model error in ITA where the viscoelastic behavior of the pipe material was not accounted for. In the first case, the transient model performance was poor and only allowed very short simulation periods before the model and measured data diverged. In the second and third cases, the instantaneous acceleration-based unsteady friction model (Bergant *et al*. 2001) was used to attempt compensation for the missing viscoelastic model component with the result being an improved, but still a poor, match between model and measured data. Covas *et al*. (2004) showed an example of a Type IIIc error where the Trikha unsteady friction model (Bergant *et al*. (2001)), which is known to produce numerical dispersion and attenuation error, was used. However, in that case viscoelastic damping dominated unsteady friction damping; thus, the numerical error produced by the Trikha model was relatively small and not overly detrimental to ITA.

In terms of ITA, it is impossible to know every parameter of the model exactly for the whole pipeline system (Type II errors). Additionally, there are often unknown local defects like air pockets, blockages, vibration, or construction changes that are not recorded (Type IIIb error). It is also unrealistic to model the pipeline system down to the smallest components, such as water connections to individual houses (Type IIIb error). For these reasons the modeler has to accept some degree of unavoidable model error. The following section outlines some approaches to minimize the effect of model errors.

Dealing with Model Error in Inverse Transient Analysis

Once some form of model error has been identified, complete correction is difficult (if not impossible in most cases). However, there are arguably two last-resort strategies that can be employed to attempt correction: (1) how the model deficiency is addressed, and (2) the application of inverse analysis under model error.

Two broad approaches may be used to address model deficiencies such as uncertainties in model parameters (Type IIa error), a missing process or poorly modeled process (Type III errors). The first approach is parameter set expansion (PSE) where certain parameters in the existing model are included in the ITA parameter set for fitting. The second approach is model error compensation (MEC) in which an artificial model (with parameters) is introduced to compensate for model deficiencies. Both approaches can cause problems due to the choice of an inappropriate MEC model or inappropriate PSE parameters interacting unfavorably with ITA parameters. The choice of PSE parameters or MEC model should be logically or physically based. Additionally, adding too many parameters to the ITA parameter set could result in over-fitting, a process that apparently improves the model because correlation coefficients are improved but the predictive ability of the model is actually degraded.

Two ITA implementation approaches under model error, after addressing the model deficiency, are pre-calibration (PC) and concurrent calibration (CC). The PC approach splits ITA into two separate tasks. The first task uses data from a known state (e.g., a leak-free state, or a historical state) to fit the parameters of the transient model. Alternatively, the unknown state data could be subtracted from the known state data and ITA applied to the difference of the two data sets. The second task uses data from the current state for the fitting of leak parameters using the now calibrated model. The PC approach can only detect changes from the known state and produces better results if the known state and current state transient events and system configurations are similar. The CC approach fits both the parameters of the transient model and the leak parameters simultaneously from the one set of measured data. The CC approach has the advantage of requiring only one set of measured data. However, care must be taken to ensure that there is little correlation between transient model parameters and leak parameters.

Wang (2002) employed a MEC-PC approach whereby a transient model for an experimental pipe network was pre-calibrated using the k_A & k_P unsteady friction model (Vítkovský 2001) to account for suspected model deficiencies from unsteady mixing, trapped air, and rubber materials used in flanged connections. Leak detection was achieved using an identical transient event as used in the pre-calibration, although it is uncertain whether the MEC model was satisfactory. Covas *et al*. (2003) employed a PSE-CC approach whereby the parameters of the viscoelastic model component were included in the ITA leak parameter set. In this case, unsteady friction and defects in the model were incorporated into the viscoelastic model.

The result was the smearing of the detected leak location, which may have been caused by lumping all model deficiencies into the viscoelastic model. Additionally, the correlation between model parameters and leak parameters was not analyzed.

EXPERIMENTAL OBSERVATIONS IN INVERSE TRANSIENT ANALYSIS

All analyses performed in this paper use experimental data from a laboratory pipeline. The following sections outline details of the experimental apparatus and the forward transient model as well as observations and improvements to ITA.

Experimental Apparatus

The experimental apparatus is a single pipeline located in the Robin Hydraulics Laboratory in the School of Civil and Environmental Engineering at the University of Adelaide (Bergant and Simpson 1995). Figure 1 shows a schematic of the pipeline. It is comprised of a 37.2 m sloping copper pipe with an inside diameter of 22.1 mm and a wall thickness of 1.6 mm. The roughness height was estimated as 0.0015 mm—a relative roughness of 7×10^{-5}—making the pipe hydraulically smooth given the low Reynolds numbers of flows that were tested.

The pipeline connects two pressurized tanks that are computer controlled using an air compressor that can supply a maximum pressure of 700 kPa. A quarter-turn ball valve is located in the pipeline next to tank 1 that is used to generate transient events. Pressure in the pipeline is measured at five equidistant locations at nodes 1, 5, 9, 13 and 17. The pressure transducers exhibit minimal noise (type Ia error). In addition, the valve position is measured using a potentiometer connected to the valve handle. The pressure regulation units at either end can be set to provide a pressure difference for the initialization of flow. Measuring the rate of change in tank levels and then converting to the flow rate into or out of the tank provides the steady-state velocity. Leaks can be positioned in the pipeline at nodes 5, 9 and 13.

Three different valve closure times are considered in the experimental tests. The valve closure speeds were t_c = 0.07, 0.70 and 1.40 s corresponding to $2.5\times L/a$, $25\times L/a$ and $50\times L/a$, respectively. The valve closure times relate to the full closure of the valve handle. The effective closure times for the ball valve are less than these values due to the geometry and action of the valve (most of the head loss through the valve occurs in the last quarter-turn of the stem). The details of the system properties for each valve closure speed are shown in Table 1. Larger initial velocities were used for the slower valve closure tests to increase the size of the transient event.

Experimental ITA results consider three leak sizes of orifice diameters 1.0, 1.5 and 2.0 mm with corresponding lumped leak coefficients calibrated as 5.0×10^{-7}, 1.1×10^{-6} and 1.7×10^{-6} m^2 respectively located at node 5. Also, a multiple leak test with two 1.0 mm leaks with calibrated lumped leak coefficients as 5.0×10^{-7} and 7.1×10^{-7} m^2 were located at nodes 5 and 17. Table 2 shows a summary of the properties of the leaks.

Forward and Inverse Transient Modeling

The transient event is modeled using the measured valve position at node 1 as the forcing input and the measured (constant) tank head at node 17. Random errors in the measured valve position dictated that these data be filtered to prevent contamination of the transient simulation results (type IIb error). A Savitsky-Golay smoothing filter (Press *et al*. 1992) was used. The effect of valve position noise was greater for the slower transient tests because the valve position was located in the near-closed region of the valve closure profile where the effect of noise on the transient simulation was the greatest.

The pipeline was divided into 32 computational reaches and a method of characteristics (MOC) simulation was performed on the resulting diamond grid. For each inverse transient analysis, 15 leak candidates were assigned at nodes along the pipeline. Additionally, global wave speed was treated as an unknown in the inverse problem, as was a multiplier for the unsteady friction. No pre-calibration of the model was performed (i.e., no dealing with model error using PC-type approaches). The leak parameters were sought in logarithmic space, allowing the minimization algorithm an efficient means to search for leaks of vastly different scale and, additionally, to limit the leak parameters to positive values. The shuffled complex evolution (SCE) algorithm was used for the minimization of the objective function. The parameter error estimates and parameter correlations were determined using the first-order second-moment (FOSM) method.

The behavior of friction in the experimental pipeline is unsteady dominant (Bergant *et al*. 2001); thus, modeling the unsteady friction effects is crucial. The weighting function-type model (Zielke 1968) was used to model the unsteady friction with the smooth-pipe turbulent flow weighting function from Vardy and Brown (2003).

A small amount of model error will exist for a number of reasons. Some of those reasons are:

1. The pipeline, although anchored at every meter, does vibrate to a small degree, thereby transmitting energy to the pipeline restraints that is lost to the flow (fluid-structure interaction). This effect is difficult to model and is neglected (type IIIb error).
2. The unsteady friction model is only approximate (type IIIa error).
3. Both steady and unsteady minor losses at the entrance and exit to the tanks are not modeled. This effect is expected to be minor and not modeled (type IIIb error).
4. The leaks in the pipelines do not exactly behave as an ideal orifice, i.e., approximately $Q \propto \sqrt{\Delta H}$ (type IIIa error).
5. The brass blocks used to contain the leaks and pressure transducers have a different impedance to the pipeline that cause small reflections. This effect is expected to be minor and not modeled (type IIIb error).
6. Other aspects (e.g., non-uniform velocity distribution, energy transmission through the pipe walls) of the process are not modeled by the water hammer equations (type IIIc error).

Impact of Transient Shape for Parameter Estimation

An important issue for the application of ITA concerns the shape of the transient used for analysis. In practice, there is a limit to the rate of pressure increase that can be generated due to practical constraints of the generating mechanism. In addition, water utilities are less likely to approve the use of overly large rapid pressure changes due to well-founded concerns about pipe bursts for large transients. This section presents an experimental investigation into the effect of transient sharpness on the ITA solution.

The transient is generated by the full closure of the in-line valve next to tank 1 using a steady change in the valve aperture. Three valve closure times are considered for analysis; they are 0.07, 0.70 and 1.40 s corresponding to $2.5 \times L/a$, $25 \times L/a$ and $50 \times L/a$ s. A single leak is located at node 5 of size $C_dA_L = 5.0 \times 10^{-7}$ m^2. Figure 2 shows the experimental and ITA solution pressure responses at node 1 (the differences are barely distinguishable). Note that the dimensionless pressure head, H^*, is used and is equal to the pressure head divided by the Joukowsky pressure head rise based on the initial velocity preceding the transient event. As observed, the pressure responses become smaller and less sharp as the valve closure times increase. Figure 3 shows the solution of the leak parameters for each valve closure speed. Only the rapid valve closure case ($t_c = 2.5 \times L/a$) determined the correct location and size of the leak. The ITA solutions for the two slower valve closure cases ($t_c = 25 \times L/a$ and $50 \times L/a$) were not correct. An analysis of each leak parameter's error (also in Figure 3) shows that the parameter error estimates for the two slower closure times are of similar size or larger than the parameter value, meaning that the slower valve closure results are not statistically well founded and little can be concluded from these results. The bandwidth of information of the slower closures is insufficient to carry the necessary information. Using frequency analysis, Lee (2005) has shown that a sharp transient contains considerably more bandwidth (i.e., information) than a slow transient.

One important property to consider when evaluating the probable success of leak detection is the sensitivity of the head to the leak area. Figure 4 shows the sensitivity of head with respect to the leak size, $\partial H/\partial C_dA_L$, (numerically calculated using measured boundary conditions) in which large magnitude sensitivities indicate likely success at correctly detecting, locating and sizing the leak. As expected, as the time of closure of the valve increases, the head becomes less sensitive to the leak, suggesting that sharper transients are better for leak detection.

Whereas Figure 4 shows the sensitivity with respect to the leak size, there is also sensitivity with respect to the leak location. This sensitivity is difficult to calculate when the transient analysis can only model leaks at discrete locations due to the discretization of the characteristic grid in which leaks can only be simulated at nodal positions. An indicator of estimated parameter confidence is to use the estimated error in the solution for leak coefficients. The error in the leak coefficients can be estimated given the residuals of the ITA fit using a first-order error analysis (Vítkovský 2001). The residuals, which represent the error between the measured and fitted data, are used as a surrogate for an "effective measurement error" in the first order analysis. Additionally, if all of the measurement data have a similar level of error, then the error in the solution parameters can be related to the error in the measurements by

$$\sigma_{C_d A_L} = K\sigma_{H^m} \tag{2}$$

where K = error transmission multiplier, $\sigma_{C_d A_L}$ = standard deviation of the parameter error and σ_{H^m} = standard deviation of the measurement error (see Vítkovský *et al*. 2003). Small values of K for a particular leak parameter correspond to low levels of error in that parameter. Figure 5 shows a plot of K for different leak positions and valve closure times. The parameter error increases markedly as the leak position approaches Tank 2. Also, the error in the parameters increases as the valve closure time increases. For simplicity, the analysis presented in Figure 5 only considers searching for a single leak with a single leak parameter in the application of ITA. This analysis says nothing about the level of correlation exhibited between parameters when applying ITA with a number of leak parameters. A similar error analysis is considered, but with two unknown leak parameters at nodes 2 and 6. Table 3 shows the results of such an analysis that, in this case, includes not only an error transmission multiplier for each parameter, but also a correlation coefficient between the error in each parameter. The correlation coefficient, ρ, between a pair of leak parameters is

$$\rho_{a,b} = \frac{\sigma_{a,b}}{\sigma_a \sigma_b} \tag{3}$$

where $\sigma_{a,b}$ = covariance of the errors in parameters *a* and *b*, and σ_a ,σ_b = standard deviations of the error in parameters *a* and *b*. If the parameters are highly correlated then a unique solution is not possible. Table 3 shows that as the valve closure time increases, the two parameters become more correlated (the correlation coefficient approaches ±1). As the valve closure time is decreased, the transient response becomes more like a damped sinusoid. As the response becomes more sinusoidal, it approaches a damped single-frequency disturbance (at the natural frequency of the pipeline). Analysis by Wang *et al*. (2002) shows that if damping information alone from a single-frequency event is used to detect and locate a leak, then no unique solution exists for both the size and location of the leak. In this respect, as the valve-closure time increases, less information is contained in the pressure response leading towards a non-unique solution as indicated in Table 3 by the extremely high correlation between the two leak parameters for the slowest valve closure considered. Thus, the observation that a fast transient provides a more accurate analysis is based firmly on both empirical observation and theoretical analysis.

Use of Measured Pressure Boundary Conditions

In many practical applications of transient modeling, and hence the application ITA, the accurate specification of boundary conditions for a transient model can be difficult. One seemingly elegant method to specify a boundary condition is to measure the pressure close to a boundary and then use that measured pressure as the boundary condition. The following example considers both independent and measured boundary condition use in ITA.

Consider the detection of a leak in the pipeline shown in Figure 1 with a fast in-line valve closure ($t_c = 2.5 \times L/a$) initiating the transient event. The leak of size $C_d A_L = 5.0 \times 10^{-7}$ m^2 is located at node 5. Two types of boundary condition specification are considered: (1) a valve with measured closure profile, the "independent" boundary condition, and (2) the head measured next to the valve, the "measured" boundary condition. For an application of ITA in a leaking system, the measured boundary condition data include the effects of the leak; they are not independent of the ITA leak coefficients. The measured boundary condition data were the measured pressure head at the valve from the experimental apparatus. Two cases are considered, being modeling with a leak at node 5 and modeling with no leak. Figure 6(a) shows the results when modeling with a leak at node 5 for the two boundary condition types. There is little difference between the two pressure responses (which are nearly identical to the experimental measured trace). In an ITA analysis a first guess may be that there are no leaks in the pipe. Figure 6(b) shows the pressure response at node 9 using both independent and measured boundary conditions. Although both models contain "no leaks," the pressure responses from each boundary-condition case are decidedly different. Figure 6(c) shows the sensitivity in the modeled pressure response to a leak using both independent and measured boundary conditions. The measured boundary condition results in less sensitivity by almost an order of magnitude. Figure 7 shows the solution ITA leak coefficients when using both independent and measured boundary conditions. The leak is successfully located using both types of boundary condition. Although ITA should still detect leaks when using measured pressure boundary conditions, leak detection under the error sources previously discussed plus low sensitivity results in low-confidence solutions. From an intuitive standpoint, the boundary conditions used for ITA should be independent of the parameters to be determined by ITA. Finally, for successful application of ITA the sensitivity of the head with respect to the parameters must be maximized; therefore, independent boundary conditions (such as a valve-closure profile) are used herein.

An Auto-Regressive Model Error Compensation Approach

Ideally, if the source of the model error can be determined, then remediation of the model is possible. However, in many cases the model error source is either unknown or known and cannot be realistically addressed. Model error is difficult to correct without knowing the exact origin and details of the error. Ultimately, if model error is allowed to persist, the ITA results are adversely affected no matter what correction is used. Different modes of model error lend themselves to certain model error compensation (MEC) strategies through the introduction of extra (latent) variables or sub-models. The solution of the partial differential equations that describe unsteady pipe flow can be expressed by an auto-regressive (AR) time series model of second order with lags based on the resonant frequency of the system (following from the free-vibrational solution, Wylie and Streeter 1993). As a result, it is reasonable that the model error may be approximated using an AR process of lag $n \times 2L/a$, where $n = 1, 2, 3, \ldots$ (i.e., the harmonics). For a single pipeline modeled using the method of characteristics with a diamond grid and measurement at the valve (see Figure 1), the lags correspond to multiples of the number of pipe reaches (Nr) used in the transient model, i.e. lags of $n \times Nr$ time steps. An example of the objective function re-written to include the AR model for the first lag ($n = 1$) is

$$E = \sum_{i=1}^{M} \left[\left(H_i^m - H_i \right) - \phi_{Nr} \left(H_{i-Nr}^m - H_{i-Nr} \right) \right]^2 \tag{4}$$

where ϕ_{Nr} = AR parameter for a lag of Nr time steps. A diagram best explains the effect of this auto-regressive MEC approach. As an example, Figure 8 shows the ITA fit for a pipeline with a leak at node 5 and an (unknown) air pocket at node 11 (pipeline as in Figure 1). However, the air pocket is not incorporated into the numerical model since only leaks are being detected and the presence of the air pocket is assumed unknown. The effect of the air pocket on the head trace is first noticed as an initial model discrepancy due to interaction of the air with the transient wave traveling away from the valve. A secondary model error occurs as the transient wave is reflected off the boundary and interacts with the air pocket when returning to the valve. Finally, the transient wave again traveling away from the valve reflects off the air pocket. These reflections occur at time intervals of $2L/a$. The transient model tries to compensate by modifying the leak size or putting leaks at incorrect (phantom) locations. The MEC approach accounts for the discrepancy between model and data in the next time period ($2L/a$ later) by not allowing the effect of the air pocket to be reinforced (by the use of the AR model). Thus, the model error does not grow. As the parameters of the MEC model are unknown prior to ITA application they must be included in the unknown parameter set (along with leak coefficients, etc.) and solved for concurrently, i.e., a CC-MEC approach.

The model error is mimicked by neglecting unsteady friction in the forward transient model and using experimental data from the pipeline shown in Figure 1. Two cases are considered for the determination of the leak parameter at node 5 using the standard ITA approach with the transient model (i) including unsteady friction and (ii) not including unsteady friction (representing systematic model error). Figure 9 shows the solution ITA leak coefficient versus amount of data used. The leak coefficient derived using ITA with model error changes with different lengths of data (a hallmark of inverse analysis affected by model error), and the ITA solution becomes increasingly incorrect as more data are used in the analysis until there is no information contained in the transient.

Even with the inclusion of unsteady friction in the forward transient model there remains an amount of model error due to factors stated earlier in this paper. The model error is addressed using the auto-regressive model error compensation (MEC) approach with a lag of 32 time steps (equivalent to a lag of $2L/a$, referred to as the AR[32] model). Figure 10 shows the ITA pressure responses for each case, both of which show seemingly good matches to the experimental data. Also shown in Figure 10 are the residual traces (measured minus the modeled data) for the same ITA fits, in which an amount of structural error is observed. The PACFs for both cases are shown in Figure 11, which shows that there is high degree of structure left in the residuals for the case in which the data contain model error (and thus the maximum likelihood derivation assumption is violated). This violation results in a solution parameter value that is not of the maximum likelihood and is incorrect. Figure 12 shows the leak parameter estimate after application of ITA, showing that the MEC approach (the AR[32] model) has an improved solution. As a confirmation of the appropriateness of the AR[32] model, the residuals and the PACF of the fit show less structure and are of less magnitude, thus, do not violate the maximum likelihood assumptions. Although not shown here, inverse analysis using AR models with non-harmonic lags resulted in very high correlations between AR model coefficients and leak parameters (ruining the ITA result). Moreover, the addition of higher harmonic lags in the AR model further reduced any structure left in the residuals.

Further experimental ITA results consider three leak sizes of orifice diameters 1.0, 1.5 and 2.0 mm with corresponding lumped leak coefficients calibrated as 5.0×10^{-7}, 1.1×10^{-6} and 1.7×10^{-6} m^2 located at node 5. Additionally, a multiple leak test with two 1.0 mm leaks with calibrated lumped leak coefficients as 5.0×10^{-7} and 7.1×10^{-7} m^2 were located at nodes 5 and

17. Only rapid valve closure speeds (0.07 s or $t_c = 2.5 \times L/a$) were considered. Figure 13 shows that ITA correctly determines both the size and location of the single leak cases and the multiple leak case.

The use of non-harmonic AR model lags interferes with the ITA leak coefficients and is not suited to this problem. This is because other non-harmonic lags are related to the timing of the leak reflections. The use of an auto-regressive model with lags based on harmonics of the pipeline system may be only applicable to the current ITA problem. Other systems, such as networks, have complicated and unevenly spaced harmonics that will generally require a different MEC approach. Finally, MEC approaches are most likely problem dependent and should not be used unless properly tested.

A Model Parsimony Approach

Traditionally, ITA has been applied by defining a large number of leak candidates and then solving for all leak parameters at once. Leak candidates with no actual leak should give near zero leak sizes. However, having a large number of parameters can produce large and complicated search spaces. Additionally, the greater the number of parameters, the harder the inverse problem becomes, increasing the computational time required for a solution. In essence, too many parameters can almost guarantee a fit to any data even though the predictive value of the fit is nil. Also, a high number of parameters promote high correlation between those parameters, potentially making the inverse problem ill-conditioned or even ill-posed. Generally, there are vastly fewer actual leaks than there are leak candidates. The following approach uses this fact to simplify and speed up ITA application based on a model parsimony argument.

The model parsimony approach assumes that a lower number of leak parameters is more likely to lead to the correct solution than a large number. The procedure is outlined schematically in Figure 14 and begins with one leak parameter tested at all possible leak candidate positions. First, the minimum objective function (E) value is used to identify the most likely single leak location. Then two simultaneous leaks are assumed to exist in the system for all possible combinations of locations. The configuration with the smallest objective function value is stored. The process is continued for larger numbers of simultaneous leaks. For a small number of possible leak candidates or suspected actual leaks, the most efficient searching method is a full enumeration. For a large number of leak candidates some algorithm could supervise the combinatorial search, e.g., a genetic algorithm. However, there are a number of leak candidates, above which there is little additional improvement in the inverse transient fit for the addition of extra leak parameters. A method to limit the number of significant parameters required to adequately model a process (and not over-fit the data) is to use information criteria. A commonly used information criterion is the Akaike's information criterion (AIC) (Shumway and Stoffer, 2000), which is defined as

$$AIC = \ln\left(\sigma_N^2\right) + \frac{2N}{M} \tag{5}$$

where σ_N^2 = variance of the fit (calculated from the residuals), N = number of parameters, and M = number of measurement data points. The AIC penalizes the fit for additional parameters and is at a minimum for an adequate number of parameters. Other information criteria based on Bayesian arguments and corrections to AIC tend to favor slightly lower-numbered parameter models or are more appropriate for higher numbers of sample points. The minimum AIC corresponds to an adequate number of leak parameters in the transient model. Although the AIC was used for determining parsimonious time series models, application to ITA could reduce a large parameter set to a more manageable smaller parameter set.

The model parsimony approach for ITA is applied to experimental data with a 1.0 mm leak located at node 5 and a rapid valve closure generates the transient event (0.07 s or t_c = $2.5\times L/a$). A total of 4.0 s of measured data was used and model error compensation was applied (an AR model with harmonic lags of 32 and 64). Table 4 shows that applying model parsimony gives a minimum Akaike's information criterion (AIC) value for three leak candidates. Consideration of greater numbers of leak candidates than three does not significantly improve the ITA solution. The model parsimony approach gives a comparable solution to the traditional ITA approach (simultaneous search for leaks at all nodes), as compared to Figure 13, with only a small number of leak candidates considered, simplifying the problem and making it more manageable. For combinations of one and two leaks, the leak size is overestimated; however, if more possible leaks are considered the computed leak size approaches the actual leak size. Additionally, the estimated parameter variances for the model parsimony approach are lower than those in Figure 13 when searching for leaks at all nodes simultaneously, suggesting fewer parameters were determined with greater confidence.

CONCLUSIONS

This paper presents experimental observations of inverse transient analysis (ITA) for leak detection in a laboratory pipeline. Both single and multiple leaks were successfully detected, located and sized. A number of considerations that promote better inverse transient performance have been identified and tested in this paper. The performance of ITA was found to improve for more rapid transients, suggesting that devices that can generate such rapid transients (but of small magnitude) should be used. An analysis has been presented that modeling using measured boundary conditions should be discouraged in favor of independent boundary conditions. Finally, a reduction of problem complexity was achieved using a model parsimony approach that favors a lower number of leak candidates making the ITA problem easier.

One disadvantage of ITA is that all boundary conditions, system properties and the transient model must be well defined. This can be an advantage in that people must get to know their system well. In some cases these properties may not be known to a sufficiently high accuracy and could severely affect the success of ITA. This suggests that a greater emphasis should be directed towards analysis of errors and strategies to deal with uncertainties in general. In particular there are many forms of model error, some of which could be included as parameters to be determined by ITA. A model error compensation approach, based on an auto-regressive function, has been presented in this paper to address the effect of model error in ITA.

Given the relative ease and precision that measurements of pressure can be made in transient systems, input error should not be a major component to the overall error. Additionally, random error can be minimized by selectively testing at night or using repeated testing. Model error is the most likely limiting factor in successful field application of ITA. Overall there appears to be a lack of rigorous verification of forward transient analysis in field pipelines and almost no verification in field networks. This knowledge gap could provide a major hurdle for inverse transient analysis. Additionally, the computational burden of applying ITA in a large network, general uncertainties in system properties and non-random environmental noise might also be limiting factors. Importantly, ITA should never be performed without standard inverse mathematics diagnostic checks and ITA results should never be presented without quantification of their uncertainty. Successful application of ITA in field pipelines and pipe networks will require rigorous study of the assumptions made in the forward model and inverse fitting.

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APPENDIX II. NOTATION

A_P = pipe cross sectional area;

a = wave speed;

C_dA_L = lumped leak coefficient;

E = objective function;

H = head;

H_i = numerically modeled head;

H_i^m = measured head;

H^* = normalised head (divided by Joukowsky pressure rise);

L = pipe length;

K	=	error transmission multiplier;
K^*	=	error transmission multiplier for normalised head;
M	=	number of measurements;
N	=	number of parameters;
Nr	=	number of computational reaches;
r	=	residual of model fit to data;
t	=	time;
t_c	=	valve closure time;
σ_a	=	standard deviation of error in parameter a;
$\sigma_{a,b}$	=	covariance of errors between parameters a and b;
$\rho_{a,b}$	=	correlation of errors between parameters a and b;
ϕ_k	=	auto-regressive parameter for k^{th} lag.

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Table 1. Experimental inverse transient analysis test details

System Property	Valve Closure Time (t_c)		
	$2.5\times L/a$	$25\times L/a$	$50\times L/a$
Closure time (s)	0.07	0.70	1.40
Initial Velocity (m/s)	0.143	0.360	0.484
Reynolds Number	3,148	7,901	10,614
Initial Head at Tank 1 (m)	26.0	26.0	26.0

Table 2. Experimental inverse transient analysis leak details

Leak Property	Leak Diameter			
	1.0 mm	1.5 mm	2.0 mm	1.0 mm*
Calibrated C_dA_L (m^2)	5.0×10^{-7}	1.1×10^{-6}	1.7×10^{-6}	7.1×10^{-7}
C_dA_L/A_P (%)	0.13	0.29	0.44	0.18
Steady Orifice Flow (L/s)	0.0113	0.0248	0.0384	0.0158
Steady Orifice Velocity (m/s)	14.4	14.1	12.2	20.1

* Second leak used for multiple leak detection.

Table 3. K^* and ρ for different sharpness events

Property	Valve Closure Time		
	2.5×L/a (s)	25×L/a (s)	50×L/a (s)
K^* for $(C_dA_L)_2$ (m^2)	1.44×10^{-7}	1.46×10^{-6}	1.82×10^{-5}
K^* for $(C_dA_L)_6$ (m^2)	2.02×10^{-7}	1.26×10^{-6}	2.07×10^{-5}
Correlation $\rho_{2,6}$	-0.8827	-0.9919	-0.9992

Table 4. ITA of experimental results using model parsimony approach (1.0 mm leak at node 5, t_c = 2.5×L/a)

Number of Leak Candidates	*AIC**	Leaking Node	Leak Parameter C_dA_L (×10^6 m^2)**
Actual	-	5	*0.5*
1	–4.2933	5	*0.606 (0.007)*
2	–4.3531	5	*0.600 (0.007)*
		15	0.123 (0.018)
3	**–4.3749**	4	0.064 (0.027)
		5	*0.533 (0.030)*
		16	0.126 (0.044)
4	–4.3747	3	0.011 (0.031)
		4	0.046 (0.057)
		5	*0.540 (0.035)*
		16	0.121 (0.046)

Estimated standard deviation of leak parameter error in brackets.
*Minimum *AIC* solution in bold. **Maximum size of leak in italics.

LIST OF FIGURES

[Note that all figures require reduction and were envisaged to exactly fit into one column of the JWRPM.]

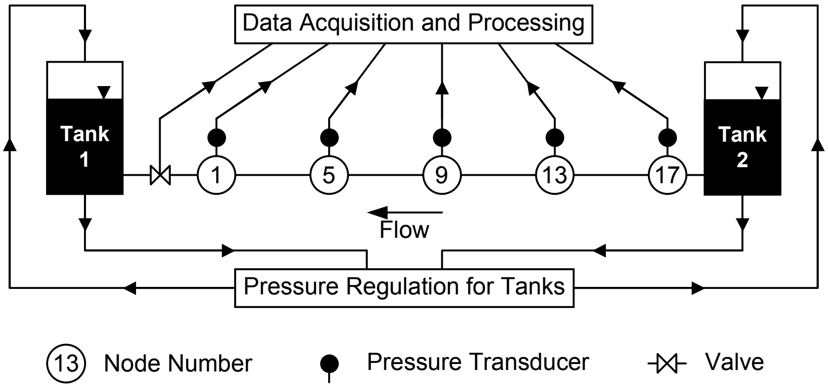

Figure 1. Experimental pipeline apparatus.

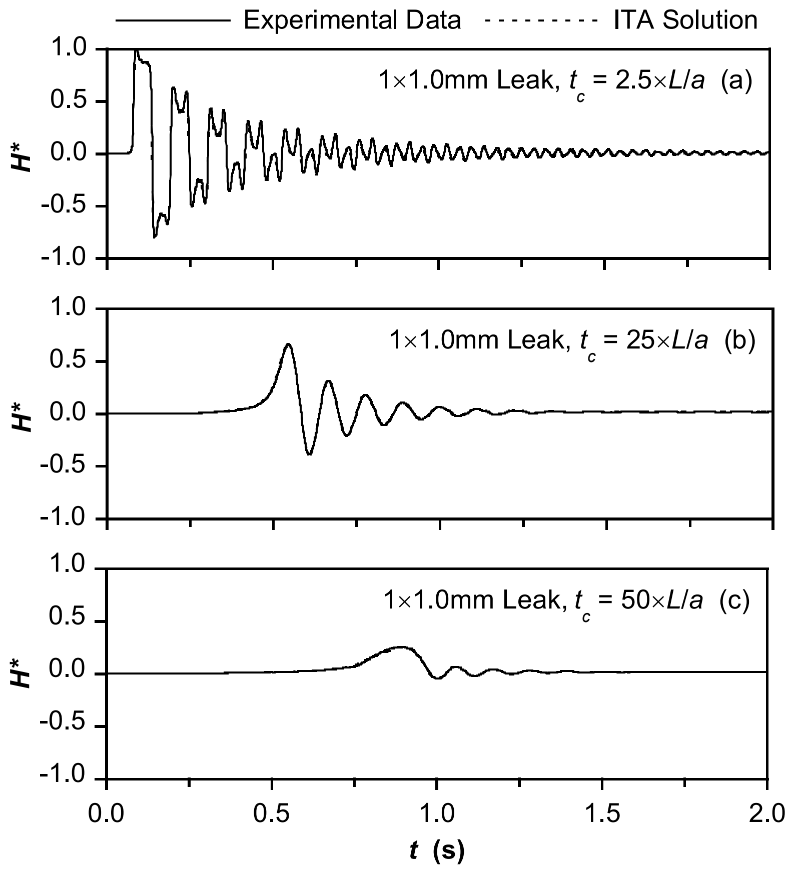

Figure 2. Transient event sharpness investigation: Experimental and ITA pressure responses.

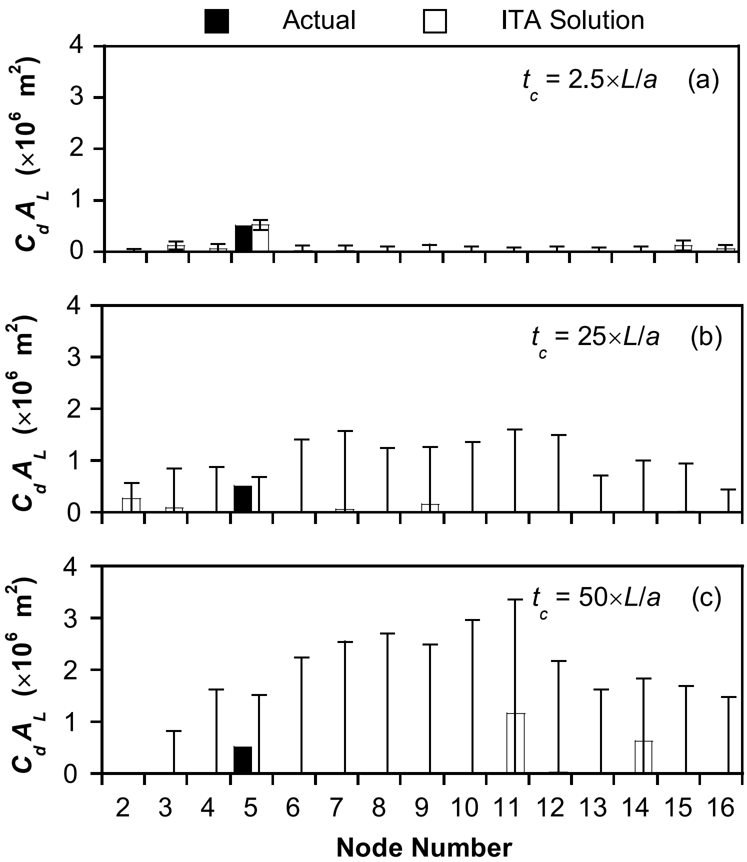

Figure 3. Transient event sharpness investigation: Experimental ITA parameter estimates. Error bars represent parameter error (±σ_a) estimated using the FOSM method.

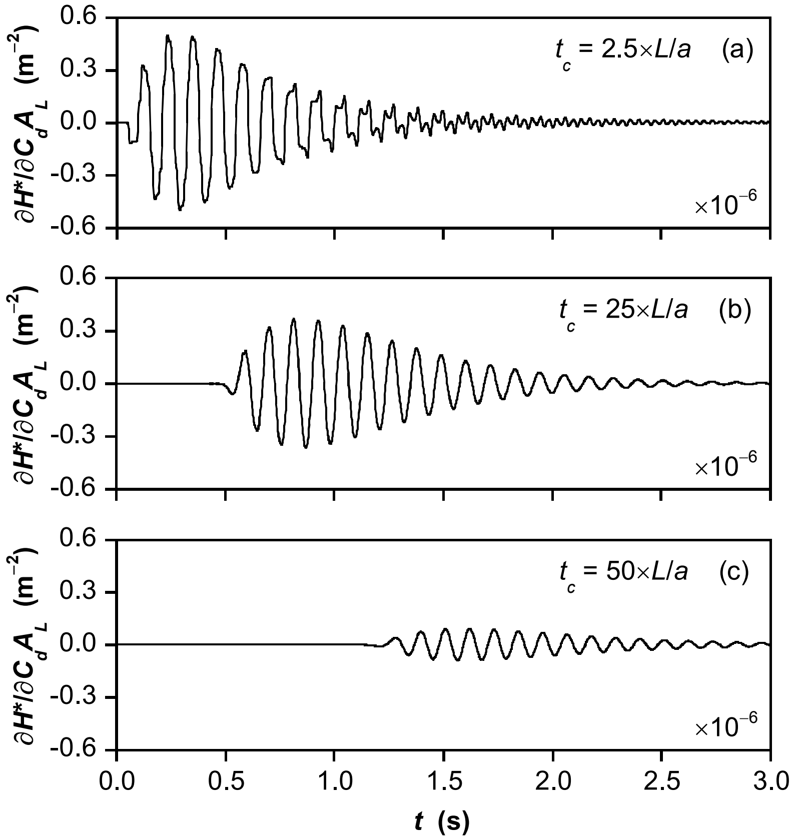

Figure 4. Transient event sharpness investigation: Head sensitivity at node 1 with respect to leak at node 5.

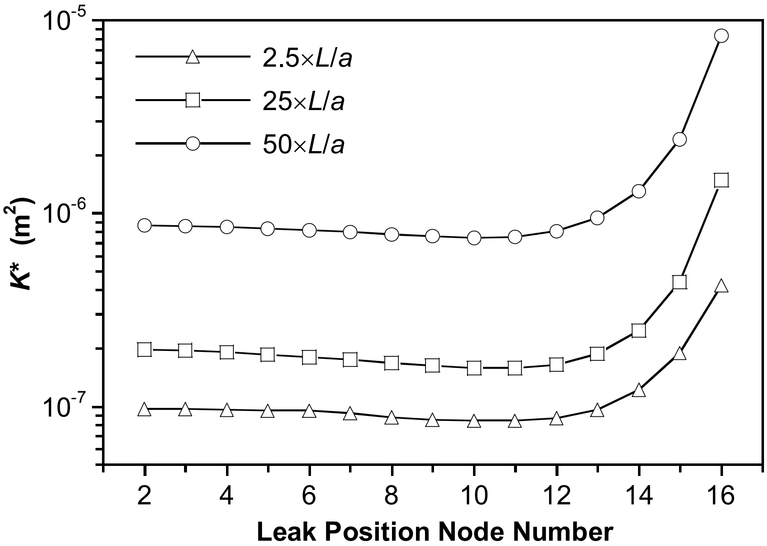

Figure 5. Transient event sharpness investigation: Error transmission multiplier for leak located along pipeline.

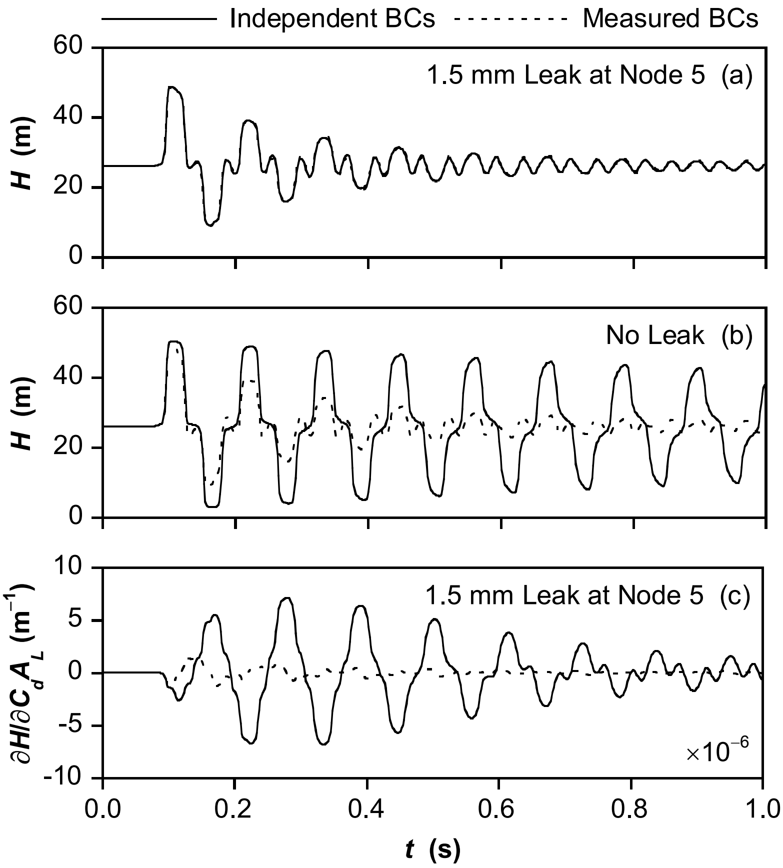

Figure 6. Independent and measured boundary condition use investigation: (a) simulation with leak at node 5, (b) simulation with no leak, and (c) head sensitivity to leak at solution.

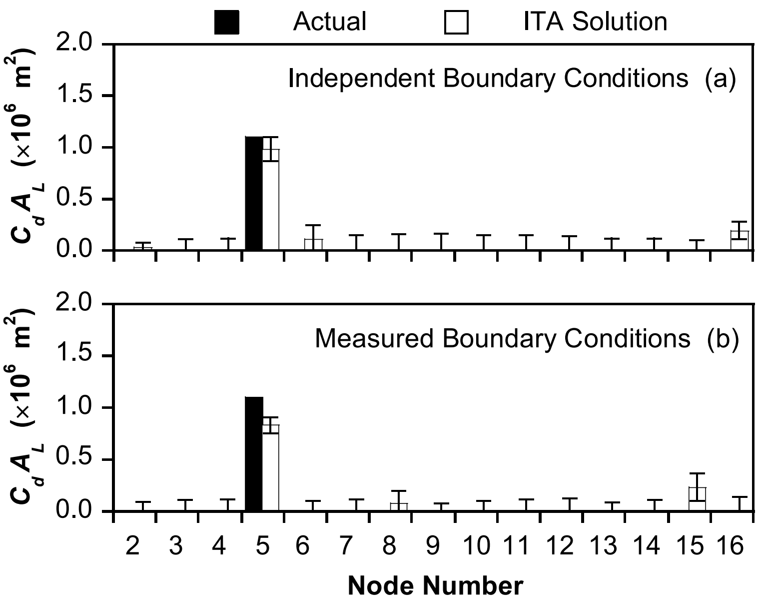

Figure 7. Independent and measured boundary condition use investigation: Experimental ITA parameter estimates. Error bars represent parameter error ($\pm\sigma_a$) estimated using the FOSM method.

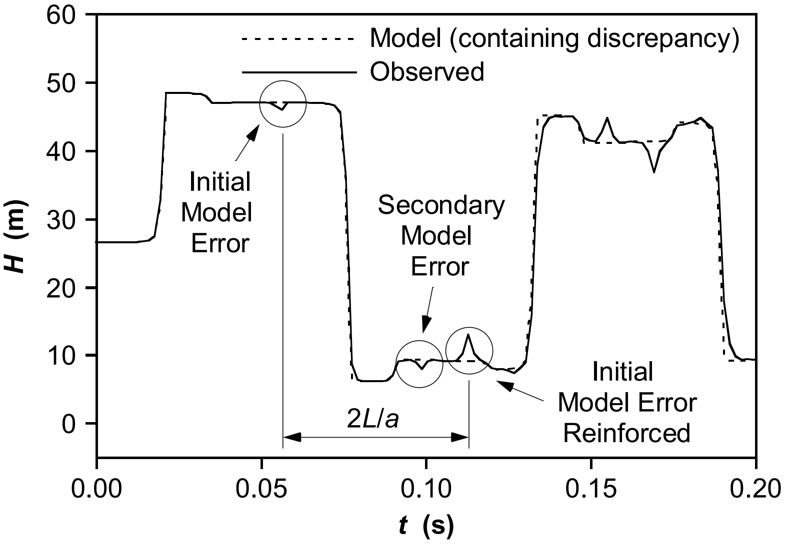

Figure 8. Example of systematic model error in a transient pipeline system.

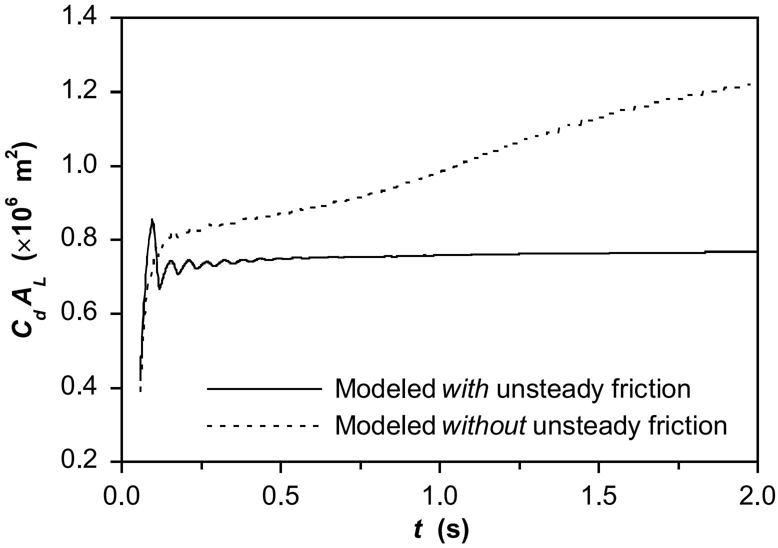

Figure 9. Model error investigation: ITA leak coefficient estimate versus data length.

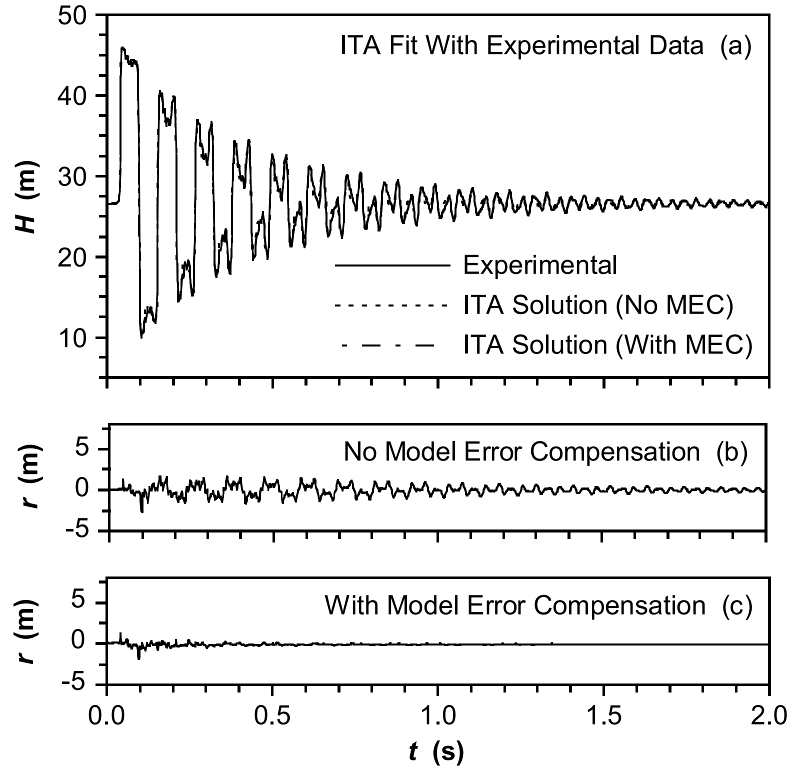

Figure 10. Model error investigation: Comparison of pressure response and residuals for ITA estimation.

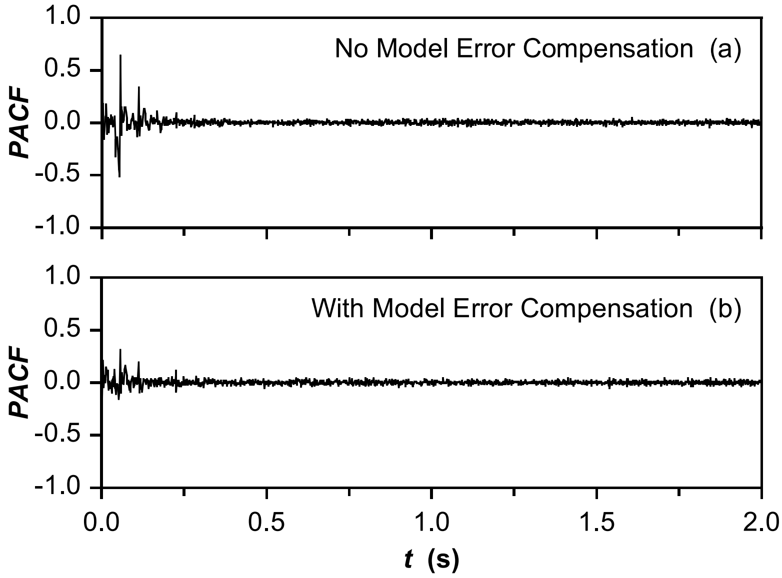

Figure 11. Model error investigation: Comparison of PACF of residuals for ITA fit.

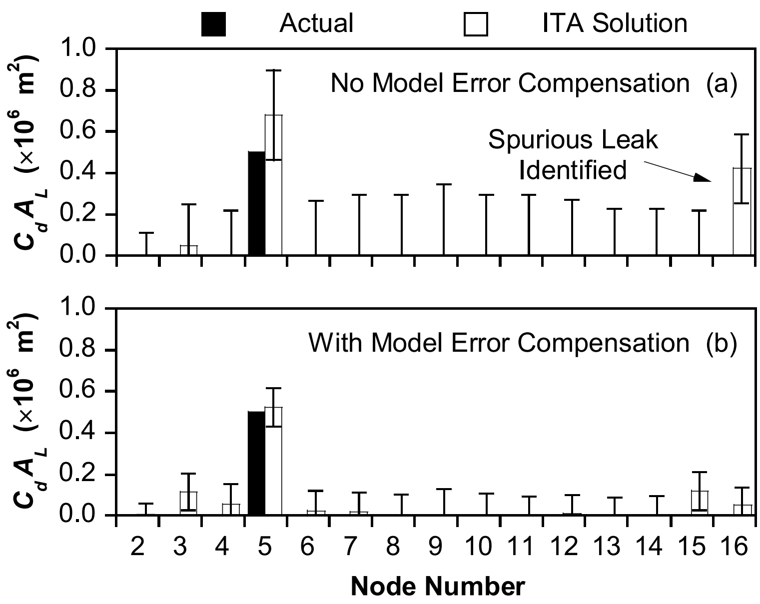

Figure 12. Model error investigation: Comparison of parameter estimates using a model error compensation approach. Error bars represent parameter error ($\pm\sigma_a$) estimated using the FOSM method.

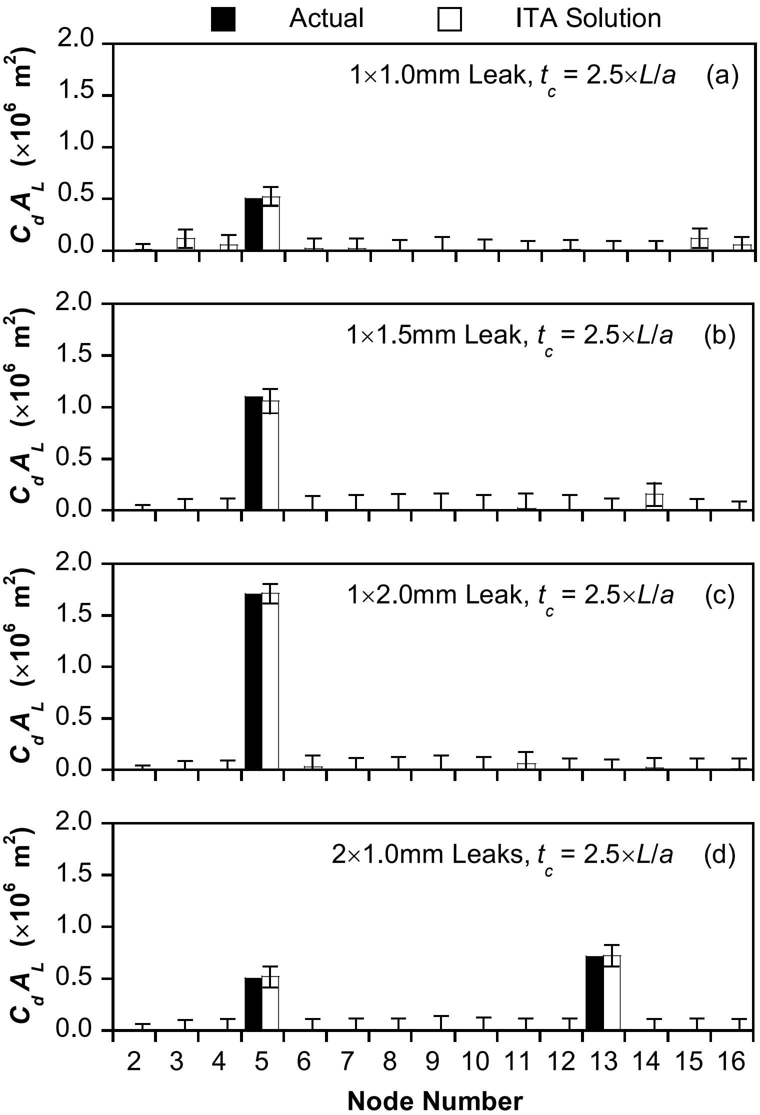

Figure 13. Experimental ITA parameter estimates using an AR model for a 2.5×L/a valve closure for single and multiple leaks. Error bars represent parameter error ($\pm\sigma_a$) estimated using the FOSM method.

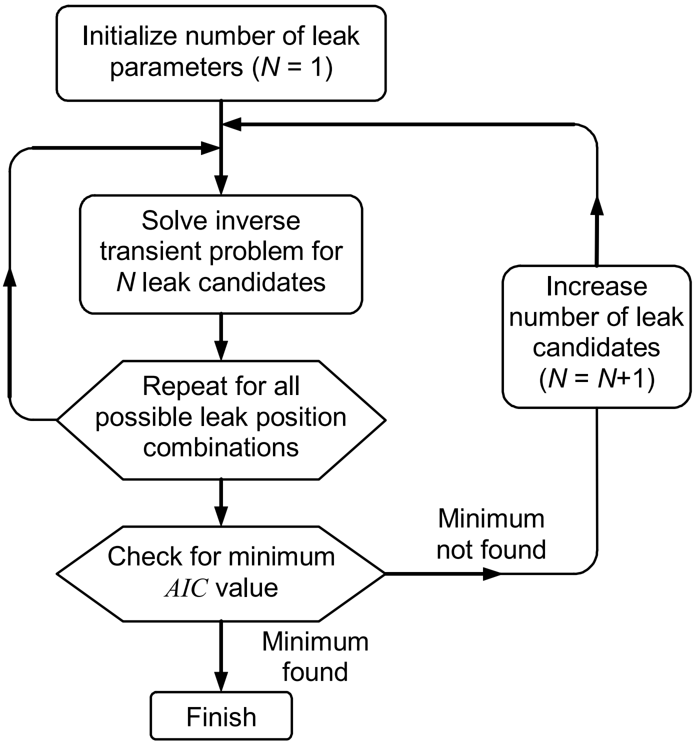

Figure 14. Schematic of the model parsimony approach (*AIC* = Akaike's information criterion).