



Kinetostatic Modelling of Compliant Micro-motion Stages with Circular Flexure Hinges

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Abstract

This thesis presents a) a scheme for selecting the most suitable flexure hinge compliance equations, and b) a simple methodology of deriving kinetostatic models of micro-motion stages by incorporating the scheme mentioned above. There were various flexure hinge equations previously derived using different methods to predict the compliances of circular flexure hinges. However, some of the analytical/empirical compliance equations provide better accuracies than others depending on the t/R ratios of circular flexure hinges. Flexure hinge compliance equations derived previously using any particular method may not be accurate for a large range of t/R ratios. There was no proper scheme developed on how to select the most suitable and accurate hinge equation from the previously derived formulations. Therefore, the accuracies and limitations of the previously derived compliance equations of circular flexure hinges were investigated, and a scheme to guide designers for selecting the most suitable hinge equation based on the t/R ratios of circular flexure hinges is presented in this thesis.

This thesis also presents the derivation of kinetostatic models of planar micro-motion stages. Kinetostatic models allow the fulfillment of both the kinematics and the statics design criteria of micro-motion stages. A precise kinetostatic model of compliant micro-motion stages will benefit researchers in at least the design and optimisation phases where a good estimation of kinematics, workspace or stiffness of micro-motion stages could be realised. The kinetostatic model is also an alternative method to the finite-element approach which uses commercially available software. The modelling and meshing procedures using finite-element software could be time consuming. The kinetostatic model of micro-motion stages was developed based on the theory of the connection of serial and parallel springs. The derivation of the kinetostatic model is simple and the model is expressed

in closed-form equations. Material properties and link parameters are variables in this model. Compliances of flexure hinges are also one of the variables in the model. Therefore the most suitable flexure hinge equation can be selected based on the scheme aforementioned in order to calculate the kinetostatics of micro-motion stages accurately.

Planar micro-motion stages with topologies of a four-bar linkage and a 3-RRR (revolute-revolute-revolute) structure were studied in this thesis. These micro-motion stages are monolithic compliant mechanisms which consist of circular flexure hinges. Circular flexure hinges are used in most of the micro-motion stages which require high positioning accuracies. This is because circular flexure hinges provide predominantly rotational motions about one axis and they have small parasitic motions about the other axes. The 3-RRR micro-motion stage studied in this thesis has three-degrees-of-freedom (DOF). The 3-RRR stage consists of three RRR linkages and each RRR linkage has three circular flexure hinges. A Pseudo-Rigid-Body-Model (PRBM), a kinetostatic model and a two-dimensional finite-element-analysis (FEA) model generated using ANSYS of micro-motion stages are presented and the results of these models were compared. Advantages of the kinetostatic model was highlighted through this comparison. Finally, experiments are presented to verify the accuracy of the kinetostatic model of the 3-RRR micro-motion stage.

Statement of Originality

To the best of my knowledge, except where otherwise referenced and cited, everything that is presented in this thesis is my own original work and has not been presented previously for the award of any other degree or diploma in any university. If accepted for the award of the degree of Doctor of Philosophy in Mechanical Engineering, I consent that this thesis be made available for loan and photocopying.

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Date

Publications

Publications arising from this thesis

Yong, Y. K., Lu, T.-F. and Handley, D. C., 2003, 'Loop-closure theory in deriving linear and simple kinematic model for a 3 DOF parallel micromanipulator', *Proceedings of SPIE on Device and Process Technologies for MEMS, Microelectronics, and Photonics III, Perth, Australia*, vol. 5276, pp. 57-66

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Other publications related to compliant micro-motion stages

Handley, D. C., Lu, T.-F. and **Yong, Y. K.**, 'A simple and efficient modelling method for planar flexure hinge compliant mechanisms', *Accepted to be published in the Precision Engineering*

Yong, Y. K., Lu, T.-F. and Minase, J. L., 2006, 'Trajectory following with a three-DOF micro-motion stage', *Australasian Conference on Robotics and Automation, ACRA, Dec. 6-8, Auckland, New Zealand*

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