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Fundamental Numerical Schemes for Parameter Estimation in Computer Vision

by

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Abstract

An important research area in computer vision is parameter estimation. Given a mathematical model and a sample of image measurement data, key parameters are sought to encapsulate geometric properties of a relevant entity. An optimisation problem is often formulated in order to find these parameters. This thesis presents an elaboration of fundamental numerical algorithms for estimating parameters of multi-objective models of importance in computer vision applications. The work examines ways to solve unconstrained and constrained minimisation problems from the view points of theory, computational methods, and numerical performance.

The research starts by considering a particular form of multi-equation constraint function that characterises a wide class of unconstrained optimisation tasks. Increasingly sophisticated cost functions are developed within a consistent framework, ultimately resulting in the creation of a new iterative estimation method. The scheme operates in a maximum likelihood setting and yields near-optimal estimate of the parameters. Salient features of the method are that it has simple update rules and exhibits fast convergence. Then, to accommodate models with functional dependencies, two variant of this initial algorithm are proposed. These methods are improved again by reshaping the objective function in a way that presents the original estimation problem in a reduced form. This procedure leads to a novel algorithm with enhanced stability and convergence properties.

To extend the capacity of these schemes to deal with constrained optimisation problems, several a posteriori correction techniques are proposed to impose the so-called ancillary constraints. This work culminates by giving two methods which can tackle ill-conditioned constrained functions. The combination of the previous unconstrained methods with these post-hoc correction schemes provides an array of powerful constrained algorithms.

The practicality and performance of the methods are evaluated on two specific applications. One is planar homography matrix computation and the other trifocal tensor estimation. In the case of fitting a homography to image data, only the unconstrained algorithms are necessary. For the problem of estimating a trifocal tensor, significant work is done first on expressing sets of usable constraints, especially the ancillary constraints which are critical to ensure that the computed object conforms to the underlying geometry. Evidently here, the post-correction schemes must be incorporated in the computational mechanism. For both of these example problems, the performance of the unconstrained and constrained algorithms is compared to existing methods. Experiments reveal that the new methods perform with high accuracy to match a state-of-the-art technique but surpass it in execution speed.

Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution. To the best of my knowledge and belief, it contains no material previously published or written by any other person, except where due reference is made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

Tony Scoleri
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Publications

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[3] T. Scoleri, W. Chojnacki and M. J. Brooks. Dimensionality Reduction for More Stable Vision Parameter Estimation. *IET Computer Vision journal*, vol 2, issue 4, December 2008. ISSN 1751-9632.

[4] T. Scoleri. Post-hoc Correction Techniques for Constrained Parameter Estimation in Computer Vision. Proceedings of the *Sixth Digital Image Computing Techniques and Applications (DICTA-08)* conference, Canberra, Australia, 1-3 December, 2008.

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$\mathbf{a} \in \mathbb{R}^p$	column vector of length p
$\mathtt{A} \in \mathbb{R}^{n \times m}$	$n \times m$ dimensional matrix
$\mathtt{I}_{n \times n}$	$n \times n$ identity matrix
$\mathtt{K}_n$	$n \times n$ commutation matrix
$\mathtt{A}^\top$	transpose of $\mathtt{A}$
$\mathtt{A}^{-1}$	inverse of $\mathtt{A}$
$\mathtt{A}^+$	Moore-Penrose inverse of $\mathtt{A}$
$\mathtt{A}_r^-$	r-truncated pseudo-inverse of $\mathtt{A}$
$\dim(\mathbf{a})$	dimensionality of $\mathbf{a}$
$\mathrm{tr}(\mathtt{A})$	trace of matrix $\mathtt{A}$
$\det(\mathtt{A})$	determinant of $\mathtt{A}$
$\mathrm{vec}(\mathtt{A})$	vectorisation of $\mathtt{A}$
$\mathrm{rank}(\mathtt{A})$	rank of $\mathtt{A}$
$\mathrm{diag}(\sigma_1, \ldots, \sigma_n)$	$n \times n$ matrix with $\sigma_1, \ldots, \sigma_n$ along the diagonal and zeros elsewhere
$\lVert \mathtt{A} \rVert_{\mathrm{F}}$	Frobenius norm of $\mathtt{A}$
$\mathbf{a} \times \mathbf{b}$	cross product of $\mathbf{a}$ and $\mathbf{b}$
$\mathtt{A} \otimes \mathtt{B}$	Kronecker product of $\mathtt{A}$ and $\mathtt{B}$
$\partial_{\boldsymbol{\theta}} J$	Jacobian (row) vector of J, $\left[\frac{\partial J}{\partial \theta_1}, \ldots, \frac{\partial J}{\partial \theta_n}\right]$
$\partial^2_{\boldsymbol{\theta}\boldsymbol{\theta}} J$	Hessian matrix of J, $\left[\frac{\partial^2 J}{\partial \theta_i \partial \theta_j}\right]_{1 \le i,j \le n}$
$\mathtt{H}_{\boldsymbol{\theta}}$	alternative notation for the Hessian matrix of J
$\mathbb{P}^2$	projective plane
$\mathbb{P}^{2*}$	dual projective plane

Vector and Matrix Sizes

$\mathbf{f}$	multi-objective constraint vector of length m
$\mathbf{f}'$	multi-objective sub-constraint vector of length r
$\boldsymbol{\Phi}$	multi-objective ancillary constraint vector of length q
$\boldsymbol{\theta}$	parameter vector of length l
$\boldsymbol{\mu}$	reduced parameter vector of length $l - m$
$\boldsymbol{\alpha}$	complementary parameter vector of length m such that $\boldsymbol{\theta} = [\boldsymbol{\mu}^\mathsf{T}, \boldsymbol{\alpha}^\mathsf{T}]^\mathsf{T}$
$\boldsymbol{\beta}$	parameter vector of length s
$\mathbf{x}_i$	$k \times 1$ element of data; $i = 1, \ldots, n$
$\mathtt{U}_i$	$l \times m$ measurement matrix made from $\mathbf{x}_i$
$\mathtt{U}'_i$	$(l - m) \times m$ reduced measurement matrix made from $\mathbf{x}_i$
$\mathrm{vec}(\mathtt{U}_i)$	vector of length ml
$\mathrm{vec}(\mathtt{U}'_i)$	vector of length $m(l - m)$
$\partial_{\mathbf{x}_i}\mathrm{vec}(\mathtt{U}_i)$	$ml \times k$ derivative matrix of $\mathrm{vec}(\mathtt{U}_i)$ with respect to $\mathbf{x}_i$
$\partial_{\mathbf{x}_i}\mathrm{vec}(\mathtt{U}'_i)$	$m(l - m) \times k$ derivative matrix of $\mathrm{vec}(\mathtt{U}'_i)$ with respect to $\mathbf{x}_i$
$\mathtt{\Lambda}_{\mathbf{x}_i}$	$k \times k$ covariance matrix of $\mathbf{x}_i$
$\mathtt{B}_i$	$ml \times ml$ propagated covariance matrix of $\mathbf{x}_i$
$\mathtt{\Sigma}_i$	$m \times m$ matrix
$\boldsymbol{\eta}_i$	Lagrange multiplier vector of length m
$\tilde{\mathtt{Z}}$	$(l - m) \times m$ "centroid" of $\mathtt{U}'_i$
$\mathtt{C}_{\boldsymbol{\theta}}$	$l \times l$ covariance matrix of $\boldsymbol{\theta}$
$\mathtt{X}_{\boldsymbol{\theta}}$	$l \times l$ matrix involved in the derivative of J_{AML} with respect to $\boldsymbol{\theta}$
$\mathbf{r}_k$	residual vector of length l
$\mathtt{J}_k$	$l \times s$ Jacobian matrix of $\mathbf{r}_k$
$\mathbf{p}_k$	search direction vector of length s