Copulas for Credit Derivative Pricing and other Applications

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Signed Statement

Declaration

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Publications Arising from this Study

Crane G.J. and van der Hoek, J., (2008) Conditional Expectation and Copulas. Australian and New Zealand Journal of Statistics **50**, 1-15.

Crane G.J. and van der Hoek, J., (2008) Using distortions of copulas to price synthetic CDOs. Insurance: Mathematics and Economics **42**, 903-908.

Abstract

Copulas are multivariate probability distributions, as well as functions which link marginal distributions to their joint distribution. These functions have been used extensively in finance and more recently in other disciplines, for example hydrology and genetics. This study has two components, (a) the development of copula-based mathematical tools for use in all industries, and (b) the application of distorted copulas in structured finance. In the first part of this study, copula-based conditional expectation formulae are described and are applied to small data sets from medicine and hydrology. In the second part of this study we develop a method of improving the estimation of default risk in the context of collateralized debt obligations. Credit risk is a particularly important application of copulas, and given the current global financial crisis, there is great motivation to improve the way these functions are applied. We compose distortion functions with copula functions in order to obtain greater flexibility and accuracy in existing pricing algorithms. We also describe an n-dimensional dynamic copula, which takes into account temporal and spatial changes.