wavefronts coming from DOAs in a narrow sector around broadside of the transformed geometry. This is the case of multibaseline interferometric SAR systems, and of phased array radar applications where only the sources present in a single azimuth resolution cell are sensed. The results obtained can be useful also when the sector of DOAs is not narrow. In wireless communication scenarios, one could preliminary select a DOA subsector by means of a beamspace algorithm and then recalculate the HCRB as outlined here. Another possible application is when the sources are clustered around some directions each well spaced apart from the others; since the estimation of the sources in one cluster is not significantly affected by the others, the HCRB derived here can be applied to each cluster separately. In the applications different from InSAR, the proper spatial decorrelation model (see e.g., [9], [10]) can be promptly adopted in place of the linear model used here for the numerical example.

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A Cholesky Factorization Based Approach for Blind FIR Channel Identification

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***Abstract*—For blind channel identification, various techniques, including transmitter-induced cyclostationarity based approaches, have been proposed in literature. To induce cyclostationarity at the transmitter, zero padding to every symbol packet can be considered. Some subspace methods are proposed for the channel estimation by exploiting the induced cyclostationarity. Due to zero padding, the covariance matrix of received signal vectors can have a special structure. Utilizing this structure, we can estimate channel impulse responses through factorization. In this correspondence, we propose a factorization based approach to estimate channel impulse responses. In general, the proposed factorization based approach can work with a small number of samples and becomes much more computationally efficient than the subspace method when the length of packets is long.**

***Index Terms*—Blind channel identification, Cholesky factorization.**

I. INTRODUCTION

It is impossible to blindly identify mixed-phase finite impulse response (FIR) using second-order statistics. If some other properties of input signals can be utilized, however, blind FIR systems identification becomes possible. For example, using high-order statistics of non-Gaussian input signals, blind identification has been studied in [6], [29]. Since the input signals to communication channels are not Gaussian signals and communication channels are usually modelled by FIR systems [19], the high-order-statistics-based approaches are successfully applied to blind FIR channel identification. In [30], [24], other approaches based on discrete-alphabet inputs in digital communication systems have been proposed. In general, the above approaches require a high computational complexity or a large number of samples for a satisfactory performance [18].

In [25] and [26], second-order-statistics-based approaches for blind channel identification are proposed using oversampling or multichannel inputs (in [27], an overview can be found). By employing the temporal oversampling techniques, phase information of FIR

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channels can be retained in second-order statistics of oversampled signals. Since this approach uses second-order statistics, it can be more computationally efficient than the approaches rely on high-order statistics. In [1], [14], other approaches based on second-order statistics are considered to identify FIR channels. Those methods have restrictions on zero locations of FIR subchannels from oversampling or multiple antennas. In [5], it is shown that those methods can fail for some practical channels which can have common zeros among the subchannels obtained by oversampling. (In [20], it is shown that a class of those channels can be identified using multiple antennas. In this case, the subchannels do not share common zeros.)

In order to overcome the above difficulty, various approaches can be considered. In [11], noncircularity of certain data symbols is exploited in conjunction with second-order statistics of signals. Transmitter-induced cyclostationarity has been considered in [7] and [28]. The methods in [7] and [28] can blindly identify FIR channels without constraints on zero locations of channels. In [28], the cyclostationarity has been obtained by repeating input blocks and it reduces the information rate by half. In order to minimize the decrease of the throughput, a different technique has been used in [7] to induce cyclostationarity. In transmitters, a precoding filterbank has been used and the associated rate reduction can be arbitrarily small. Cyclostationary-based approaches are applicable to various systems, including orthogonal-frequency-division multiplexing (OFDM) [12] and code-division multiple access (CDMA) [21]. In [13], virtual carriers (unmodulated carriers) are utilized for blind channel estimation in OFDM systems as cyclostationary is induced using virtual carriers. Various approaches for blind channel estimation and equalization, including the approaches discussed above, are reviewed in [8].

In order to induce cyclostationarity at the transmitter, a simple approach is to append zeros or null signals at the end of each data packet. This simple method provides a special structure of the covariance matrix of received signal vectors. Using the special structure, various subspace methods are proposed in [4], [17], and [21]. A generalization of the subspace method has been considered in [23]. In this correspondence, we propose a blind channel estimation method that can provide a reasonably good estimate of the channel impulse response with a relatively small number of data blocks using the (modified) Cholesky factorization of the covariance matrix. We can observe that the upper triangular matrix (obtained by the Cholesky factorization) retains a full information of the channel impulse response. A parameterization technique is devised to estimate the channel impulse response from the upper triangular matrix. It is noteworthy that the computational complexity of the proposed method depends on the length of channel impulse response, while the computational complexity of the subspace method depends on the length of packet. Thus, when the length of packet is long, the proposed method becomes much more computationally efficient.

II. System Model

Consider a block of data symbols

$$\mathbf{x}[k] = [x[kM]\, x[kM-1] \ldots x[kM-M+1]]^T$$

where $\{x[t]\}$ denotes the data symbol sequence and M denotes the block of (encoded) data symbols. For zero padding, $(N-M)$ zeros $(N > M)$ are appended and the resulting packet to be transmitted is given as follows:

$$\begin{aligned}\mathbf{y}[k] &= [y[kN]\, y[kN-1] \ldots y[kN-N+1]]^T \\ &= [x[kM]\, x[kM-1] \ldots x[kM-M+1] \quad \underbrace{0 \ldots 0}_{N-M \text{ xzeros}} \quad]^T \end{aligned} \tag{1}$$

where $\mathbf{y}[k]$ denotes the kth extended data packet and N denotes its length. Note that by inserting zeros or null signals, cyclostationarity can be induced [7]. The information rate can be defined by the ratio M/N. For convenience, denote by $(N-M)/N$ the redundancy ratio. The information rate decreases with the redundancy ratio. The received signal (in the discrete-time domain) is written as

$$z[t] = \sum_{\ell=0}^{L-1} h[\ell] y[t-\ell] + n[t]$$

where $\{h[\ell]\}$ denotes the FIR of the channel, L denotes the length of the channel impulse response, and $n[t]$ denotes the background noise. Stacking $z[t]$, we have the following vector: $\mathbf{z}[k] = [z[kN]\, z[kN-1] \ldots z[kN-N+1]]^T$. For channel identification, we now consider a truncated version of $\mathbf{z}[k]$ which is defined as $\bar{\mathbf{z}}[k] = [z[kN]\, z[kN-1] \ldots z[kN-N+L]]^T$. Another representation of the vector $\bar{\mathbf{z}}[k]$ can be found as follows:

$$\bar{\mathbf{z}}[k] = \mathcal{H}\mathbf{y}[k] + \mathbf{n}[k] = \mathcal{H} \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{0}_{(N-M)\times 1} \end{bmatrix} + \mathbf{n}[k] \tag{2}$$

where $\mathcal{H}$ is the channel filtering matrix of size $N-(L-1) \times N$

$$\mathcal{H} = \begin{bmatrix} h[0] & \ldots & h[L-1] & & 0 \\ & \ddots & & \ddots & \\ 0 & & h[0] & \ldots & h[L-1] \end{bmatrix} \tag{3}$$

and $\mathbf{n}[k] = [n[kN]\, n[kN-1] \ldots n[kN-N+L]]^T$. For convenience, let $Q = N-(L-1)$. Using the vector $\bar{\mathbf{z}}[n]$, we can derive various factorization methods for the blind channel identification.

In OFDM, cyclic prefixing or zero padding can be used to avoid the intersymbol interference (ISI) [16]. The received signal in (2) is valid for OFDM symbols with zero padding. In this case, $\mathbf{x}[k]$ is the inverse discrete Fourier transform (IDFT) of signal vector (of size $M \times 1$). Let $\mathbf{v}[k]$ denote the kth data symbol vector whose elements are data symbols. Then, $\mathbf{x}[k]$ is given as follows:

$$\mathbf{x}[k] = \mathbf{F}\mathbf{v}[k] \tag{4}$$

where $\mathbf{F}$ is the IDFT matrix with $[\mathbf{F}]_{p,q} = \frac{1}{\sqrt{M}} e^{j2\pi(p-1)(q-1)/M}$.

III. Channel Identification Via Cholesky Factorization

In this section, we show that how FIR channels can be identified via factorization. The following conditions are necessary for the channel identifiability.

C1) $M \geq L$: Although the exact length of channel impulse response is not required to identify channel vectors, its upper bound shall be known to decide M.

C2) $Q > M$: Since $Q = N-(L-1)$, this condition implies that $N \geq M+L$. If the variance of the background noise is known, this condition can be relaxed as $Q \geq M$.

A. Second-Order Statistics

Second-order statistics are utilized for blind channel identification. The following assumptions on second-order statistics are necessary.

A) $\mathbf{R}_{\mathbf{x}} = E[\mathbf{x}[k]\mathbf{x}^H[k]] = \sigma^2\mathbf{I}$ and $E[\mathbf{x}[k]] = \mathbf{0}; \mathbf{R}_{\mathbf{n}} = E[\mathbf{n}[k]\mathbf{n}^H[k]] = \sigma_n^2\mathbf{I}$ and $E[\mathbf{n}[k]] = \mathbf{0}; h[0] \neq 0$. Consider the covariance matrix of $\bar{\mathbf{z}}[k], \mathbf{R}_{\bar{\mathbf{z}}} = E[\bar{\mathbf{z}}[k]\bar{\mathbf{z}}^H[k]]$. From A), we have

$$\mathbf{R}_{\bar{\mathbf{z}}} = \sigma^2 \mathcal{H}\mathbf{B}\mathcal{H}^H + \sigma_n^2\mathbf{I} \tag{5}$$

where $\mathbf{B} = \begin{bmatrix} \mathbf{I}_{M\times M} & \mathbf{0}_{M\times(N-M)} \\ \mathbf{0}_{(N-M)\times M} & \mathbf{0}_{(N-M)\times(N-M)} \end{bmatrix}$. Consider a submatrix of $\mathbf{R}_{\bar{\mathbf{z}}}$ as follows: $\mathbf{R} = [\mathbf{R}_{\bar{\mathbf{z}}}]_{1:M,1:M}$, where $[\mathbf{A}]_{1:M,1:M}$ stands for the submatrix of $\mathbf{A}$ which has the same elements of $\mathbf{A}$, $a_{i,k}$, for $i,k = 1,2,\ldots,M$. It follows that

$$\mathbf{R} = \sigma^2 \mathbf{H}\mathbf{H}^H + \sigma_n^2 \mathbf{I} \tag{6}$$

where

$$\mathbf{H} = \begin{bmatrix} h[0] & h[1] & \ldots & \\ 0 & h[0] & h[1] & \\ \vdots & & \ddots & \\ 0 & & \ldots & h[0] \end{bmatrix}. \tag{7}$$

Under C1), the matrix $\mathbf{H}$ contains a full information of the channel vector $\mathbf{h} = [h[0]\ h[1]\ \ldots\ h[L-1]]^T$. Clearly, if $\mathbf{H}$ is available from $\mathbf{R}$, the channel vector can be found from its elements.

B. Modified Cholesky Factorization

The matrix $\mathbf{R}$ in (6) has an interesting structure. Consider

$$\bar{\mathbf{R}} = \mathbf{R} - \sigma_n^2 \mathbf{I} = \sigma^2 \mathbf{H}\mathbf{H}^H. \tag{8}$$

Obviously, we can see that the matrix $\mathbf{H}$ can be obtained via factorization. Using a modification of the Cholesky factorization, we can find the matrix $\mathbf{H}$ uniquely from $\bar{\mathbf{R}}$ [9].

Property 1: Let

$$\bar{\mathbf{R}} = \mathbf{G}\mathbf{G}^H \tag{9}$$

where $\mathbf{G}$ is the modified Cholesky triangular matrix. Under **A)**, the matrix $\mathbf{H}$ is uniquely identified (up to complex scalar ambiguity) from the following relation:

$$\mathbf{G} = \alpha \mathbf{H} \tag{10}$$

where α is a complex scalar.

In above, we show that the channel vector can be blindly estimated using the modified Cholesky factorization. A similar approach that uses the Cholesky factorization was considered in [15] to estimate the channel impulse response in OFDM systems with cyclic prefix. For the factorization, we need to know $\bar{\mathbf{R}}$. To obtain $\bar{\mathbf{R}}$, we need to find $\mathbf{R}_{\bar{\mathbf{z}}}$ and σ_n^2. Those are all second-order statistics. Thus, the factorization method to blindly identify single-channel vectors can be considered as one of the second-order-statistics-based approaches.

C. Parameterization Technique From Sample Covariance Matrix

From finite samples, say K, we need to have the estimate of the covariance matrix $\mathbf{R}_{\bar{\mathbf{z}}}$ as $\hat{\mathbf{R}}_{\bar{\mathbf{z}}} = 1/K \sum_{k=1}^{K} \bar{\mathbf{z}}[k]\bar{\mathbf{z}}^H[k]$. To estimate $\bar{\mathbf{R}}$ from $\hat{\mathbf{R}}_{\bar{\mathbf{z}}}$, we shall estimate the variance of the background noise, σ_n^2. Since the rank of $\mathcal{H}\mathbf{B}\mathcal{H}^H$ is M, we can estimate σ_n^2 using the eigendecomposition under Condition C2). [Since $Q > M$ (to estimate σ_n^2), the minimum redunancy ratio can be given by L/N (with $Q = M+1$). The redundancy ratio can approach zero as N increases.] Once σ_n^2 is estimated, an estimate of $\bar{\mathbf{R}}$ is available using (8). From C2), we can consider the average of the $Q-M > 0$ smallest eigenvalues of $\hat{\mathbf{R}}_{\bar{\mathbf{z}}}$ as an estimate of σ_n^2, $\hat{\sigma}_n^2 = 1/Q - M \sum_{\ell=M+1}^{Q} \hat{\lambda}_\ell$, where $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \ldots \geq \hat{\lambda}_Q$ are the eigenvalues of $\hat{\mathbf{R}}_{\bar{\mathbf{z}}}$ and $\hat{\mathbf{e}}_\ell$ denotes the eigenvector corresponding to $\hat{\lambda}_\ell$. For the uniqueness of the factorization, the estimate of $\bar{\mathbf{R}}$ should be positive definite according to Theorem 1. From the eigendecomposition of $\hat{\mathbf{R}}_{\bar{\mathbf{z}}}$, we can find an estimate of $\bar{\mathbf{R}}$ which is nonnegative definite:

$$\hat{\bar{\mathbf{R}}} = \hat{\mathbf{S}}_1 \left(\hat{\mathbf{\Lambda}}_S - \hat{\sigma}_n^2 \mathbf{I} \right) \hat{\mathbf{S}}_1^H \tag{11}$$

where $\hat{\mathbf{S}}_1 = [\hat{\mathbf{S}}]_{1:M,1:M}, \hat{\mathbf{S}} = [\hat{\mathbf{e}}_1\ \hat{\mathbf{e}}_2 \ldots \hat{\mathbf{e}}_M]$, and $\hat{\mathbf{\Lambda}}_S = \mathrm{diag}(\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_M)$. Using the estimate in (11), we can obtain the estimate of the channel vector uniquely through the factorization. Note that if $\hat{\mathbf{e}}_\ell$ and $\hat{\lambda}_\ell$ approach $\mathbf{e}_\ell$ and λ_ℓ, respectively, $\hat{\bar{\mathbf{R}}}$ also approaches $\bar{\mathbf{R}}$. Here, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_Q$ are the eigenvalues of $\mathbf{R}_{\bar{\mathbf{z}}}$ and $\mathbf{e}_\ell$ are the corresponding eigenvectors. This shows that the estimated channel impulse response approaches the true one as K increases.

Note that C2) can be modified as C2′) $Q \geq M$ if σ_n^2 is known (because the estimate of σ_n^2 is not necessary). With a sufficiently high SNR, the noise variance can be negligible in (11) and $\hat{\bar{\mathbf{R}}} \simeq \hat{\mathbf{R}}_{\bar{\mathbf{z}}}$. In this case, the eigendecomposition in (11) is not required.

The channel vector can be estimated from the estimate $\hat{\bar{\mathbf{R}}}$. Let $\hat{\mathbf{H}}$ denote the estimate of $\mathbf{H}$ from $\hat{\bar{\mathbf{R}}}$ using the modified Cholesky factorization. Although $\mathbf{H}$ is Toeplitz, its estimate, $\hat{\mathbf{H}}$, may not be Toeplitz from estimation errors. This leads to a $M(M+1)/2$ to M mapping problem. Using Toeplitz structure of $\mathbf{H}$, we can have a parameterization technique to estimate the channel vector $\mathbf{h}$ from $\hat{\mathbf{H}}$. Using the relation in (10) with assuming that $h[0]$ is real, we can consider the following optimization problem:

$$\min_{\mathbf{h}} \|\hat{\mathbf{G}} - \mathbf{G}(\mathbf{h})\|_F \tag{12}$$

where $\mathbf{h} = [h[0]\ h[1] \ldots h[M-1]]^T$ and

$$\mathbf{G}(\mathbf{h}) = \begin{bmatrix} h[0] & h[1] & \ldots & h[M-1] \\ 0 & h[0] & \ldots & h[M-2] \\ & & \ddots & \\ 0 & 0 & \ldots & h[0] \end{bmatrix}.$$

In (12), we assume that the length of the channel vector is $M (\geq L)$. If the length of the channel vector is known, we can have a smaller number of the parameters to be estimated (because we assume that $L \leq M$).

Property 2: The solution vector $\mathbf{H}$ in (12) can be found using the vectorization operation and least squares (LS) solution. The solution of (12) is given as follows:

$$\hat{\mathbf{h}} = (\mathbf{Q}^H \mathbf{Q})^{-1} \mathbf{Q}^H \underline{\hat{\mathbf{g}}} \tag{13}$$

where $\underline{\hat{\mathbf{G}}} = \mathrm{vec}(\hat{\mathbf{G}})$ and the matrix $\mathbf{Q}$ of size $M^2 \times M$ is a selection matrix which will be given below. In addition, the rank of $\mathbf{Q}$ is M (it means that the inverse of $\mathbf{Q}^H\mathbf{Q}$ exists).

Proof: Through the vectorization of the upper triangular matrix $\mathbf{G}$, it follows that

$$\underline{\mathbf{g}} = \mathrm{vec}(\mathbf{G}(\mathbf{h})) = \mathbf{Q}\mathbf{h} \tag{14}$$

where $\mathbf{Q}$ is the selection matrix of size $M^2 \times L$. For convenience, assume that $M = L$. If $L < M$, the vector $\mathbf{h}$ can be modified as $[\mathbf{h}^T \underbrace{0 \ldots 0}_{(M-L)\times}]^T$. Then, the selection matrix $\mathbf{Q}$ becomes $\mathbf{Q} = \left[\mathbf{J}_1^T\ \mathbf{J}_2^T \ldots \mathbf{J}_M^T\right]^T$, where $M \times M$ matrices $\mathbf{J}_k$ are defined as

$$[\mathbf{J}_k]_{s,t} = \begin{cases} 1, & \text{if } s+t = k-1 \\ 0, & \text{otherwise.} \end{cases}$$

Using (14), we have $\|\hat{\mathbf{G}} - \mathbf{G}(\mathbf{H})\|_F = \|\underline{\hat{\mathbf{g}}} - \mathbf{Q}\mathbf{H}\|$. Thus, the solution vector in (12) is the LS solution and given in (13).

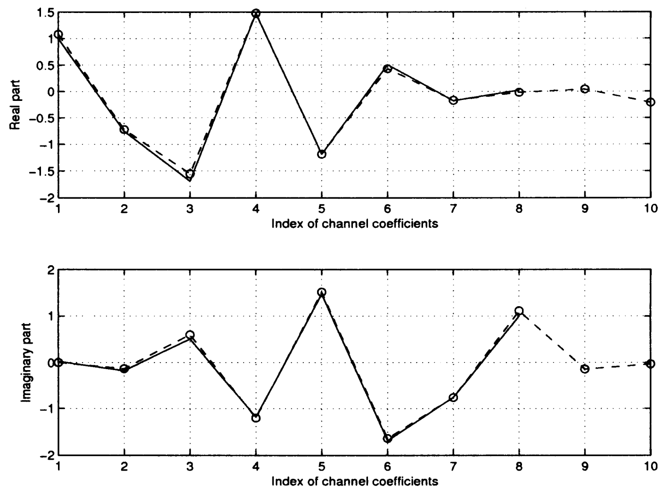

Fig. 1. Results of the channel estimation when $M = 10 > L = 8 (N = 50, K = 400$ and SNR = 20 dB).

D. Complexity and Other Issues

The proposed approach for the blind channel identification can be seen as a two-step approach. In the first step, the Cholesky factorization is performed with the truncated covariance matrix in (8). In the second step, the channel impulse response is extracted from the upper triangular matrix through the minimization in (12). The second step requires a matrix inversion in (13). Since $\mathbf{Q}$ is known and fixed, $(\mathbf{Q}^H\mathbf{Q})^{-1}\mathbf{Q}^H$ can be computed *a priori*. In addition, the parameterization technique in Section III-C can be avoided if the diagonal elements of $\mathbf{G}$ can be averaged to estimate $\mathbf{h}$, as follows:

$$\hat{h}[p] = \frac{1}{M-p}\sum_{q=1}^{M-p}[\hat{\mathbf{G}}]_{q,q+p},\ p = 0, 1, 2, \ldots, L-1.$$

Consequently, the second step does not incur a high computational complexity, and the compuational complexity would be dominated by the first step, in particular, the Cholesky factorization. The complexity of the Cholesky factorization is $O(M^2)$. However, if the length of the channel impulse response, L, or its upper bound is known a priori, the complexity can be reduced. Suppose that $M \gg L$ (this is the case for a high information rate, M/N). From (8), we can see that $\bar{\mathbf{R}}$ is banded (i.e., $[\bar{\mathbf{R}}]_{p,q} = 0$ for $q > p + L$ and $p > q + L$). Then, the complexity of the Cholesky factorization becomes $O(ML^2)$ [9]. Alternatively, the following $\bar{L} \times \bar{L}$ submatrix of $\bar{\mathbf{R}}$ for the Cholesky factorization can be considered, where $\bar{L}$ is an upper bound of the length of the channel impulse response and $M \gg \bar{L} \geq L$:

$$[\mathbf{\Phi}]_{p,q} = [\bar{\mathbf{R}}]_{p+Q-\bar{L},q+Q-\bar{L}},\ p, q = 1, 2, \ldots, \bar{L}. \quad (15)$$

We can see that $\mathbf{Phi} = \sigma^2\mathbf{H}_{\bar{L}}\mathbf{H}_{\bar{L}}^H$, where

$$\mathbf{H}_{\bar{L}} = \begin{bmatrix} h[0] & h[1] & \ldots & h[\bar{L}-1] \\ 0 & h[0] & \ldots & h[\bar{L}-2] \\ & & \ddots & \\ 0 & 0 & \cdots & h[0] \end{bmatrix}.$$

From this, we can see that the complexity is not significantly high even though M is large as long as $\bar{L}(\geq L)$ is small.

The proposed approach is applicable to OFDM systems with zero padding. If the data symbols in $\mathbf{v}[k]$ in (4) are independent and identically distributed (i.i.d.), we can show that $\mathbf{R_x} = \mathbf{F}E[\mathbf{v}[k]\mathbf{v}^H[k]]\mathbf{F}^H = \sigma_v^2\mathbf{I}$, where $E[\mathbf{v}[k]\mathbf{v}^H[k]] = \sigma_v^2\mathbf{I}$ and σ_v^2 denotes the variance of the data symbols in $\mathbf{v}[k]$. Thus, A) is valid to OFDM systems with zero padding, and the proposed approach can be used to estimate the channel impulse response in OFDM systems.

Generally, in the subspace methods in [4], [21], the number of data blocks, K, should be sufficiently large so that a reasonably good estimate of $\mathbf{G}$ is available. For fast fading channels, however, it is difficult to have a sufficiently large number of data blocks as the coherence time can be short. As shown above, the proposed method does not use the noise subspace. Thus, the number of data blocks, K, can be small. There are other subspace methods that can provide good performance even though K is small [17], [23] . In particular, a generalized subspace method is proposed and its performance is excellent (performs better than the proposed factorization based approach as will be shown in Section IV). However, the computational complexity becomes prohibitively high when the length of packet N is long as a singular value decomposition (SVD) of a matrix of size (approximately) $N \times N$ is required. On the other hand, the complexity of the proposed factorization-based approach depends on the length of the channel impulse response L (provided that L is known). Thus, when $N \gg L$, the proposed factorization-based approach becomes much more computationally efficient than the subspace method in [23].

IV. Numerical Results

In this section, we show some numerical results for the proposed approach with statistical performance.

We consider an FIR channel that has common zeros. The real part of the channel has the zeros at $0.2, \pm 1.5, \pm j0.5, 0.2783 \pm j0.3488$. The length of the channel impulse response, L, is 8. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = ||\mathbf{h}||^2/\sigma_n^2$. If the length of the channel impulse response, M, is not known, an overestimate of M can be used. Fig. 1 shows simulation results of the proposed approach when $M = 10 > L = 8$ (we assume that $N = 20, K = 400$, and SNR = 20 dB). We can observe that the estimates of the last two coefficients are close to zero.

For performance comparison, the mean squared error (MSE) of the channel estimate (after correcting scalar ambiguity) is defined as

$$\text{MSE}(\hat{\mathbf{h}}) = \frac{E[\min_\alpha ||\mathbf{h} - \alpha\hat{\mathbf{h}}||^2]}{||\mathbf{h}||^2}. \quad (16)$$

For OFDM signals (of size $N = 64$), simulations are carried out with the proposed approach and the subspace approach in [21]. We assume that $h[l]$ is independent zero-mean complex Gaussian random variable (i.e., Rayleigh fading is assumed) and $E[|h[l]|^2] = 1/L$ for the normalization purpose. The SNR is defined as $\text{SNR} = 1/\sigma_n^2$ with QPSK signaling of signal alphabet $\{(\pm 1 \pm j)/\sqrt{2}\}$. Once the channel is estimated, the minimum mean square error (MMSE) equalizer is used to detect the data symbols.

Fig. 2 shows the results for various values of SNR when $L = 5, M = Q - 1$, and $K = 10$. We consider the two subspace methods in [21] and [23]. Note that for the subspace method in [23], a smoothing factor (for convenience, we denote by Q_s) has to be decided. According to [23], $Q_s \geq N - 1/K - 1$, and we set $Q_s = 7$ for simulations. For the subspace method in [21], since $K < N$, a good sample covariance matrix for the noise subspace estimation cannot be obtained. This results in a poor performance of the subspace method in [21]. This problem was solved in [23] and a much better performance can be achieved as shown in Fig. 2. The proposed method can also provide a reasonably good performance when $K < N$, but its performance is worse than that of the subspace method in [23]. However, as mentioned in Section III, the proposed method can be more computationally efficient than the subspace method in [23] as N increases.

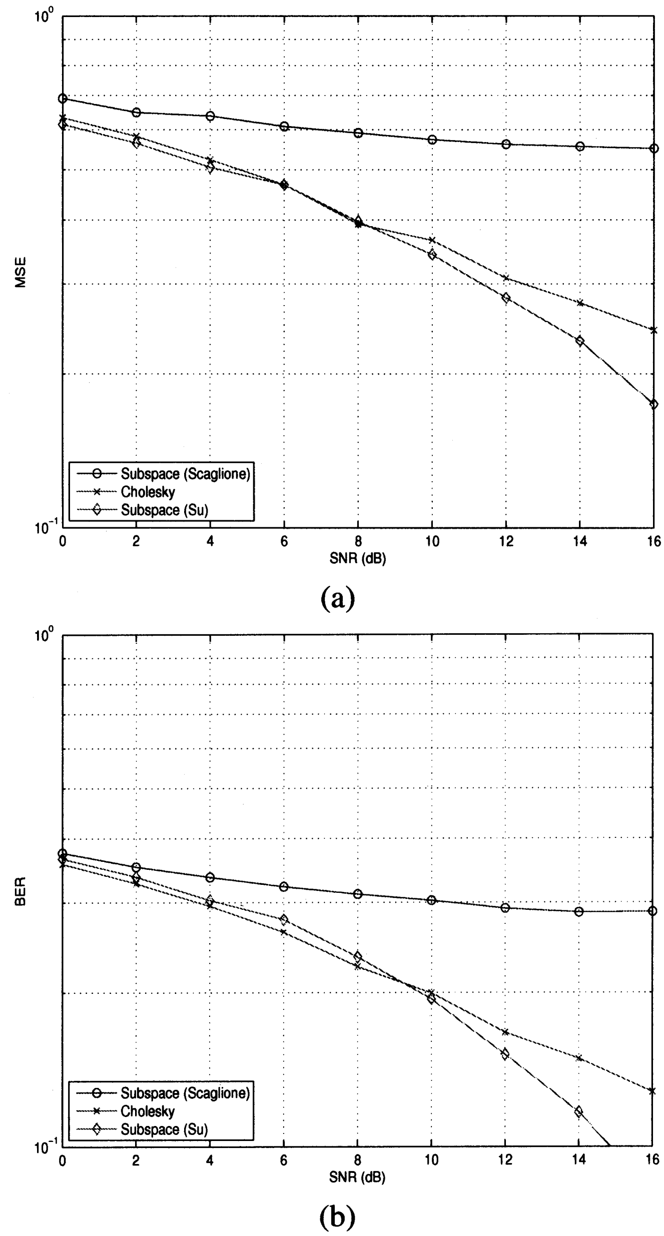

Fig. 2. MSE and BER for various SNRs $(N = 64, K = 10)$.

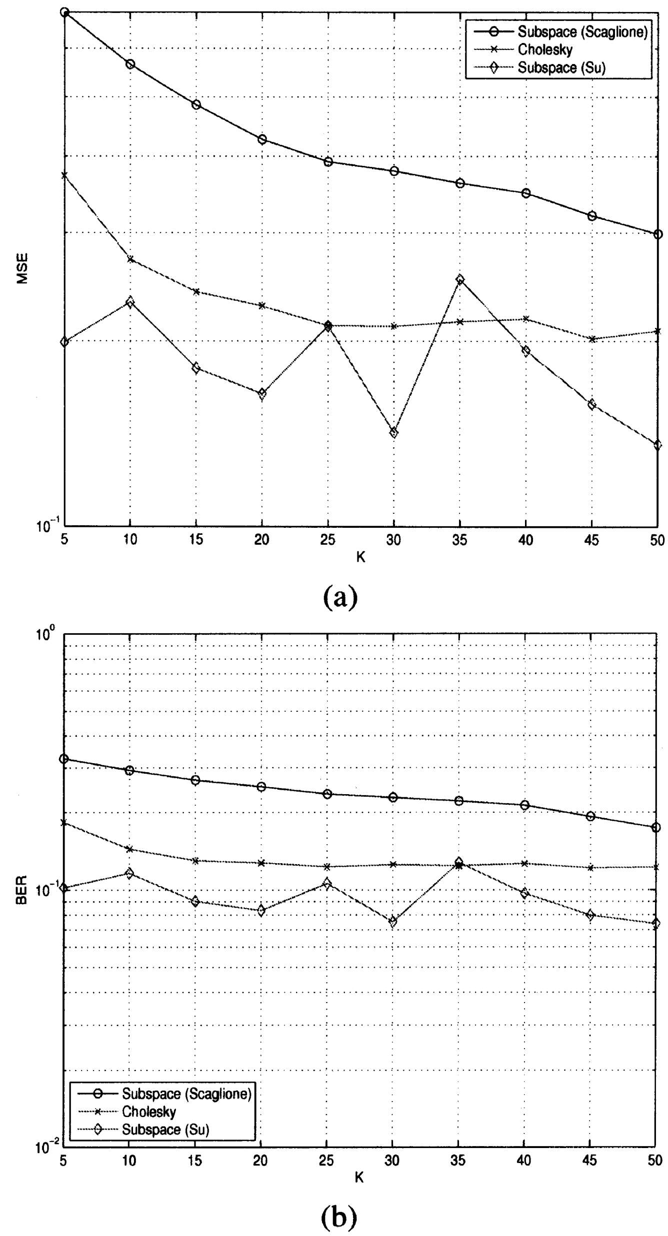

Fig. 3. MSE and BER for various values of $K(N = 64$ and SNR = 14 dB).

Fig. 3 shows the results for various values of K with SNR $= 14$ dB. For the subspace method in [23], we set $Q_s = \lceil N - 1/K - 1 \rceil$. In general, the subspace method in [23] outperforms. Note that it is shown that the MSE of the proposed method decreases slowly with K in Fig. 3. Generally, the MSE of the sample covariance matrix is proportional to $1/K$ and the MSE of the subspace method is also proportional to $1/K$ [22]. Unfortunately, this is not valid in the proposed method. As shown in [3], the error matrix of the Cholesky factor $\mathbf{G}$ is proportional to the condition number of the covariance matrix $\bar{\mathbf{R}}$. In general, the condition number of $\bar{\mathbf{R}}$ depends on the spectrum of the channel impulse response [10]. If there are spectral nulls, the conditional number becomes infinity. This implies that the performance of the proposed method depends on not only the number of data blocks, K, but also the spectrum of the channel impulse response. Although K is large, if the spectrum of the channel has nulls, the performance cannot be satisfactory. This is the reason why the MSE performance is slowly improved as K increases as shown in Fig. 3.

V. Concluding Remarks

A Cholesky factorization based approach for blind channel identification was proposed with a parametric technique to estimate the channel impulse response from the upper triangular matrix which retains a full information of the channel impulse response. Through simulations, it was shown that the proposed method can be effective even if the number of data blocks is small. This becomes attractive for time-varying channels. In addition, the proposed method becomes much more computationally efficient than the subspace method as the length of packet increases.

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MMSE WL Equalizer in Presence of Receiver IQ Imbalance

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***Abstract*—In this correspondence, with reference to the transmission over a linear time-dispersive channel, we address the problem of the in-phase and quadrature-phase (IQ) imbalance compensation when a single-carrier modulation scheme is used. Low-cost fabrication technologies and high data-rate transmissions render the conventional receivers very sensitive to the imperfections of their analog stage and, hence, proper countermeasures need to be adopted. Since the IQ imbalance renders the received signal rotationally variant, we propose to resort to the widely linear (WL) filtering in the synthesis of the receiver. More specifically, the synthesis of the WL receiver is performed by assuming perfect knowledge of the IQ imbalance parameters, but a new blind algorithm for IQ imbalance compensation, which outperforms the existing blind technique, is also proposed.**

***Index Terms*—Blind equalization, IQ imbalance, widely linear.**

I. Introduction

Most of the communication receivers include an analog stage where frequency conversion (or sometimes the low-pass signal extraction) takes place. An ideal model for such an analog stage is often employed for receiver synthesis and for its analysis. The conversion stage, as well as any other stage, is subject to the unpredictable imperfections of its analog components. In particular, the conversion stage suffers from the imbalance between the two periodic signals in the in-phase and quadrature (IQ) branches of the converter: $|c_1^I| \neq |c_1^Q|$ and/or

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