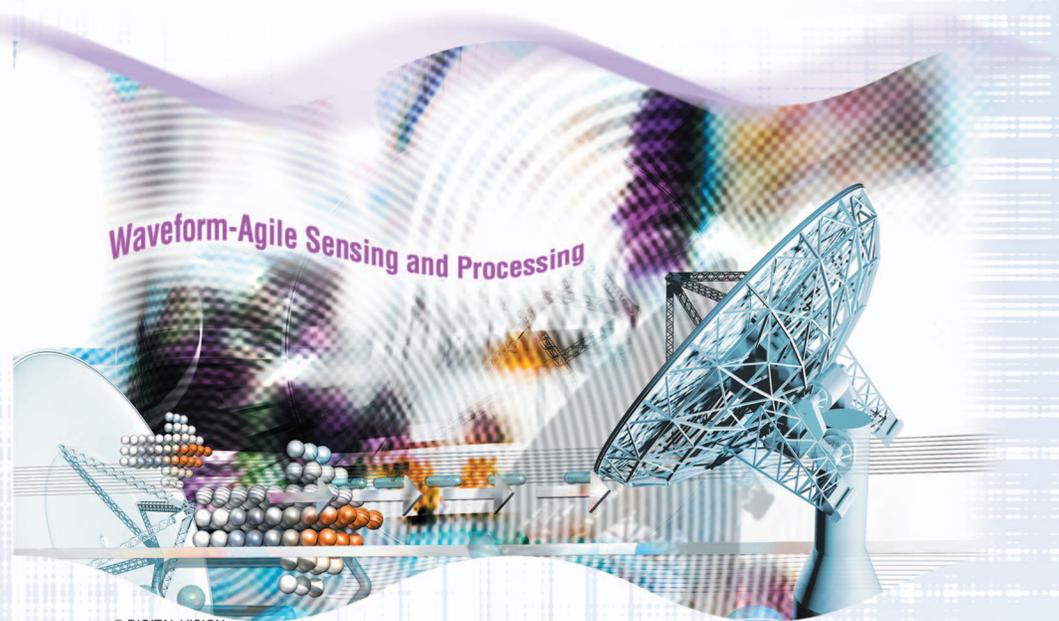


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Reprinted from IEEE Signal Processing Magazine, 2009; 26 (1):12-21

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# Waveform Libraries

[ Measures of effectiveness for radar scheduling ]

**A**ctive radars transmit sequences of electromagnetic pulses to illuminate a scene. Energy from these pulses scattered by objects in the scene is captured at one or more receivers for processing. Much attention has been devoted to development of effective receiver design and operating principles, including algorithms for extracting essential information about the scene from the received signals. Development of suitable waveforms to serve as the transmitted pulses has also been a topic of ongoing interest, though this has played a less central role than receiver design in the historical advancement of radar technology. One reason for this is that radar receiver technology has aggressively capitalized on advances in digital computing by incorporating increasingly sophisticated digital signal processing algorithms, in particular, for adaptive processing. Until recently, radar transmitters have not enjoyed the same degree of software-driven agility. Rather, they have typically relied on a few preprogrammed waveforms or waveform sequences that were adapted to evolving scenarios only in very coarse ways (e.g., the waveform changed when the radar was switched from search mode to tracking mode).

Over the past decade, new radar transmitter concepts have emerged that incorporate highly agile, software driven waveform generators. Implementation of prototype radar systems, and anticipation of

*Digital Object Identifier 10.1109/MSP.2008.930415*

operational systems, offering vastly increased transmitter agility has fueled a surge of research to identify ways to exploit agile transmitter capabilities in practice. Although few demonstrations in real radars or even test-bed systems have been carried out to date, a substantial corpus of theoretical and simulation results clearly indicates that waveform agility, if suitably applied, can yield highly significant performance gains across a variety of radar application scenarios. Many of the schemes introduced for exploiting waveform agility involve closed-loop waveform scheduling, a simple form of which is depicted in Figure 1. In a processing epoch, the transmitter sends a waveform or sequence of waveforms selected according to some criterion that depends on knowledge about the scene accumulated in prior processing epochs. Under ideal circumstances, one can envision that the waveform scheduler has access to a state estimate encompassing everything of interest that can be deduced about the scene from prior epochs. If the scene is dynamic, as in target tracking for example, the scheduler would also have available a means to update the scene estimate to account for evolution between the time of the last measurement and the time at which the next measurement will be taken. Then, a waveform or waveform sequence could be designed to optimize acquisition of information needed to update the scene estimate with respect to some performance metric. At least one key issue can make this scheme impractical for many actual radar applications: even assuming the availability of an algorithm to produce a custom-designed waveform that optimizes the performance criterion, typical epoch durations allow only a very short time (less than 100 ms—often much less) for waveform design. In view of this, an appealing approach is to design a library of waveforms offline and require the waveform scheduler only to select the most suitable waveform from the library rather than undertake the full design in real time. Elaborating on this approach, the issues it entails, and the status of its development is the focus of this article.

Many researchers have contributed to this circle of ideas over the past few years and, although some are cited here, space does not permit inclusion of a comprehensive survey of relevant work. The example presented to illustrate the performance of waveform scheduling using libraries in target tracking applications draws on the authors' own work, but is intended to exemplify a broader set of contributions by numerous researchers.

### WAVEFORM SCHEDULING

Modern radar systems perform many functions across a variety of application regimes. While waveform scheduling is of interest for most of these, perhaps the most natural setting in which to explain the fundamental approaches and issues is in multitarget tracking. Development of radar waveform

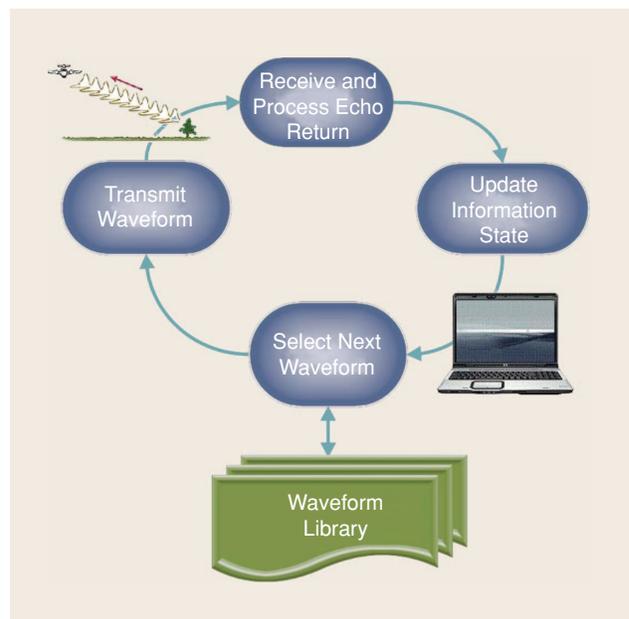
**THE WAVEFORM SELECTOR USES THE INFORMATION STATE TO DETERMINE, IN REAL TIME, WHICH WAVEFORM OR SEQUENCE OF WAVEFORMS FROM THE LIBRARY TO TRANSMIT.**

scheduling to support tracking has received a lot of attention over the past few years, so it is also a suitable regime in which to illustrate performance gains achieved to date by waveform scheduling. Additionally, tracking is an especially well suited application for waveform sched-

uling because the tracker's state encapsulates information accumulated from past measurements and its prediction step propagates this information forward to provide at least a partial description of the scene to be interrogated by the next transmitted waveform.

To understand why one might anticipate that closed-loop dynamic waveform scheduling should offer performance enhancement compared to repeated use of the same waveform in each measurement epoch, regardless of how well designed that waveform might be, it is useful to consider how pulse-Doppler radars work. For this purpose, it is sufficient to assume that transmitter and receiver are collocated (i.e., a monostatic radar) and to adopt a simplified model of the propagation and scattering of the electromagnetic waveform emitted by the transmitter. In this model, the echo corresponding to a particular target that arrives at the receiver is a replicate of the transmitted waveform that is:

- time delayed according to the distance between the radar platform and the target
- Doppler shifted due to the relative velocities of the target and radar
- attenuated and embedded in additive interference from noise in the transmission channel and superposition of returns from other scatterers.



**[FIG1]** Radar waveform scheduling entails a closed-loop process.

The time delay is an essential quantity for the radar to estimate accurately because it describes the range (i.e., the distance between the radar platform and the target). Because the waveform is transmitted on a carrier, it is common to assume (the so-called narrowband assumption) that the Doppler effect is well approximated by a frequency shift. Measurement of the Doppler shift is also important, especially in tracking, because it encodes information about the velocity of the target. The target echo is typically very weak compared to the noise and other interference present in the received signal, so the signal is processed, at least conceptually, with a bank of matched filters [7] corresponding to physically viable intervals of time delay and Doppler shift values at which the target echo might be present. The result of this receiver processing is generally portrayed as a two-dimensional (2-D) range-Doppler diagram, with time delay (range) on one axis, Doppler shift on the other, and the magnitude of the corresponding matched filter output appearing at each point of this range-Doppler plane. It is essentially an estimate of a portion of the magnitude cross-ambiguity function for the transmitted waveform and the received signal.

**OVER THE PAST DECADE, NEW RADAR TRANSMITTER CONCEPTS HAVE EMERGED THAT INCORPORATE HIGHLY AGILE, SOFTWARE DRIVEN WAVEFORM GENERATORS.**

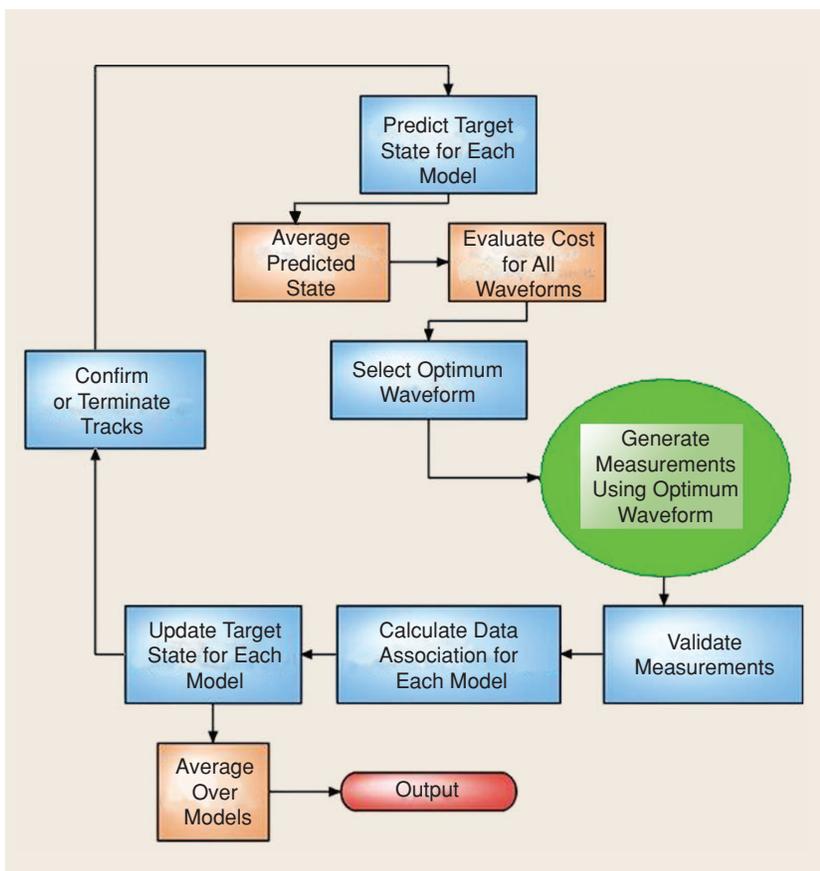
An ideal waveform would unerringly render a point reflector in the scene as a single point in the range-Doppler plot. If such a waveform existed, there would be no reason to change the transmitted waveform from epoch to epoch. But it is well known that there is no such

ideal waveform, even without imposing other desirable properties such as a constant power. Moyal's identity (see, for example, [13]) implies that, for any waveform and any form of linear processing of the received signal, the range-Doppler plot will provide a blurred image of the point scatterers in the scene being illuminated. Further, this blurring is a well-defined function of the illuminating waveform and the specifics of the processing performed on the received signal. This blurring of point scatterers is seriously detrimental to a radar's performance. For example, a moving target of interest can be obscured by the smearing resulting from a large static reflector. Frequently, such sources of clutter manifest much stronger signatures in the range-Doppler plot than those of targets of interest; their smeared images can completely obscure the target return even if they are separated from the target in both the range and Doppler dimensions.

Different choices of waveforms yield different smearing patterns, and this is a key source for the utility of waveform scheduling in radar. In the tracking problem, the range and Doppler vicinity of a target as well as significant sources of clutter is often predictable from past measurements. Although these predictions will typically be uncertain (e.g., have associated covariances), it is often possible to tell in advance of transmission whether a particular waveform is likely to manifest a smearing pattern that causes the target to be obscured by clutter (or two target signatures to interfere with one another, etc.) This perspective leads naturally to the problem of quantifying, given an information state for a radar scene (e.g., means and covariance matrices for the states of each target and significant interference source in a scene), the likelihood that a particular waveform will optimize a given performance criterion.

**ONLINE MEASURES OF EFFECTIVENESS**

Figure 2 shows a schematic of a typical waveform scheduler. Although this represents a typical high-level design, it is not the only approach. The main components of this design are:



**[FIG2]** Components of a typical radar waveform scheduler.

- 1) a transmitter/receiver to send and collect radio frequency (RF) signals. In a monostatic radar system, these are colocated and often share the same antenna configuration (usually a phased array). To accommodate waveform scheduling, this component of the system is software driven to allow control over the transmitted waveform.
- 2) post-receiver return processing that controls the received beam pattern, matched filtering, and other processing of signals received by the antenna.
- 3) a waveform library from which transmit waveforms may be selected. In principle, it is possible that the library could be replaced by a program that generates waveform designs to meet criteria posed in real time. This is seldom viable in view of the fast operating tempo of modern radars, so libraries of digital waveforms that are essentially ready to transmit are stored in a memory-resident database.
- 4) a waveform selector that determines, based on input distilled from previous measurements as well as models of such things as target and clutter dynamics, which waveform or sequence of waveforms from the library to schedule for transmission.

To appreciate the issues in design of a waveform library, it is useful to consider the role of the waveform selector. This component of the scheduler has at its disposal an information state for the radar scene of interest. As noted above, ideally the information state encapsulates everything that is known about the scene at the time the next waveform will be transmitted as a result of prior measurements and underlying models. The latter are not restricted to target dynamics and wave propagation conditions, but increasingly include such scenario knowledge as positions of roads and buildings in urban environments or wind and sea state in maritime applications [6]. Dynamic scenes, such as in target tracking, will require forward propagation of target and perhaps clutter state vectors, by means of dynamical system models, from the time of the last measurement to the time of the next transmission. In any case, the information state will include information about uncertainty in knowledge of the scene's parameters, such as target position and velocity covariance estimates. The waveform selector uses the information state to determine, in real time, which waveform or sequence of waveforms from the library to transmit.

The waveform selector relies on a measure of effectiveness (MOE): a function of the information state and waveform that measures expected operational merit of transmitting that waveform in the current epoch. In practice, the same radar is usually used for a spectrum of different operational functions, each of which may have different performance goals. For example, an MOE might be designed to assess a waveform's usefulness in maintaining accurate tracks of the current targets. It

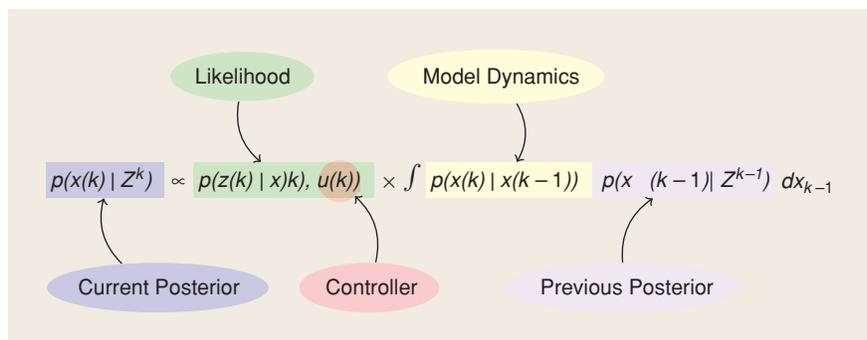
could be focused on mean track accuracy, or worst case, or some combination of these. It might be much more concerned with some targets than others and could also seek to identify new targets entering the scene as well as maintaining tracks for known targets.

Design of real-time MOEs is a very application-specific undertaking, and a radar system might employ a different effectiveness measure in each of several operating modes. This challenging area of research is tangential to the main points of this article. If attention is restricted to one-step-ahead scheduling (i.e., a greedy or myopic scheme), the waveform selector needs only to evaluate the MOE function on each waveform in the library to determine which is optimal to transmit. In practice, however, this can still present issues, foremost among which is computational complexity. As already mentioned, the operating tempo of modern radars is high, so the MOE may have to be evaluated for each waveform in the library many times per second. The following section summarizes some ideas and issues connected with multistep-ahead (i.e., nonmyopic) radar waveform scheduling. A central tenet in both the myopic and nonmyopic settings is that keeping the library small is important for maintaining computational feasibility.

Before turning attention to nonmyopic waveform scheduling, an additional comment is needed regarding the role of information gathered from previous measurements in selecting the next waveform for transmission. The perspective taken in the preceding discussion, in which prior measurements are used in conjunction with underpinning models to estimate an information state for the scene at the time of the next measurement, is prevalent in sensor scheduling literature. Typical of the information state update mechanisms used is the Bayes recursion depicted in Figure 3. Consideration of so-called separation theorems in stochastic control raises some question as to whether this approach constitutes the best use of information from past measurements (see [21]).

### NONMYOPIC SCHEDULING

In the broader literature of sensor management, nonmyopic sensor scheduling receives considerable attention. Probably the most well-developed approach for addressing multistep-ahead sensor scheduling problems involves their formulation in terms of partially observed Markov decision processes



**[FIG3] The Bayes recursion for tracking with scheduling.**

(POMDPs). An overview of POMDP theory in the context of sensor management is given in [2], and practical aspects of implementing POMDP-based sensor schedulers are discussed in [3]. Other approaches based on multiarmed bandits [12] have also been proposed for nonmyopic sensor scheduling [11], [20].

Despite considerable research activity in the area of nonmyopic sensor scheduling, especially in recent years, essentially none of this work has found manifestation in radar waveform scheduling. The only published work the authors know of that demonstrates the merits of multipulse radar waveform scheduling is [17], where it is shown that optimization over two pulses results both in a different waveform sequence and in better performance in a tracking application than greedy scheduling. Some of this work is summarized in the final section. One reason nonmyopic scheduling is less attractive for radar waveform scheduling than for other sensor scheduling applications is the need in radar for very expeditious waveform selection. In the POMDP approach, optimal scheduling policies are almost always extremely hard or impossible to determine exactly. Approximate solution methods, using reinforcement learning or Q-learning, for example, are available but have not been applied in this context, possibly because even these approximate approaches present severe computational demands for practical implementation.

Ultimately, if nonmyopic scheduling is desired, the MOE will have to be computed for a combinatorially large set of possible waveform sequences. In this case, even more so than with greedy waveform scheduling, keeping the library small and the MOE computationally simple are two key weapons in the battle to keep the problem computationally feasible.

**A SURROGATE MOE DOES NOT SEEK TO MEASURE THE EXPECTED OPERATION PAYOFF DIRECTLY; RATHER IT ASSIGNS A VALUE TO POTENTIAL MEASUREMENT BASED ON SOME MORE GENERIC MEASURE.**

### **INFORMATION-THEORETIC MOEs**

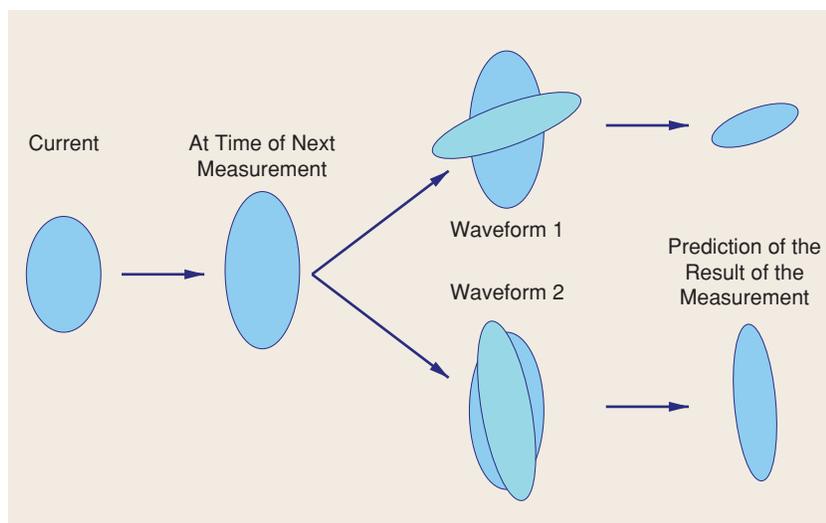
As previously discussed, the ultimate goal in designing MOEs for radar waveform scheduling is to quantify the payoff associated with transmitting a particular waveform given an information state. A highly simplified view of this is illustrated schematically in

Figure 4. In this example, operational performance is taken to be target position estimate accuracy as encompassed by the size of the error covariance ellipsoid (e.g., the determinant or trace of the error covariance matrix of the position estimate). The information state at the far left of the diagram is the result of estimation following the previous measurement, and it evolves according to dynamical equations modeling target state evolution between the times of measurements. The MOE for waveform 1 is determined by the predicted error ellipse that would be obtained if it were used in the next measurement. Waveform 2 is evaluated in the same way. Waveform 1 achieves a higher MOE (smaller predicted error ellipse) and is hence chosen for use in the next measurement epoch.

In this illustration, the waveform selector has explicit knowledge of both the current information state of the scene and an MOE that precisely quantifies the objective to be addressed by the next measurement. In practice, quantitatively encapsulating information from a scene, as implied from previous measurements, with sufficient fidelity to support optimal waveform selection, is a challenging problem in all but the simplest cases. And, even if this is accomplished, formulating an MOE that exactly captures the value of a particular waveform in addressing the objective given the information state is still difficult. For these reasons, several authors have proposed the use of surrogate MOEs in sensor scheduling applications (e.g., [8] and [10]). A surrogate MOE does not seek to measure the expected operational payoff directly; rather it assigns a value to a potential measurement based on some more generic measure, usually information-theoretic in nature, of the amount of information that will be gained from that measurement.

In exchange for accepting a good choice of measurement selection rather than one that is certifiably optimal, surrogate MOEs can offer relative computational simplicity and make effective use of coarser quantifications of the information state. In much of the literature on scheduling of waveforms for tracking, for example, the information state available is just the track-error covariance.

It is important to note that MOEs have a dual role. They are used both in real-time waveform scheduling and in waveform library design. In the latter case, the



**[FIG4] One-step-ahead prediction in scheduling a pair of waveforms for tracking.**

library designer does not have in-hand an explicit information state for which each waveform is to be optimized. In fact, if a radar serves multiple functions (e.g., surveillance and tracking), the designer of the waveform library will not even have a single specific operational performance criterion with which to optimize the design of the waveforms in the library. These considerations make information-theoretic MOEs especially important for waveform library design. As a general rule, surrogate MOEs used in this role are even more generic than those used in real-time scheduling because less is usually known about both information state and operational objectives during the offline library design process than in real-time waveform selection.

Pioneering work of Bell [1] provides strong motivation for the merits of surrogate information theoretic MOEs in sensor scheduling, and contributions of many others, a good sampling of which are summarized and cited [8], have cemented this approach in the sensor scheduling arsenal. Bell's justification of mutual information as a surrogate for operational measures of effectiveness is particularly appealing. It can be illustrated in concept by considering the target state as a single random variable (RV)  $X$ , the measurement as another RV  $Y$ , and the measurement process as a channel with input  $X$  and output  $Y$ . Taking the perspective that a continuous parameter estimation problem is a limiting case of a classification problem, Bell argues that the problem of optimizing estimation of  $X$  using one of a family of measurements  $Y_\mu$  is equivalent to maximizing the number of equiprobable classes into which  $X$  can be classified given  $Y_\mu$ . Application of Shannon's theorem to the noisy channel from  $X$  to  $Y_\mu$  implies that this is, with arbitrarily small probability of error,  $N = \lceil \exp I(X; Y_\mu) \rceil$  where  $I$  denotes mutual information [5]. Thus better estimation results from greater mutual information between  $X$  and  $Y_\mu$ ; i.e.,  $\mu$  should be chosen so that the mutual information between  $X$  and  $Y_\mu$  is maximized. The relationship between mutual information and entropy  $\mathcal{H}$  is given by  $I(X; Y) = \mathcal{H}(X) - \mathcal{H}(X|Y)$ , so choosing  $Y$  to maximize mutual information is equivalent to minimizing the conditional entropy  $\mathcal{H}(X|Y)$ .

An important realization for waveform scheduling for tracking occurred in the work of Kershaw and Evans [9, p. 1519], who recognized that, in a tracking context, information performance of a waveform could be encapsulated in the Hessian (i.e., the matrix of second-order partial derivatives)  $\mathbf{R}_w$  of the squared modulus of the auto-ambiguity function of the waveform. As described by van Trees [19, p. 300], this is the Fisher information for estimation of the range-Doppler position of the target from the radar measurement, and serves as an approximate covariance matrix for the error in the measurement of target parameters. As noted above, the target state covariance

**...IT IS OFTEN POSSIBLE TO TELL IN ADVANCE OF TRANSMISSION WHETHER A PARTICULAR WAVEFORM IS LIKELY TO MANIFEST A SMEARING PATTERN THAT CAUSES THE TARGET TO BE OBSCURED BY CLUTTER...**

matrix  $\mathbf{P}$  is often taken as the information state in tracking applications. Given an information state in this form, and assuming Bell's perspective, a good MOE to maximize in waveform selection is the mutual information between the target state and measurement. This mutual information can be expressed in terms of  $\mathbf{R}_w$  as  $\log \det(\mathbf{I} + \mathbf{R}_w^{-1}\mathbf{P})$ .

### WAVEFORM LIBRARY DESIGN

Lack of real-time knowledge about the information state and specificity of the performance goal is mitigated by creating a library sufficiently rich that it is highly likely to contain a waveform providing good performance for every real-time performance criterion and every information state that will be encountered in actual real-time operation. The goal of offline waveform library design is thus to produce a waveform library that is parsimonious, yet sufficient to provide a choice of high-performance waveform against operational measure of effectiveness that vary across a range of possible environments and unknown target states.

Accordingly, it is desirable to design to an MOE that is generic, and hence the information-theoretic MOE concept is at the foundation of the theory of waveform libraries for target tracking applications set forth in this section. This approach cannot, of course, guarantee optimal performance of a given library in most specific scenarios. Still, it supports a principled approach to designing waveform libraries that will provide good performance in practice.

Once mutual information is accepted as an appropriate generic MOE, it is straightforward to arrive at a measure of utility for a waveform library. The approach is to calculate the expected information obtained from measurement with a particular waveform, given the information state. As synopsis above, this is taken to be the mutual information between the target state covariance and the processed (e.g., matched-filtered) radar return resulting from the use of the waveform. With this set-up, a measure of utility for a waveform library  $\mathcal{L}$  emerges immediately as the expected value, over a distribution  $F$  on the possible state covariance matrices (i.e., all positive definite matrices  $\mathbf{P}$ ), of the maximum of this expected information over all waveforms in  $\mathcal{L}$  [13]; i.e.,

$$G_F(\mathcal{L}) = \int_{\mathbf{P} > 0} \max_{w \in \mathcal{L}} \log \det(\mathbf{I} + \mathbf{R}_w^{-1}\mathbf{P}) dF(\mathbf{P}). \quad (1)$$

Prior information about the class of state covariance matrices encountered in actual operation of the tracking system can be encoded in the distribution  $F(\mathbf{P})$ . If no such information is available, the Jeffries prior  $dF(\mathbf{P}) = d\mathbf{P}/(\det \mathbf{P})^{(m+1)/2}$ , where  $m$  is the dimension of  $\mathbf{P}$ , is an appropriate choice. It might appear that lack of knowledge of the nature of the distribution

$F(P)$  renders this concept of utility impractical. However, the structure of this formalism enables some general statements about waveform libraries that apply to all possible distributions  $F$ .

The utility metric (1) can be used to compare libraries of waveforms. In particular, an element of a library is redundant if it does not contribute to the utility; i.e., if the library has the same utility with or without that member. The evident fact that a redundant waveform can be removed without reduction in performance provides a means for constructing parsimonious libraries; i.e., libraries with no redundancy.

To illustrate the power of this concept, consider a library consisting of a family of (perhaps all) linear frequency modulated (LFM) waveforms of fixed time duration  $T$  and with a chirp rate lying between a minimum value  $\lambda_{\min}$  and a maximum value  $\lambda_{\max}$ . Such a range of chirp rates might, for instance, result from constraints in the transmitter hardware. Using the concept of utility, it can be shown that, regardless of the distribution  $F$ , any library containing the maximum and minimum chirp-rate waveforms has no greater utility than the waveform library containing only these two waveforms [16]. In other words, for an LFM library with a time-duration constraint, there is no point in having more than two waveforms. A similar result applies for a library in which the time-bandwidth product of the LFM waveforms is fixed, rather than the time duration. At this time, the only published source explaining the derivation of these surprising results is [16].

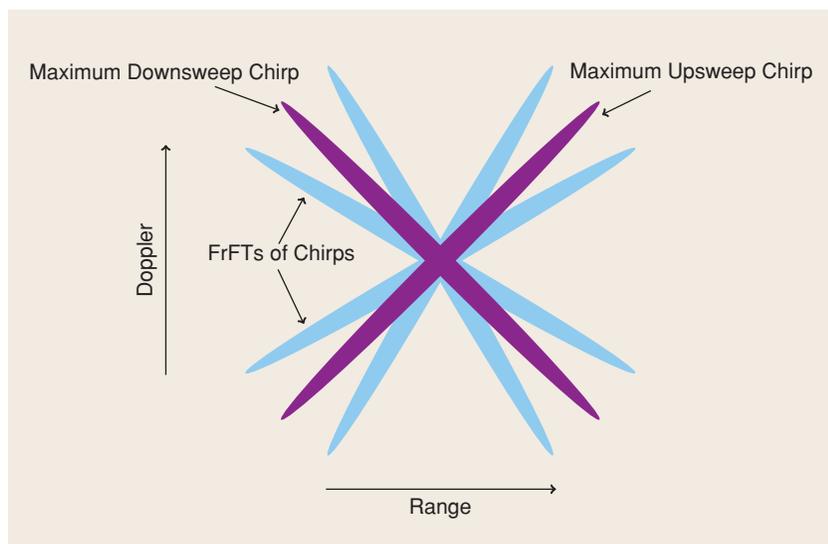
With the availability of the utility metric, it is natural to consider how the utility of a library can be increased. To date, there is no general methodology for this, but the utility concept provides a framework for addressing the problem in specific cases. This is illustrated by the LFM example just

described, in which insertion of more LFM waveforms between minimum and maximum chirp rates does not improve utility. One way to increase utility in this case is to employ the fractional Fourier transform, a tool whose importance in designing waveform libraries for tracking applications is established [15]. The fractional Fourier transform  $\mathcal{F}_\theta$  is parametrized by an angle  $0 \leq \theta < 2\pi$ , with the value  $\theta = \pi/2$  corresponding to the usual Fourier transform. Application of  $\mathcal{F}_\theta$  to a waveform has the effect of rotating its ambiguity function through an angle  $\theta$  in the range-Doppler plane. Given a particular state covariance matrix  $P$ , an ideal choice of waveform  $w$  would result in a covariance matrix  $P_w$  for which the major axis of the associated error ellipse is perpendicular to the major axis of the error ellipse for  $P$ . This can be seen by considering the effect of rotating the measurement covariance ellipses in Figure 4. With this in mind, Figure 5 illustrates the use of the fractional Fourier transform to construct a waveform library. Inclusion of fractional Fourier transforms of the maximum and minimum chirp rate waveforms yields a library with substantially higher utility than one containing only LFM chirps.

#### IMPLEMENTATION AND PERFORMANCE: WAVEFORM SCHEDULING IN A TRACKING APPLICATION

The waveform scheduling ideas presented earlier are illustrated in this section through the results of a simulation of a fairly realistic scheduling problem. In the simulated scenario, which is similar to scenarios reported in greater detail in [17] and [18], a radar tracks multiple maneuvering targets while undertaking surveillance for new targets. The waveform, the beam direction, and the revisit time to each known target are all available to be scheduled. Gains from reduction in revisit time by scheduling permit more surveillance time.

A brief synopsis of the simulation scenario is as follows. An S-band (3 GHz) radar system with a fixed pulse-repetition frequency (PRF) of 10 kHz is used to support surveillance and tracking in a region whose shape is a quarter sector of a disk of radius 15 km. The radar is assumed to integrate 16 pulses in each coherent processing interval and to employ a 32-element phased array antenna. This last characteristic is used to postulate target state covariance matrix based on the beam pattern of such an array. The simulation for which specific results are shown here involved two crossing targets, as depicted in Figure 6. A Swerling type-1 reflectivity model [19] and a maximum velocity of 50 m/s are assumed for both targets. Target maneuvers are implemented by switching between motion models for constant velocity, acceleration, deceleration, and coordinated turns to the left or right according to a Markov chain driven switching



**[FIG5]** A waveform library, with waveforms represented by the error ellipses of their measurement covariance matrices, consisting of maximum upswEEP and minimum downswEEP LFM waveforms and four other waveforms generated from these via the fractional Fourier transform.

model. The spatial clutter density is modeled by generating false detections based on a Gaussian mixture distribution that produces an average of two false detections per square kilometer per measurement interval.

Many existing trackers have the functionality to handle such a scenario to a greater or lesser degree. In this simulation, the so-called linear multitarget integrated probabilistic data association (LMIPDA) tracker of Mušicki [14] is used. It handles maneuvers using an integrated multiple model (IMM) approach in which the trajectory of the target is assumed to be described at any time by one of the dynamical models listed above. Switching between the tracker dynamical models is also governed by a Markov chain with a known transition matrix that is similar to, but not exactly the same as, the one that generates the target maneuvers in the simulation. Multiple possible maneuvers are considered at each epoch and are accepted with some probability based on the previous estimated state and the current measurement. For the remainder of the tracker model, Gaussian process and measurement noise are assumed. This particular tracker also estimates clutter density based on measurements. The minimum track update time is 0.5 s, with the maximum provided by the scheduler, but never in excess of 10 s. This track update interval is sufficiently long to allow many measurement dwells of the radar, so it is assumed that the radar can accommodate essentially simultaneous revisits to multiple targets.

As described in [14], the LMIPDA tracker models the dynamics of track existence probability using a Markov chain, thus providing a probability of track existence that supports the use of a track-before-detect approach. False alarm tracks are maintained until the probability of track existence falls below a threshold. In this simulation, the tracker is used to maintain tracks of known targets to within a specified accuracy, as determined by the determinant of the track error covariance matrix. Other cost functions could be used.

In cluttered environments, as simulated in this example, potential measurements result from both targets and clutter scatterers in each epoch. Target measurements are unidentified, but assumed to be present with a position, clutter, and target dependent probability of detection. It may be that more than one target is in the beam during a particular radar dwell, in which case the measurement is processed using the LMIPDA-IMM algorithm described in [14]. This recursive algorithm combines a multitarget data association technique (i.e., the LMIPDA) with the IMM approach. For each track, and at each epoch, the LMIPDA-IMM filter provides an a posteriori probability of target existence, used to confirm or terminate tracks, a track state estimate and estimate covariance, and filter inputs for recursion at the next epoch.

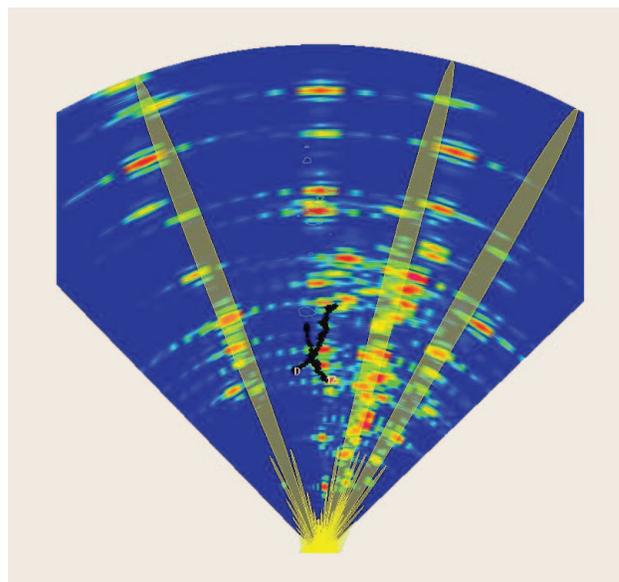
Waveforms are chosen from a library of six waveforms constructed as outlined in the previous section. The choice of waveform influences the measurement process through the covariance matrix of the measurement noise, as also described in the preceding section. Specifically, the waveforms used are an upsweep LFM chirp and a downsweep LFM chirp, together with

fractional Fourier transforms of these waveforms of order 0.4 for the upsweep chirp and 1.6 for the downsweep chirp. In addition, a continuous-wave (CW) pulse and a Golay pair are included in the waveform library. Each waveform has duration 12.8  $\mu\text{s}$ , and the bandwidth of the chirps is 5 MHz.

To schedule revisit times, a list is maintained of potential revisit intervals representing the possible number of epochs between measurements of any of the known targets. In this example the list consists of all time durations between 0.5–10 s in steps of 0.5 s. It is assumed that during any of these revisit intervals the target dynamics do not change, though the simulation of the targets permits maneuvers on an epoch-by-epoch basis.

The scheduling algorithm determines which target to measure (i.e., where to point the beam) and which waveform to use as follows. For each known target and each waveform, the track error-covariance is propagated forward using the dynamic model equations. This is done for each of the potential revisit intervals and each waveform in the library. Evidently, the number of combinations grows exponentially in the number of steps ahead, and soon becomes impractical for implementation. Here attention is restricted to myopic and two-step-ahead scheduling. The track-error covariance matrix is calculated for all possible combinations of sensor modes and the optimal sensor mode (waveform) chosen for each target to be the one that gives the longest revisit time, while constraining the absolute value of the determinant of the error covariance matrix to be smaller than a prescribed upper limit.

Once a target is measured, its revisit time is recalculated. In the absence of measurements, current knowledge is used to predict forward and update the covariance matrix, dynamic model probability density, and probability of track existence. Note that it is possible that there is no solution to the scheduling problem that maintains the track errors within the acceptable limits, but



**[FIG6]** The scenario for used in this illustrative example involved two crossing targets.

this does not occur in the simulations here. As an encouragement for the tracker to undertake surveillance in all directions, a number of virtual targets spaced across the azimuthal range are spaced across the azimuthal range introduced with tracks that degrade over time. This artifice is described in detail in [18].

**KEEPING THE LIBRARY SMALL  
AND THE MOE COMPUTATIONALLY  
SIMPLE ARE TWO KEY WEAPONS  
IN THE BATTLE.**

The (averaged) results of one collection of simulations comparing two-step ahead and one-step ahead scheduling with an unscheduled approach for one of the two are summarized targets in Figure 7. The scheduler seeks

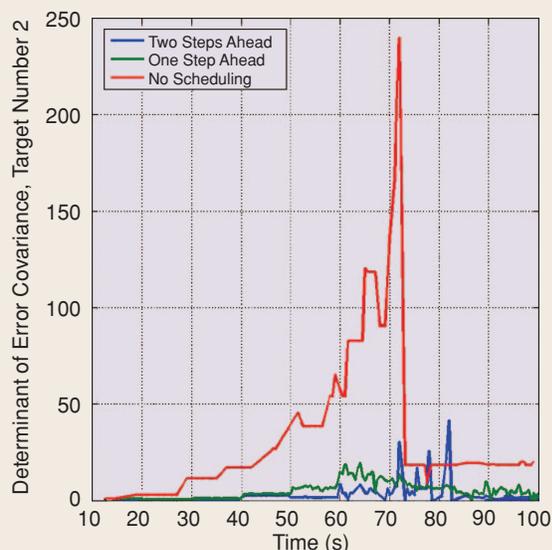
to maximize revisit time while constraining track accuracy, in terms of the determinant of the target state covariance matrix, to a specified level. In the unscheduled case, no constraint was imposed on the track accuracy and the revisit time was fixed. Note that revisit times are significantly reduced by scheduling and reduced somewhat more by scheduling two steps ahead rather than one. Thus, acceptable limits of track error are being achieved with far fewer measurements through the use of waveform scheduling from a predesigned library.

**CONCLUDING REMARKS**

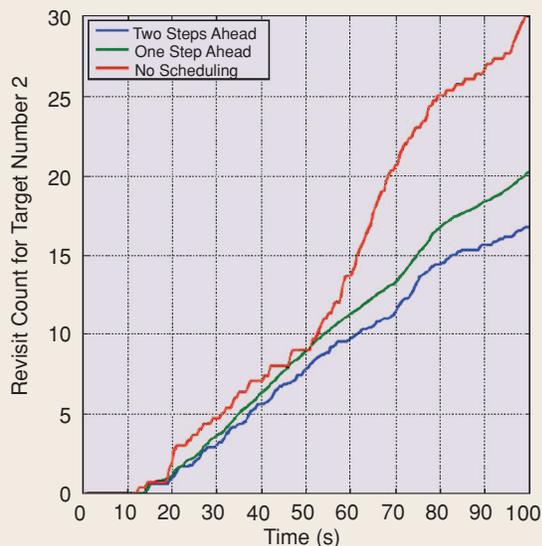
Our goal was to provide an overview of a circle of emerging ideas in the area of waveform scheduling for active radar. Principled scheduling of waveforms in radar and other active sensing modalities is motivated by the nonexistence of any single waveform that is ideal for all situations encountered in typical operational scenarios. This raises the possibility of achieving operationally significant performance gains through closed-loop waveform scheduling. In principle, the waveform transmitted in each epoch should be optimized with respect to a metric of desired performance using all information available from prior measurements in conjunction with models of scenario dynamics. In practice, the operational tempo of the system may preclude such on-the-fly waveform design, though further research into fast adaption of waveforms could possibly attenuate such obstacles in the future.

The focus in this article has been on the use of pre-designed libraries of waveforms from which the scheduler can select in lieu of undertaking a real-time design. The use of information-theoretic measures of effectiveness has been discussed as a means to provide surrogate assessments of waveform performance that are meaningful, if not optimal, general predictors of how well a waveform will serve in informing a spectrum of application-specific performance metrics. The concept of utility of a waveform library was introduced as a means for assessing the value of an entire library of waveforms, in terms of an information-theoretic measure of effectiveness, with respect to a distribution on possible information states that could be encountered in real-time application. Finally, the kind of operational performance gains attainable has been illustrated in a nontrivial target tracking scenario.

The approach synopsised in this article is only exemplary of recent research surrounding the emerging area of waveform scheduling in active sensing. Despite promising results, such as the performance gains shown in the tracking example presented here, many challenges remain



(a)



(b)

**[FIG7] Tracking error for Target 2, represented by (a) target-state covariance matrix determinant, and (b) revisit count for unscheduled operation and for one-step-ahead and two-steps-ahead beam and waveform scheduling. The scheduler seeks to minimize revisit count while constraining tracking error to a preset threshold.**

to be addressed to bring the power of waveform scheduling to the level of maturity needed to manifest major impact as a standard component of civilian and military radar systems.

## AUTHORS

**Douglas Cochran** (cochran@asu.edu) received his S.M. and Ph.D. degrees in applied mathematics from Harvard University, and degrees in mathematics from Massachusetts Institute of Technology and the University of California, San Diego. He is currently program manager in applied mathematics at the U.S. Air Force Office of Scientific Research. Previously, he held positions at Arizona State University, the U.S. Defense Advanced Research Projects Agency (DARPA), and at BBN Systems and Technologies, Inc. He was general cochair of the IEEE International Conference for Acoustics, Speech, and Signal Processing (ICASSP) in 1999 as well as cochair of the 1997 U.S.–Australia Workshop on Defense Signal Processing. His research interests are in applied harmonic analysis and statistical signal processing. He is a Senior Member of the IEEE.

**Sofia Suworova** (suworova@ee.unimelb.edu.au) holds a first degree from Moscow State University and a Ph.D. from Flinders University of South Australia. She is a senior research fellow at the University of Melbourne and was previously a research scientist at the Australian Defence Science and Technology Organisation. She also previously held a research position at the Australian Centre for Sensor Signal and Information Processing. Her research interests are sensor signal processing, with a particular emphasis on sensor scheduling for detection, and tracking applications involving radar and other sensing modalities.

**Stephen D. Howard** (stephen.howard@dsto.defence.gov.au) graduated in 1982 from La Trobe University, Melbourne, Australia. He received his M.S. and Ph.D. degrees in mathematics from La Trobe University in 1984 and 1990, respectively. He joined the Australian Defence Science and Technology Organisation (DSTO) in 1991, where he has been involved in research in the area electronic surveillance and radar systems. He led the DSTO research effort into the development of algorithms in all areas of electronic surveillance, including radar pulse-train deinterleaving, precision radar parameter estimation and tracking, estimation of radar intrapulse modulation, and advanced geolocation techniques. Since 2003, he has led the DSTO long-range research program in radar resource management and waveform design.

**Bill Moran** (b.moran@ee.unimelb.edu.au) is a professor of electrical engineering at the University of Melbourne, Australia, where he is the technical director of the Melbourne Systems Laboratory. Previously, he was a professor of mathematics at both the University of Adelaide and Flinders University. He also serves as a consultant to the Australian Department of Defence through the Defence Science and Technology Organisation. His research interests are signal processing, particularly with radar applications,

waveform design and radar theory, and sensor management. He also works in various areas of mathematics, including harmonic analysis and number theory and has published widely in these areas. He is a Member of the IEEE.

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