DC RESISTIVITY MODELLING AND SENSITIVITY ANALYSIS IN ANISOTROPIC MEDIA

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APPENDIX A. Dyadic Representation of Electrical Conductivity and Euler Angle Rotations

Consider a right Cartesian system O, with unit vectors $\vec{e}_1, \vec{e}_2$ and $\vec{e}_3$ along the respective axes x_1, x_2 and x_3 i.e. (x, y, z). Let (E_1, E_2, E_3) represent the electric field vector $\vec{E}$ and (j_1, j_2, j_3) represent the current density vector $\vec{J}$. Also, σ_{ij} represents the 2nd rank conductivity tensor, a linear operator transforming the electric field into the current density vector in the same system. The above definitions imply:

$$\vec{J} = j_1\vec{e}_1 + j_2\vec{e}_2 + j_3\vec{e}_3 \qquad \vec{E} = E_1\vec{e}_1 + E_2\vec{e}_2 + E_3\vec{e}_3 \tag{A1}$$

and

$$\begin{aligned} J_1 &= \sigma_{11}E_1 + \sigma_{12}E_2 + \sigma_{13}E_3 \\ J_2 &= \sigma_{21}E_1 + \sigma_{22}E_2 + \sigma_{23}E_3 \\ J_3 &= \sigma_{31}E_1 + \sigma_{32}E_2 + \sigma_{33}E_3 \end{aligned} \qquad \text{(A2}$$

Now rewriting equation (A2) in an alternative form that involves the vectors rather than their components:

$$\begin{aligned} \vec{J} &= J_1\vec{e}_1 + J_2\vec{e}_2 + J_3\vec{e}_3 \\ \vec{E} &= E_1\vec{e}_1 + E_2\vec{e}_2 + E_3\vec{e}_3 \end{aligned} \tag{A3}$$

and defining the vectors

$$\begin{aligned} \vec{\sigma}_1 &= \sigma_{11}\vec{e}_1 + \sigma_{12}\vec{e}_2 + \sigma_{13}\vec{e}_3 \\ \vec{\sigma}_2 &= \sigma_{21}\vec{e}_1 + \sigma_{22}\vec{e}_2 + \sigma_{23}\vec{e}_3 \\ \vec{\sigma}_3 &= \sigma_{31}\vec{e}_1 + \sigma_{32}\vec{e}_2 + \sigma_{33}\vec{e}_3 \end{aligned} \tag{A4}$$

it then follows that

$$\vec{J} = \vec{e}_1\left(\vec{\sigma}_1 \cdot \vec{E}\right) + \vec{e}_2\left(\vec{\sigma}_2 \cdot \vec{E}\right) + \vec{e}_3\left(\vec{\sigma}_3 \cdot \vec{E}\right) \tag{A5}$$

Here a dot implies a scalar dot product. Alternatively, this can be written symbolically as

$$\vec{J} = \left(\vec{e}_1\vec{\sigma}_1 + \vec{e}_2\vec{\sigma}_2 + \vec{e}_3\vec{\sigma}_3\right) \cdot \vec{E} \tag{A6}$$

or simply

$$\vec{J} = \Im \cdot \vec{E} \tag{A7}$$

Substituting for $\vec{\sigma}_1, \vec{\sigma}_2$ and $\vec{\sigma}_3$ from equation A4 the explicit expression for $\Im$ in terms of unit vectors is:

$$\Im = \sum_{i,j=1}^{3} \sigma_{ij}\vec{e}_i\vec{e}_j \tag{A8}$$

The entity $\Im$ is known as a dyadic and each of its elements a dyad. The representation in equation A8 is known as the nonion form of the dyadic. We may write $\sigma_{ij}\vec{e}_i\vec{e}_j = \vec{e}_i\vec{e}_j\sigma_{ij}$ but the order of the unit vectors cannot be changed for the most general dyadic.

Every dyadic $\Im$ has a scalar and a vector associated with it, which we denote by σ and $\langle\Im\rangle$, respectively.

$\sigma = \sigma_{11} + \sigma_{22} + \sigma_{33}$ where σ is also known as the trace of $\Im$.

(A9)

$$\langle\Im\rangle = \left(\sigma_{23} - \sigma_{32}\right)\vec{e}_1 + \left(\sigma_{31} - \sigma_{13}\right)\vec{e}_2 + \left(\sigma_{12} - \sigma_{21}\right)\vec{e}_3$$

Consider now the dyadic $R(\vec{e}, \omega)$, which describes a rotation of space about $\vec{e}$ by an angle ω. Such a rotation aligns the geographic frame with the natural frame of the rock. In addition to the representation of a rotation by the vector $\vec{\varpi} = \vec{e}\,\varpi$, it can also be represented by the three Eulerian angles (α, β, γ):

$$R(\vec{\varpi}) = R(\vec{e}, \omega) = R_1\left(\vec{e}_z, \alpha\right) \cdot R_2\left(\vec{e}_y \beta\right) \cdot R_3\left(\vec{e}_z \gamma\right) \tag{A10}$$

where the component rotations are given by:

$$R_1 \equiv \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 \equiv \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \quad R_3 \equiv \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A11}$$

The dyadic R_1 represents the rotation of space (known as active rotation in contradistinction to a rotation of the axis, which is called passive rotation) relative to the fixed axis (x,y,z) about the z axis by an angle α. This is followed by a rotation β about the new y axis terminated by a rotation γ about the new z axis. Since the dyadic *R* in A10 is orthogonal, it follows that the components of the conductivity tensor σ_{ij} in the geographic frame (x,y,z) can be written in terms of the principal values $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ in the natural frame and the component rotations *R* so that $\sigma_{ij} = R^T \hat{\sigma} R$.

$$\sigma_{ij} = \begin{bmatrix} r_{11}^2\hat{\sigma}_1 + r_{21}^2\hat{\sigma}_2 + r_{31}^2\hat{\sigma}_3 & r_{11}r_{12}\hat{\sigma}_1 + r_{21}r_{22}\hat{\sigma}_2 + r_{31}r_{32}\hat{\sigma}_3 & r_{11}r_{13}\hat{\sigma}_1 + r_{21}r_{23}\hat{\sigma}_2 + r_{31}r_{33}\hat{\sigma}_3 \\ r_{12}r_{11}\hat{\sigma}_1 + r_{21}r_{22}\hat{\sigma}_2 + r_{31}r_{32}\hat{\sigma}_3 & r_{12}^2\hat{\sigma}_1 + r_{22}^2\hat{\sigma}_2 + r_{32}^2\hat{\sigma}_3 & r_{12}r_{13}\hat{\sigma}_1 + r_{23}r_{22}\hat{\sigma}_2 + r_{33}r_{32}\hat{\sigma}_3 \\ r_{11}r_{13}\hat{\sigma}_1 + r_{21}r_{23}\hat{\sigma}_2 + r_{31}r_{33}\hat{\sigma}_3 & r_{12}r_{13}\hat{\sigma}_1 + r_{23}r_{22}\hat{\sigma}_2 + r_{33}r_{32}\hat{\sigma}_3 & r_{13}^2\hat{\sigma}_1 + r_{23}^2\hat{\sigma}_2 + r_{33}^2\hat{\sigma}_3 \end{bmatrix} \tag{A12}$$

Here the individual elements r_{ij} are given by:

$$\begin{aligned} & r_{11} = \cos\gamma\cos\beta\cos\alpha - \sin\gamma\sin\alpha \qquad r_{22} = -\sin\gamma\cos\beta\sin\alpha + \cos\gamma\cos\alpha \qquad r_{33} = \cos\beta \\ & r_{12} = \cos\gamma\cos\beta\cos\alpha + \sin\gamma\cos\alpha \qquad r_{21} = -\sin\gamma\cos\beta\cos\alpha - \cos\gamma\sin\alpha \\ & r_{13} = -\cos\gamma\sin\beta \qquad r_{31} = \sin\beta\cos\alpha \\ & r_{23} = \sin\gamma\sin\beta \qquad r_{32} = \sin\beta\sin\alpha \end{aligned} \tag{A13}$$

A special case of a 2.5D Tilted Transversely Isotropic (TTI) case in which $\alpha = \gamma = 0$ and arbitrary dip angle β, the conductivity tensor takes on the form:

$$\sigma_{ij} = R^T\hat{\sigma}R = \begin{bmatrix} \cos^2\beta\hat{\sigma}_1 + \sin^2\beta\hat{\sigma}_3 & \cos^2\beta\hat{\sigma}_1 & -\cos\beta\sin\beta\hat{\sigma}_1 + \cos\beta\sin\beta\hat{\sigma}_3 \\ \cos^2\beta\hat{\sigma}_1 & \cos^2\beta\hat{\sigma}_1 + \hat{\sigma}_2 & -\cos\beta\sin\beta\hat{\sigma}_1 \\ -\cos\beta\sin\beta\hat{\sigma}_1 + \cos\beta\sin\beta\hat{\sigma}_3 & -\cos\beta\sin\beta\hat{\sigma}_1 & \sin^2\beta\hat{\sigma}_1 + \cos^2\beta\hat{\sigma}_3 \end{bmatrix} \tag{A14}$$

Appendix B

Derivation of the θ and ϕ Spatial Derivatives

Equation (6.38) involves spatial derivatives terms of the polar angles θ and ϕ which specify, along with radial distance R, the subsurface point (x,y,z) at which the Frechet derivative is to be computed. Here we derive equations for these derivatives.

For the θ derivatives we use the relation $\cos\theta = z/R = z/(x^2 + y^2 + z^2)^{1/2}$ (B1)

to obtain

$$-\sin\theta \frac{\partial\theta}{\partial x} = \frac{\partial}{\partial x}\left(\frac{z}{(x^2+y^2+z^2)^{1/2}}\right) = \frac{-xz}{(x^2+y^2+z^2)^{3/2}}$$

$$-\sin\theta \frac{\partial\theta}{\partial y} = \frac{-yz}{(x^2+y^2+z^2)^{3/2}} \qquad \text{(B2)}$$

$$-\sin\theta \frac{\partial\theta}{\partial z} = \frac{\partial}{\partial z}\left(\frac{z}{(x^2+y^2+z^2)^{1/2}}\right) = \frac{x^2+y^2}{(x^2+y^2+z^2)^{3/2}}$$

But $\sin\theta = (1-\cos^2\theta)^{1/2} = (1 - z^2/R^2)^{1/2} = (x^2+y^2)^{1/2}/R$ (B3)

This then yields for the θ derivatives:

$$\frac{\partial\theta}{\partial x} = \frac{xz}{R^2.(x^2+y^2)^{1/2}}$$

$$\frac{\partial\theta}{\partial y} = \frac{yz}{R^2.(x^2+y^2)^{1/2}} \qquad \text{(B4)}$$

$$\frac{\partial\theta}{\partial z} = \frac{(x^2+y^2)^{1/2}}{R^2}$$

For the ϕ derivatives we use the relation $\tan\phi = y/x$

to obtain

$$\frac{\partial\phi}{\partial z} = 0$$

and (B5)

$$\sec^2\phi.\frac{\partial\phi}{\partial x} = \frac{-y}{x^2}, \qquad \sec^2\phi.\frac{\partial\phi}{\partial y} = \frac{1}{x}$$

with

$$\sec^2\phi = 1 + \tan^2\phi = 1 + (y^2/x^2) \tag{B6}$$

Giving

$$\begin{aligned}\frac{\partial\phi}{\partial x} &= -\frac{y}{x^2 + y^2}\\ \frac{\partial\phi}{\partial y} &= \frac{1}{x.(1 + y^2/x^2)}\end{aligned} \tag{B7}$$

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