DC RESISTIVITY MODELLING AND SENSITIVITY ANALYSIS IN ANISOTROPIC MEDIA

by

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Contents

Abstract

In this thesis I present a new numerical scheme for 2.5-D/3-D direct current resistivity modelling in heterogeneous, anisotropic media. This method, named the 'Gaussian quadrature grid' (GQG) method, co-operatively combines the solution of the Variational Principle of the partial differential equation, Gaussian quadrature abscissae and local cardinal functions so that it has the main advantages of the spectral element method. The formulation shows that the GQG method is a modification of the spectral element method and does not employ the constant elements and require the mesh generator to match the earth's surface. This makes it much easier to deal with geological models having a 2-D/3-D complex topography than using traditional numerical methods. The GQG technique can achieve a similar convergence rate to the spectral element method. It is shown that it transforms the 2.5-D/3-D resistivity modelling problem into a sparse and symmetric linear equation system, which can be solved by an iterative or matrix inversion method.

Comparison with analytic solutions for homogeneous isotropic and anisotropic models shows that the error depends on the Gaussian quadrature order (abscissae number) and the sub-domain size. The higher order or smaller the subdomain size employed, the more accurate the solution. Several other synthetic examples, both homogeneous and inhomogeneous, incorporating sloping, undulating and severe topography are presented and found to yield results comparable to finite element solutions involving a dense mesh.

The thesis also presents for the first time explicit expressions for the Fréchet derivatives or sensitivity functions in resistivity imaging of a heterogeneous and fully anisotropic earth. The formulation involves the Green's functions and their gradients, and is developed both from a formal perturbation analysis and by means of a numerical (finite element) method. A critical factor in the equations is the derivative of the electrical conductivity tensor with respect to the principal conductivity values and the angles defining the axes of symmetry; these are given analytically. The Fréchet derivative expressions are given for both the 2.5-D and the 3-D problem using both constant point and constant block model parameterisations. Special cases like the isotropic earth and tilted transversely isotropic (TTI) media are shown to emerge from the general solutions. Numerical examples are presented for the various sensitivities as functions of the dip angle and strike of the plane of stratification in uniform TTI media.

In addition, analytic solutions are derived for the electric potential, current density and Fréchet derivatives at any interior point within a 3-D transversely isotropic homogeneous medium having a tilted axis of symmetry. The current electrode is assumed to be on the surface of the Earth and the plane of stratification given arbitrary strike and dip. Profiles can be computed for any azimuth. The equipotentials exhibit an elliptical pattern and are not orthogonal to the current density vectors, which are strongly angle dependent. Current density reaches its maximum value in a direction parallel to the longitudinal conductivity direction. Illustrative examples of the Fréchet derivatives are given for the 2.5-D problem, in which the profile is taken perpendicular to strike. All three derivatives of the Green's function with respect to longitudinal conductivity, transverse resistivity and dip angle of the symmetry axis ($dG/d\sigma_l, dG/d\sigma_t, dG/d\theta_0$) show a strongly asymmetric pattern compared to the isotropic case. The patterns are aligned in the direction of the tilt angle. Such sensitivity patterns are useful in real time experimental design as well as in the fast inversion of resistivity data collected over an anisotropic earth.

Statement

This work contains no material which has been accepted for the award of any degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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