PATTERNS IN CORRELATION MATRICES ARISING IN WINE-TASTING AND OTHER EXPERIMENTS

by

Christopher J. Brien
B. Sc. Agric. (Sydney)

Thesis submitted for the Degree of Master of Agricultural Science

in The University of Adelaide,
Biometry Section.

May, 1980

CONTENTS

Page

SUMMARY

There are two distinct areas of research on which the work in this thesis impinges. They are methods for the analysis of patterns in correlation matrices and the analysis of taster performance in wine-tasting experiments in which the wines are scored.

For the analysis of patterns in correlation matrices, least squares procedures are developed to examine patterns under certain equal correlation hypotheses. The procedures are applied to the z-transforms of the elements of correlation matrices that can be based on either a single group of variables, or variables that can be cross-indexed by two factors such as the multitrait-multimethod matrices given by Campbell and Fiske (1959). The procedures are of the analysis of variance type, being investigative in the sense that, in the event that the correlation matrix is judged to depart from the hypothesised pattern, alternative models to be pursued further are indicated. The associated statistics are calculated directly from closed-form expressions, rather than requiring the iterative solution of some estimation function as is the case with some alternative methods.

The procedures are used to analyse the data from a number of wine-tasting and other experiments. The results obtained are shown to be similar, in many instances, to those obtained with maximum likelihood procedures applied to variance-covariance matrices; in other instances, large differences occur between the methods. The test for the hypothesis of equal correlation between all variables developed here is also shown to give similar answers to Lawley's (1963) test for the same hypothesis, in a number of cases.

For the analysis of taster performance in wine-tasting experiments in which the wines are scored, the method of examining patterns in correlation matrices can be applied to multitaster and multitaster-multisession

correlation matrices. Certain conditions to be fulfilled by multitaster-multisession matrices are specified; the extent to which they are met in a particular experiment can be ascertained from the results of these analyses. The data from several wine-tasting experiments are analysed and the results provide further substantive evidence of the lack of agreement and differences in reliability that can occur between tasters in such experiments.

As the technique is applied to data from a single experiment, it can be used, particularly when session replicates are included, to select tasters on the basis of their performance in the experiment under consideration - a highly desirable approach. Four duplicate-evaluation wine-tasting experiments, that were aimed at determining the effect of several treatments on wine quality, fit into this category and so are analysed in more detail. A group of less heterogeneous tasters is selected, where possible, for each of the experiments using the results of the analysis of the multitaster-multisession correlation matrices.

Compared with other techniques for selecting tasters on the basis of their results in a wine-tasting experiment, the analysis of multitaster-multisession correlation matrices has the advantages that both reliability and agreement are measured and that the measures are correlation coefficients.

However, even the subsets of selected tasters do not behave in a manner that would justify a single analysis for mean differences for each subset. Because this is likely to be a common phenomenon, it is recommended that wine-tasting experiments be designed to include session replicates and the scores of each taster be analysed for mean differences separately. The results of the analysis of the multitaster-multisession matrix can then be used to determine the confidence to be attached to the results of individual tasters in drawing inferences from the experiment.

SIGNED STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university and, to the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except when due reference is made in the text of the thesis.

C.J. Brien

ACKNOWLEDGEMENTS

It is with much appreciation that I acknowledge the guidance and assistance that I received from Professor A.T. James and Drs O. Mayo and W.N. Venables. In particular, Professor A.T. James suggested the approach taken in Chapters 3 and 4.

I am grateful to Dr P. May for providing me with the original problem, for encouraging me to undertake a higher degree and for his indulgence while I carried out the work.

The work was begun while I was employed by the CSIRO Division of Horticultural Research and completed after I moved to Roseworthy Agricultural College. I thank these institutions for their support.

I am greatly indebted to Mrs H.M. Simpson who performed the demanding task of typing the majority of this manuscript. I also wish to thank Mrs B. Adlard who typed appendices D-H.

Messrs D. Armstrong, J. Brown, A. Hoey, A. Hood, P. John, G. Kerridge, N. Lindsay, R. Moody, G. Prass, M. Tummel, B. Tyson, C. Weeks and P. Weste played a vital role in the work presented in this thesis. They were the tasters in the four wine-tasting experiments described in section 2.3.

Finally, I thank Ellen, Jamie and Melissa who gave up more than they might have so that I could do this work.

1. INTRODUCTION

In this thesis, using a method developed herein, I examine the patterns in correlation matrices arising from wine-tasting experiments, in order to investigate the tasters' performances in such experiments. In particular, the reliability of and agreement between the tasters will be studied. As will be argued below it is necessary to have tasters who are both reliable and who agree.

The analysis of a number of other experiments using the new procedure will also be presented to demonstrate the wider applicability of the procedure and to compare the conclusions of the new procedures with those of previous analyses.

The work in this thesis arose from the author's involvement in the design and analysis of a number of wine-tasting experiments conducted by the CSIRO Division of Horticultural Research. These experiments consisted of presenting a group of expert tasters with wines to be evaluated; the tasters assign subjectively-determined numerical scores to the wines on the basis of their perceived quality. To confirm these judgements, the wines were then presented to the tasters a second time to be re-evaluated. The primary purpose of the experiments was to evaluate the effects on wine quality of a number of treatments applied either to the vines or to the harvested fruit.

Such scores are usually analysed by analysis of variance techniques (see e.g. Marie *et al.* (1962) and Amerine and Roessler (1976)). Ough and Baker (1961), Marie *et al.* (1962) and Ough and Winton (1976), having investigated the problem, concluded that the distributions of scores are approximately normal for most tasters, although considerable experience with some scoring methods may be needed to achieve this. However, they also found that tasters' error variances are not always homogeneous. Furthermore, it is generally recognised that even expert tasters can differ in their assessment of a wine (see e.g. Baker and Amerine (1953), Amerine and Roessler (1976, p.7) and Ough and Winton (1976)).

The reasons for expert tasters differing in their assessments include the occurrence of psychologically induced errors (halo effects, contrast errors, positional effects and so on) and differences between tasters in

(i) inherent sensitivity to the various stimuli provided by wine;

(ii) opinions of the relative importance of the perceived stimuli;

(iii) experience, training, motivation, memory and concentration; and

(iv) adaptation

(Amerine *et al.* (1965, ch.5) and Amerine and Roessler (1976, ch.3)). Within a particular experiment these phenomena can be either persistent (i.e. phenomena that affect all evaluations of the wine (for example, differences in inherent sensitivity)) or transitory (i.e. phenomena that vary during the evaluation of the wine (for example, psychologically induced errors)). A particular phenomenon may, in different experiments, be either persistent or transitory. Thus, when the evaluations of a wine are completed in one or two days, experience and training may be persistent. But, when the evaluations take several weeks, experience and training may give rise to transitory effects. In general, persistently differing assessments will give rise to wine-taster interactions in the analysis of wine-tasting data by analysis of variance. On the other hand, transitory differences in assessments will contribute to the unexplained or random variability.

In spite of the difficulties mentioned above, it is not unusual, when analysing wine-tasting data, for an analysis including all the tasters to be performed and to use an error term that includes second- and higher-order wine-taster interactions. Sometimes even first-order interactions are included, as when a set of wines are each scored once by several tasters. For published examples of such analyses see Baker and Amerine (1953), Ough *et al.* (1969), Ough and Berg (1971) and Amerine and Roessler (1976).

When a set of wines is scored once by several tasters, it is impossible to obtain independent estimates of the wine-taster interaction and uncontrolled variability; these two sources of variability contribute to the error term. Thus, it is not possible to determine whether or not a wine-taster interaction has occurred and so the validity of any conclusions about wine differences, drawn from the overall wine means, is subject to the assumption that there was no wine-taster interaction. Further, the tasters' error variances must also be homogeneous if such conclusions are to be valid. As both wine-taster interaction and heterogeneity of tasters' error variances are not unlikely in wine tasting experiments, the validity of analyses such as those just described must be suspect without evidence of the absence of these phenomena.

Under these circumstances, I considered it desirable to examine the tasters' performances in wine-tasting experiments prior to any overall analysis of variance for the treatment differences. As stated previously, a method has been developed for this purpose; it is a least squares procedure applied to the z-transforms of the elements of correlation matrices.

For the CSIRO wine-tasting experiments, each of the variables on which a correlation matrix is based is a taster's scores in a particular session. This matrix is then of the same form as the multitrait-multimethod matrix of Campbell and Fiske (1959). The reliability and agreement of the tasters can be ascertained from an examination of each of the following three classes of correlations of which such matrices are comprised:

(i) there are the correlations between a taster's wine scores given on one occasion with those for the same set of wines but given on a subsequent occasion;

(ii) there are the correlations between two tasters' wine scores for the same set of wines given on the same occasion (or at the same position in a sequence of repetitions); and

(iii) the correlations between two tasters' scores for the same set of wines given on different occasions (or at different positions in the sequence of repetitions).

The first group of correlations provide a measure of the tasters' reliabilities and the other two groups provide a measure of the agreement between the tasters.

It should be recognised that the correlation coefficients do not in themselves provide irrefutable evidence of either agreement or reliability in that a good correlation between two tasters' scores or between a taster's scores on two different occasions may result from the consistent scoring of entirely different aspects of wine. However, relatively low correlations in a particular instance are indicative of a relative lack of consistency in comparison with situations in which relatively high correlations are obtained.

In Chapter 2, the experiments to be analysed are described. In Chapter 3, the large-sample, joint distribution theory of Fisher's z-transforms of the elements of a correlation matrix is obtained and this used to develop, in Chapter 4, a procedure for testing certain patterns in correlation matrices. In Chapter 5, the results of analyses, using the procedure developed in Chapter 4, are presented and discussed. In Chapter 6, the results of these analyses, as tests of equal-correlation type patterns, are compared with those obtained using maximum likelihood procedures applied to the variance-covariance matrix (Swain (1975) and Joreskog (1978)) and Lawley's test of equal correlation (Lawley (1963)). In Chapter 7, I discuss the procedure developed here and the results obtained with it. I also conjecture on its generalisation.

2. EXPERIMENTS AND DATA

In this chapter we describe a five-year viticultural field experiment, a single evaluation wine experiment and four double-evaluation wine experiments. The data from these experiments will be analysed using the method described in chapter 4 and the results of these analyses presented in chapter 5. We will also give, in chapter 5, the results of the analyses of (i) an experiment on the body composition of calves and (ii) the administration of five primary ability tests on each of two occasions. The correlation matrices and analyses of these last two examples have been presented previously (Swain (1975) and McDonald (1975), respectively).

2.1 FIVE-YEAR VITICULTURAL FIELD EXPERIMENT

May *et al.* (1973) report an experiment to investigate the effects of three "fixed" levels of pruning, three rootstock treatments and three trellises. The experiment was laid out in a split-split-plot arrangement, the trellis treatments being randomised to the main plots, the levels of pruning to the sub-plots and the rootstocks to the sub-sub-plots. The main plots were arranged in six blocks and altogether we are concerned with 162 vines.

Each vine received the same treatment for the five years 1969-73 and in each of these years the yield of each vine was measured. (The data for 1973 were not presented by May *et al.* (1973) as they were unavailable.)

The analysis of the results for any one year would follow the conventional analysis of variance for a split-split-plot (see for example Cochran and Cox (1957 sec. 7.2)), while an overall analysis of the results of all five years would normally be accomplished by including an additional factor for years and regarding the years as defining an additional split of the sub-sub-plots. This latter analysis is valid if the results in one year display the same variability as those in other years and the correlation between the results in two different years is the same for all pairs of years (Hunyh and Feldt (1970) and Rouanet and Lepine (1970)).

2.2 SINGLE-EVALUATION WINE EXPERIMENT

Baker and Amerine (1953) report an experiment in which five tasters assessed 13 Cabernet and 17 Riesling commercial wines. The taster scored each wine for ten components and these are summed to give a total score which is taken as a measure of a wine's quality. All the tasters were "highly trained" and "had extensive acquaintance with both Cabernet and Riesling wines".

"In order to determine the consistency of the ratings of the five tasters", Baker and Amerine (*loc. cit.*) analysed the total scores using a two-way analysis of variance. The calculated F values are shown in Table 2.1. Thus, for both wine types, the tasters detected a significant difference between the wines but there was no significant difference between the overall means of each taster's scores.

Table 2.1: Calculated F values from the two-way analysis of variance (including Tukey's test for non-additivity) performed on the scores given for Cabernet and Riesling wines.

Source	Cabernet	Riesling
Wines	4.34**	4.12**
Tasters	0.96 n.s.	1.90 n.s.
Non-additivity	1.09 n.s.	7.19 **

However, for the two-way analysis of variance to be valid, it is required that the wines and tasters are additive in their effects, i.e. that there is no wine-taster interaction. Baker and Amerine (*loc.cit.*) comment that "some tasters....are more consistent than others and have much greater sensitivity to varying chemical and physical properties of of the wine." This might lead one to conclude that it is likely that there will be a wine-taster interaction. To investigate this possibility, Tukey's one-degree-of-freedom for non-additivity was calculated and the resulting F-values are also presented in Table 2.1. This latter test indicates that there is no significant wine-taster interaction for the Cabernets but it would appear that there is a significant interaction for the Rieslings. However, examination of the residuals from the two-way analysis reveals that the magnitudes of the residual effects for wines 16 and 17 when scored by taster 2 are much larger than any of the other residuals; taster 2 gave these two wines inordinately low scores. It is well known that outliers such as these will cause Tukey's test for non-additivity to be significant (see e.g. Bliss (1967, sec. 11.5a)). Replacement of the score given by taster 2 for wine 16 with a missing value estimate lowers the Wine x Taster mean square from 39.76 to 20.50 and the F-value for Tukey's test of non-additivity is .002 and thus non-significant. (Replacement of both low scores by missing values further reduces the Wine x Taster mean square to 13.03 and results in a smaller F-value for Tukey's test of non-additivity.) Clearly, the significant result for Tukey's test of non-additivity obtained when all the scores for the Riesling wines were analysed can be attributed to the scores given by taster 2 to wines 16 and 17. It would appear, on the basis of Tukey's test, that the effects for wine and taster are additive for the other scores.

2.3 DUPLICATE-EVALUATION WINE EXPERIMENTS

Four wine-tasting experiments were conducted between 1972 and 1976 as part of the experimental programmes of the CSIRO Division of Horticultural Research. In these programmes wines were produced, using the small-scale wine-making procedures described by Antcliff and Kerridge (1976), from the grapes harvested from field trials involving a number of different treatments.

The primary purpose of the wine-tasting experiments was to compare the effects of the treatments on wine quality. To assess these effects the wines from a field trial were evaluated by a panel of tasters using the Australian Wine Show Judging System (Rankine, 1974). In this system, wines are allotted points for colour (0 to 3), bouquet (i.e. aspects of smell) (0 to 7) and palate (i.e. aspects of taste) (0 to 10). The points for the three components are summed to give a total score (out of 20) which is used as a measure of wine quality. This system was used because the tasters, experienced wine-industry personnel, were familiar with it and because it is the industry standard.

All the tastings except one, consisted of two sessions at each of which all the wines were evaluated, so that every wine was evaluated twice. A session was subdivided into several sittings at each of which 8 or 12 wines were presented for simultaneous evaluation by each taster. The same wines were presented to all tasters at a sitting, but the wines were coded and presented in a different randomised order to each taster. The use of several sittings was necessary because of the number of wines to be evaluated.

Tables 2.2 and 2.3 compare the four wine-tasting experiments and the wines evaluated in these experiments.

We note that,

(i) for all experiments except the first, batches of grapes from different portions of the field were kept separate during the wine-making and evaluation processes;

(ii) for all experiments, replicates of the wine-making process were made from the grapes from each field replicate of each treatment;

(iii) for all experiments, the wines were tasted twice, the two samples coming from the same bottle; however, in the fourth experiment, wines were presented in clear glasses at one sitting and in black glasses at the subsequent sitting;

Table 2.2: Comparison of four duplicate-evaluation wine-experiments

			Number of				
Experiment	Year	Variety	Scores (TxSxW)	Wines per sitting	Tasters	Sessions	Wines
1	1972	Shiraz	288	12	6	2	24
2	1973	Shiraz	576	8	6	2	48
3	1974	Crouchen	576	8	6	2	48
4	1976	Shiraz	384	8	8	2	24

Table 2.3: Comparison of wines evaluated in four wine-tasting experiments

	Number of			
Experiment	Field replicates	Treatments	Wine-making replicates	Bottle replicates
1	1	4	3	2
2	2	4	6	1
3	4	4	3	1
4	2	6	2	1

(iv) for the first experiment, each taster evaluated two bottles from the same field replicate.

As mentioned above, the tasters were experienced wine-industry personnel. A different set of tasters was used in each experiment although some tasters participated in more than one experiment. Altogether, 13 tasters were used and Table 2.4 shows the experiments in which each of these tasters participated.

Table 2.4: Participation of tasters

	Taster												
Experiment	A	B	C	D	E	F	G	H	I	J	K	L	M
1	✓	✓	✓			✓	✓		✓				
2	✓	✓	✓	✓	✓			✓					
3	✓	✓	✓	✓	✓	✓							
4	✓			✓		✓	✓			✓	✓	✓	✓

The scores from the four experiments are given in Appendix A. These are total scores (out of 20) for all but the fourth experiment where the sum of points for bouquet and palate only are given. As points for colour cannot be given when the wine is scored in black glasses, these are not included in the sum for clear glasses to make the results from the two types of glass comparable.

The results of all four experiments have been published. May (1977) presents the results from experiments 1 and 2, May *et al.* (1976) present the results from experiment 3, and Hale and Brien (1978) present the results from experiment 4. In all experiments, the total scores were analysed by analysis of variance; separate analyses were carried out on the scores of each taster because of the suspected heterogeneity of the tasters.

3. LARGE-SAMPLE JOINT DISTRIBUTION OF FISHER'S z-TRANSFORMS OF ELEMENTS OF CORRELATION MATRICES

Fisher (1921) gives the z-tranformation of the correlation coefficient viz.

$$z = g(r)$$
$$= \tfrac{1}{2} \log \frac{1+r}{1-r} .$$

Fisher demonstrated that the transformation results in a quantity that is "sensibly normal for all but the smallest possible samples". He showed the mean and variance of the quantity to be as follows:

$$\mu_1' = g(\rho) + \frac{\rho}{2(n-1)} \{1 + \frac{5+\rho^2}{4(n-1)} + \ldots\}$$

and

$$\mu_2 = \frac{1}{n-1} \{1 + \frac{4-\rho^2}{2(n-1)} + \ldots\}$$
$$= \frac{1}{n-1} + 0(n^{-2}) \qquad (3.1)$$

where ρ is the population correlation coefficient.

This transformation can be derived as a variance stabilising transformation (see Rao, 1965 (Sec. 6g.4)).

In this chapter, we first investigate the joint distribution of the z-transforms of the elements of a correlation matrix, obtaining asymptotic expressions for the variances and covariances of the z-transforms. Secondly, expressions for the variances and covariances in some particular cases are established. The simplicity of these expressions will be exploited in Chapter 4.

3.1 JOINT DISTRIBUTION OF THE z-TRANSFORMS OF THE ELEMENTS OF A CORRELATION MATRIX

In this section, the joint distribution of the z-transforms of the elements of a correlation matrix, R say, is investigated.

Let $\underset{\sim}{z}$ be the vector containing the $N = \frac{1}{2} p(p-1)$ observed z-transforms of the above diagonal element of the correlation matrix of order p,

and $\underset{\sim}{Z}$ be the vector of random variables underlying an observed $\underset{\sim}{z}$.

Also,

$$\underset{\sim}{\xi} = E[\underset{\sim}{Z}] = g(\underset{\sim}{\rho}) + O(n^{-1}) \qquad (3.2)$$

and $V(\underset{\sim}{\rho})$ = variance-covariance matrix of $\underset{\sim}{Z}$

where $\underset{\sim}{\rho}$ = the vector of population correlation coefficients.

Then, the distribution of $\underset{\sim}{Z} - \underset{\sim}{\xi}$ is asymptotically multinormal with expectation $\underset{\sim}{0}$ and variance $V(\underset{\sim}{\rho})$. This can be deduced from the fact that the variance-covariance matrix from which the correlation matrix R is obtained is asymptotically multinormal. By theorem (iii) of section 6a.2 of Rao (1965) $\underset{\sim}{Z}$, a function of the variances and covariances of the observations, is multinormal.

The purpose of the next two theorems is to obtain expressions for the elements of $V(\underset{\sim}{\rho})$ to second order of n. To do this we first establish, in theorem 3.1, expressions for the variances and covariances of the observed correlation coefficients for a set of multinormal variables. We use the same method as Kendall (1952, p. 211) used to obtain the variance of the correlation coefficient for bivariate normal variables. Rao (1965, section 6a.2) calls this method the δ method. In theorem 3.2, expressions for the asymptotic variances and covariances of the z-transforms are derived.

Theorem 3.1: If r_{xy} is the observed correlation between the xth and yth variables of a set of multinormal variables, then

i) $$\text{cov}(r_{ij},r_{k\ell}) = \frac{1}{n'}\{(\rho_{ik}\rho_{j\ell}+\rho_{i\ell}\rho_{jk})-\rho_{k\ell}(\rho_{ik}\rho_{jk}+\rho_{i\ell}\rho_{j\ell})$$
$$-\rho_{ij}(\rho_{ik}\rho_{i\ell}+\rho_{jk}\rho_{j\ell})$$
$$+\tfrac{1}{2}\rho_{ij}\rho_{k\ell}(\rho_{ik}^2+\rho_{i\ell}^2+\rho_{jk}^2+\rho_{j\ell}^2)\} + 0(n^{-2})$$

where ρ_{xy} is the population correlation coefficient

and n' = degrees of freedom of the original variance-covariance matrix.

ii) In particular, $$\text{cov}(r_{ij},r_{i\ell}) = \frac{1}{n'}\{\rho_{i\ell}(1-\rho_{ij}^2)(1-\rho_{i\ell}^2)$$
$$-\tfrac{1}{2}\rho_{ij}\rho_{i\ell}(1-\rho_{i\ell}^2-\rho_{ij}^2-\rho_{j\ell}^2+2\rho_{ij}\rho_{i\ell}\rho_{j\ell})\} + 0(n^{-2})$$

and $\text{var}(r_{ij})$

$$= \frac{(1-\rho_{ij}^2)^2}{n'} + 0(n^{-2})$$

PROOF

The proof of this theorem is based on the δ method (Rao (1965, Sec. 6a.2), Kendall (1952, p.208)).

i) The expression for the covariance of two correlation coefficients is obtained firstly for the general case.

Now, $$dr_{ij} = d\left(\frac{s_{ij}}{\sqrt{s_{ii}s_{jj}}}\right)$$
$$= \frac{ds_{ij}}{\sqrt{s_{ii}s_{jj}}} - \tfrac{1}{2}.\frac{s_{ij}ds_{ii}}{s_{ii}^{3/2}\sqrt{s_{jj}}} - \tfrac{1}{2}\frac{s_{ij}ds_{jj}}{s_{jj}^{3/2}\sqrt{s_{ii}}}$$
$$= \frac{1}{\sqrt{s_{ii}s_{jj}}}\left(ds_{ij} - \frac{s_{ij}ds_{ii}}{2s_{ii}} - \frac{s_{ij}ds_{jj}}{2s_{jj}}\right)$$

and $$dr_{k\ell} = d\left(\frac{s_{k\ell}}{\sqrt{s_{kk}s_{\ell\ell}}}\right)$$

$$= \frac{1}{\sqrt{s_{kk}s_{\ell\ell}}} \left(ds_{k\ell} - \frac{s_{k\ell}ds_{kk}}{2s_{kk}} - \frac{s_{k\ell}ds_{\ell\ell}}{2s_{\ell\ell}}\right)$$

Hence, $dr_{ij}dr_{k\ell} = d\left(\frac{s_{ij}}{\sqrt{s_{ii}s_{jj}}}\right) \; d\left(\frac{s_{k\ell}}{\sqrt{s_{kk}s_{\ell\ell}}}\right)$

$$= \frac{1}{\sqrt{s_{ii}s_{jj}s_{kk}s_{\ell\ell}}} \Big[ds_{ij}ds_{k\ell} - \frac{s_{k\ell}ds_{ij}ds_{kk}}{2s_{kk}} - \frac{s_{k\ell}ds_{ij}ds_{\ell\ell}}{2s_{\ell\ell}}$$

$$- \frac{s_{ij}ds_{ii}ds_{k\ell}}{2s_{ii}} + \frac{s_{ij}s_{k\ell}ds_{ii}ds_{kk}}{4s_{ii}s_{kk}} + \frac{s_{ij}s_{k\ell}ds_{ii}ds_{\ell\ell}}{4s_{ii}s_{\ell\ell}}$$

$$- \frac{s_{ij}ds_{jj}ds_{k\ell}}{2s_{jj}} + \frac{s_{ij}s_{k\ell}ds_{jj}ds_{kk}}{4s_{jj}s_{kk}} + \frac{s_{ij}s_{k\ell}ds_{jj}ds_{\ell\ell}}{4s_{jj}s_{\ell\ell}}\Big]$$

Thus,

$$\mathrm{cov}(r_{ij},r_{k\ell}) = \frac{1}{(\sigma_{ii}\sigma_{jj}\sigma_{kk}\sigma_{\ell\ell})^{\frac{1}{2}}} \Big[\mathrm{cov}(s_{ij},s_{k\ell}) - \tfrac{1}{2}\Big\{\frac{\sigma_{k\ell}\mathrm{cov}(s_{ij},s_{kk})}{\sigma_{kk}}$$

$$+ \frac{\sigma_{k\ell}}{\sigma_{\ell\ell}}\mathrm{cov}(s_{ij},s_{\ell\ell}) + \frac{\sigma_{ij}}{\sigma_{ii}}\mathrm{cov}(s_{ii},s_{k\ell}) + \frac{\sigma_{ij}}{\sigma_{jj}}\mathrm{cov}(s_{jj},s_{k\ell})\Big\}$$

$$+ \tfrac{1}{4}\Big\{\frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_{ii}\sigma_{kk}}\mathrm{cov}(s_{ii},s_{kk}) \; \frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_{ii}\sigma_{\ell\ell}}\mathrm{cov}(s_{ii},s_{\ell\ell}) + \frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_{jj}\sigma_{kk}}\mathrm{cov}(s_{jj},s_{kk})$$

$$+ \frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_{jj}\sigma_{\ell\ell}}\mathrm{cov}(s_{jj},s_{\ell\ell})\Big\}\Big]$$

It can be shown that $\mathrm{cov}(s_{ij},s_{k\ell}) = \frac{1}{n'}(\sigma_{ik}\sigma_{j\ell} + \sigma_{i\ell}\sigma_{jk})$ (Anderson (1958), p. 161 equation 14), and so,

$$\mathrm{cov}(r_{ij},r_{k\ell}) = \frac{1}{n'\sqrt{\sigma_{ii}\sigma_{jj}\sigma_{kk}\sigma_{\ell\ell}}} \{(\sigma_{ik}\sigma_{j\ell} + \sigma_{i\ell}\sigma_{jk})$$

$$- \left[\frac{\sigma_{k\ell}}{\sigma_{kk}}\sigma_{ik}\sigma_{jk} + \frac{\sigma_{k\ell}}{\sigma_{\ell\ell}}\sigma_{i\ell}\sigma_{j\ell} + \frac{\sigma_{ij}}{\sigma_{ii}}\sigma_{ik}\sigma_{i\ell} + \frac{\sigma_{ij}}{\sigma_{jj}}\sigma_{jk}\sigma_{j\ell}\right]$$

$$+ \tfrac{1}{2} \quad \frac{\sigma_{kj}\sigma_{k\ell}}{\sigma_{ii}\sigma_{kk}} \sigma_{ik}^2 + \frac{\sigma_{kj}\sigma_{k\ell}}{\sigma_{ii}\sigma_{\ell\ell}} \sigma_{i\ell}^2 + \frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_{jj}\sigma_{kk}} \sigma_{jk}^2 + \frac{\sigma_{ij}\sigma_{k\ell}}{\sigma_{jj}\sigma_{\ell\ell}} \sigma_{j\ell}^2 \quad \}$$

$$= \frac{1}{n'} \{ (\rho_{ik}\rho_{j\ell} + \rho_{i\ell}\rho_{jk}) - \rho_{k\ell}(\rho_{ik}\rho_{jk} + \rho_{i\ell}\rho_{j\ell})$$

$$- \quad \rho_{ij}(\rho_{ik}\rho_{i\ell} + \rho_{jk}\rho_{j\ell}) + \tfrac{1}{2}\rho_{ij}\rho_{k\ell}(\rho_{ik}^2 + \rho_{i\ell}^2 + \rho_{jk}^2 + \rho_{j\ell}^2) \}$$

ii) In particular, setting $i = k$, $\rho_{ii} = 1$ gives the simpler special case where one variable is in common.

$$\text{cov}(r_{ij}, r_{i\ell}) = \frac{1}{n'} \{ (\rho_{j\ell} + \rho_{i\ell}\rho_{ij}) - \rho_{i\ell}(\rho_{ij} + \rho_{i\ell}\rho_{j\ell}) - \rho_{ij}(\rho_{i\ell} + \rho_{ij}\rho_{j\ell})$$

$$+ \quad \tfrac{1}{2}\rho_{ij}\rho_{i\ell}(1 + \rho_{i\ell}^2 + \rho_{ij}^2 + \rho_{j\ell}^2) \}$$

$$= \frac{1}{n'} \{ \rho_{j\ell}(1 - \rho_{i\ell}^2 - \rho_{ij}^2 + \tfrac{1}{2}\rho_{i\ell}\rho_{ij}\rho_{j\ell}) - \tfrac{1}{2}\rho_{i\ell}\rho_{ij}(1 - \rho_{i\ell}^2 - \rho_{ij}^2) \}$$

If, in addition, we put $j = \ell$ we obtain the familiar result

$$\text{var}(r_{ij}) = \frac{1}{n'} (1 - 2\rho_{ij}^2 + \rho_{ij}^4)$$

$$= \frac{(1-\rho_{ij}^2)^2}{n'}$$

<u>Theorem 3.2:</u> If $z_{xy} = g(r_{xy})$, then

i) $$\text{cov}(z_{ij}, z_{k\ell}) = \frac{\text{cov}(r_{ij}, r_{k\ell})}{(1-\rho_{ij}^2)(1-\rho_{k\ell}^2)} + O(n^{-2})$$

ii) In particular,

$$\text{cov}(z_{ij}, z_{i\ell}) = \frac{1}{n'} \{ \frac{\rho_{j\ell}(1-\rho_{i\ell}^2 - \rho_{ij}^2 + \tfrac{1}{2}\rho_{i\ell}\rho_{ij}\rho_{j\ell}) - \tfrac{1}{2}\rho_{i\ell}\rho_{ij}(1-\rho_{i\ell}^2 - \rho_{ij}^2)}{(1-\rho_{ij}^2)(1-\rho_{i\ell}^2)} \}$$

$$+ O(n^{-2})$$

and, $$\text{var}(z_{ij}) = \frac{1}{n'} + O(n^{-2}).$$

PROOF: We note that $g'(r) = \frac{1}{1-r^2}$

i) $$\text{cov}(z_{ij}, z_{k\ell}) \simeq g'(r_{ij})\Big|\Big|_{r_{ij}=\rho_{ij}} \; g'(r_{k\ell})\Big|\Big|_{r_{k\ell}=\rho_{k\ell}} \; \text{cov}(r_{ij}, r_{k\ell})$$

$$= \frac{\text{cov}(r_{ij}, r_{k\ell})}{(1-\rho_{ij}^2)(1-\rho_{k\ell}^2)} + 0(n^{-2})$$

ii) Similarly,

$$\text{cov}(z_{ij}, z_{i\ell}) = \frac{\text{cov}(r_{ij}, r_{i\ell})}{(1-\rho_{ij}^2)(1-\rho_{i\ell}^2)}$$

$$= \frac{1}{n'} \left\{ \frac{\rho_{j\ell}(1-\rho_{i\ell}^2 - \rho_{ij}^2 + \tfrac{1}{2}\rho_{i\ell}\rho_{ij}\rho_{j\ell}) - \tfrac{1}{2}\rho_{i\ell}\rho_{ij}(1-\rho_{i\ell}^2 - \rho_{ij}^2)}{(1-\rho_{ij}^2)(1-\rho_{i\ell}^2)} \right\}$$

$$+ \; 0(n^{-2})$$

and, $$\text{var}(z_{ij}) = \frac{\text{var}(r_{ij})}{(1-\rho_{ij}^2)^2}$$

$$= \frac{1}{n'} + 0(n^{-2})$$

Then the variance-covariance matrix of the z-transform $V(\underset{\sim}{\rho})$ is a function of the elements of $\underset{\sim}{\rho}$ only.

This expression for the variance of a z-transform agrees with that given by Fisher (1921) (see equation (3.1)).

3.2 SOME PARTICULAR JOINT DISTRIBUTIONS

3.2.1 INDEPENDENCE

The form of $V(\underset{\sim}{\rho})$ under the hypothesis of independence between all the variables $(\underset{\sim}{\rho} = 0)$ is trivial, being

$$V(\underset{\sim}{\rho}) = \frac{1}{n'} I$$

Thus,

$$\underset{\sim}{Z} \sim N(\underset{\sim}{0}, \frac{1}{n'} I)$$

That is, the z-transforms are also independent and have variance $\frac{1}{n'}$.

3.2.2 EQUAL CORRELATION

Under the hypothesis of equal correlation between all the variables (i.e. $\underset{\sim}{\rho} = \underset{\sim}{1}\rho$), the expressions for the covariance of the z-transforms and hence the form of $V(\underset{\sim}{\rho})$, simplify to a great extent

$$\begin{aligned}
\mathrm{var}(z_{ij}) &= \frac{1}{n'} \\
&= v_1 \\
\mathrm{cov}(z_{ij}, z_{il}) &= \{\rho(1-2\rho^2 + \tfrac{1}{2}\rho^3) - \tfrac{1}{2}\rho^2(1-2\rho^2)\}/\{n'(1-\rho^2)^2\} \\
&= \frac{\rho(3\rho + 2)}{n' 2(1+\rho)^2} \\
&= v_2 \\
\mathrm{cov}(z_{ij}, z_{kl}) &= \frac{2\rho^2(1-\rho)^2}{n'(1-\rho^2)^2} \\
&= \frac{2\rho^2}{n'(1+\rho)^2} \\
&= v_3
\end{aligned}$$

Thus, in this case, the variance-covariance matrix, V say, of the z-transforms contains only three distinct values and we can write the matrix as:

$$V = v_1 I + v_2 A + v_3 B \quad . \tag{3.3}$$

where I is the N x N identity matrix,

A is the N x N incidence matrix for covariance between two correlations with one variable in common,

and B is the N x N incidence matrix for covariance between two correlations with no variables in common.

That is, $$\underset{\sim}{Z} \sim N(\underset{\sim}{1}\xi, V) \tag{3.4}$$

where $\xi \doteq g(\rho)$.

3.2.3 A MORE GENERAL EQUAL CORRELATION PATTERN

Sometimes the variables can be cross-indexed by two factors. For example, in experiments to assess the quality of a number of wines, such as those described in Chapter 2, p tasters score a number of wines for quality. All the wines are scored once at each of m sessions by each taster. A taster's score at a particular session is a variable and this set of mp variables is indexed by the factors Tasters and Sessions. The pairwise correlations for these variables can be arranged in a correlation matrix of the following general form, where A and B are the two factors indexing the variables.

$$
\begin{array}{cc|cccc|cccc|c|cccc}
{}^{A}/_{B} & & & 1 & & & & 2 & & & & & m & & \\
 & & 1 & 2 & \cdots & p & 1 & 2 & \cdots & p & \cdots & 1 & 2 & \cdots & p \\
\hline
 & 1 & 1 & r^{11}_{12} & \cdots & r^{11}_{1p} & r^{12}_{11} & \cdots & & r^{12}_{1p} & & r^{1m}_{11} & \cdots & & r^{1m}_{1p} \\
 & 2 & & 1 & & \vdots & \vdots & & & \vdots & & \vdots & & & \vdots \\
1 & \vdots & & & \ddots & \vdots & \vdots & & \ddots & \vdots & \cdots & \vdots & & \ddots & \vdots \\
 & p & & & & 1 & r^{12}_{p1} & \cdots & & r^{12}_{pp} & & r^{1m}_{p1} & \cdots & & r^{1m}_{pp} \\
\hline
 & 1 & & & & & 1 & r^{22}_{12} & \cdots & r^{22}_{1p} & & r^{2m}_{11} & \cdots & & r^{2m}_{1p} \\
 & 2 & & & & & & 1 & & \vdots & & \vdots & & & \vdots \\
2 & \vdots & & & & & & & \ddots & \vdots & \cdots & \vdots & & \ddots & \vdots \\
 & p & & & & & & & & 1 & & r^{2m}_{p1} & \cdots & & r^{2m}_{pp} \\
\hline
 & \vdots & & & & & & & & & & & \vdots & & \\
\hline
 & 1 & & & & & & & & & & 1 & r^{mm}_{12} & \cdots & r^{mm}_{1p} \\
 & 2 & & & & & & & & & & & 1 & & \vdots \\
m & \vdots & & & & & & & & & & & & \ddots & \vdots \\
 & p & & & & & & & & & & & & & 1
\end{array}
$$

In the context of psychological testing, matrices of this form have been called multitrait-multimethod matrices by Campbell and Fiske (1959).

There are $\frac{1}{2}\, mp(mp-1)$ distinct correlations in this matrix which we denote by

$r_{ij}^{k\ell}$ = the correlation between, on the one hand, the variable at the i^{th} level of B and the k^{th} level of A and, on the other hand, the variable at the j^{th} level of B and the ℓ^{th} level of A.

(Where necessary, we use the notation $r_{(ik)(j\ell)}$ for $r_{ij}^{k\ell}$.)

In this section, we derive expressions for some of the variances and covariances between the z-transforms of the elements of such matrices for a population correlation matrix of the following form:

$$
\begin{array}{cc|cccc|cccc|c|cccc}
 & & & 1 & & & & 2 & & & & & m & & \\
A/B & & 1 & 2 & \cdots & p & 1 & 2 & \cdots & p & \cdots & 1 & 2 & \cdots & p \\
\hline
 & 1 & 1 & \rho_1 & \cdots & \rho_1 & \rho_3 & \rho_2 & \cdots & \rho_2 & & \rho_3 & \rho_2 & \cdots & \rho_2 \\
 & 2 & & 1 & & \rho_1 & \rho_2 & \rho_3 & & \rho_2 & & \rho_2 & \rho_3 & & \rho_2 \\
1 & \vdots & & & \ddots & \vdots & \vdots & & \ddots & \vdots & \cdots & \vdots & & \ddots & \vdots \\
 & & & & & \rho_1 & & & & \rho_2 & & & & & \rho_2 \\
 & p & & & & 1 & \rho_2 & \rho_2 & \cdots\ \rho_2 & \rho_3 & & \rho_2 & \rho_2 & \cdots\ \rho_2 & \rho_3 \\
\hline
 & 1 & & & & & 1 & \rho_1 & \cdots & \rho_1 & & \rho_3 & \rho_2 & \cdots & \rho_2 \\
 & 2 & & & & & & 1 & & \rho_1 & & \rho_2 & \rho_3 & & \rho_2 \\
2 & \vdots & & & & & & & \ddots & \vdots & \cdots & \vdots & & \ddots & \vdots \\
 & & & & & & & & & \rho_1 & & & & & \rho_2 \\
 & p & & & & & & & & 1 & & \rho_2 & \rho_2 & \rho_2 & \rho_3 \\
\hline
 & \vdots & & & & & & & & & \ddots & \vdots & & & \\
\hline
 & 1 & & & & & & & & & & 1 & \rho_1 & \cdots & \rho_1 \\
 & 2 & & & & & & & & & & & 1 & & \rho_1 \\
m & \vdots & & & & & & & & & & & & \ddots & \vdots \\
 & & & & & & & & & & & & & & \rho_1 \\
 & p & & & & & & & & & & & & & 1
\end{array}
$$

That is, the population correlation matrix is given by

$$
\begin{aligned}
R = {} & I_m \otimes \{(1-\rho_1+\rho_2-\rho_3)I_p + (\rho_1-\rho_2)\,J_p\} \\
& + J_m \otimes \{(\rho_3-\rho_2)\,I_p + \rho_2\,J_p\}
\end{aligned}
\qquad (3.5)
$$

where $\otimes$ denotes the Kronecker product of matrices.

Define $H_{(k-1)\times k}$ to be a matrix containing k-1 orthonormal rows such that $H_{(k-1)\times k}\, J_k = 0$.

Thus,

$$H_{(k-1)\times k}\, H'_{(k-1)\times k} = I_{k-1}$$

If we let

$$C_A = H_{(m-1)\times m} \otimes J_{1\times p},$$

$$C_B = J_{1\times m} \otimes H_{(p-1)\times p},$$

$$C_{AB} = H_{(m-1)\times m} \otimes H_{(p-1)\times p}$$

and note that

$$(A \otimes B)(C \otimes D) = AC \otimes BD$$

and

$$(A \otimes B)' = A' \otimes B'$$

then it is straightforward to show that

$$C_A R C_A' = p(1 + (p-1)\rho_1 - (p-1)\rho_2 - \rho_3)\, I_{m-1},$$

$$C_B R C_B' = m(1 - \rho_1 - (m-1)\rho_2 + (m-1)\rho_3)\, I_{p-1},$$

and

$$C_{AB} R C_{AB}' = (1 - \rho_1 + \rho_2 - \rho_3)\, I_{(m-1)(p-1)}.$$

That is each of these last three matrices is spherical and so it is clear that the condition that the underlying correlation matrix is of the form specified by (3.5), together with the assumption of homoscedastic variances and equality of the variance-covariance matrix for each observation of the variables, is sufficient to ensure the validity of the F-tests in an overall analysis of variance of the observations (i.e. including factors A and B). However, as Hunyh and Feldt (1970) and Rouanet and Lepine (1970) point out it is not necessary that all of these homogeneity conditions be met for the F-tests in an overall analysis of variance to be valid.

Let $\underset{\sim}{z}'$ be the vector of the z-transforms of the observed correlation matrix,

with $\underset{\sim}{z}' = (\underset{\sim}{z}_1' \; \underset{\sim}{z}_2' \; \underset{\sim}{z}_3')$ (3.6)

where $\underset{\sim}{z}_1' = (z_{12}^{11}, z_{13}^{11}, \ldots, z_{1p}^{11}, z_{23}^{11}, \ldots, z_{12}^{22}, z_{13}^{22}, \ldots\ldots, z_{12}^{mm}, z_{13}^{mm}, \ldots),$

$$\underset{\sim}{z}_2' = (z_{11}^{12}, z_{11}^{13}, \ldots, z_{11}^{1m}, z_{11}^{23}, \ldots z_{22}^{12}, z_{22}^{13}, \ldots\ldots, z_{pp}^{12}, z_{pp}^{13}, \ldots),$$

and $$\underset{\sim}{z}_3' = (z_{12}^{12}, z_{13}^{12}, \ldots, z_{23}^{12}, \ldots\ldots, z_{21}^{12}, z_{31}^{12}, \ldots, z_{32}^{12}, \ldots\ldots, z_{12}^{13}, z_{13}^{13}, \ldots, z_{23}^{13}, \ldots)$$

That is,

i) $\underset{\sim}{z}_1'$ contains the m sets of $\frac{1}{2}p(p-1)$ transforms of diagonal blocks of the observed correlation matrix;

ii) $\underset{\sim}{z}_2'$ contains the p sets of $\frac{1}{2}m(m-1)$ transforms of the diagonal elements of the off-diagonal blocks; and

iii) $\underset{\sim}{z}_3'$ contains the $\frac{1}{2}m(m-1)$ sets of $p(p-1)$ transforms of the off-diagonal elements of the off-diagonal blocks.

Clearly,
$$E[\underset{\sim}{Z}_1] \doteq \{g(\rho_1)\}\ \underset{\sim}{1} = \xi_1\ \underset{\sim}{1}\ ,$$
$$E[\underset{\sim}{Z}_3] \doteq \{g(\rho_2)\}\ \underset{\sim}{1} = \xi_2\ \underset{\sim}{1}\ ,$$
$$E[\underset{\sim}{Z}_2] \doteq \{g(\rho_3)\}\ \underset{\sim}{1} = \xi_3\ \underset{\sim}{1}\ .$$

The variance-covariance matrix of $\underset{\sim}{z}$ can be similarly partitioned, i.e.

$$V(\underset{\sim}{\rho}) = \begin{bmatrix} V_1 & V_{12} & V_{13} \\ V_{21} & V_2 & V_{23} \\ V_{31} & V_{32} & V_3 \end{bmatrix} \qquad (3.7)$$

where V_1, V_2, V_3 are the variance-covariance matrices of the elements of $\underset{\sim}{z}_1$, $\underset{\sim}{z}_2$ and $\underset{\sim}{z}_3$, respectively, and V_{ij} are the covariances between the elements of $\underset{\sim}{z}_i$ and $\underset{\sim}{z}_j$.

We will only derive expressions for the elements of V_1, V_2 and V_3, this being sufficient to allow the formulation of a useful analysis.

<u>Expressions for the elements of V_1</u>

V_1 is the variance-covariance matrix of the elements of $\underset{\sim}{z}_1$, the transforms of observed correlation coefficients each of which involves only one level of A. The general formula for the covariances between such transforms is, from theorem 3.2,

$$\mathrm{cov}(z_{ij}^{kk}, \; z_{ef}^{\ell\ell}) = \frac{1}{n'} \{(\rho_{ie}^{k\ell}\rho_{jf}^{k\ell} + \rho_{if}^{k\ell}\rho_{je}^{k\ell}) - \rho_1(\rho_{ie}^{k\ell}\rho_{je}^{k\ell} + \rho_{if}^{k\ell}\rho_{jf}^{k\ell} + \rho_{ie}^{k\ell}\rho_{if}^{k\ell} + \rho_{je}^{k\ell}\rho_{jf}^{k\ell})$$

$$+ \tfrac{1}{2}\rho_1^2 \; (\rho_{ie}^{k\ell}\rho_{ie}^{k\ell} + \rho_{if}^{k\ell}\rho_{if}^{k\ell} + \rho_{je}^{k\ell}\rho_{je}^{k\ell} + \rho_{jf}^{k\ell}\rho_{jf}^{k\ell})\}/(1 - \rho_1^2)^2$$

However, there are only six distinct values of this expression, these being as follows:

i) Same levels of A and the two levels of B in common

$$\mathrm{var}(z_{ij}^{kk}) = \frac{1}{n'}$$

$$= v_1$$

ii) Same levels of A and one level of B in common

$$\mathrm{cov}(z_{ij}^{kk}, z_{if}^{kk}) = \frac{\rho_1(3\rho_1 + 2)}{n' \; 2(1+\rho_1)^2} \qquad \text{for } j \neq f.$$

$$= v_2$$

iii) Same levels of A and no levels of B in common

$$\mathrm{cov}(z_{ij}^{kk}, z_{ef}^{kk}) = \frac{2\rho_1^2}{n'(1+\rho_1)^2}$$

$$= v_3$$

iv) Different levels of A and both levels of B in common

$$\mathrm{cov}(z_{ij}^{kk}, z_{ij}^{\ell\ell}) = \frac{1}{n'(1-\rho_1^2)^2}\{(1+\rho_1^2)(\rho_2^2 + \rho_3^2) - 4\rho_1\rho_2\rho_3\}$$

$$= v_4$$

v) Different levels of A and one level only of B in common

$$\mathrm{cov}(z_{ij}^{kk}, z_{if}^{\ell\ell}) = \frac{\rho_2(1-2\rho_1)(\rho_2 + \rho_3) + \tfrac{1}{2}\rho_1^2(\rho_3^2 + 3\rho_2^2)}{n'(1 - \rho_1^2)^2} \qquad \text{for } j \neq f.$$

$$= v_5$$

vi) Different levels of A and no levels of B in common

$$\mathrm{cov}(z_{ij}^{kk}, z_{ef}^{\ell\ell}) = \frac{2\rho_2^2}{n'(1+\rho_1)^2} \qquad \text{for } i \neq e,\ j \neq f$$

$$= v_6$$

Thus, if V_1 is the variance-covariance matrix of $\underset{\sim}{Z}$ then

$$V_1 = \sum_{i=1}^{6} v_i A_i \tag{3.8}$$

where $A_1, \ldots, A_6$ are the incidence matrices for z-transforms for which the correspondences between the levels of A and B are as given in i),...,vi) above, respectively.

The A_i can be expressed in terms of the I, A and B of equation (3.3) and J as follows:

Let $p_1 = \frac{1}{2}p(p-1)$.

Then $A_1 = I_m \otimes I_{p_1}$,

$A_2 = I_m \otimes A_{p_1}$,

$A_3 = I_m \otimes B_{p_1}$,

$A_4 = (J_m - I_m) \otimes I_{p_1}$,

$A_5 = (J_m - I_m) \otimes A_{p_1}$,

and $A_6 = (J_m - I_m) \otimes B_{p_1}$.

<u>Expressions for the elements of V_2</u>

The situation here is structurally the same as that outlined for the elements of $\underset{\sim}{z}_1$ with factors A and B, and correlations ρ_1 and ρ_3, interchanged. Thus,

$$V_2 = \sum_{i=1}^{6} v_i A_i \tag{3.9}$$

where $v_1 = \frac{1}{n'}$,

$v_2 = \frac{\rho_3(3\rho_3 + 2)}{n'2(1+\rho_3)^2}$,

$$v_3 = \frac{2\rho_3^2}{n'(1+\rho_3)^2}$$

$$v_4 = \frac{1}{n'(1-\rho_3^2)^2}\{(1+\rho_3^2)(\rho_1^2+\rho_2^2) - 4\rho_1\rho_2\rho_3\},$$

$$v_5 = \frac{\rho_2(1-2\rho_3)(\rho_2+\rho_1) + \tfrac{1}{2}\rho_3^2(\rho_1^2+3\rho_2^2)}{n'(1-\rho_3^2)^2}$$

and $$v_6 = \frac{2\rho_2^2}{n'(1+\rho_3)^2}.$$

Expressions for the elements of V_3

V_3 is the variance-covariance matrix of the elements of $\underset{\sim}{z}_3$, the transforms of observed correlation coefficients each of which has two different levels of A and two different levels of B. The general formula for the covariances between such transforms is, from theorem 3.2,

$$\text{cov}(r_{ij}^{k\ell}, r_{ef}^{gh}) = \frac{1}{n'}\{(\rho_{ie}^{kg}\rho_{jf}^{\ell h} + \rho_{if}^{kh}\rho_{je}^{\ell g}) - \rho_2(\rho_{ie}^{kg}\rho_{je}^{\ell g} + \rho_{if}^{kh}\rho_{jf}^{\ell h} + \rho_{ie}^{kg}\rho_{if}^{kh} + \rho_{je}^{\ell g}\rho_{jf}^{\ell h})$$

$$+ \tfrac{1}{2}\rho_2^2(\rho_{ie}^{kg}\rho_{ie}^{kg} + \rho_{jf}^{\ell h}\rho_{jf}^{\ell h} + \rho_{if}^{kh}\rho_{if}^{kh} + \rho_{je}^{\ell g}\rho_{je}^{\ell g})\}/(1-\rho_2^2)^2$$

$$\text{for } i \neq j,\ k \neq \ell,\ e \neq f,\ g \neq h.$$

Other than n' and ρ_2, this formula involves only ρ_{ie}^{kg}, $\rho_{jf}^{\ell h}$, ρ_{if}^{kh} and $\rho_{je}^{\ell g}$, possible values for which are 1, ρ_1, ρ_2 and ρ_3. If we consider the first two correlations, ρ_{ie}^{kg} and $\rho_{jf}^{\ell h}$, to form a set and the last two, ρ_{if}^{kh} and $\rho_{je}^{\ell g}$, to form another set, it is clear from an examination of the formula that the elements of V_3 depend only on which of 1, ρ_1, ρ_2 and ρ_3 are grouped together. There are 14 unique elements of V_3, each one being associated with a particular correspondence between the levels of A and the levels of B involved in the two z-transforms. Table 3.1 details the form of the two z-transforms, the composition of the sets and the correspondences between the levels of A and B for each of the distinct elements.

Table 3.1: Distinct Elements of V_3

Element of V_3	Form of z-transforms	Sets		Correspondence between levels of A & B: No. levels in common A	B	Association of each common level of B
v_1	$z_{ij}^{k\ell}, z_{ij}^{k\ell}$	1,1	ρ_2,ρ_2	2	2	at same level of A
v_2	$z_{ij}^{k\ell}, z_{ji}^{k\ell}$	$\rho_1,\rho_{\substack{2\\1}}$	ρ_3,ρ_3	2	2	at different levels of A
v_3	$z_{ij}^{k\ell}, z_{ie}^{k\ell}$	$1, \rho_1$	ρ_2,ρ_2	2	1	at same level of A
v_4	$z_{ij}^{k\ell}, z_{fi}^{k\ell}$	ρ_2,ρ_3	ρ_1,ρ_1	2	1	at different levels of A
v_5	$z_{ij}^{k\ell}, z_{ef}^{k\ell}$	ρ_1,ρ_1	ρ_2,ρ_2	2	0	–
v_6	$z_{ij}^{k\ell}, z_{ij}^{kh}$	$1, \rho_3$	$\rho_2\ \rho_2$	1	2	one at same level of A
v_7	$z_{ij}^{k\ell}, z_{ji}^{kh}$	ρ_1,ρ_2	ρ_3,ρ_3	1	2	at different levels of A
v_8	$z_{ij}^{k\ell}, z_{if}^{kh}$	$1, \rho_2$	ρ_2,ρ_2	1	1	at same level of A
v_9	$z_{ij}^{k\ell}, z_{fi}^{kh}$	ρ_1,ρ_2	ρ_2,ρ_3	1	1	at different levels of A, one of which is the common B level
v_{10}	$z_{ij}^{k\ell}, z_{if}^{g\ell}$	ρ_1,ρ_3	ρ_2,ρ_2	1	1	at different levels of A,none of which is the common B level
v_{11}	$z_{ij}^{k\ell}, z_{ef}^{kh}$	ρ_1,ρ_2	ρ_2,ρ_2	1	0	–
v_{12}	$z_{ij}^{k\ell}, z_{ij}^{gh}$	ρ_3,ρ_3	ρ_2,ρ_2	0	2	–
v_{13}	$z_{ij}^{k\ell}, z_{if}^{gh}$	ρ_2,ρ_3	ρ_2,ρ_2	0	1	–
v_{14}	$z_{ij}^{k\ell}, z_{ef}^{gh}$	ρ_2,ρ_2	ρ_2,ρ_2	0	0	–

If A_i is the incidence matrix for pairs of z-transforms whose levels of A and B correspond as described for the i^{th} element of V_3, then

$$V_3 = \sum_{i=1}^{14} v_i A_i \tag{3.10}$$

The expression for each v_i can be obtained by substituting, into the general formula, the elements of the first (or second) set associated with v_i for ρ_{ie}^{kg} and $\rho_{jf}^{\ell h}$, and the elements of the second (or first) set for ρ_{if}^{kh} and $\rho_{je}^{\ell g}$.

Also, the A_i can be expressed in terms of I, A and B of equation (3.3) and J as follows:

Let $p_1 = \frac{1}{2}p(p-1)$ and $m_1 = \frac{1}{2}m(m-1)$.

$$\begin{aligned}
\text{Then } A_1 &= I_{m_1} \otimes I_2 \otimes I_{p_1}, \\
A_2 &= I_{m_1} \otimes (J_2 - I_2) \otimes I_{p_1}, \\
A_3 + A_4 &= I_{m_1} \otimes J_2 \otimes A_{p_1}, \\
A_5 &= I_{m_1} \otimes J_2 \otimes B_{p_1}, \\
A_6 + A_7 &= A_{m_1} \otimes J_2 \otimes I_{p_1}, \\
A_8 + A_9 + A_{10} &= A_{m_1} \otimes J_2 \otimes A_{p_1}, \\
A_{11} &= A_{m_1} \otimes J_2 \otimes B_{p_1}, \\
A_{12} &= B_{m_1} \otimes J_2 \otimes I_{p_1}, \\
A_{13} &= B_{m_1} \otimes J_2 \otimes A_{p_1}, \\
\text{and } A_{14} &= B_{m_1} \otimes J_2 \otimes B_{p_1}.
\end{aligned} \tag{3.11}$$

4. PATTERNS IN CORRELATION MATRICES

In the previous chapter we noted that asymptotically $\underset{\sim}{Z} \sim N(\underset{\sim}{\xi}, V(\underset{\sim}{\rho}))$ where $V(\underset{\sim}{\rho})$ is much simplified for some specific hypotheses concerning $\underset{\sim}{\rho}$. In this chapter, a form of least squares procedure that utilises these results will be derived for investigating patterns in correlation matrices.

By equation 3b4.7 of section 3b4 of Rao (1965),

$$\underset{\sim}{Z}'V^{-1}(\underset{\sim}{\rho})\ \underset{\sim}{Z} \sim \chi^2(\tfrac{1}{2}n(n-1), \underset{\sim}{\xi}'V^{-1}(\underset{\sim}{\rho})\underset{\sim}{\xi})$$

where n is the order of the underlying correlation matrix.

We will obtain an expression for $\underset{\sim}{Z}'V^{-1}(\underset{\sim}{\rho})\underset{\sim}{Z}$ which involves its partition into independent components that are distributed as χ^2's (in some cases as central χ^2's). Two situations will be investigated, i) a set of variables assumed to be equally correlated, and ii) a set of variables cross-indexed by two factors and whose correlations are assumed to display the pattern given by equation (3.5). In the first situation, the algebra resulting from the symmetries in V will be analysed and then it will be demonstrated that the analysis of variance technique is a convenient alternative method of analysing the algebra. In the second situation, only the analysis of variance technique will be used to analyse the algebra.

4.1 AN ANALYSIS FOR THE EQUALLY CORRELATED SET

In the situation in which the variables are assumed to be equally correlated, V is a highly patterned matrix for which I, A and B form a basis (see equation (3.3)). The problem of inverting V can be simplified using algebraic theory. Gleeson and McGilchrist (1978) provide a convenient summary of the aspects of this theory needed in the ensuing discussion.

V is clearly invariant under permutation of the variables, i.e., if P is a permutation matrix representing such a transformation, then

$$P'VP = V$$

and as P is orthogonal (i.e., $P^{-1} = P'$), it follows that

$$PV = VP \tag{4.1}$$

The set of all such permutation matrices {P} form a group; hence the set of all matrices {V} which, according to equation 4.1, commute with all matrices of this group form the commuting algebra of this group.

As the algebra is generated by symmetric matrices (i.e. by I, A and B), one of which is the identity matrix, the algebra is semi-simple. Such an algebra has a unique decomposition into the direct sum of minimum two-sided ideals each of which has a unit element or idempotent that generates the ideal. Thus, V can, in general, be written as the linear combination of the idempotents of the minimum two-sided ideals, i.e.

$$V = \lambda_1 E_1 + \lambda_2 E_2 + \ldots + \lambda_\ell E_\ell$$

where E_i are the idempotents of the minimum two-sided ideals, with

$$I = E_1 + E_2 + \ldots + E_\ell$$

and

$$E_i E_j = \delta_{ij} E_i.$$

It follows that

$$V^{-1} = \lambda_1^{-1} E_1 + \lambda_2^{-1} E_2 + \ldots + \lambda_\ell^{-1} E_\ell$$

Thus,

$$\underset{\sim}{Z}' V^{-1} \underset{\sim}{Z} = \lambda_1^{-1} \underset{\sim}{Z}' E_1 \underset{\sim}{Z} + \lambda_2^{-1} \underset{\sim}{Z}' E_2 \underset{\sim}{Z} + \ldots + \lambda_\ell^{-1} \underset{\sim}{Z}' E_\ell \underset{\sim}{Z} \tag{4.2}$$

where

$$\lambda_i^{-1} \underset{\sim}{Z}' E_i \underset{\sim}{Z} \sim \chi^2(p_i, \gamma_i)$$

with

$$p_i = tr(E_i)$$

and

$$\gamma_i = \underset{\sim}{\xi}' E_i \underset{\sim}{\xi}.$$

The idempotents will be found by constructing the left regular representation of an ℓ-dimensional abstract algebra isomorphic to the commuting algebra. This will lead to the abstract counterparts of the E_i, that is to the idempotents of the abstract algebra, and hence to the E_i themselves. The ranges of the E_i are the invariant subspaces of the vector space in which the algebra is represented.

The development just outlined parallels that given by James (1957) where I, A and B are analagous to his relationship matrices. James showed that a set of relationship matrices determines an analysis of variance in the sense that (i) the sums of squares in an analysis of variance are the lengths of the orthogonal projections of a data vector into orthogonal, invariant and irreducible subspaces, and (ii) an analysis of the algebra generated by the relationship matrices leads to expressions for these orthogonal projection operators (E_i). A convenient and concise method of specifying the orthogonal projection operators, and hence the associated subspaces, is to write down the formulae for the sums of squares in an appropriate analysis of variance.

Expressions for the E_i will be found using two essentially different methods:

(a) The Algebraic Technique: The algebra generated by I, A and B will be analysed as outlined above. Clearly, this will determine an analysis of variance of an observed $\underset{\sim}{z}$ that corresponds to the relationships implicit in I, A and B.

(b) The Analysis of Variance Technique: From the symmetries inherent in the particular form of V, model subspaces for several hypothetical linear models for $E[\underset{\sim}{Z}]$ will be conjectured. The orthogonal projection matrices for these subspaces will be obtained by using least squares principles to derive formulae for the sums of squares in an analysis of variance corresponding to the hypothesised model subspaces. It will be seen that these projection matrices are the E_i from method (a).

The disadvantage of the latter technique is that it relies partly on conjecture, but its advantages are its convenience and that it greatly elucidates the inferential interpretation (to be discussed below) of the components in the canonical reduction of $\underset{\sim}{Z}'V^{-1}\underset{\sim}{Z}$ in equation (4.2) above.

The following theorem (theorem 4.1) specifies the decomposition of an observed $\underset{\sim}{z}$ into three independent components whose underlying distribution will be shown to be a χ^2. The proof of this theorem by the algebraic technique is given in section 4.1.1 and the proof by the analysis of variance technqiue is given in section 4.1.2. Section 4.1.3 contains a convenient synopsis of the analysis procedure.

<u>THEOREM 4.1</u>: Let $\underset{\sim}{z}$ be the vector containing the $\frac{1}{2}p(p-1)$ z-transforms of the elements of the observed correlation matrix of the variables, i.e.

$$\underset{\sim}{z}' = (z_{12}, z_{13}, \ldots, z_{1p}, z_{23}, z_{24}, \ldots, z_{2p}, \ldots\ldots, z_{p-1,p})$$

is an observation of the vector of random variables $\underset{\sim}{Z}$.

Then the total variation in the observed z-transform can be partitioned, under the null hypothesis that the underlying distribution of $\underset{\sim}{z}$ is as given in equation (3.4), into three orthogonal components as follows:

$$\underset{\sim}{z}'V^{-1}\underset{\sim}{z} = \lambda_1^{-1}\underset{\sim}{z}'E_1\underset{\sim}{z} + \lambda_2^{-1}\underset{\sim}{z}'E_2\underset{\sim}{z} + \lambda_3^{-1}\underset{\sim}{z}'E_3\underset{\sim}{z}$$

$$= \frac{\frac{2}{p(p-1)}\, z_{++}^2}{v_1 + 2(p-2)v_2 + \frac{1}{2}(p-1)(p-2)v_3}$$

$$+ \frac{\frac{1}{p-2}\sum_{i=1}^{p} z_{i+}^2 - \frac{4}{p(p-2)}\, z_{++}^2}{v_1 + (p-4)v_2 - (p-3)v_3}$$

$$+ \frac{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} z_{ij}^2 - \frac{1}{(p-2)}\sum_{i=1}^{p} z_{i+}^2 + \frac{2}{p(p-1)(p-2)}\, z_{++}^2}{v_1 - 2v_2 + v_3}$$

where $$z_{++} = \sum_{i=1}^{p-1}\sum_{j=i+1}^{p} z_{ij}$$

and $$z_{i+} = \sum_{j=1}^{i-1} z_{ji} + \sum_{j=i+1}^{p} z_{ij}.$$

Under the null hypothesis, the underlying distributions of the three components are independent, being

i) $\lambda_1^{-1}\underset{\sim}{Z}'E_1\underset{\sim}{Z} \sim \chi^2\left(1, \frac{\frac{1}{2}p(p-1)\xi^2}{v_1+2(p-2)v_2+\frac{1}{2}(p-2)(p-3)v_3}\right) (=\chi^2(1) \text{ if } \xi = 0)$,

ii) $\lambda_2^{-1}\underset{\sim}{Z}'E_2\underset{\sim}{Z} \sim \chi^2(p-1)$, and

iii) $\lambda_3^{-1}\underset{\sim}{Z}'E_3\underset{\sim}{Z} \sim \chi^2(\frac{1}{2}p(p-3))$.

4.1.1 PROOF OF THEOREM 4.1 BY THE ALGEBRAIC TECHNIQUE

The proof consists of three parts. Firstly, the abstract algebra, isomorphic to the algebra generated by the three incidence matrices I, A and B, is analysed to obtain expressions for the basis elements of the abstract algebra (and hence the commuting algebra) in terms of the idempotents of the algebra and *vice versa*. Secondly, expressions for the incidence matrices in terms of convenient calculation matrices and for the idempotents in terms of the calculation matrices is obtained. Finally, summation formulae, degrees of freedom and distributions for the proposed test statistics are given.

i) Incidence matrices in terms of the idempotents and *vice versa*

The multiplication table of the abstract algebra isomorphic to the commuting algebra is, with some computation, shown to be

X	i	a	b
i	i	a	b
a	a	2(p-2)i+(p-2)a+4b	(p-3)a+2(p-4)b
b	b	(p-3)a+2(p-4)b	$\frac{(p-3)(p-2)i+(p-3)(p-4)a+(p-4)(p-5)b}{2}$

Now, the algebra is commutative, semi-simple and of dimension 3. Thus, the algebra must be isomorphic to a direct sum of complete matrix algebras. As the algebra of all 2 x 2 matrices or matrices of higher order is not commutative, the algebra must be isomorphic to the direct sum of three one-dimensional complete matrix algebras. To find the three idempotent elements, e_1, e_2 and e_3, which generate these one-dimensional matrix algebras, we first construct the left regular representation.

$$[i \quad a \quad b] \overset{a}{\rightarrow} [a \quad a^2 \quad ab] = [i \quad a \quad b] \begin{bmatrix} 0 & 2(p-2) & 0 \\ 1 & (p-2) & (p-3) \\ 0 & 4 & 2(p-4) \end{bmatrix}$$

$$[i \quad a \quad b] \overset{b}{\rightarrow} [b \quad ba \quad b^2] = [i \quad a \quad b] \begin{bmatrix} 0 & 0 & \frac{1}{2}(p-2)(p-3) \\ 0 & (p-3) & \frac{1}{2}(p-3)(p-4) \\ 1 & 2(p-4) & \frac{1}{2}(p-4)(p-5) \end{bmatrix}$$

Hence the left regular representation is established by the correspondences

$$i \longleftrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \varepsilon$$

$$a \longleftrightarrow \begin{bmatrix} 0 & 2(p-2) & 0 \\ 1 & p-2 & p-3 \\ 0 & 4 & 2(p-4) \end{bmatrix} = \alpha$$

$$b \longleftrightarrow \begin{bmatrix} 0 & 0 & \frac{1}{2}(p-2)(p-3) \\ 0 & p-3 & \frac{1}{2}(p-3)(p-4) \\ 1 & 2(p-4) & \frac{1}{2}(p-4)(p-5) \end{bmatrix} = \beta$$

Since the algebra is commutative, ε, α and β have the same eigenvectors. If the column eigenvectors are $\underset{\sim}{c}_1$, $\underset{\sim}{c}_2$ and $\underset{\sim}{c}_3$ and the corresponding row eigenvectors $\underset{\sim}{r}_1'$, $\underset{\sim}{r}_2'$ and $\underset{\sim}{r}_3'$, then the idempotents will correspond to the matrices ε_i, i = 1,2,3 i.e.,

$$e_i \leftrightarrow \varepsilon_i = \underset{\sim}{c}_i \underset{\sim}{r}_i' / \underset{\sim}{r}_i' \underset{\sim}{c}_i \quad \text{for } i = 1,2,3 \tag{4.3}$$

A little calculation shows that α has eigenvalues

$$\lambda_1 = 2(p-2), \quad \lambda_2 = (p-4) \text{ and } \lambda_3 = -2,$$

column eigenvectors that are the columns of

$$\gamma = \begin{bmatrix} 1 & 2(p-2) & (p-2)(p-3) \\ 1 & p-4 & -(p-3) \\ 1 & -4 & 2 \end{bmatrix}$$

and row eigenvectors that are the rows of

$$\delta = \begin{bmatrix} 2 & 4(p-2) & (p-2)(p-3) \\ 1 & p-4 & -(p-3) \\ 1 & -2 & 1 \end{bmatrix}$$

Further,

$$\beta\gamma = \begin{bmatrix} \tfrac{1}{2}(p-2)(p-3) & 0 & 0 \\ 0 & -(p-3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which establishes the corresponding eigenvalues of β as

$$\mu_1 = \tfrac{1}{2}(p-2)(p-3), \quad \mu_2 = -(p-3), \quad \mu_3 = 1.$$

The expressions for each of the basis elements in terms of the idempotents are given by the eigenvalues of the left regular representation of the corresponding element, i.e.,

$$\begin{bmatrix} i \\ a \\ b \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2(p-2) & p-4 & -2 \\ \tfrac{1}{2}(p-2)(p-3) & -(p-3) & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{4.4}$$

To find expressions for the idempotent elements (e_1, e_2 and e_3) in terms of the basis elements (i, a and b) consider a corresponding equation of the form

$$\varepsilon_i = \theta_{i1}\,\varepsilon + \theta_{i2}\,\alpha + \theta_{i3}\,\beta.$$

Comparing coefficients in the first column of this sum, we see that $(\theta_{i1}, \theta_{i2}, \theta_{i3})'$ is precisely the first column of ε_i, which can be calculated using (4.3). We note that

$$\delta\gamma = \begin{bmatrix} p(p-1) & 0 & 0 \\ 0 & p(p-2) & 0 \\ 0 & 0 & (p-1)(p-2) \end{bmatrix}$$

where the diagonal elements give the divisors in (4.3). The calculations yield

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{p(p-1)} & \frac{2}{p(p-1)} & \frac{2}{p(p-1)} \\ \frac{2}{p} & \frac{p-4}{p(p-2)} & \frac{-4}{p(p-2)} \\ \frac{p-3}{p-1} & \frac{-(p-3)}{(p-1)(p-2)} & \frac{2}{(p-1)(p-2)} \end{bmatrix} \begin{bmatrix} i \\ a \\ b \end{bmatrix} \qquad (4.5)$$

Identical connexions to those given in (4.4) and (4.5) exist between the incidence matrices I, A and B and the orthogonal projection matrices E_1, E_2 and E_3 onto the orthogonal invariant subspaces of V, the vector space in which the commuting algebra is represented.

The algebra analysed above is precisely the same as the association algebra for a partially balanced incomplete block design with a triangular association scheme. This association algebra has been analysed by Ogawa and Ishii (1965) and equation (4.5) above is the same as their equation (2.10).

ii) <u>Incidence and idempotent matrices in terms of calculation matrices</u>

Matrices which provide a convenient basis for calculation are introduced. These are given by

$$\begin{bmatrix} I \\ K \\ J \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I \\ A \\ B \end{bmatrix}$$

where K is the matrix operator that replaces each observation in the vector $\underset{\sim}{z}$ by

$$z^*_{ij} = \sum_{k=1}^{i-1} z_{ki} + \sum_{k=i+1}^{p} z_{ik} + \sum_{\ell=1}^{j-1} z_{\ell j} + \sum_{\ell=j+1}^{p} z_{j\ell}$$

J is the matrix of ones and,

I is the unit matrix

Thus,

$$\begin{bmatrix} I \\ A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} I \\ K \\ J \end{bmatrix}$$

and

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{p(p-1)} & \frac{2}{p(p-1)} & \frac{2}{p(p-1)} \\ \frac{2}{p} & \frac{(p-4)}{p(p-2)} & \frac{-4}{p(p-2)} \\ \frac{p-3}{p-1} & \frac{-(p-3)}{(p-1)(p-2)} & \frac{2}{(p-1)(p-2)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} I \\ K \\ J \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \frac{2}{p(p-1)} \\ 0 & \frac{1}{(p-2)} & \frac{-4}{p(p-2)} \\ 1 & \frac{-1}{(p-2)} & \frac{2}{(p-1)(p-2)} \end{bmatrix} \begin{bmatrix} I \\ K \\ J \end{bmatrix}$$

iii) Summation formulae, degrees of freedom and distributions of test statistics

Now $V = v_1 I + v_2 A + v_3 B = \{v_1 + 2(p-2)\, v_2 + \tfrac{1}{2}(p-2)(p-3)\, v_3\}E_1$

$$+ \{v_1 + (p-4)\, v_2 - (p-3)\, v_3\}E_2 + \{v_1 - 2v_2 + v_3\}E_3$$

Thus,

$$V^{-1} = \frac{E_1}{v_1 + 2(p-2)v_2 + \tfrac{1}{2}(p-2)(p-3)v_3} + \frac{E_2}{v_1 + (p-4)v_2 - (p-3)v_3}$$

$$+ \frac{E_3}{v_1 - 2v_2 + v_3}$$

and

$$\underset{\sim}{z}'V^{-1}\underset{\sim}{z} = \frac{\underset{\sim}{z}'\, E_1 \underset{\sim}{z}}{v_1 + 2(p-2)v_2 + \tfrac{1}{2}(p-2)(p-3)v_3} + \frac{\underset{\sim}{z}'\, E_2 \underset{\sim}{z}}{v_1 + (p-4)v_2 - (p-3)v_3}$$

$$+ \frac{\underset{\sim}{z}'\, E_3 \underset{\sim}{z}}{v_1 - 2v_2 + v_3}$$

$$= \frac{\underset{\sim}{z}'\, \frac{2J}{p(p-1)}\, \underset{\sim}{z}}{v_1 + 2(p-2)v_2 + \tfrac{1}{2}(p-2)(p-3)v_3}$$

$$+ \frac{\underset{\sim}{z}'\{\frac{K}{(p-2)} - \frac{4J}{p(p-2)}\}\, \underset{\sim}{z}}{v_1 + (p-4)v_2 - (p-3)v_3}$$

$$+ \frac{\underset{\sim}{z}'\{I - \frac{K}{(p-2)} + \frac{2J}{(p-1)(p-2)}\}\, \underset{\sim}{z}}{v_1 - 2v_2 + v_3}$$

But, $\underset{\sim}{z}'I\,\underset{\sim}{z} = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij}^2$

$\underset{\sim}{z}'K\,\underset{\sim}{z} = \sum_{i=1}^{p} z_{i+}^2$ where $z_{i+} = \sum_{j=1}^{i-1} z_{ji} + \sum_{j=i+1}^{p} z_{ij}$, and $\underset{\sim}{z}'J\,\underset{\sim}{z} = (\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij})^2 = z_{++}^2.$

Therefore,

$$\underset{\sim}{z}'V^{-1}\underset{\sim}{z} = \frac{\frac{2}{p(p-1)}\, z_{++}^2}{v_1 + 2(p-2)v_2 + \frac{1}{2}(p-2)(p-1)v_3}$$

$$+ \frac{\frac{1}{p-2}\sum_{i=1}^{p} z_{i+}^2 - \frac{4}{p(p-2)}\, z_{++}^2}{v_1 + (p-4)v_2 - (p-3)v_3}$$

$$+ \frac{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} z_{ij}^2 - \frac{1}{p-2}\sum_{i=1}^{p} z_{i+}^2 + \frac{2}{(p-1)(p-2)}\, z_{++}^2}{v_1 - 2v_2 + v_3} \qquad (4.6)$$

Each of the terms in this expression is a test statistic. Their degrees of freedom are given by the trace of the corresponding idempotent matrix (i.e. $tr(E_i)$). With $tr(I) = \frac{p(p-1)}{2}$, $tr(A) = tr(B) = 0$ and using (4.5) we obtain

$$tr(E_1) = 1,\ tr(E_2) = p-1 \text{ and } tr(E_3) = \tfrac{1}{2}p(p-3) \qquad (4.7)$$

Under the null hypothesis that $\underset{\sim}{\xi} = \xi\underset{\sim}{1}$ and to the extent that $\underset{\sim}{Z} \sim N(\xi\underset{\sim}{1}, V)$, the underlying distributions of the test statistics are clearly as follows (see Rao (1965, (vii) of section 3b4)).

$$\frac{\underset{\sim}{Z}'E_1\underset{\sim}{Z}}{v_1 + 2(p-2)v_2 + \frac{1}{2}(p-2)(p-3)v_3} \sim \chi^2\left(1, \frac{\xi^2\frac{1}{2}p(p-1)}{v_1 + 2(p-2)v_2 + \frac{1}{2}(p-2)(p-3)v_3}\right),$$

$$\frac{\underset{\sim}{Z}'E_2\underset{\sim}{Z}}{v_1 + (p-4)v_2 - (p-3)v_3} \sim \chi^2\,(p-1), \quad \text{and}$$

$$\frac{\underset{\sim}{Z}'E_3\underset{\sim}{Z}}{v_1 - 2v_2 + v_3} \sim \chi^2\,(\tfrac{1}{2}p(p-3)).$$

Under the hypothesis that $\rho = 0$ (and here $\xi = 0$), the formula for each test statistic reduces to

$$\frac{1}{v_1} \underset{\sim}{Z}' E_i \underset{\sim}{Z}$$

and these have central χ^2 distributions.

4.1.2 PROOF OF THEOREM 4.1 VIA THE ANALYSIS OF VARIANCE TECHNIQUE

Consider the three linear models

$$M_1 : E[Z_{ij}] = \mu$$

$$M_2 : E[Z_{ij}] = \mu + \alpha_i + \alpha_j$$

$$M_3 : E[Z_{ij}] = \mu + \alpha_i + \alpha_j + \gamma_{ij} \qquad (\gamma_{ij} = \gamma_{ji})$$

Let $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$ $(= R^{\frac{1}{2}p(p-1)})$ denote the corresponding model sub-spaces. Clearly, $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ and $\dim(\mathcal{M}_1) = 1$, $\dim(\mathcal{M}_2) = p$ and $\dim(\mathcal{M}_3) = \frac{1}{2}p(p-1)$.

The analysis of variance (i.e. a partition of $\underset{\sim}{z}'\underset{\sim}{z}$) corresponding to the sequential fitting of these models is established, producing an analysis for which the orthogonal projections operators are the idempotents E_1, E_2 and E_3 of section 4.1.1. Thus the analysis of variance technique provides an alternative method of obtaining the partition of $\underset{\sim}{z}'V^{-1}\underset{\sim}{z}$ and, further, will enable the statistical significance of the terms in this partition to be established.

The correspondence between the models and V is that they are both invariant under permutation of the variables. Thus, the algebra generated by the basis elements of V and that generated by the relationship matrices corresponding to the models (viz. I + A + B, I + A and I) have the same orthogonal, invariant subspaces in the vector space in which they are represented.

i) Summation formulae for SSQ of an orthogonal partition

The total sums of squares will be partitioned as follows:

$$\text{Total SSQ} = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij}^2$$

$$= \text{Mean SSQ} + \text{Variable SSQ} + \text{Variable Interaction SSQ}$$

where,

Mean SSQ is the length of the orthogonal projection of $\underset{\sim}{z}$ onto M_1

Variable SSQ is the length of the orthogonal projection of $\underset{\sim}{z}$ onto $M_2 \theta M_1$,

Variable Interaction SSQ is the length of the orthogonal projection of $\underset{\sim}{z}$ onto $M_3 \theta M_2$,

and $A\theta B$ denotes the subspace of A orthogonal to B.

The length of the projection of $\underset{\sim}{z}$ onto M_1 is clearly

$$\text{Mean SSQ} = \frac{2z_{++}^2}{p(p-1)} \tag{4.8}$$

$$= \underset{\sim}{z}' E_1 \underset{\sim}{z}$$

To obtain the Variable SSQ (i.e. the projection onto $M_2 \theta M_1$), we first consider the projection onto M_2 which we obtain by least squares.

Let X be such that $R(X) = M_2$.

It is not difficult to show that

$X'\underset{\sim}{z}$ is the vector whose p elements are the sums of z_{i+} $(i = 1,\ldots p)$,

$X'X = (p-2)\, I_p + J_p$ and hence,

$(X'X)^{-1} = \frac{1}{p-2}\, (I_p - \frac{1}{2(p-1)}\, J_p)$.

It readily follows that

$$\underset{\sim}{z}'X(X'X)^{-1}X'\underset{\sim}{z} = \frac{1}{p-2}\, (\sum_{i=1}^{p} z_{i+}^2 - \frac{2}{p-1}\, z_{++}^2) \tag{4.9}$$

The Variable SSQ is then given by subtracting (4.8) from (4.9), viz.

$$\text{Variable SSQ} = \frac{1}{p-2} \sum_{i=1}^{p} z_{i+}^2 - \frac{4}{p(p-2)}\, z_{++}^2$$

$$= \underset{\sim}{z}' E_2 \underset{\sim}{z}.$$

Note that this component may be deduced by conjecturing that it is of the form

$$a\left(\sum_{i=1}^{p} z_{i+}^2 - bz_{++}^2\right)$$

and choosing a and b such that

(i) Variable SSQ = 0 if $\underset{\sim}{z} \in M_1$, and

(ii) the trace of the matrix of the quadratic form in $\underset{\sim}{z} = \dim(M_2 \theta M_1) = p-1$.

The Variables Interaction SSQ is found by difference

$$\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij}^2 - \frac{1}{p-2} \sum_{i=1}^{p} z_{i+}^2 + \frac{2}{(p-1)(p-2)} z_{++}^2$$

$$= \underset{\sim}{z}' E_3 \underset{\sim}{z}$$

In summary, the decomposition of the Total SSQ is

$$\sum_{i=j}^{p-1} \sum_{j=i+1}^{p} z_{ij}^2 = \underset{\sim}{z}'\underset{\sim}{z}$$

$$= \frac{2}{p(p-1)} z_{++}^2 + \left\{\frac{1}{p-2} \sum_{i=1}^{p} z_{i+}^2 - \frac{4}{p(p-2)} z_{++}^2 \right\}$$

$$+ \left\{ \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij}^2 - \frac{1}{p-2} \sum_{i=1}^{p} z_{i+}^2 + \frac{2}{(p-1)(p-2)} z_{++}^2 \right\}$$

$$= \underset{\sim}{z}' E_1 \underset{\sim}{z} + \underset{\sim}{z}' E_2 \underset{\sim}{z} + \underset{\sim}{z}' E_3 \underset{\sim}{z}$$

To obtain expression of the orthogonal projection operators, E_1, E_2 and E_3, in terms of the calculation matrices we recall that

$$\underset{\sim}{z}' K \underset{\sim}{z} = \sum_{i=1}^{p} z_{i+}^2$$

and $$\underset{\sim}{z}' J \underset{\sim}{z} = z_{++}^2.$$

Thus,

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{p(p-2)} & 0 & 0 \\ \frac{-4}{p(p-2)} & \frac{1}{p-2} & 0 \\ \frac{2}{(p-1)(p-2)} & \frac{-1}{p-2} & 1 \end{bmatrix} \begin{bmatrix} J \\ K \\ I \end{bmatrix}$$

whence

$$\begin{bmatrix} J \\ K \\ I \end{bmatrix} = \begin{bmatrix} \frac{1}{2}p(p-1) & 0 & 0 \\ 2(p-1) & p-2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

ii) Calculation matrices and idempotents in terms of incidence matrices and *vice versa*.

We now derive expressions for the calculation matrices, J, K and I, and the idempotent matrices, E_1, E_2 and E_3, in terms of the incidence matrices I, A and B, and *vice versa*

Firstly, it is clear that

$$\begin{bmatrix} J \\ K \\ I \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ A \\ B \end{bmatrix}$$

and so

$$\begin{bmatrix} I \\ A \\ B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} J \\ K \\ I \end{bmatrix}$$

Then,

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{p(p-1)} & 0 & 0 \\ \frac{-4}{p(p-1)} & \frac{1}{p-2} & 0 \\ \frac{2}{p(p-1)(p-2)} & \frac{-1}{p-2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ A \\ B \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{p(p-1)} & \frac{2}{p(p-1)} & \frac{2}{p(p-1)} \\ \frac{2}{p} & \frac{p-4}{p(p-2)} & \frac{-4}{p(p-2)} \\ \frac{p-3}{p-1} & \frac{-(p-3)}{(p-1)(p-2)} & \frac{2}{(p-1)(p-2)} \end{bmatrix} \begin{bmatrix} I \\ A \\ B \end{bmatrix}$$

Similarly, by multiplying the appropriate matrices, we obtain

$$\begin{bmatrix} I \\ A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2(p-2) & p-4 & -2 \\ \frac{1}{2}(p-2)(p-3) & -(p-3) & 1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

iii) Variance-covariance matrix and test statistics in terms of the idempotents

$$\begin{aligned} \text{But} \quad V &= v_1 I + v_2 A + v_3 B \\ &= \{v_1 + 2(p-2)v_2 + \tfrac{1}{2}(p-2)(p-3)v_3\} E_1 \\ &+ \{v_1 + (p-4)v_2 - (p-3)v_3\} E_2 \\ &+ \{v_1 - 2v_2 + v_3\} E_3. \end{aligned}$$

$$\text{with} \quad \underset{\sim}{z}' V^{-1} \underset{\sim}{z} = \frac{\underset{\sim}{z}' E_1 \underset{\sim}{z}}{v_1 + 2(p-2)v_2 + \frac{1}{2}(p-2)(p-3)v_3} + \frac{\underset{\sim}{z}' E_2 \underset{\sim}{z}}{v_1 + (p-4)v_2 - (p-3)v_3}$$

$$+ \frac{\underset{\sim}{z}' E_3 \underset{\sim}{z}}{v_1 - 2v_2 + v_3}$$

Rewriting this formula in terms of summations of z_{ij}'s leads to precisely the same formula as given in (4.6) and the remainder of the proof of theorem

4.1 is straightforward.

4.1.3 ANALYSIS OF VARIANCE FOR THE EQUALLY-CORRELATED SET

The analysis appropriate under the hypothesis of equal correlation can be summarised in the form of an analysis of variance table as follows:

Source	DF	Sums of squares	Divisor
Mean	1	$\frac{2}{p(p-1)} z_{++}^2$	v_1
Variables	p-1	$\frac{1}{p-2} \sum_{i=1}^{p} z_{i+}^2 - \frac{4}{p(p-2)} z_{++}^2$	$v_1 + (p-4)v_2 - (p-3)v_3$
Variable interaction	$\frac{1}{2}p(p-3)$	$\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij}^2 - \frac{1}{p-2} \sum_{i=1}^{p} z_{i+}^2 + \frac{2}{(p-1)(p-2)} z_{++}^2$	$v_1 - 2v_2 + v_3$

The χ^2 values for the above table are formed by dividing the sums of squares for each line by the corresponding divisor. These values are only approximately distributed as χ^2 since the divisors are a function of ρ, the unknown population correlation coefficient between any two variables under M_1. A natural estimate for ρ would be the inverse transform of the generalised least squares estimate of ξ, viz.

$$\hat{\xi} = \underset{\sim}{1}'V^{-1}\underset{\sim}{z}/\underset{\sim}{1}'V^{-1}\underset{\sim}{1}$$

However, since

$$V^{-1} = \frac{1}{\lambda_1} E_1 + \frac{1}{\lambda_2} E_2 + \frac{1}{\lambda_3} E_3$$

and

$$\underset{\sim}{1}'E_2 = \underset{\sim}{1}'E_3 = \underset{\sim}{0}'$$

it readily follows that

$$\hat{\xi} = 1'E_1\underset{\sim}{z}/\underset{\sim}{1}'E_1\underset{\sim}{1}$$

$$= \bar{z}_{++}$$

and so we use

$$\hat{\rho} = g^{-1}(\bar{z}_{++}).$$

The attainment of a significant result will depend on (a) the variability of the observed z-transforms and (b) the magnitude of the correlations as these affect the sums of squares and divisors, respectively.

The decomposition of the total sums of squares given above and the linear model associated with it are the same as that for the analysis of the results of a plant breeding experiment in which all possible reciprocal or diallel crosses are made between plants that are self sterile and if the effect of sex of the parents is ignored. That is, it corresponds to the analysis for diagonal sums given by Yates (1947) for such an experiment (see also Giffing (1956a,b)). Wilkinson (1970) describes the analysis of diallel experiments using sweeps.

The Mean line provides a test of whether or not ρ is significantly different from zero, a significant χ^2 for this line indicating that ρ is significantly different from zero.

Examining the significance of the χ^2's for the Variables and Variables Interaction lines leads to conclusions concerning the pattern that exists in the observed z-transforms and hence the observed correlation matrix (the z-transformation being a strictly increasing monotonic function).

If both the Variables and Variables Interaction lines are not significant, then one would retain the hypothesis of equal correlation. If either the Variables or Variables Interaction lines is significant, then one would reject the null hypothesis of equal correlation.

If only the Variables lines is significant, this indicates that the pattern in the z-transforms would be adequately described by the linear model

$$E[Z_{ij}] = \mu + \alpha_i + \alpha_j \qquad (4.10)$$

The partition of χ^2 based upon an overall null hypothesis, viz. ρ constant, is valuable because it supplies a more sensitive test of the first alternative hypothesis, viz. variation between tasters, and even in the presence of this, an approximate test of the second null hypothesis, viz. interaction between tasters. Some simulation experiments have tended to confirm this approximate property (W.N. Venables, *personal communication*).

This implies that some variables tend to be relatively well correlated with the other variables while another group of variables tend to be relatively poorly correlated with the remaining variables. That is, the α_i's reflect the general strength of correlations between the i^{th} variable and the other variables. In the situation under discussion, some α_i's will be larger than others; the variables with larger α_i's tend to be better correlated with the other variables than those with smaller α_i's.

If the Variables Interaction line is significant, then, to describe the pattern in z-transforms adequately, the following linear model would be necessary:

$$E[Z_{ij}] = \mu + \alpha_i + \alpha_j + \gamma_{ij} \qquad (\gamma_{ij} = \gamma_{ji}) \qquad (4.11)$$

That is, the magnitude of the correlation of one variable with any other variable depends on the particular pair of variables involved. The γ_{ij}'s reflect the specific strength of the correlation between the i^{th} and j^{th} variables.

The justification for these models lies in the fact that, in general, z_{ij}'s for which equation (4.10) is an adequate description will contribute to the Variables SSQ but not appreciably as a rule in practice to the Variables Interaction SSQ. On the other hand, z_{ij}'s for which equation (4.11) is required to provide an adequate description can have components in both sums of squares.

this paragraph here

More detailed information about the pattern in the z-transforms is obtained from smoothed means estimated from the z-transforms. The formulae for these are derived using the expressions in which the idempotent matrices are given in terms of the calculation matrices.

Let $\bar{J} = \dfrac{2J}{p(p-1)}$,

and $\bar{K} = \dfrac{K}{p-1}$.

Then,

$$E_1 = \bar{J}$$

$$E_2 = \frac{p-1}{p-2}(\bar{K} - 2\bar{J}),$$

and

$$E_3 = I - \frac{p-1}{p-2}(\bar{K} - 2\bar{J}) - \bar{J}.$$

Clearly, the estimated effects are given by (cf. Yates (1947))

$$\hat{\mu} = \frac{2z_{++}}{p(p-1)},$$

$$\hat{\alpha}_i = \frac{p-1}{p-2}\left(\frac{z_{i+}}{p-1} - \hat{\mu}\right),$$

and

$$\hat{\gamma}_{ij} = z_{ij} - \hat{\mu} - \hat{\alpha}_i - \hat{\alpha}_j.$$

And so the smoothed means are given by

$$\hat{\mu} = \frac{2z_{++}}{p(p-1)},$$

$$\widehat{\mu+\alpha_i} = \frac{z_{i+}}{(p-2)} - \frac{2z_{++}}{p(p-1)(p-2)}$$

and $\widehat{\mu+\alpha_i+\alpha_j+\gamma_{ij}} = z_{ij}.$

4.2 ANALYSIS FOR VARIABLES CROSS-INDEXED BY TWO FACTORS

In this section we derive, for the special case of m = 2, an analysis of the z-transforms of an observed correlation matrix of the form given in section 3.2.3. That is the analysis of the vector $\underset{\sim}{z}$ given in equation (3.6). However, instead of a single analysis of the elements of $\underset{\sim}{z}$, we present separate analyses for each of $\underset{\sim}{z}_1$, $\underset{\sim}{z}_2$ and $\underset{\sim}{z}_3$.

The matrices V_1, V_2 and V_3 of equation (3.7) are invariant under permutation of either the levels of A or the levels of B. Thus, each of the sets of matrices $\{V_1\}$,$\{V_2\}$ and $\{V_3\}$ forms the commuting algebra of the respective permutation groups. The analysis of these algebras will lead to expressions for each V_i matrix in terms of the idempotents of the corresponding algebra. This will enable us to express the inverse of each V_i in terms of these idempotents which leads to partition of the chi-square statistics, $\chi_i^2 = \underset{\sim}{Z}_i' V_i^{-1} \underset{\sim}{Z}_i$.

We will use the analysis of variance technique to establish idempotents that specify partitions of $\underset{\sim}{Z}_i' V_i^{-1} \underset{\sim}{Z}_i$, as was done in section 4.1.2 for the case of equally correlated variables.

The theorems that specify the partitions of chi-square statistics are first stated (theorems 4.2.1 - 4.2.3). Their proofs follow in sections 4.2.1 - 4.2.3. An indication of the proof of the theorems using the algebraic technique is given in section 4.2.4. A synopsis of the analysis procedure is given in section 4.2.5.

In theorem 4.2.3 we only consider the case m = 2, however, the analysis could be applied in the case m > 2 to any particular off-diagonal block. The full algebra generated by the variance-covariance matrix of $\underset{\sim}{z}_3$, in the general case (m > 2), has not been fully analysed. The particular case of m = 2 suffices for the applications discussed in chapter 5.

THEOREM 4.2.1: Let $\underset{\sim}{z}_1$ be the vector containing the m sets of $\frac{1}{2}p(p-1)$ z-transforms of the off-diagonal elements of the diagonal blocks of the observed correlation matrix (i.e. $\underset{\sim}{z}_1$ is as given in equation (3.6)).

Under the hypothesis that the correlation matrix is of the form given by equation (3.5), the total variation of the z-transforms, $\underset{\sim}{z}_1' V_1^{-1} \underset{\sim}{z}_1$, can be partitioned into independent components as follows:

$$\begin{aligned}
\underset{\sim}{z}_1' V_1^{-1} \underset{\sim}{z}_1 &= \sum_{i=1}^{6} \lambda_i^{-1} \underset{\sim}{z}_1' E_i^* \underset{\sim}{z}_1 \\
&= \frac{1}{\lambda_1}\left\{\frac{2}{mp(p-1)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_2}\left\{\frac{1}{m(p-2)} \sum_{i=1}^{p} z_{i++}^2 - \frac{4}{mp(p-2)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_3}\left\{\frac{1}{m} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij+}^2 - \frac{1}{m(p-2)} \sum_{i=1}^{p} z_{i++}^2 + \frac{2}{m(p-1)(p-2)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_4}\left\{\frac{2}{p(p-1)} \sum_{k=1}^{m} z_{++k}^2 - \frac{2}{mp(p-1)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_5}\left\{\frac{1}{p-2} \sum_{k=1}^{m} \sum_{i=1}^{p} z_{i+k}^2 - \frac{1}{m(p-2)} \sum_{i=1}^{p} z_{i++}^2 - \frac{4}{p(p-2)} \sum_{k=1}^{m} z_{++k}^2 \right.\\
&\quad \left. + \frac{4}{mp(p-2)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_6}\left\{ \sum_{k=1}^{m} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ijk}^2 - \frac{1}{p-2} \sum_{k=1}^{m} \sum_{i=1}^{p} z_{i+k}^2 - \frac{1}{m} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij+}^2 \right.\\
&\quad \left. + \frac{1}{m(p-2)} \sum_{i=1}^{p} z_{i++}^2 + \frac{2}{(p-1)(p-2)} \sum_{k=1}^{m} z_{++k}^2 - \frac{2}{m(p-1)(p-2)} z_{+++}^2\right\}
\end{aligned}$$

where

$$\lambda_1 = v_1 + 2v_2(p-2) + \tfrac{1}{2}v_3(p-2)(p-3) + v_4(m-1) + 2v_5(m-1)(p+1) + \tfrac{1}{2}v_6(m-1)(p-2)(p-3),$$

$$\lambda_2 = v_1 + v_2(p-4) - v_3(p-3) + v_4(m-1) + v_5(m-1)(p-4) - v_6(m-1)(p-3),$$

$$\lambda_3 = v_1 - 2v_2 + v_3 + (m-1)\, v_4 - 2v_5(m-1) + v_6(m-1),$$

$$\lambda_4 = v_1 + 2v_2(p-2) + \tfrac{1}{2}v_3(p-2)(p-3) - v_4 - 2v_5(p-2) - \tfrac{1}{2}v_6(p-2)(p-3),$$

$$\lambda_5 = v_1 + v_2(p-4) - v_3(p-3) - v_4 - v_5(p-4) + v_6(p-3),$$

$$\lambda_6 = v_1 - 2v_2 + v_3 - v_4 + 2v_5 - v_6,$$

$v_1,\ldots,v_6$ are the distinct elements of V_1 from equation (3.8),

and

$$z_{+++} = \sum_{k=1}^{m} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ijk},$$

$$z_{ij+} = \sum_{k=1}^{m} z_{ijk},$$

$$z_{i++} = \sum_{j=1}^{i-1} z_{ji} + \sum_{j=i+1}^{p} z_{ij+},$$

$$z_{++k} = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ijk},$$

$$z_{i+k} = \sum_{j=1}^{i=1} z_{jik} + \sum_{j=i+1}^{p} z_{ijk}, \quad \text{and}$$

$$z_{ijk} = z_{ij}^{kk}.$$

If $E[Z_{ijk}] = \xi_1$, the components are independent and distributed as follows:

$$\lambda_1^{-1} \underset{\sim}{Z}_1' E_1^* \underset{\sim}{Z}_1 \sim \chi^2(1, \lambda_1^{-1} \tfrac{1}{2} mp(p-1)\xi_1^2) \; (= \chi^2(1) \text{ if } \xi_1 = 0),$$

$$\lambda_2^{-1} \underset{\sim}{Z}_1' E_2^* \underset{\sim}{Z}_1 \sim \chi^2(p-1),$$

$$\lambda_3^{-1} \underset{\sim}{Z}_1' E_3^* \underset{\sim}{Z}_1 \sim \chi^2(\tfrac{1}{2}p(p-3)),$$

$$\lambda_4^{-1} \underset{\sim}{Z}_1' E_4^* \underset{\sim}{Z}_1 \sim \chi^2(m-1),$$

$$\lambda_5^{-1} \underset{\sim}{Z}_1' E_5^* \underset{\sim}{Z}_1 \sim \chi^2((m-1)(p-1)),$$

$$\lambda_6^{-1} \underset{\sim}{Z}_1' E_6^* \underset{\sim}{Z}_1 \sim \chi^2(\tfrac{1}{2}p(m-1)(p-3)).$$

THEOREM 4.2.2: Let $\underset{\sim}{z}_2$ be the vector containing the p sets of $\frac{1}{2}m(m-1)$ z transforms of the diagonal elements of the off-diagonal blocks of the observed correlation matrix (i.e. $\underset{\sim}{z}_2$ is as given in equation (3.6)).

Under the hypothesis that the correlation matrix is of the form given by equation (3.5), the total variation in the z-transforms, $\underset{\sim}{z}_2' V_2^{-1} \underset{\sim}{z}_2$, can be partitioned into independent components as follows:

$$
\begin{aligned}
\underset{\sim}{z}_2' V_2^{-1} \underset{\sim}{z}_2 &= \sum_{i=1}^{6} \lambda_i^{-1} \underset{\sim}{z}_2' E_i^* \underset{\sim}{z}_2 \\
&= \frac{1}{\lambda_1}\left\{\frac{2}{mp(m-1)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_2}\left\{\frac{1}{p(m-2)} \sum_{k=1}^{m} z_{k++}^2 - \frac{4}{mp(m-2)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_3}\left\{\frac{1}{p} \sum_{k=1}^{m-1} \sum_{\ell=k+1}^{m} z_{k\ell+}^2 - \frac{1}{p(m-2)} \sum_{k=1}^{m} z_{k++}^2 + \frac{2}{p(m-1)(m-2)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_4}\left\{\frac{2}{m(m-1)} \sum_{i=1}^{p} z_{++i}^2 - \frac{2}{mp(m-1)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_5}\left\{\frac{1}{m-2} \sum_{i=1}^{p} \sum_{k=1}^{m} z_{k+i}^2 - \frac{1}{p(m-2)} \sum_{k=1}^{m} z_{k++}^2 - \frac{4}{m(m-2)} \sum_{i=1}^{p} z_{i++}^2 \right. \\
&\qquad \left. + \frac{4}{mp(m-2)} z_{+++}^2\right\} \\
&+ \frac{1}{\lambda_6}\left\{ \sum_{i=1}^{p} \sum_{k=1}^{m-1} \sum_{\ell=k+1}^{m} z_{k\ell i}^2 - \frac{1}{m-2} \sum_{i=1}^{p} \sum_{k=1}^{m} z_{k+i}^2 - \frac{1}{p} \sum_{k=1}^{m-1} \sum_{\ell=k+1}^{m} z_{k\ell+}^2 \right. \\
&\qquad \left. + \frac{1}{p(m-2)} \sum_{k=1}^{m} z_{k++}^2 + \frac{2}{(m-1)(m-2)} \sum_{i=1}^{p} z_{++i}^2 - \frac{2}{p(m-1)(m-2)} z_{+++}^2\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda_1 &= v_1 - 2v_2(m-2) + \tfrac{1}{2}v_3(m-2)(m-3) + v_4(p-1) + 2v_5(p-1)(m+1) + \tfrac{1}{2}v_6(p-1)(m-2)(m-3), \\
\lambda_2 &= v_1 + v_2(m-4) - v_3(m-3) + v_4(p-1) + v_5(p-1)(m-4) - v_6(p-1)(m-3), \\
\lambda_3 &= v_1 - 2v_2 + v_3 + v_4(p-1) - 2v_5(p-1) + v_6(p-1), \\
\lambda_4 &= v_1 + 2v_2(m-2) + \tfrac{1}{2}v_3(m-2)(m-3) - v_4 - 2v_5(m-2) - \tfrac{1}{2}v_6(m-2)(m-3), \\
\lambda_5 &= v_1 + v_2(m-4) - v_3(m-3) - v_4 - v_5(m-4) + v_6(m-3), \\
\lambda_6 &= v_1 - 2v_2 + v_3 - v_4 + 2v_5 - v_6,
\end{aligned}
$$

$v_1,\ldots,v_6$ are the distinct elements of V_2 from equation (3.9),

and

$$z_{+++} = \sum_{i=1}^{p} \sum_{k=1}^{m-1} \sum_{\ell=k+1}^{m} z_{k\ell i},$$

$$z_{k\ell +} = \sum_{i=1}^{p} z_{k\ell i},$$

$$z_{k++} = \sum_{\ell=1}^{k-1} z_{\ell k+} + \sum_{\ell=k+1}^{m} z_{k\ell +},$$

$$z_{++i} = \sum_{k=1}^{m-1} \sum_{\ell=k+1}^{m} z_{k\ell i},$$

$$z_{k+i} = \sum_{\ell=1}^{k-1} z_{\ell ki} + \sum_{\ell=k+1}^{m} z_{k\ell i},$$

$$z_{k\ell i} = z_{ii}^{k\ell} .$$

If $E[Z_{k\ell i}] = \xi_2$, the components are independent and distributed as follows:

$$\lambda_1^{-1}\, \underset{\sim}{Z}_2'\, E_1^*\, \underset{\sim}{Z}_2 \sim \chi^2(1, \lambda_1^{-1}\, \tfrac{1}{2}mp(m-1)\xi_2^2) \quad (=\chi^2(1) \text{ if } \xi_2 = 0),$$

$$\lambda_2^{-1}\, \underset{\sim}{Z}_2'\, E_2^*\, \underset{\sim}{Z}_2 \sim \chi^2(m-1),$$

$$\lambda_3^{-1}\, \underset{\sim}{Z}_2'\, E_3^*\, \underset{\sim}{Z}_2 \sim \chi^2(\tfrac{1}{2}m(m-3)),$$

$$\lambda_4^{-1}\, \underset{\sim}{Z}_2'\, E_4^*\, \underset{\sim}{Z}_2 \sim \chi^2(p-1),$$

$$\lambda_5^{-1}\, \underset{\sim}{Z}_2'\, E_5^*\, \underset{\sim}{Z}_2 \sim \chi^2((m-1)(p-1)),$$

$$\lambda_6^{-1}\, \underset{\sim}{Z}_2'\, E_6^*\, \underset{\sim}{Z}_2 \sim \chi^2(\tfrac{1}{2}m(p-1)(m-3)).$$

THEOREM 4.2.3: Let $\underset{\sim}{z}_3$ be the vector containing the p(p-1) z-transforms of the off-diagonal elements of the off-diagonal blocks of an observed correlation matrix for the case when m = 2. (i.e. $\underset{\sim}{z}_3$ is as given in equation (3.6) when m = 2).

Under the hypothesis that the correlation matrix is of the form given by equation (3.5), the total variation of the z-transforms, $\underset{\sim}{z}_3' V_3^{-1} \underset{\sim}{z}_3$ can be partitioned into independent components as follows:

$$
\begin{aligned}
\underset{\sim}{z}_3' V_3^{-1} \underset{\sim}{z}_3 &= \sum_{i=1}^{5} \lambda_i^{-1} \underset{\sim}{z}_3' E_i^* \underset{\sim}{z}_3 \\
&= \frac{1}{\lambda_1} \left\{ \frac{1}{p(p-1)} z^2_{(++)(++)} \right\} \\
&+ \frac{1}{\lambda_2} \left\{ \frac{1}{2(p-2)} \sum_{i=1}^{p} z^2_{(i+)(++)} - \frac{2}{p(p-2)} z^2_{(++)(++)} \right\} \\
&+ \frac{1}{\lambda_3} \left\{ \tfrac{1}{2} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z^2_{(i+)(j+)} - \frac{1}{2(p-2)} \sum_{i=1}^{p} z^2_{(i+)(++)} \right. \\
&\qquad \left. + \frac{1}{(p-1)(p-2)} z^2_{(++)(++)} \right\} \\
&+ \frac{1}{\lambda_4} \left\{ \frac{1}{p} \sum_{k=1}^{2} \sum_{i=1}^{p} z^2_{(ik)(++)} - \frac{1}{2p} \sum_{i=1}^{p} z^2_{(i+)(++)} \right\} \\
&+ \frac{1}{\lambda_5} \left\{ \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \left(z^2_{(i1)(j2)} + z^2_{(j1)(i2)} \right) - \frac{1}{p} \sum_{k=1}^{2} \sum_{i=1}^{p} z^2_{(ik)(++)} \right. \\
&\qquad \left. - \tfrac{1}{2} \sum_{i=1}^{p} \sum_{j=i+1}^{p} z^2_{(i+)(j+)} + \frac{1}{2p} \sum_{i=1}^{p} z^2_{(i+)(++)} \right\}
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda_1 &= v_1 + v_2 + 2v_3(p-2) + 2v_4(p-2) + v_5(p-2)(p-3) \\
\lambda_2 &= v_1 + v_2 + v_3(p-4) + v_4(p-4) - 2v_5(p-3), \\
\lambda_3 &= v_1 + v_2 - 2v_3 - 2v_4 + v_5, \\
\lambda_4 &= v_1 - v_2 + v_3(p-2) - v_4(p-2), \\
\lambda_5 &= v_1 - v_2 - 2v_3 + 2v_4,
\end{aligned}
$$

$v_1,\ldots,v_5$ are the distinct values of V_3 from equation (3.10), and

$$z_{(++)(++)} = \sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(z_{(i1)(j2)} + z_{(ji)(i2)}),$$

$$z_{(i+)(++)} = \sum_{\substack{j=1\\ j\neq i}}^{p}(z_{(i1)(j2)} + z_{(j1)(i2)}),$$

$$z_{(i+)(j+)} = z_{(i1)(j2)} + z_{(j1)(j2)},$$

$$z_{(ik)(++)} = \sum_{\substack{j=1\\ j\neq i}}^{p} z_{(ik)(j\ell)}.$$

If $E[Z_{(ik)(j\ell)}] = \xi_3$, the components are independent and distributed as follows:

$$\lambda_1^{-1}\, \underset{\sim}{Z}_3'\, E_1^*\, \underset{\sim}{Z}_3 \sim \chi^2(1, \lambda_1^{-1}\, p(p-1)\xi_3^2) \quad (=\chi^2(1) \text{ if } \xi_3 = 0),$$

$$\lambda_2^{-1}\, \underset{\sim}{Z}_3'\, E_2^*\, \underset{\sim}{Z}_3 \sim \chi^2(p-1),$$

$$\lambda_3^{-1}\, \underset{\sim}{Z}_3'\, E_3^*\, \underset{\sim}{Z}_3 \sim \chi^2(\tfrac{1}{2}p(p-3)),$$

$$\lambda_4^{-1}\, \underset{\sim}{Z}_3'\, E_4^*\, \underset{\sim}{Z}_3 \sim \chi^2(p-1),$$

$$\lambda_5^{-1}\, \underset{\sim}{Z}_3'\, E_5^*\, \underset{\sim}{Z}_3 \sim \chi^2(\tfrac{1}{2}(p-1)(p-2)).$$

4.2.1 PROOF OF THEOREM 4.2.1 VIA THE ANALYSIS OF VARIANCE TECHNIQUE

i) Models and summation formulae for SSQ of an orthogonal partition

For convenience, we write z_{ij}^{kk} as z_{ijk}.

The form of the six incidence matrices that are a basis for V_1 (see equation (3.8)) leads us to conjecture the following set of linear models for z_{ijk} as an appropriate set to generate the invariant subspaces of V_1.

$$M_1 : E[Z_{ijk}] = \mu$$

$$M_2 : E[Z_{ijk}] = \mu + \beta_i + \beta_j$$

$$M_3 : E[Z_{ijk}] = \mu + \beta_i + \beta_j + \gamma_{ij} \qquad (\gamma_{ij} = \gamma_{ji})$$

$$M_4 : E[Z_{ijk}] = \mu + \alpha_k$$

$$M_5 : E[Z_{ijk}] = \mu + \alpha_k + \beta_i + \beta_j + (\alpha\beta)_{ik} + (\alpha\beta)_{jk}$$

$$M_6 : E[Z_{ijk}] = \mu + \beta_i + \beta_j + \gamma_{ij} + \alpha_k + (\alpha\beta)_{ik} + (\alpha\beta)_{jk} + (\alpha\gamma)_{ijk}$$

Let $\mathcal{M}_i$ denote the model subspace corresponding to the model M_i. We require expressions for the idempotents E_1^* to E_6^* which project orthogonally onto the subspaces $\mathcal{M}_1$, $\mathcal{M}_2 \ominus \mathcal{M}_1$, $\mathcal{M}_3 \ominus \mathcal{M}_2$, $\mathcal{M}_4 \ominus \mathcal{M}_1$ $\mathcal{M}_5 \ominus (\mathcal{M}_2 + \mathcal{M}_4)$ and $\mathcal{M}_6 \ominus \mathcal{M}_5$, respectively. That is, we need to establish the analysis of variance corresponding to the sequential fitting of the models.

On noting the similarities between the models given here and those given in section 4.1.2, it is clear that the idempotents $E_1^*,\ldots,E_6^*$ are related to E_1, E_2 and E_3 of section 4.1.2 as follows:

Let $p_1 = \tfrac{1}{2}p(p-1)$.

Then
$$E_1^* = \frac{1}{m} J_m \otimes E_{1p_1},$$

$$E_2^* = \frac{1}{m} J_m \otimes E_{2p_1},$$

$$E_3^* = \frac{1}{m} J_m \otimes E_{3p_1},$$

$$E_4^* = (I_m - \frac{1}{m} J_m) \otimes E_{1p_1},$$

$$E_5^* = (I_m - \frac{1}{m} J_m) \otimes E_{2p_1}, \text{ and}$$

$$E_6^* = (I_m - \frac{1}{m} J_m) \otimes E_{3p_1}. \qquad (4.12)$$

It is also clear that these matrices are mutually annihilating, symmetric and sum to I, and hence specify a complete analysis of variance. It remains to show that V_1 is expressible as a linear combination of them.

We first express these matrices in terms of convenient calculation matrices. To do this, we substitute expressions for E_1, E_2 and E_3 in terms of J, K and I, the calculation matrices defined in section 4.1.2.

Thus,
$$E_1^* = \frac{1}{m} J_m \otimes \frac{1}{p_1} J_{p_1},$$

$$E_2^* = \frac{1}{m} J_m \otimes (\frac{1}{p-2} K_{p_1} - \frac{4}{p(p-2)} J_{p_1}),$$

$$E_3^* = \frac{1}{m} J_m \otimes (I_{p_1} - \frac{1}{p-2} K_{p_1} + \frac{2}{(p-1)(p-2)} J_{p_1}),$$

$$E_4^* = (I_m - \frac{1}{m} J_m) \otimes \frac{1}{p_1} J_{p_1},$$

$$E_5^* = (I_m - \frac{1}{m} J_m) \otimes (\frac{1}{p-2} K_{p_1} - \frac{4}{p(p-2)} J_{p_1}),$$

$$E_6^* = (I_m - \frac{1}{m} J_m) \otimes (I_{p_1} - \frac{1}{p-2} K_{p_1} + \frac{2}{(p-1)(p-2)} J_{p_1}).$$

If we let
$$J = J_m \otimes J_{p_1},$$

$$K_1 = J_m \otimes K_{p_1},$$

$$K_2 = J_m \otimes I_{p_1},$$

$$K_3 = I_m \otimes J_{p_1},$$

$$K_4 = I_m \otimes K_{p_1},$$

$$I = I_m \otimes I_{p_1},$$

then,

$$\begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \\ E_4^* \\ E_5^* \\ E_6^* \end{bmatrix} = \begin{bmatrix} \frac{2}{mp(p-1)} & 0 & 0 & 0 & 0 & 0 \\ \frac{-4}{mp(p-2)} & \frac{1}{m(p-2)} & 0 & 0 & 0 & 0 \\ \frac{2}{m(p-1)(p-2)} & \frac{-1}{m(p-2)} & \frac{1}{m} & 0 & 0 & 0 \\ \frac{-2}{mp(p-1)} & 0 & 0 & \frac{2}{p(p-1)} & 0 & 0 \\ \frac{4}{mp(p-2)} & \frac{-1}{m(p-2)} & 0 & \frac{-4}{p(p-2)} & \frac{1}{(p-2)} & 0 \\ \frac{-2}{m(p-1)(p-2)} & \frac{1}{m(p-2)} & \frac{-1}{m} & \frac{2}{(p-1)(p-2)} & \frac{-1}{(p-2)} & 1 \end{bmatrix} \begin{bmatrix} J \\ K_1 \\ K_2 \\ K_3 \\ K_4 \\ I \end{bmatrix}$$

whence,

$$\begin{bmatrix} J \\ K_1 \\ K_2 \\ K_3 \\ K_4 \\ I \end{bmatrix} = \begin{bmatrix} \frac{mp(p-1)}{2} & 0 & 0 & 0 & 0 & 0 \\ 2m(p-1) & m(p-2) & 0 & 0 & 0 & 0 \\ m & m & m & 0 & 0 & 0 \\ \frac{p(p-1)}{2} & 0 & 0 & \frac{p(p-1)}{2} & 0 & 0 \\ 2(p-1) & (p-2) & 0 & 2(p-1) & (p-2) & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \\ E_4^* \\ E_5^* \\ E_6^* \end{bmatrix}$$

But,

$$\underset{\sim}{z}'\, J \underset{\sim}{z} = z_{+++}^2$$

$$\underset{\sim}{z}'\, K_1 \underset{\sim}{z} = \sum_{i=1}^{p} z_{i++}^2$$

$$\underset{\sim}{z}'\, K_2 \underset{\sim}{z} = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij+}^2$$

$$\underset{\sim}{z}'\, K_3 \underset{\sim}{z} = \sum_{k=1}^{m} z_{++k}^2$$

$$\underset{\sim}{z}'\, K_4 \underset{\sim}{z} = \sum_{k=1}^{m} \sum_{i=1}^{p} z_{i+k}^2$$

$$\underset{\sim}{z}'\, I \underset{\sim}{z} = \sum_{k=1}^{m} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ijk}^2$$

where z_{+++}, z_{i++}, z_{ij+}, z_{++k}, z_{i+k} and z_{ijk} are as defined in the statement of theorem 4.2.1.

The partition of the total sums of squares is as follows:

$$\text{Total SSQ} = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \sum_{k=1}^{m} z_{ijk}^2 \tag{4.13}$$

$$= \text{Mean SSQ} + \text{B SSQ} + \text{B Interaction SSQ} + \text{A SSQ}$$

$$+ \text{B.A SSQ} + \text{B Interaction.A SSQ}$$

$$= \sum_{i=1}^{6} \underset{\sim}{z}_1' E_i^* \underset{\sim}{z}_1$$

where $$\text{Mean SSQ} = \frac{2}{mp(p-1)} z_{+++}^2,$$

$$\text{B SSQ} = \frac{1}{m(p-2)} \sum_{i=1}^{p} z_{i++}^2 - \frac{4}{mp(p-2)} z_{+++}^2,$$

$$\text{B Interaction SSQ} = \frac{1}{m} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij+}^2 - \frac{1}{m(p-2)} \sum_{i=1}^{p} z_{i++}^2 + \frac{2}{m(p-1)(p-2)} z_{+++}^2,$$

$$\text{A SSQ} = \frac{2}{p(p-1)} \sum_{k=1}^{m} z_{++k}^2 - \frac{2}{mp(p-1)} z_{+++}^2$$

$$\text{B.A SSQ} = \frac{1}{p-2} \sum_{i=1}^{p} \sum_{k=1}^{m} z_{i+k}^2 - \frac{4}{p(p-2)} \sum_{k=1}^{m} z_{++k}^2 - \frac{1}{m(p-2)} \sum_{i=1}^{p} z_{i++}^2$$

$$+ \frac{4}{mp(p-2)} z_{+++}^2$$

$$\text{B Interaction.A SSQ} = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \sum_{k=1}^{m} z_{ijk}^2 - \frac{1}{p-2} \sum_{i=1}^{p} \sum_{k=1}^{m} z_{i+k}^2 + \frac{2}{(p-1)(p-2)} \sum_{k=1}^{m} z_{++k}^2$$

$$- \frac{1}{m} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z_{ij+}^2 + \frac{1}{m(p-2)} \sum_{i=1}^{p} z_{i++}^2 - \frac{2}{m(p-1)(p-2)} z_{+++}^2.$$

ii) Variance-covariance matrix in terms of idempotents

Firstly, it is clear that

$$\begin{bmatrix} J \\ K_1 \\ K_2 \\ K_3 \\ K_4 \\ I \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix}$$

where $A_1,\ldots,A_6$ are the incidence matrices given in equation (3.8).

Then

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & -2 & 0 & -1 & 2 \\ 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} J \\ K_1 \\ K_2 \\ K_3 \\ K_4 \\ I \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 2(p-2) & (p-4) & -2 & 2(p-2) & (p-4) & -2 \\ \tfrac{1}{2}(p-2)(p-3) & -(p-3) & 1 & \tfrac{1}{2}(p-2)(p-3) & -(p-3) & 1 \\ m-1 & m-1 & m-1 & -1 & -1 & -1 \\ 2(m-1)(p-2) & (m-1)(p-4) & -2(m-1) & -2(p-2) & -(p-4) & 2 \\ \tfrac{1}{2}(m-1)(p-2)(p-3) & -(m-1)(p-3) & (m-1) & -\tfrac{1}{2}(p-2)(p-3) & p-3 & -1 \end{bmatrix} \begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \\ E_4^* \\ E_5^* \\ E_6^* \end{bmatrix}$$

From equation (3.8) we have

$$V_1 = \sum_{i=1}^{6} v_i A_i$$

$$= [v_1 \; v_2 \; v_3 \; v_4 \; v_5 \; v_6] \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix}$$

$$\begin{aligned} = \; & \{v_1 + 2v_2(p-2) + \tfrac{1}{2}v_3(p-2)(p-3) + v_4(m-1) + 2v_5(m-1)(p-1) \\ & + \tfrac{1}{2}v_6(m-1)(p-2)(p-3)\} \; E_1^* \\ + \; & \{v_1 + v_2(p-4) - v_3(p-3) + v_4(m-1) + v_5(m-1)(p-4) \\ & - v_6(m-1)(p-3)\} \; E_2^* \\ + \; & \{v_1 - 2v_2 + v_3 + (m-1)\, v_4 - 2v_5(m-1) + v_6(m-1)\} \; E_3^* \\ + \; & \{v_1 + 2v_2(p-2) + \tfrac{1}{2}v_3(p-2)(p-3) - v_4 - 2v_5(p-2) \\ & - \tfrac{1}{2}v_6(p-2)(p-3)\} \; E_4^* \\ + \; & \{v_1 + v_2(p-4) - v_3(p-3) - v_4 - v_5(p-4) + v_6(p-3)\} \; E_5^* \\ + \; & \{v_1 - 2v_2 + v_3 - v_4 + 2v_5 - v_6\} \; E_6^* \end{aligned}$$

$$= \sum_{i=1}^{6} \lambda_i E_i^*$$

The idempotents E_i^* are clearly the idempotents of the commuting algebra as,

(i) the incidence matrices, A_i, can be expressed as a non-singular set of linear combination of them, and

(ii) V_1 can be expressed as a linear combination of them with distinct coefficient (λ_i).

iii) Summation formulae, degrees of freedom and distributions of test statistics

Clearly, $\underset{\sim}{z}_1' V_1^{-1} \underset{\sim}{z}_1 = \sum_{i=1}^{6} \frac{1}{\lambda_i} \underset{\sim}{z}_1' E_i^* \underset{\sim}{z}_1$

Substitution of the summation formulae expressions for $\underset{\sim}{z}_1' E_i^* \underset{\sim}{z}_1$ given in equation (4.13) and the values of λ_i given above lead to the expressions for the partition of $\underset{\sim}{z}_1' V_1^{-1} \underset{\sim}{z}_1$ given in the statement of theorem 4.2.1.

The degrees of freedom of each of the terms $\underset{\sim}{z}_1' E_i^* \underset{\sim}{z}_1$ is equal to $tr(E_i^*)$.

On noting that $tr(A \otimes B) = tr(A)\, tr(B)$, it immediately follows from equations (4.7) and (4.12) that

$$\begin{aligned} tr(E_1^*) &= 1, \\ tr(E_2^*) &= p-1, \\ tr(E_3^*) &= \tfrac{1}{2}p(p-3), \\ tr(E_4^*) &= m-1, \\ tr(E_5^*) &= (m-1)(p-1), \\ tr(E_6^*) &= \tfrac{1}{2}p(m-1)(p-3). \end{aligned}$$

To the extent that $\underset{\sim}{Z}_1 \sim N(\xi_1 \underset{\sim}{1}, V_1)$, the underlying distributions of the test statistics are clearly as follows (see Rao (1965), (vii) of sec. 3b4).

$$\begin{aligned} \lambda_1^{-1} \underset{\sim}{Z}_1' E_1^* \underset{\sim}{Z}_1 &\sim \chi^2(1, \lambda_1^{-1}\, \tfrac{1}{2}mp(p-1)\xi_1^2), \\ \lambda_2^{-1} \underset{\sim}{Z}_1' E_2^* \underset{\sim}{Z}_1 &\sim \chi^2(p-1), \\ \lambda_3^{-1} \underset{\sim}{Z}_1' E_3^* \underset{\sim}{Z}_1 &\sim \chi^2(\tfrac{1}{2}p(p-3), \\ \lambda_4^{-1} \underset{\sim}{Z}_1' E_4^* \underset{\sim}{Z}_1 &\sim \chi^2(m-1), \\ \lambda_5^{-1} \underset{\sim}{Z}_1' E_5^* \underset{\sim}{Z}_1 &\sim \chi^2((m-1)(p-1)), \\ \lambda_6^{-1} \underset{\sim}{Z}_1' E_6^* \underset{\sim}{Z}_1 &\sim \chi^2(\tfrac{1}{2}p(m-1)(p-3)). \end{aligned}$$

Under the hypothesis that $\rho_1 = 0$ (and hence $\xi_1 = 0$), the test statistics reduce to

$$\frac{1}{v_1} \underset{\sim}{Z}_1' E_i^* \underset{\sim}{Z}_1$$

and these have central χ^2 distributions.

4.2.2 PROOF OF THEOREM 4.2.2 VIA THE ANALYSIS OF VARIANCE TECHNIQUE

The proof of theorem 4.2.2 is precisely the same as that of theorem 4.2.1 except that the factors A and B are interchanged and the basis for V_2 is as prescribed in equation (3.9). However, some simplication of the formulae is obtained when there are only two levels of A (i.e., m=2). For convenience we write z_{ii}^{12} as z_i. That is, there are only p correlations and, in this situation, only the Mean and B SSQ are relevant. Substitution in the appropriate formulae of the previous section gives

$$V_2 = [v_1\ v_2\ v_3\ v_4\ v_5\ v_6] \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ p-1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_1^* \\ E_4^* \end{bmatrix}$$

Thus, $$\underset{\sim}{z}_2' V_2^{-1} \underset{\sim}{z}_2 = \frac{\underset{\sim}{z}_2' E_1^* \underset{\sim}{z}_2}{v_1 + (p-1)v_4} + \frac{\underset{\sim}{z}_2' E_4^* \underset{\sim}{z}_2}{v_1 - v_4}$$

where $$\underset{\sim}{z}_2' E_1^* \underset{\sim}{z}_2 = \frac{\left(\sum_{i=1}^{p} z_i\right)^2}{p}$$

and $$\underset{\sim}{z}_2' E_4^* \underset{\sim}{z}_2 = \sum_{i=1}^{p} z_i^2 - \frac{\left(\sum_{i=1}^{p} z_i\right)^2}{p} .$$

4.2.3 PROOF OF THEOREM 4.2.3 VIA THE ANALYSIS OF VARIANCE TECHNIQUE

i) Models and summation formulae for SSQ of an orthogonal partition

For the case where factor A has only two levels (i.e. $m = 2$), the variance-covariance matrix, V_3, contains only the five distinct values $v_1,\ldots,v_5$ of Table 3.1 and

$$V_3 = \sum_{i=1}^{5} v_i A_i$$

where $A_1,\ldots,A_5$ are the incidence matrices corresponding to $A_1,\ldots,A_5$ of equations (3.10) and (3.11).

Given the form of these incidence matrices, we propose the following linear models for $z_{(ik)(j\ell)}$ as an appropriate set to generate the invariant subspaces of V_3.

$$M_1 : \quad E[Z_{(ik)(j\ell)}] = \mu$$

$$M_2 : \quad E[Z_{(ik)(j\ell)}] = \mu + \beta_i + \beta_j$$

$$M_3 : \quad E[Z_{(ik)(j\ell)}] = \mu + \beta_i + \beta_j + \gamma_{ij} \qquad (\gamma_{ij} = \gamma_{ji})$$

$$M_4 : \quad E[Z_{(ik)(j\ell)}] = \mu + \beta_i + \beta_j + \theta_{ik} + \theta_{j\ell}$$

$$M_5 : \quad E[Z_{(ik)(j\ell)}] = \mu + \beta_i + \beta_j + \gamma_{ij} + \theta_{ik} + \theta_{j\ell} + \phi_{(ik)(j\ell)}$$

Let $\mathcal{M}_i$ denote the model subspace corresponding to the model M_i. We require expressions for the idempotents E_1^* to E_5^* which are the orthogonal projection operators onto the subspaces $\mathcal{M}_1$, $\mathcal{M}_2 \ominus \mathcal{M}_1$, $\mathcal{M}_3 \ominus \mathcal{M}_2$, $\mathcal{M}_4 \ominus \mathcal{M}_2$ and $\mathcal{M}_5 \ominus (\mathcal{M}_3 + \mathcal{M}_4)$, respectively. We note that $\mathcal{M}_4 \ominus \mathcal{M}_2$ is clearly orthogonal to $\mathcal{M}_3$ (and hence $\mathcal{M}_3 \ominus \mathcal{M}_2$) as $\mathcal{M}_4 \ominus \mathcal{M}_2$ is a skew-symmetric model and $\mathcal{M}_3$ is the general symmetric, non-additive model. Any skew-symmetric model is orthogonal to any symmetric model.

Thus, we wish to establish the analysis of variance corresponding to the fitting of these models. It will be seen that the analysis is in fact the same as that given by Yates (1947) for the analysis of all reciprocal crosses between a set of lines, the two parents in each cross being of different sexes. However, the form of the models given above is different to that used by Yates (1947) but it will allow us to comment on the general model later on.

It is clear that if $p_1 = \frac{1}{2}p(p-1)$

$$E_1^* = \tfrac{1}{2}J_2 \otimes E_{1p_1} = \tfrac{1}{2}J_2 \otimes \frac{1}{p} J_{p_1},$$

$$E_2^* = \tfrac{1}{2}J_2 \otimes E_{2p_1} = \tfrac{1}{2}J_2 \otimes \left(\frac{1}{p-2} K_{p_1} - \frac{4}{p(p-2)} J_{p_1}\right),$$

$$E_3^* = \tfrac{1}{2}J_2 \otimes E_{3p_1} = \tfrac{1}{2}J_2 \otimes \left(I_{p_1} - \frac{1}{p-2} K_{p_1} + \frac{2}{(p-1)(p-2)} J_{p_1}\right).$$

Let

$$J = J_2 \otimes J_{p_1},$$

$$K_1 = J_2 \otimes K_{p_1},$$

$$K_2 = J_2 \otimes I_{p_1}.$$

Then,

$$\begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \end{bmatrix} \quad \begin{bmatrix} \frac{1}{p(p-1)} & 0 & 0 \\ \frac{-2}{p(p-2)} & \frac{1}{2(p-2)} & 0 \\ \frac{1}{(p-1)(p-2)} & \frac{-1}{2(p-2)} & \frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} J \\ K_1 \\ K_2 \end{bmatrix}$$

But,

$$\underset{\sim}{z}_3' \, J \, \underset{\sim}{z}_3 = z^2_{(++)(++)},$$

$$\underset{\sim}{z}_3' \, K_1 \underset{\sim}{z}_3 = \sum_{i=1}^{p} z^2_{(i+)(++)},$$

$$\underset{\sim}{z}_3' \, K_2 \underset{\sim}{z}_3 = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z^2_{(i+)(j+)}$$

where $z_{(++)(++)} = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (z_{(i1)(j2)} + z_{(j1)(i2)}),$

$$z_{(i+)(++)} = \sum_{\substack{j=1 \\ j\neq i}}^{p} (z_{(i1)(j2)} + z_{(j1)(i2)}),$$

and

$$z_{(i+)(j+)} = z_{(i1)(j2)} + z_{(j1)(i2)}$$

Thus, Mean SSQ $= \underset{\sim}{z}_3' E_1^* \underset{\sim}{z}_3$

$$= \frac{1}{p(p-1)} z^2_{(++)(++)} \qquad (4.14)$$

B SSQ $= \underset{\sim}{z}_3' E_2^* \underset{\sim}{z}_2$

$$= \frac{1}{2(p-2)} \sum_{i=1}^{p} z^2_{(i+)(++)} - \frac{2}{p(p-2)} z^2_{(++)(++)}. \qquad (4.15)$$

B Interaction SSQ $= \underset{\sim}{z}_3' E_3^* \underset{\sim}{z}_3$

$$= \tfrac{1}{2} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} z^2_{(i+)(j+)} - \frac{1}{2(p-2)} \sum_{i=1}^{p} z^2_{(i+)(++)}$$

$$+ \frac{1}{(p-1)(p-2)} z^2_{(++)(++)} \qquad (4.16)$$

To obtain expressions for E_4^* we note that, if X is a design matrix such that $R(X) = M_4$ then $X' \underset{\sim}{z}_3$ is the vector whose 2p elements are the sums $z_{(ik)(++)}$ $(i = 1,\ldots,p;\ k = 1,2)$,

where

$$z_{(i1)(++)} = \sum_{\substack{j=1 \\ j\neq i}}^{p} z_{(i1)(j2)}$$

and

$$z_{(i2)(++)} = \sum_{\substack{j=1 \\ j\neq i}}^{p} z_{(j1)(i2)} .$$

Now, $X'X = pI_{2p} + J_{2p} - J_2 \otimes I_p - I_2 \otimes J_p$

and $(X'X)^- = \frac{1}{p} I_{2p} + \frac{1}{p(p-2)} J_2 \otimes I_p - \frac{1}{4(p-1)(p-2)} J_{2p}.$

Thus, $$\underset{\sim}{z}_3' X(X'X)^- X'\underset{\sim}{z}_3 = \frac{1}{p}\sum_{k=1}^{2}\sum_{i=1}^{p} z^2_{(ik)(++)} + \frac{1}{p(p-2)}\sum_{i=1}^{p} z^2_{(i+)(++)}$$

$$- \frac{1}{(p-1)(p-2)} z^2_{(++)(++)}$$

and $$\text{A.B. SSQ} = \underset{\sim}{z}_3' E_4^* \underset{\sim}{z}_3$$

$$= \underset{\sim}{z}_3' X(X'X)^- X'\underset{\sim}{z}_3 - \text{B SSQ} - \text{Mean SSQ}$$

$$= \frac{1}{p}\sum_{k=1}^{2}\sum_{i=1}^{p} z^2_{(ik)(++)} - \frac{1}{2p}\sum_{i=1}^{p} z^2_{(i+)(++)}. \qquad (4.17)$$

Finally,

$$\text{B Interaction.A SSQ} = \underset{\sim}{z}_3' \underset{\sim}{z}_3 - \sum_{i=1}^{4} \underset{\sim}{z}_3' E_i^* z_3$$

$$= \sum_{i=1}^{p-1}\sum_{j=i+1}^{p} (z^2_{(i1)(j2)} + z^2_{(j1)(i2)}) - \frac{1}{p}\sum_{k=1}^{2}\sum_{i=1}^{p} z^2_{(ik)(++)}$$

$$- \tfrac{1}{2}\sum_{i=1}^{p}\sum_{j=i+1}^{p} z^2_{(i+)(j+)} + \frac{1}{2p}\sum_{i=1}^{p} z^2_{(i+)(++)} \qquad (4.18)$$

Let $$\underset{\sim}{z}_3' K_3 \underset{\sim}{z}_3 = \sum_{k=1}^{2}\sum_{i=1}^{p} z^2_{(ik)(++)}.$$

Then, $$\begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \\ E_4^* \\ E_5^* \end{bmatrix} = \begin{bmatrix} \frac{1}{p(p-1)} & 0 & 0 & 0 & 0 \\ \frac{-2}{p(p-2)} & \frac{1}{2(p-2)} & 0 & 0 & 0 \\ \frac{1}{(p-1)(p-2)} & \frac{-1}{2(p-2)} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{-1}{2p} & 0 & \frac{1}{p} & 0 \\ 0 & \frac{1}{2p} & -\frac{1}{2} & -\frac{1}{p} & 1 \end{bmatrix} \begin{bmatrix} J \\ K_1 \\ K_2 \\ K_3 \\ I \end{bmatrix}.$$

Note that

$$\begin{bmatrix} J \\ K_1 \\ K_2 \\ K_3 \\ I \end{bmatrix} = \begin{bmatrix} p(p-1) & 0 & 0 & 0 & 0 \\ 4(p-1) & 2(p-2) & 0 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 2(p-1) & p-2 & 0 & p & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \\ E_4^* \\ E_5^* \end{bmatrix}$$

ii) Variance-covariance matrix in terms of idempotents

Firstly,

$$\begin{bmatrix} J \\ K_1 \\ K_2 \\ K_3 \\ I \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix}$$

where $A_1, \ldots, A_5$ are as given in equation (3.10).

Then

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 1 & -2 & -1 & 2 \\ 1 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} J \\ K_1 \\ K_2 \\ K_3 \\ I \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ 2(p-2) & p-4 & -2 & p-2 & -2 \\ 2(p-2) & p-4 & -2 & -(p-2) & 2 \\ (p-2)(p-3) & -2(p-3) & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1^* \\ E_2^* \\ E_3^* \\ E_4^* \\ E_5^* \end{bmatrix}$$

But, as was noted at the beginning of this section,

$$V_3 = \sum_{i=1}^{5} v_i A_i$$

and so

$$V_3 = \{v_1 + v_2 + 2v_3(p-2) + 2v_4(p-2) + v_5(p-2)(p-3)\}\ E_1^*$$

$$+ \{v_1 + v_2 + v_3(p-4) + v_4(p-4) - 2v_5(p-3)\}\ E_2^*$$

$$+ \{v_1 + v_2 - 2v_3 - 2v_4 + 2v_5\}\ E_3^*$$

$$+ \{v_1 - v_2 + v_3(p-2) - v_4(p-2)\}\ E_4^*$$

$$+ \{v_1 - v_2 - 2v_3 + 2v_4\}\ E_5^*$$

$$= \sum_{i=1}^{5} \lambda_i E_i^* .$$

The idempotents E_i^* are clearly the idempotents of the commuting algebra as the incidence matrices $A_1,\ldots,A_5$ are a non-singular set of linear combinations of them and V_3 can be expressed as a linear combination of them with distinct coefficients.

iii) Summation formulae, degrees of freedom and distributions of test statistics

Clearly, $\underset{\sim}{z}_3' V_3^{-1} \underset{\sim}{z}_3 = \sum_{i=1}^{5} \frac{1}{\lambda_i} \underset{\sim}{z}_3' E_i^* \underset{\sim}{z}_3$.

Substitution of the summation formulae expressions for $\underset{\sim}{z}_3' E_i^* \underset{\sim}{z}_3$ given in equations (4.14) - (4.18) and the values of λ_i given above lead to the expression for the partition of $\underset{\sim}{z}_3' V_3^{-1} \underset{\sim}{z}_3$ given in the statement of theorem 4.2.3.

The degrees of freedom of each of the terms $\underset{\sim}{z}_3' E_i^* \underset{\sim}{z}_3$ is equal to $tr(E_i^*)$.

Thus,

$$tr(E_1^*) = 1,$$

$$tr(E_2^*) = p-1,$$

$$tr(E_3^*) = \tfrac{1}{2}p(p-3),$$

$$tr(E_4^*) = p-1,$$

and

$$tr(E_5^*) = \tfrac{1}{2}(p-1)(p-2) .$$

To the extent that $\underset{\sim}{Z}_3 \sim N(\xi_3 \underset{\sim}{1}, V_3)$, the underlying distributions of the test statistics, $\underset{\sim}{z}_3' E_i^* \underset{\sim}{z}_3$, are clearly χ^2's as specified in the statement of theorem 4.2.3.

4.2.4 PROOF OF THEOREMS 4.2.1 - 4.2.3 VIA THE ALGEBRAIC TECHNIQUE

In this section we give an indication of the proof of the theorems using the algebraic technique. In each case, the multiplication table of an abstract algebra is established. This would then be analysed, leading to the idempotents of the algebra. Expressions for the idempotents in terms of convenient calculation matrices and for the variance-covariance matrix in terms of the idempotents would be derived. From these expressions we would obtain the partitions of the total variability given in the statement of the theorems.

PROOF OF THEOREM 4.2.1

We will now examine the abstract algebra generated by the incidence matrices $A_1, \ldots, A_6$ introduced in equation (3.8). Let

$$A_1 \to i,\ A_2 \to a,\ A_3 \to b,\ A_4 \to i^+,\ A_5 \to a^+ \text{ and } A_6 \to b^+.$$

It is convenient to define the following elements:

$$i^* = \frac{1}{m}(i + i^+),\ a^* = \frac{1}{m}(a + a^+),\ b^* = \frac{1}{m}(b + b^+),$$

$$i^{**} = \frac{1}{m}[(m-1)i-i^+],\ a^{**} = \frac{1}{m}[(m-1)a-a^+)],\ b^{**} = \frac{1}{m}[(m-1)b-b^+]$$

and to express the abstract algebra in terms of these elements rather than the original elements corresponding to the incidence matrices. Then the linear associative algebra generated by the incidence matrices is isomorphic to the abstract algebra generated by i^*, a^*, b^*, i^{**}, a^{**}, and b^{**}. The multiplication table of this algebra is as follows:

x	i^*	a^*	b^*	i^{**}	a^{**}	b^{**}
i^*	i^*	a^*	b^*	0	0	0
a^*	a^*	$2(p-2)i^*+(p-2)a^*+4b^*$	$(p-3)a^*+2(p-4)b^*$	0	0	0
b^*	b^*	$(p-3)a^*+2(p-4)b^*$	$\frac{1}{2}[(p-3)(p-2)i^*+(p-3)(p-4)a^*+(p-4)(p-5)b^*]$	0	0	0
i^{**}	0	0	0	i^{**}	a^{**}	b^{**}
a^{**}	0	0	0	a^{**}	$2(p-2)i^{**}+(p-2)a^{**}+4b^{**}$	$(p-3)a^{**}+2(p-4)b^{**}$
b^{**}	0	0	0	b^{**}	$(p-3)a^{**}+2(p-4)b^{**}$	$\frac{1}{2}[(p-3)(p-2)i^{**}+(p-3)(p-4)a^{**}+(p-4)(p-5)b^{**}]$

That is, the abstract algebra, and hence the algebra generated by the incidence matrices $A_1,\dots,A_6$, is the direct sum of two 3-dimensional algebras both of which are isomorphic to the abstract algebra given in section 4.1.1 for the equally correlated set of variables.

The sub-algebra generated by i*, a* and b* corresponds to an analysis, as outlined in section 4.1, of the sums over the levels of A for each pairwise combination of the levels of B. The sub-algebra generated by i**, a** and b** deals with the components of the observations orthogonal to the first analysis.

The remainder of the proof would utilise the results of section 4.1.1 to obtain expressions for the idempotents of the algebra and then follow the same procedure as was used in section 4.1.1 to obtain expressions for the idempotents in terms of calculation matrices and V_1 in terms of the idempotents.

PROOF OF THEOREM 4.2.2

The proof of theorem 4.2.2 for $m > 2$ is precisely the same as that for theorem 4.2.1 except $A_1,\dots,A_6$ are the incidence matrices given in equation (3.9). The multiplication table for an abstract algebra isomorphic to the algebra generated by the incidence matrices is obtained as described above but with m and p interchanged.

For the case when $m = 2$ the variance-covariance matrix, V_2, is given by

$$V_2 \;=\; v_1A_1 + v_2A_4 .$$

Let $A_1 \to i,\ A_4 \to i^+$

and $i^* \;=\; \frac{1}{p}(i + i^+),\ i^{**} = \frac{1}{p}[(p-1)i - i^+]$.

Then the multiplication table of the abstract algebra generated by i* and i** is as follows:

x	i^*	i^{**}
i^*	i^*	0
i^{**}	0	i^{**}

That is, the idempotents of the algebra are i^* and i^{**}.

Thus,

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{p} & \frac{1}{p} \\ \frac{p-1}{p} & -\frac{1}{p} \end{bmatrix} \begin{bmatrix} A_1 \\ A_4 \end{bmatrix}$$

and the remainder of the proof proceeds straightforwardly.

PROOF OF THEOREM 4.2.3

We will now examine the abstract algebra generated by the incidence matrices $A_1,\ldots,A_5$ introduced in equation (3.10). Let

$$A_1 \to i,\ A_2 \to h,\ A_3 \to c,\ A_4 \to g,\ A_5 \to d.$$

Then the linear associative algebra generated by these incidence matrices is isomorphic to the abstract algebra generated by i, h, c, g and d, the multiplication table of this algebra being as follows:

x	i	c	d	h	g
i	i	c	d	h	g
c	c	2(p-2)i+(p-3)a+2d+g	(p-3)(c+g)+2(p-4)d	g	c+(p-3)g+2d +2(p-2)h
d	d	(p-3)(c+g)+2(p-4)d	(p-2)(p-3)(i+h) +(p-3)(p-4)(c+g) +(p-4)(p-5)d	d	(p-3)(c+g)+2(p-4)d
h	h	g	d	i	c
g	g	c+(p-3)g+2d+(p-2)h	(p-3)(c+g)+2(p-4)d	c	2(p-2)i+(p-3)c+2d+g

Clearly, this algebra also corresponds to the association algebra for a simple rectangular lattice for p(p-1) varieties discussed by Nair (1951).

Two mutually annihilating idempotents are immediately clear, namely

$$i^* = \tfrac{1}{2}(i + h)$$

and $$i^{**} = \tfrac{1}{2}(i - h)$$

(The corresponding matrices, $\tfrac{1}{2}(I + H)$ and $\tfrac{1}{2}(I - H)$, as operators on $\underset{\sim}{z}_3$, have the effect of replacing each element by the mean and half-difference, respectively of symmetrically opposite elements.)

Now put

$$a^* = \tfrac{1}{2}(c + g) \qquad (= i^*c = i^*g)$$

$$b^* = \tfrac{1}{2}d \qquad (= i^*d)$$

and $$a^{**} = \tfrac{1}{2}(c - g) \qquad (= i^{**}c).$$

The algebra now becomes

x	i*	a*	b*	i**	a**
i*	i*	a*	b*	0	0
a*	a*	2(p-2)i*+(p-2)a*+4b*	(p-3)a*+2(p-4)b*	0	0
b*	b*	(p-3)a*+2(p-4)b*	½[(p-2)(p-3)i* + (p-3)(p-4)a* + (p-4)(p-5)b*]	0	0
i**	0	0	0	i**	a**
a**	0	0	0	a**	2(p-2)i**+(p-4)a**

That is, the algebra is the direct sum of a 3-dimensional and a 2-dimensional algebra. The 3-dimensional algebra is isomorphic to the abstract algebra given in section 4.1.1 for the equally correlated set of variables.

Thus, we can utilise the results of section 4.1.1 to write down three of the five idempotents of the algebra generated by $A_1,\ldots,A_5$. The remaining two idempotents can be found by analysing the 2-dimensional sub-algebra generated by i** and a**. The proof of the theorem is then completed using precisely the same procedure as was used in section 4.1.1.

4.2.5 ANALYSIS OF VARIANCE FOR VARIABLES CROSS-INDEXED BY TWO FACTORS

The partition specified by theorems 4.2.1, 4.2.2 and 4.2.3 can be summarized in an analysis of variance table the form of which when m = 2 is as follows:

SOURCE	DF
BETWEEN B WITHIN A	
Mean	1
B	p-1
B Interaction	$\frac{1}{2}p(p-3)$
A	m-1
A.B	(m-1)(p-1)
A.B Interaction	$\frac{1}{2}p(p-3)(m-1)$
Corrected Total	$\frac{1}{2}mp(p-1)-1$
BETWEEN A WITHIN B	
Mean	1
B	p-1
Corrected Total	p-1
BETWEEN A AND B	
Mean	1
B	p-1
B Interaction	$\frac{1}{2}p(p-3)$
A.B	(p-1)
A.B Interaction	$\frac{1}{2}(p-1)(p-2)$
Corrected Total	p(p-1)-1

The divisors for the Mean lines are all equal to v_1 (= 1/n'), this being the appropriate divisor under the hypothesis that $\xi_i = 0$.

The other divisors (the λ_i's of theorems 4.2.1 - 4.2.3) are functions of the unknown population correlation coefficients ρ_1, ρ_2 and ρ_3 of equation (3.5). We use estimates of them given by

$$\hat{\rho}_1 = g^{-1}(\frac{1}{n_1}\ \underset{\sim}{z}_1'\ \underset{\sim}{1}),\quad \hat{\rho}_3 = g^{-1}(\frac{1}{n_2}\ \underset{\sim}{z}_2'\ \underset{\sim}{1}),\quad \hat{\rho}_2 = g^{-1}(\frac{1}{n_3}\ \underset{\sim}{z}_3'\ \underset{\sim}{1})$$

where n is the order of $\underset{\sim}{z}_i$ and $\frac{1}{n}\ \underset{\sim}{z}_i'\underset{\sim}{1}$ is the generalised least squares estimate of ξ_i (see section 4.1.3).

The interpretation of this analysis is analagous to that described for the analysis of an equally correlated set of variables (see section 4.1.3). That is, the significance of various terms will indicate which terms need to be included in a linear model for the z-transforms if the pattern in the transforms is to be adequately described.

The three Mean lines in the analysis are tests of whether or not ρ_1, ρ_3 and ρ_2 respectively are significantly different from zero. A significant χ^2 indicates that the corresponding ρ_i is significantly different from zero.

The B and B interaction lines (in the first and third segments of the analysis) test the general and specific strengths, respectively, of correlations between variables with different levels of B, these effects being independent of A. The B line in the second segment of the analysis tests whether or not there is a significant difference between the correlations involved, the variables from which these correlations are formed differing only in the levels of B.

The A line in the first segment of the analysis tests whether or not there is a significant overall difference between the sets of correlations, each set being comprised of all the correlations at a particular level of A.

The A.B and A.B Interaction lines test whether or not the general and specific strengths, respectively, of correlations between variables with different levels of B depend on A.

The decomposition of the total sums of squares for the third segment of the analysis corresponds to that for the analysis of the results of a diallel breeding experiment in which the parents are of different sexes and are self-sterile. The analysis for such experiments has been described by Yates (1947) and the decomposition of the total sums of squares given in his Table 5 is equivalent to that obtained here. His model is also equivalent but is expressed in a slightly different form, viz:

$$y_{ij} = m + c_i + c_j + \varepsilon_{ij} + d_i - d_j + \delta_{ij},$$

with $$\varepsilon_{ij} = \varepsilon_{ji}$$

We see that the first four terms are symmetrical in i and j, while the d_i's specify a skew-symmetric component. Of course, δ_{ij} is also a non-symmetric term.

More detailed information about the pattern in the observed correlations can be obtained from smoothed estimates from the observed transforms. Formulae for these are derived using expressions for the idempotent matrices given in terms of the calculation matrices. These formulae will be used throughout chapter 5 in interpreting the analysis of the examples.

BETWEEN B WITHIN A

Let $$\bar{J} = \frac{2J}{mp(p-1)},$$

$$\bar{K}_1 = \frac{K_1}{m(p-1)},$$

$$\bar{K}_2 = \frac{K_2}{m},$$

$$\bar{K}_3 = \frac{2K_3}{p(p-1)},$$

$$\bar{K}_4 = \frac{K_4}{p-1} .$$

Then,

$$E_1^* = \bar{J},$$

$$E_2^* = \frac{p-1}{p-2}(\bar{K}_1 - 2\bar{J}),$$

$$E_3^* = \bar{K}_2 - \frac{p-1}{p-2}(\bar{K}_1 - 2\bar{J}) - \bar{J},$$

$$E_4^* = \bar{K}_3 - \bar{J},$$

$$E_5^* = \frac{p-1}{p-2}\,[\bar{K}_4 - \bar{K}_1 - 2(\bar{K}_3 - \bar{J})]$$

$$E_6^* = I - \frac{p-1}{p-2}\,[\bar{K}_4 - \bar{K}_1 - 2(\bar{K}_3 - \bar{J})] - \bar{K}_3 - \bar{K}_2 - \bar{J} .$$

The estimated effects are given by:

$$\hat{\mu} = \frac{2z_{+++}}{mp(p-1)},$$

$$\hat{\beta}_i = \frac{p-1}{p-2}\left(\frac{z_{i++}}{p-1} - \hat{\mu}\right),$$

$$\gamma_{ij} = \frac{z_{ij+}}{m} - \hat{\mu} - \hat{\beta}_i - \hat{\beta}_j,$$

$$\hat{\alpha}_k = \frac{2z_{++k}}{p(p-1)} - \hat{\mu},$$

$$(\widehat{\alpha\beta})_{ik} = \frac{p-1}{p-2}\left(\frac{z_{i+k}}{p(p-1)} - \frac{z_{i++}}{p-1} - \hat{\alpha}_k\right),$$

$$(\widehat{\alpha\gamma})_{ijk} = z_{ijk} - \frac{z_{ij+}}{m} - (\widehat{\alpha\beta})_{ik} - (\widehat{\alpha\beta})_{jk} - \hat{\alpha}_k.$$

Thus, the smoothed means are as follows:

$$\hat{\mu} = \frac{2z_{+++}}{p(p-1)},$$

$$\widehat{\mu+\beta_i} = \frac{z_{i++}}{p-2} - \frac{2z_{+++}}{mp(p-1)(p-2)},$$

$$\widehat{\mu+\beta_i+\beta_j+\gamma_{ij}} = \frac{z_{ij+}}{m} ,$$

$$\widehat{\mu+\alpha_k} = \frac{2z_{++k}}{p(p-1)},$$

$$\widehat{\mu+\beta_i+\alpha_k+(\alpha\beta)_{ik}} = \frac{z_{i+k}}{p(p-2)} - \frac{z_{++k}}{p(p-1)(p-2)},$$

$$\widehat{\mu+\beta_i+\beta_j+\gamma_{ij}+\alpha_k+(\alpha\beta)_{ik}+(\alpha\beta)_{jk}+(\alpha\gamma)_{ijk}} = z_{ijk}$$

BETWEEN A WITHIN B

The formulae for the smoothed means for the present case can be derived from those for the analysis BETWEEN B WITHIN A by interchanging m, i and j with p, k and ℓ, respectively.

BETWEEN A AND B

Let
$$\bar{J} = \frac{J}{p(p-1)},$$

$$\bar{K}_1 = \frac{K_1}{2(p-1)},$$

$$\bar{K}_2 = \frac{K_2}{2},$$

$$\bar{K}_3 = \frac{K_3}{(p-1)}.$$

Then,

$$E_1^* = \bar{J},$$

$$E_2^* = \frac{p-1}{p-2}(\bar{K}_1 - 2\bar{J}),$$

$$E_3^* = \bar{K}_2 - \frac{p-1}{p-2}(\bar{K}_1 - 2\bar{J}) - \bar{J},$$

$$E_4^* = \frac{p-1}{p}(\bar{K}_3 - \bar{K}_1),$$

$$E_5^* = I - \frac{p-1}{p}(\bar{K}_3 - \bar{K}_1) - \bar{K}_2.$$

The estimated effects are given by:

$$\hat{\mu} = \frac{z_{(++)(++)}}{p(p-1)} ,$$

$$\hat{\beta}_i = \frac{z_{(i+)(++)}}{2(p-2)} - \frac{z_{(++)(++)}}{p(p-2)} ,$$

$$\hat{\gamma}_{ij} = \frac{z_{(i+)(j+)}}{2} - \hat{\mu} - \hat{\beta}_i - \hat{\beta}_j ,$$

$$\hat{\theta}_{ik} = \frac{z_{(ik)++}}{p} - \frac{z_{(i+)(++)}}{2p} ,$$

$$\phi_{(ik)(j\ell)} = z_{(ik)(j\ell)} - \hat{\mu} - \hat{\beta}_i - \hat{\beta}_j - \hat{\gamma}_{ij} - \hat{\theta}_{ik} - \hat{\theta}_{j\ell} .$$

Thus the smoothed means are given by:

$$\hat{\mu} = \frac{z_{(++)(++)}}{p(p-1)} ,$$

$$\widehat{\mu+\beta_i} = \frac{z_{(i+)(++)}}{2(p-2)} - \frac{z_{(++)(++)}}{p(p-1)(p-2)} ,$$

$$\widehat{\mu+\beta_i+\beta_j+\gamma_{ij}} = \frac{z_{(i+)(j+)}}{2} ,$$

$$\widehat{\mu+\beta_i+\theta_{ik}} = \frac{z_{(ik)(++)}}{p} + \frac{z_{(i+)(++)}}{p(p-2)} - \frac{(p^2-3p+4)\, z_{(++)(++)}}{2p(p-1)(p-2)} ,$$

$$\widehat{\mu+\beta_i+\beta_j+\gamma_{ij}+\theta_{ik}+\theta_{j\ell}+\theta_{ij}^{k\ell}} = z_{(ij)(k\ell)} .$$

5. RESULTS AND INTERPRETATION OF THE ANALYSES OF z-TRANSFORMS

In this chapter, the analysis procedure presented in chapter 4 is used to investigate the pattern in correlation matrices calculated from the data from the experiments discussed in chapter 2. Sections 5.1 and 5.2 deal with experiments in which a test of the hypothesis of equal correlation is to be performed, while sections 5.3, 5.4 and 5.5 deal with experiments in which a test for the more general equal correlation pattern given in section 3.2.3 is to be performed.

In most instances, the whole analysis of variance table and the full set of tables of smoothed means is presented for completeness; of course, as is generally the case with analysis of variance procedures, much of this information is of limited interest in the final interpretation of the results. The analyses were performed using a set of GENSTAT (Alvey *et al.* (1977)) macros which implement the procedures and which are listed in Appendix B.

5.1 FIVE-YEAR VITICULTURAL FIELD EXPERIMENT

To investigate the agreement between the yields of the vines over the course of the five-year viticultural field experiment discussed in section 2.1, we take the yields from the same year as the observations of a variable. The correlation matrix of order 5 with the years as the variables is calculated from (i) the yields, and (ii) the residuals, corresponding to the sub-sub-plot error or "error (c)" (Cochran and Cox (1957, section 7.2)), from the analysis of the observations from a single year. These two correlation matrices, together with the z-transforms of their elements, are given in Appendix C.

The z-transforms were analysed under the assumption of equal correlation in both cases; that is, using the procedure of section 4.1. The analysis of variance tables and associated smoothed means are given in Table 5.1.

From these analyses I conclude that, while the original yields are not equally correlated, the residuals are with the correlation being approximately .11. So that, provided the variances of the residuals are homogeneous, an overall analysis of the results from all five years for treatment differences would not be invalidated by the behaviour of the residuals at the sub-sub-plot level.

Table 5.1: Analysis of variance tables and smoothed means for the analysis of the z-transforms of the elements of the correlation matrices calculated from the yields and residuals (corresponding to error (c)) from a five-year viticultural field experiment.

I. ANALYSIS OF VARIANCE TABLES

		YIELDS			RESIDUALS		
Source	DF	SSQ	DIV	χ^2	SSQ	DIV	χ^2
Mean	1	2.24	.006	360.60***	.11	.011	10.04**
Years	4	.17	.006	28.73***	.09	.012	7.38 n.s.
Years Interaction	5	.02	.003	6.76n.s.	.02	.009	1.77 n.s.
Corrected Total	9	.19	-	35.48	.10	-	9.15

II. TABLES OF SMOOTHED MEANS

	Yields 1	2	3	4	5	Residuals 1	2	3	4	5
Grand Mean			.47					.11		
Years	1	2	3	4	5	1	2	3	4	5
	.33	.49	.37	.59	.58	-.02	.11	.11	.22	.12

5.2 SINGLE-EVALUATION WINE EXPERIMENT

To investigate the agreement between the tasters' assessments in the single-evaluation wine-experiment described in section 2.2, the scores of the two types of wines were analysed using the procedure of section 4.1. For this analysis, the tasters were taken as defining the variables. The correlation and z-transform matrices based on these variables, are given in Appendix D. The analysis of variance tables and smoothed means for the analysis of the z-transforms are given in Table 5.2.

Table 5.2: Analysis of variance tables and smoothed means from the analysis of the single-evaluation wine experiments in which wines made from Cabernet and Riesling were assessed.

I. ANALYSIS OF VARIANCE TABLES

		CABERNET			RIESLING		
Source	DF	SSQ	DIV	χ^2	SSQ	DIV	χ^2
Mean	1	2.15	.08	25.85***	4.46	.06	71.30***
Tasters	4	.85	.08	10.34*	.21	.06	3.80 n.s.
Tasters Interaction	5	.58	.04	14.25*	.04	.02	1.75 n.s.
Corrected Total	9	1.42	-	24.59	.26	-	5.55

II. TABLES OF SMOOTHED MEANS

	CABERNET					RIESLING				
Grand Mean			.46					.67		
Taster	1	2	3	4	5	1	2	3	4	5
	.69	.20	.41	.79	.24	.81	.52	.74	.53	.74

From the results of these analyses we would conclude that the correlations between tasters' scores, for the Rieslings, were the same for all pairs of tasters, this correlation being about .58 (= $g^{-1}(.67)$). One explanation for these results is that the tasters agree in the scores they assigned to the Riesling wines, within the limits of the variability inherent in a taster's score for a wine. This result is reasonably consistent with that obtained with Tukey's test for non-additivity (see section 2.2), which was significant only because one of the tasters assigned extremely low scores to two of the wines.

For the Cabernet wines, the results of the analysis indicates that the correlations between two tasters' scores depends upon the pair of tasters involved. That is, there is poor agreement between some tasters and better agreement between others (e.g. from the correlation matrix in Appendix C, we see that tasters 1 and 2 are poorly correlated, while tasters 1 and 4 are well correlated). The results of analysis of the z-transforms are not consistent with Tukey's one degree-of-freedom for non-additivity (see section 2.2) as the latter test was non-significant indicating that there was no wine-taster interaction.

5.3 DUPLICATE-EVALUATION WINE EXPERIMENTS

For each of the duplicate-evaluation wine experiments described in section 2.3, we shall investigate the agreement and reliability of the tasters over the sessions in which the wines were tasted. Thus, the correlation matrix for each experiment, and the associated matrix of z-transforms, will be of the form described in section 3.2.3. Following Campbell and Fiske (1959), these matrices will be termed multitaster-multisession matrices. The variables on which the matrix is based are indexed by the two factors, Sessions (or Types), of which there are m(=2), and Tasters, of which there are p (= 6 or 8).

The estimated correlation and z-transform multitaster-multisession matrices for each of the experiments are given in Appendices E1 and E2. The matrices in Appendix E1 have been calculated from the original scores. Those in Appendix E2 have been calculated using residuals which were the calculated effects for Bottles within Wine replicates for experiment 1 and for Wine replicates within Plots for the other three experiments. These effects were calculated separately for the scores of each taster at each session or with each glass type.

The matrices for experiment 4 were also calculated after uniform random variation over the interval $[-\frac{1}{2},\frac{1}{2}]$ had been added to both the bouquet and palate scores. This was necessary because Taster F had a correlation of exactly 1.0 between his scores for the two different types of glass, a consequence of the discrete nature of the data. The z-transformation is undefined for this value and so uniform random variation was added as an empirical correction for continuity.

As was discussed in chapter 1, multitaster-multisession correlation matrices are comprised of three classes of correlation coefficients. Firstly, there are the correlations between a taster's wine scores on one occasion and those for the same set of wines on a subsequent occasion; these correlations measure taster reliability. Secondly, there are correlations between two tasters' scores for the same set of wines on the same occasion, and thirdly, correlations between two tasters' scores for the same set of wines on different occasions. These latter two groups provide a measure of the agreement between the tasters.

Campbell and Fiske (1959) proposed several desirable conditions to be met by the elements of multitrait-multimethod matrices when the validity of various methods of measuring a number of traits is being investigated. A similar set of desiderata for the elements of multi-taster-multisession correlation matrices is as follows. Firstly, the correlations from the first group should, in each case, be significantly greater than zero and sufficiently large if the taster is to be considered reliable. Secondly, for a particular taster, the magnitude of his correlations from the first group should be significantly greater than that of his correlations from the other two groups. That is, a taster should correlate more highly with himself than with the other tasters. Thirdly, the magnitude of all of the correlations between the same two tasters from the second and third groups should be approximately equal. That is, the correlation between a pair of tasters should be independent of the sessions involved. Finally, the correlations from the second and third groups should be significantly greater than zero and sufficiently large if the tasters are to be considered in agreement. Clearly, the second and third conditions would not be met if there are factors operating that result in the differences between tasters' scores given on the same occasion being less variable than those between scores given on different occasions. If this is the case, then the magnitude of the correlations in the second group would, in general, be greater than those in the third group and the magnitude of those from the first group may be less than those in the second and third groups.

The analysis procedure described in section 4.2 has been applied to the z-transforms of the elements of the multitaster-multisession correlation matrices calculated from the original scores, and to those calculated from the residuals, of each of the four double-evaluation wine experiments described in section 2.3. The three segments of this analysis yield information about three aspects of taster performance and each segment corresponds to one of the three classes of correlation coefficients mentioned above, viz:

(i) BETWEEN TASTERS WITHIN SESSIONS - corresponds to the second class of correlations and analyses the agreement between tasters within sessions and its stability over the sessions.

(ii) BETWEEN SESSIONS WITHIN TASTERS - corresponds to the first class of correlations and analyses the reliability of the tasters, i.e. the agreement between a taster's scores in different sessions.

(iii) BETWEEN SESSIONS AND TASTERS - corresponds to the third class of correlation and analyses the agreement between tasters' scores in different sessions.

The results of these analyses for the full sets of tasters in each experiment are presented in section 5.3.1, whilst those for the subsets of tasters satisfying desiderata 1 and 4 above are given in section 5.3.2. It will be seen that the analyses performed on z-transforms based on original scores provide evidence of the tenability of the four desiderata. The analyses performed on z-transforms based on residuals are relevant to the consideration of the validity of an analysis of variance for treatment differences.

5.3.1 ANALYSIS OF FULL SETS OF TASTERS

The analysis of variance tables calculated from the results of the four experiments are given in Appendix F. A summary of the results of the significance tests is given in Table 5.3.

The significance tests indicate that the correlations for experiment 1, whether based on the original scores or the residuals are completely heterogeneous; the magnitude of the correlation depends on the tasters and sessions involved. However, in all three segments of the two analyses there is a significant additive taster effect, indicating that, in spite of the overall heterogeneity of the z-transforms, the tasters differ in the overall magnitude of their z-transforms.

For experiment 2, the results of the analysis based on the original scores indicate that the magnitude of the correlation is independent of the sessions involved, but that it does depend on the tasters involved. Further, there are no significant additive taster effects and so there are no overall differences between the tasters in the magnitude of their correlations. On the other hand, there are no significant lines in the analysis based on the residuals, other than Mean lines, indicating that the correlations based on the residuals are homogeneous. This is the only experiment for which the Tasters line in the second segment of the analysis is not significant. That is, it is the only experiment where there are no significant differences in the tasters' ability to repeat,

Table 5.3: Summary of results of tests of significance in the analyses of variance of the z-transforms of the elements of the taster-session correlation matrices.

	EXPERIMENT 1		2		3		4†	
	Original	Residual	Original	Residual	Original	Residual	Original	Residual
BETWEEN TASTERS WITHIN SESSIONS								
Mean	c	b	c	c	c	c	c	–
Tasters	c	–	–	–	a	–	c	–
Tasters Interaction	–	c	c	–	–	–	a	–
Sessions	–	a	–	–	–	–	–	–
Sessions.Tasters	a	b	–	–	–	–	–	–
Sessions.Tasters Interaction	c	c	–	–	–	–	–	–
BETWEEN SESSIONS WITHIN TASTERS								
Mean	c	c	c	b	c	c	c	–
Tasters	c	b	–	–	c	a	c	a
BETWEEN TASTERS AND SESSIONS								
Mean	c	–	c	c	c	c	c	–
Tasters	c	b	–	–	a	–	c	–
Tasters Interaction	–	c	b	–	–	–	c	–
Sessions.Tasters	a	c	–	–	–	–	–	–
Sessions.Tasters Interaction	c	b	a	–	–	–	–	–

† For experiment 4, the factor Type should be substituted for Sessions.
a chi-square value is significant at the 5% level of significance;
b chi-square value is significant at the 1% level of significance;
c chi-square value is significant at the 0.1% level of significance;
– chi-square is not significant.

in the second session, the judgements made in the first session.

Experiment 3 is the most homogeneous of the four. However, for both the original and residual scores the tasters differ significantly in their ability to produce similar scores in the two sessions. Also, the tasters differ significantly in the average level of their correlations between tasters within sessions, when these are calculated from the original scores. That is, some tasters scores, in general, correlate well with the other tasters while for other tasters the correlations are poor.

For the original scores of experiment 4, all the lines involving Tasters but not Glass types are significant, while none of those involving Glass types are significant. That is, the magnitude of a correlation is independent of the glass types involved but depends on the tasters involved. The significance of the line Tasters in all three segments indicates that there are differences between the tasters in the magnitude of their agreement with the other tasters and in the similarity of a taster's scores for the two different glass types. Differences between the tasters in the latter case could be due to either, differences in ability to repeat on a second occasion judgements made previously, or to use clear and black glasses equally effectively. The only differences between correlations based on residuals are in the correlations measuring the agreement between a taster's scores for the two different glass types.

Recalling the four desiderata proposed above, we see that:

(i) the third desideratum is met by the full set of tasters from experiments 2, 3 and 4 as (a) the analyses for the full sets indicate that the magnitudes of the correlations from the second and third groups are independent of the sessions (or glass-types) involved and (b) corresponding elements in the tables of Taster Interaction smoothed means from the first and from the third segments of the analyses are approximately equal.

(ii) the other desiderata are tenable for some of the tasters in each experiment, since in all four experiments there are significant Taster or Taster Interaction lines in the analyses. Further examination of the tables of Taster Interaction smoothed means and Taster smoothed means from the second segment of the analysis reveals that in all cases some tasters' correlations are relatively large.

In section 5.3.2, we attempt to obtain subsets of tasters from each experiment that meet the four desiderata in each case.

5.3.2 ANALYSIS OF SUBSETS OF THE TASTERS

In an attempt to obtain a set of homogeneous reliable tasters, the tables of smoothed means from the analyses based on the original scores were examined. Tasters were excluded if they had a relatively lower smoothed mean for the Tasters line in the first and third segments of the analysis of the original scores or a smoothed mean of less than 0.55 (i.e. a correlation of less than 0.5) in the second segment of the analysis of the original scores. The first condition excludes tasters who are, in general, in poor agreement with the other tasters, while the second condition excludes tasters who have low repeatability.

It would also be advantageous if the analysis of the residuals for any subset showed that the corresponding correlation matrix conformed to the more general equal correlation pattern introduced in section 3.2.3. If this were the case, then an analysis of variance for treatment differences on the data of the subset may be valid.

Experiment 1

The tables of smoothed means for the analysis of the full set of tasters that participated in experiment 1 (Tasters A,B,C,F,G,I) are given in Table 5.4.

Clearly, Taster I is in poor agreement with the other tasters, while Taster G is rather unreliable. A summary of the analysis with these

Table 5.4: Tables of smoothed means for experiment 1 calculated from the z-transforms of both the original scores and residuals, and corresponding to the lines in the analysis of variance table.

	Original						Residual					
BETWEEN TASTERS WITHIN SESSIONS												
Mean	.50						.14					
Tasters												
	A	B	C	F	G	I	A	B	C	F	G	I
	.57	.60	.72	.71	.51	-.11	-.10	.16	.33	.25	.10	.10
Tasters Interaction												
	A	B	C	F	G	I	A	B	C	F	G	I
A		.74	.67	.75	.64	-.03		.19	-.20	-.15	.42	-.52
B			.90	.72	.64	-.12			.14	.01	.25	.19
C				1.12	.71	-.02				.81	.19	.53
F					.55	.21					-.11	.58
G						.02						-.21
I												
Sessions												
	1	2					1	2				
	.54	.46					.28	.00				
Sessions.Tasters												
	A	B	C	F	G	I	A	B	C	F	G	I
1	.69	.64	.67	.67	.47	.09	.36	.25	.36	.29	.16	.27
2	.45	.56	.77	.76	.55	-.30	-.56	.07	.30	.22	.04	-.06
BETWEEN SESSIONS WITHIN TASTERS												
Mean	1.02						.52					
Tasters												
	A	B	C	F	G	I	A	B	C	F	G	I
	.96	.80	1.55	1.39	.42	1.02	.34	-.13	1.19	.68	.44	.61

/contd...

Table 5.4 - cont'd

	Original						Residual					
BETWEEN TASTERS AND SESSIONS												
Mean				.52					.09			
Tasters												
	A	B	C	F	G	I	A	B	C	F	G	I
	.55	.61	.72	.70	.65	-.12	-.18	.07	.36	.12	.16	-.02
Tasters Interaction												
	A	B	C	F	G	I	A	B	C	F	G	I
A		.70	.61	.69	.85	-.10		.05	-.17	-.24	.49	-.76
B			.99	.72	.68	-.14			.25	-.10	.17	.00
C				1.04	.78	-.01				.71	.14	.60
F					.70	.17					-.02	.21
G						.12						-.04
I												
Sessions.Tasters												
	A	B	C	F	G	I	A	B	C	F	G	I
1	.58	.64	.68	.72	.52	-.02	-.14	.00	.23	.26	-.06	.22
2	.53	.57	.77	.68	.79	-.22	-.22	.15	.49	-.03	.38	-.26

Table 5.5: Summary of results of the analyses of variance of z-transforms of the original scores and residuals from experiment 1.

I. TESTS OF SIGNIFICANCE

	Tasters			
	All		A,B,C.F	
	Original	Residual	Original	Residual
BETWEEN TASTERS WITHIN SESSIONS				
Mean	c	b	c	–
Tasters	c	–	–	–
Tasters Interaction	–	c	–	c
Sessions	–	a	–	b
Sessions.Tasters	a	b	a	c
Sessions.Tasters Interaction	c	c	–	a
BETWEEN SESSIONS WITHIN TASTERS				
Mean	c	c	c	c
Tasters	c	b	b	b
BETWEEN SESSIONS AND TASTERS				
Mean	c	–	c	–
Tasters	c	b	–	a
Tasters Interaction	–	c	–	b
Sessions.Tasters	a	c	–	–
Sessions.Tasters Interaction	c	b	–	b

II. STANDARD DEVIATIONS

BETWEEN TASTERS WITHIN SESSIONS	.43	.47	.23	.53
BETWEEN SESSIONS WITHIN TASTERS	.41	.44	.35	.56
BETWEEN TASTERS AND SESSIONS	.44	.47	.21	.43

*This procedure for making comparisons between groups of tasters is only descriptive. A more rigorous treatment would involve an appropriate partition of the χ^2 components in the analysis. This comment applies to the other similar situations on pages 96, 99 and 103.

two tasters excluded, together with that for the full set, is given in Table 5.5. The analyses indicate that the heterogeneity of the original scores of the subset of tasters is less than that of the full set of tasters. However, there is still significant deviation of the z-transforms of the original scores from the homogeneity pattern. The residuals of the subset are as heterogeneous as those of the full set.

These conclusions are supported by the standard deviations of the z-transforms (= (Corrected Total SSQ/Corrected Total df)$^{\frac{1}{2}}$) for the three segments of each analysis, these also being given in Table 5.5.*

It is of interest to note that tasters C and F were, in this experiment, the most reliable tasters, having correlations of .91 and .88 respectively, between their scores in the two sessions. They are also in good agreement with each other having an average correlation of .81 between their scores in the same session. Thus they fulfil the four desiderata given previously. In addition, their residuals are reliable and in agreement (see table of smoothed means for residuals), indicating further the credibility of the two tasters' performances.

Experiment 2

The tables of smoothed means from the analysis of the full set of tasters who participated in experiment 2 (Tasters A,B,C,D,E,H) are given in Table 5.6.

In this experiment, none of the Tasters lines are significant but both the Tasters Interaction lines are significant. That is, there were no tasters tending to have higher correlations than other tasters. This, together with the fact that the correlations calculated from the original scores were much lower in this experiment (<0.5 in most cases), suggested that it would not be possible to find a subgroup of the tasters that were more homogeneous than the full set.

The standard deviations of the z-transforms, given in Table 5.7, are also lower than for the other experiments.

Table 5.6: Tables of smoothed means for experiment 2 calculated from the z-transforms of both the original scores and residuals, and corresponding to the lines in the analysis of variance table.

	Original						Residual					
BETWEEN TASTERS WITHIN SESSIONS												
Mean			.28						.18			
Tasters												
	A	B	C	D	E	H	A	B	C	D	E	H
	.30	.19	.30	.27	.37	.27	.19	.06	.23	.18	.18	.23
Tasters Interaction												
	A	B	C	D	E	H	A	B	C	D	E	H
A		.07	.21	.55	.41	.25		.05	.16	.37	.07	.30
B			.25	.11	.23	.37			.07	.00	.11	.19
C				.19	.48	.35				.27	.37	.24
D					.38	.12					.11	.13
E						.27						.23
H												
Sessions												
		1	2						1	2		
		.33	.24						.15	.20		
Sessions.Tasters												
	A	B	C	D	E	H	A	B	C	D	E	H
1	.40	.25	.34	.26	.38	.37	.22	.10	.19	.05	.10	.23
2	.21	.13	.27	.27	.37	.17	.16	.02	.27	.30	.25	.22
BETWEEN SESSIONS WITHIN TASTERS												
Mean			.46						.17			
Tasters												
	A	B	C	D	E	H	A	B	C	D	E	H
	.47	.43	.45	.37	.64	.42	.21	.06	.39	−.05	.08	.34

/contd...

Table 5.6 - cont'd

	Original						Residual					
BETWEEN TASTERS AND SESSIONS												
Mean			.27						.18			
Tasters												
	A	B	C	D	E	H	A	B	C	D	E	H
	.24	.24	.24	.27	.42	.21	.14	.13	.19	.15	.29	.18
Tasters Interaction												
	A	B	C	D	E	H	A	B	C	D	E	H
A		.09	.24	.36	.51	.04		.08	.30	.04	.31	.01
B			.21	.27	.27	.40			.02	.21	.13	.25
C				.13	.38	.28				.10	.26	.26
D					.49	.08					.35	.08
E						.30						.29
H												
Sessions.Tasters												
	A	B	C	D	E	H	A	B	C	D	E	H
1	.31	.25	.28	.20	.36	.23	.19	.14	.22	.12	.27	.15
2	.17	.24	.20	.33	.48	.19	.10	.12	.16	.17	.32	.21

Table 5.7: Standard deviations of z-transforms for both the original scores and residuals of Experiment 2.

	Original	Residual
BETWEEN TASTERS WITHIN SESSIONS	.17	.17
BETWEEN SESSIONS WITHIN TASTERS	.09	.17
BETWEEN SESSIONS AND TASTERS	.19	.16

Experiment 3

The tables of smoothed means from the analysis of the full set of tasters that participated in experiment 3 (Tasters A,B,C,D,E,F), are given in Table 5.8. Examination of the smoothed means for the three Taster lines reveals that taster B is both unreliable and in poor agreement with the other tasters while taster D is unreliable. Summaries of the analysis for the full set and after tasters B and D have been deleted are given in Table 5.9. This subset is almost completely homogeneous as only the Sessions.Tasters line in the first segment of the analysis is significant. This statement is supported by the relatively small standard deviations for this subset (see Table 5.9). The results of the analysis for this subset and examination of the tables of smoothed means for the Taster Interaction line in the first and third segments and the Taster line in the second segment of the analysis, lead to the conclusion that the results for Tasters A,C,E and F in this experiment, essentially meet the desiderata given previously.

Table 5.8: Tables of smoothed means for experiment 3, calculated from the z-transforms of both the original scores and residuals and corresponding to the lines in the analysis of variance table.

	Original						Residual					
BETWEEN TASTERS WITHIN SESSIONS												
Mean				.56						.56		
Tasters												
	A	B	C	D	E	F	A	B	C	D	E	F
	.59	.43	.56	.61	.68	.51	.57	.51	.49	.56	.67	.55
Tasters Interaction												
	A	B	C	D	E	F	A	B	C	D	E	F
A		.45	.58	.57	.75	.58		.57	.45	.54	.73	.57
B			.45	.47	.46	.46			.51	.39	.53	.62
C				.58	.68	.52				.52	.55	.51
D					.86	.51					.85	.47
E						.55						.58
F												
Sessions												
			1	2					1	2		
			.55	.57					.59	.52		
Sessions.Tasters												
	A	B	C	D	E	F	A	B	C	D	E	F
1	.65	.40	.45	.62	.73	.50	.65	.60	.37	.66	.74	.54
2	.55	.47	.67	.59	.64	.52	.50	.43	.62	.45	.60	.56
BETWEEN SESSIONS WITHIN TASTERS												
Mean				.64						.57		
Tasters												
	A	B	C	D	E	F	A	B	C	D	E	F
	.99	.17	.67	.43	.86	.74	.90	.28	.55	.35	.73	.64

/cont'd...

Table 5.8 - cont'd

	Original						Residual					
BETWEEN TASTERS AND SESSIONS												
Mean			.50						.46			
Tasters												
	A	B	C	D	E	F	A	B	C	D	E	F
	.58	.34	.47	.50	.59	.51	.50	.40	.38	.43	.54	.51
Tasters Interaction												
	A	B	C	D	E	F	A	B	C	D	E	F
A		.46	.45	.60	.78	.54		.49	.26	.49	.69	.51
B			.34	.36	.31	.39			.39	.38	.35	.45
C				.49	.62	.48				.39	.48	.46
D					.55	.51					.46	.47
E						.59						.63
F												
Sessions.Tasters												
	A	B	C	D	E	F	A	B	C	D	E	F
1	.63	.28	.42	.52	.67	.47	.49	.35	.28	.52	.62	.49
2	.54	.40	.52	.48	.51	.53	.50	.45	.48	.34	.45	.54

Table 5.9: Summary of the results of the analyses of the z-transforms of the original scores and residuals from experiment 3.

I. TESTS OF SIGNIFICANCE

	Tasters			
	All		A,C,E,F	
	Original	Residual	Original	Residual
BETWEEN TASTERS WITHIN SESSIONS				
Mean	c	c	c	c
Tasters	a	–	–	–
Tasters Interaction	–	–	–	–
Sessions	–	–	–	–
Sessions.Tasters	–	–	b	b
Sessions.Tasters Interaction	–	–	–	–
BETWEEN SESSIONS WITHIN TASTERS				
Mean	c	c	c	c
Tasters	c	a	–	–
BETWEEN SESSIONS AND TASTERS				
Mean	c	c	c	c
Tasters	a	–	–	–
Tasters Interaction	–	–	–	–
Sessions.Tasters	–	–	–	–
Sessions.Tasters Interaction	–	–	–	–

II. STANDARD DEVIATIONS

BETWEEN TASTERS WITHIN SESSIONS	.15	.17	.15	.18
BETWEEN SESSIONS WITHIN TASTERS	.30	.23	.14	.15
BETWEEN TASTERS AND SESSIONS	.15	.16	.14	.18

Experiment 4

The tables of smoothed means from the analysis of the full set of tasters that participated in experiment 4 (Tasters A,D,F,G,J,K,L,M), are given in Table 5.10.

In this experiment, tasters G,K and L are excluded because of their low smoothed means for the Tasters line in the first and third segments of the analysis (i.e. poor agreement with the other tasters). Taster J is excluded because of his low smoothed mean in the second segment of the analysis (i.e. unreliability). The original scores of the tasters not excluded are more homogeneous than those of the full group as is evidenced by the results of the tests of significance and the reduced standard deviations of the z-transforms calculated from the original scores (see Table 5.11). However, the original scores still deviate significantly from the homogeneity pattern. But, except for the tendency of the agreement between tasters to be higher when clear glasses were used rather than black glasses, the results obtained with Tasters A,D,F and M meet the four desiderata given previously in that they are a group of relatively reliable tasters who are, to some extent, in agreement.

There appears to have been little change in the heterogeneity of the residuals as a result of taking only a subgroup of the tasters. Of particular note is the comparatively high smoothed mean for Taster A in the second segment of the analysis.

Table 5.10: Tables of smoothed means for experiment 4 calculated from the z-transforms of both the original scores and residuals, and corresponding to the lines in the analysis of variance table.

	Original								Residual							

BETWEEN TASTERS WITHIN GLASS TYPES

Mean

	Original	Residual
Mean	.55	.03

Tasters

	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M
	.72	.66	.77	.53	.62	.44	.07	.60	.19	-.05	.09	.00	.10	-.10	-.03	.05

Tasters Interaction

	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M
A		.75	1.26	.74	.92	.38	.09	.70		.18	.48	.18	.21	-.13	.11	.14
D			.84	.65	.73	.60	.25	.71			-.05	.11	.22	-.27	-.38	-.05
F				.78	.92	.55	.05	.77				.13	.02	-.13	.05	.10
G					.54	.45	.03	.51					.03	-.13	-.12	-.17
J						.41	.08	.65						-.06	.07	.18
K							.22	.56							.10	.08
L								.24								.05
M																

Glass Types

	Original Clear	Original Black	Residual Clear	Residual Black
	.59	.51	.10	-.03

Glass Types.Tasters

	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M
Clear	.78	.70	.83	.58	.61	.49	.01	.73	.27	-.19	.17	.16	.16	-.07	.04	.24
Black	.65	.63	.71	.47	.63	.39	.13	.46	.10	.09	.02	-.17	.05	-.12	-.09	-.15

BETWEEN GLASS TYPES WITHIN TASTERS

Mean

	Original	Residual
Mean	.87	.13

Tasters

	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M
	1.48	.83	1.43	.57	.51	.68	.56	.94	.90	-.19	.03	.45	-.37	.30	-.13	.11

/cont'd...

Table 5.10 - cont'd

	Original								Residual							
BETWEEN TASTERS AND GLASS TYPES																
Mean	.55								.06							
Tasters																
	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M
	.70	.64	.79	.56	.60	.44	.08	.60	.18	.05	.11	−.08	.09	−.07	.03	.13
Tasters Interaction																
	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M
A		.63	1.23	.70	.95	.34	.09	.80		.06	.28	−.09	.44	−.20	.18	.49
D			.93	.77	.61	.51	.23	.71			.15	.14	.02	−.39	.16	.25
F				.78	.95	.57	.06	.75				.03	.16	.02	−.11	.21
G					.70	.26	.12	.56					.17	−.18	−.36	−.12
J						.41	.04	.49						−.22	.19	−.18
K							.37	.71							.30	.29
L								.14								−.10
M																
Glass Types.Tasters																
	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M
Clear	.67	.68	.74	.60	.54	.45	.06	.66	.13	−.01	.07	−.03	.19	−.14	.07	.18
Black	.73	.60	.83	.51	.66	.42	.10	.55	.23	.12	.16	−.12	−.01	.00	.00	.08

Table 5.11: Summary of the results of the analyses of the z-transforms for both the original scores and residuals from experiment 4.

I. TESTS OF SIGNIFICANCE

	Tasters			
	All		A,D,F,M	
	Original	Residual	Original	Residual
BETWEEN TASTERS WITHIN GLASS TYPES				
Mean	c	–	c	–
Tasters	c	–	b	–
Tasters Interaction	a	–	a	–
Types	–	–	a	–
Types.Tasters	–	–	–	–
Types.Tasters Interaction	–	–	–	–
BETWEEN GLASS TYPES WITHIN TASTERS				
Mean	c	–	c	–
Tasters	c	a	a	a
BETWEEN TASTERS AND GLASS TYPES				
Mean	c	–	c	b
Tasters	c	–	a	–
Tasters Interaction	c	–	a	–
Types.Tasters	–	–	–	–
Types.Tasters Interaction	–	–	–	–

II. STANDARD DEVIATIONS

BETWEEN TASTERS WITHIN GLASS TYPES	.32	.29	.25	.27
BETWEEN GLASS TYPES WITHIN TASTERS	.38	.41	.33	.43
BETWEEN TASTERS AND GLASS TYPES	.32	.30	.23	.28

5.3.3 DISCUSSION

Choosing tasters on the basis of the magnitude of their smoothed means for the Tasters line in the first and third segments of the analysis requires caution when the Tasters Interaction is significant. In such circumstances, as the smoothed means for a particular taster depends upon that taster's correlation with the other tasters, the smoothed mean could change dramatically when some tasters are excluded. Thus, the Tasters Interaction smoothed means should also be examined to ensure that the appropriate subset is being selected.

The correlations between a taster's scores from the different sessions (or glass types) are given in Table 5.12 for all four experiments.

Table 5.12: Correlations between a taster's scores in the different sessions (or glass types) for the original scores of all four experiments

	TASTERS												
Experiment	A	B	C	D	E	F	G	H	I	J	K	L	M
1	.74	.66	.91			.88	.40		.77				
2	.44	.41	.42	.35	.56			.40					
3	.76	.17	.58	.41	.70	.63							
4	.90			.68		.89	.52			.47	.59	.51	.74

Over experiments one, three and four, tasters A and F were consistently reliable. As many of the other tasters have been observed in only one or two tastings, it is difficult to assess their reliability. However, tasters C, E and M show good potential.

In respect of the tasters' performances in the four wine-tasting experiments under discussion, the following conclusions are drawn:

(i) Experiment 1: The group of tasters involved in this experiment were extremely heterogeneous in their assessment of the wines and there was no stability between sessions. Tasters C and F were both reliable and in good agreement; tasters G and I needed to be excluded.

(ii) Experiment 2: The correlations were virtually independent of the sessions but the tasters involved in this experiment were heterogeneous in their assessment of the wines. The correlations in this experiment were, overall, smaller than in other experiments, probably because the wines were young and made from Shiraz grapes and the tasters found them difficult to evaluate. (Note that in this experiment taster A did not perform as reliably as in the other experiments (see Table 5.12).)

(iii) Experiment 3: Tasters A, C, E and F form a reasonably homogeneous subset. The magnitude of the correlations calculated from the residuals is larger in this experiment than in the other experiments and is similar to the magnitude of the correlations calculated from the original scores. In the other experiments, the magnitude of the correlations calculated from the residuals is smaller than that of the correlations calculated from the original scores. An explanation of this phenomenon is that in this experiment the tasters are responding to differences between the wine-making replicates of the grapes harvested from the single group of vines forming a field replicate of a treatment. May *et al.* (1976) reported that there were differences between the wine replicates which they ascribed to differential oxidation of the wines, possibly due to inadequate sealing of some flagons.

(iv) Experiment 4: The correlations calculated from residuals are extremely small and so, in many instances, the residuals of two tasters are independent. Tasters A,D,F and M are a more homogeneous group of tasters than the full set, although they are still not homogeneous.

5.4 BODY COMPOSITION OF CALVES

Swain (1975) analyses an experiment taken from Tudor (1971) in which 58 calves were allocated at birth to one of sixteen nutrition treatment groups, and slaughtered on reaching approximately 400 kg liveweight. The data analysed were the weights (in grams) recorded for five muscles dissected from the left and right hind legs.

I have analysed the z-transforms of the elements of the correlation matrix calculated from the measurements taken from both left and right hind legs. (The correlation and z-transform matrices are given in Appendix G.) These matrices are of the form given in section 3.2.3, being indexed by the factors Side (left vs. right) and Muscle (A, B, C, D, E). Thus the method of section 4.2 is applicable for testing whether or not the relationship between the muscles is the same on both sides of the animal and whether or not correlations between sides are the same for all muscles.

The analysis of variance table for this analysis is presented in Table 5.13 and the tables of smoothed means in Table 5.14. All Muscle lines are significant and all Muscle Interaction lines are non-significant. This indicates that there is a general difference in the magnitudes of the correlations for the different muscles rather than the magnitude of the correlation depending, specifically, on the pair of muscles involved. Examination of the smoothed means reveals that muscle E is poorly correlated with the other muscles and that agreement between the left and right side is less for this muscle than the other muscles.

Re-analysis of the z-transforms after muscle E has been excluded leads to the second analysis shown in Table 5.13. From the analysis we conclude that the correlation between the measurements of the same muscle on different sides is the same for all the muscles A to D (approximately .93) and that muscles on the same side are equally correlated but the magnitude of the latter correlation is different for the

Table 5.13: Analysis of variance table for the analysis of z-transforms from body composition experiment i) for the five hind leg muscles and ii) for the first four hind leg muscles (A-D).

	Muscles A - E				Muscles A - D			
SOURCE	DF	SSQ	DIV	χ^2	DF	SSQ	DIV	χ^2
BETWEEN MUSCLES WITHIN SIDES								
Mean	1	5.11	.03	194.25***	1	6.20	.03	235.77***
Muscles	4	1.39	.04	31.23***	3	.04	.03	1.34n.s.
Muscles Interaction	5	.16	.02	7.78n.s.	2	.03	.02	1.69n.s.
Sides	1	.17	.02	11.13***	1	.07	.01	7.05*
Sides.Muscles	4	.02	.01	2.70n.s.	3	.01	.01	1.36n.s.
Sides.Muscles Interaction	5	.01	.01	4.26n.s.	2	.01	.01	4.84n.s.
Corrected Total	19	1.75	-	57.09n.s.	11	.17	-	16.27
BETWEEN SIDES WITHIN MUSCLES								
Mean	1	10.71	.03	407.04***	1	10.99	.03	417.51***
Muscles	4	.78	.02	34.17***	3	.03	.02	1.44n.s.
Corrected Total	4	.78	-	34.17	3	.03	-	1.44
BETWEEN MUSCLES AND SIDES								
Mean	1	4.62	.03	175.70***	1	5.61	.03	213.03***
Muscles	4	1.26	.04	29.04***	3	.05	.03	1.70n.s.
Muscles Interaction	5	.13	.02	6.49n.s.	2	.02	.02	1.46n.s.
Sides.Muscles	4	.26	.01	22.27***	3	.01	.01	1.19n.s.
Sides.Muscles Interaction	6	.02	.53	5.97n.s.	3	.01	.01	3.62n.s.
Corrected Total	19	1.66	-	63.77	11	.10	-	7.98

Table 5.14: Tables of smoothed means from the analysis of the z-transforms from the body composition experiment involving the measurement of five muscles on the left and right hind legs of calves.

BETWEEN MUSCLES WITHIN SIDES

Mean

Muscles

A	B	C	D	E
.32	.63	.56	.62	.08

Muscles Interaction

	A	B	C	D	E
A		.75	.81	.74	.11
B			.55	.72	.38
C				.75	.09
D					.17
E					

Sides

Left	Right
.41	.60

Sides.Muscles

	A	B	C	D	E
Left	.53	.54	.51	.52	-.04
Right	.73	.71	.61	.73	.20

BETWEEN MUSCLES AND SIDES

Mean

Muscles

A	B	C	D	E
.61	.58	.53	.61	.08

Muscles Interaction

	A	B	C	D	E
A		.70	.76	.73	.12
B			.51	.68	.34
C				.72	.08
D					.17
E					

Sides.Muscles

	A	B	C	D	E
Left	.67	.59	.59	.63	-.06
Right	.54	.58	.49	.58	.21

BETWEEN SIDES WITHIN MUSCLES

Mean 1.46

Muscles

A	B	C	D	E
1.60	1.58	1.80	1.60	..69

two sides (the correlation for muscles on the left is 0.57 and the right is 0.66).

Swain (1975) noticed the smaller left-right correlation for Muscle E which he attributes to a higher measurement error associated with this muscle because it is smaller and harder to dissect out than the others. However, Swain did not mention the difference in the correlation for the two sides as his analysis does not allow this comparison.

5.5 PRIMARY ABILITY TESTS ON TWO OCCASIONS

Corballis and Traub (1970) present the correlation matrix calculated from the data obtained by Meyer and Bendig when they administered the five Thurstone Primary Mental Ability Tests to 110 schoolchildren on each of two occasions separated by three and a half years. McDonald (1975) analysed these results, using maximum likelihood estimates of parameters and an asymptotic likelihood ratio test to test the hypothesis that "the correlations have not changed from the first occasion to the next". His test yielded a χ^2 of 11.72 on 10 degrees of freedom and he concluded that no significant change in the structure of the correlations has occurred from the first occasion to the second.

I have analysed the z-transforms of this correlation matrix (the correlation and z-transform matrices are given in Appendix H) using the method of section 4.2. The analysis of variance table for this analysis is given in Table 5.15 and the smoothed means in Table 5.16.

From this analysis we conclude that, as the lines in the analysis involving Occasions are all non-significant, or nearly so, the structure of the correlations is independent of Occasion. However, as the Occasions.Test Interaction line in the first segment of the analysis is just significant, there is a tendency for the correlation between tests administered on the same occasion to depend on both the pair of tests and the occasion involved. Also, as the Tests line in the second segment of the analysis is significant, the reliability of the tests differs. As the fifth test has a much smaller smoothed mean for this line, its results for the two occasions are less consistent than those of the other tests.

The sum of the χ^2's for the three lines involving Occasions in the first segment of the analysis presented in Table 5.14 is equivalent to McDonald's test in that it tests the same hypothesis; this sum is equal to 14.54 and, as it is a χ^2 on 10 degrees of freedom, is also non-significant.

Table 5.15: Analysis of variance tables for the analysis of the z-transforms of the elements of the Meyer-Bendig correlation matrix for 5 Primary Ability Tests administered on two occasions.

SOURCE	DF	SSQ	DIV	χ^2
BETWEEN TESTS WITHIN OCCASIONS				
Mean	1	2.02	.0092	220.03***
Tests	4	.42	.0135	31.27***
Tests Interaction	5	.09	.0072	12.33*
Occasions	1	.00	.0101	.17n.s.
Occasions.Tests	4	.02	.0056	2.77n.s.
Occasions.Tests Interaction	5	.04	.0035	11.60*
Corrected Total	19	.57	-	58.13
BETWEEN OCCASIONS WITHIN TESTS				
Mean	1	3.67	.0092	400.17***
Tests	4	.26	.0086	30.03***
Corrected Total	4	.26	-	30.03
BETWEEN OCCASIONS AND TESTS				
Mean	1	1.77	.0092	193.25***
Tests	4	.39	.0135	28.91***
Tests Interaction	5	.08	.0070	11.26*
Occasions.Tests	4	.04	.0074	5.25n.s.
Occasions.Tests Interaction	6	.03	.0034	9.45n.s.
Corrected Total	19	.54	-	54.87

Table 5.16: Tables of smoothed means from the analysis of the z-transforms of the elements of the Meyer-Bendig correlation matrix for 5 Primary Ability Tests administered on two occasions.

BETWEEN TESTS WITHIN OCCASIONS

Mean .32

Tests

1	2	3	4	5
.49	.18	.37	.37	.18

Tests Interaction

	1	2	3	4	5
1		.38	.47	.61	.32
2			.26	.10	.13
3				.50	.20
4					.20
5					

Occasions

1	2
.33	.31

Occasions.Tests

	1	2	3	4	5
1	.50	.22	.36	.35	.21
2	.47	.15	.39	.38	.15

BETWEEN TESTS AND OCCASIONS

Mean .30

Tests

1	2	3	4	5
.47	.17	.37	.30	.17

Tests Interaction

	1	2	3	4	5
1		.36	.49	.55	.29
2			.27	.05	.13
3				.43	.22
4					.17
5					

Occasions.Tests

	1	2	3	4	5
1	.43	.21	.35	.32	.19
2	.51	.14	.40	.29	.16

BETWEEN OCCASIONS WITHIN TESTS

Mean

Tests

1	2	3	4	5
1.14	.78	.97	.93	.46

5.6 CONCLUSIONS

The techniques developed in chapter 4 have been used to investigate the patterns in the correlation matrices calculated from the results of wine-tasting experiments.

For wine-tasting experiments, these procedures were used to examine the tasters' performances as a prelude to an analysis to investigate treatment differences. It was found that, for the experiments analysed, the reliability of the tasters' assessments varied and that it was difficult to obtain a group of tasters that are homogeneous in their evaluation.

The analysis of the five-year viticultural field experiment confirmed that the residuals calculated from the measurements in the different years were equally correlated between years. Thus, provided the variability of the yields is the same in each year, the sub-sub-plot residuals fulful conditions that make the use of an overall analysis of variance feasible.

The analysis of the calf body composition leads to similar conclusions to those obtained by Swain (1975) and the analysis of the primary ability test data gave similar results to that performed by McDonald (1975).

6. COMPARISON WITH OTHER TESTS FOR PATTERNS IN CORRELATION MATRICES

The procedures derived in chapter 4 provide tests for (i) equal correlation, and (ii) the more general equal correlation pattern described in section 3.2.3. In addition, they are investigative procedures in that they suggest which alternative models should be pursued further in the event that the observed matrix is judged not to be consistent with the hypothesised pattern.

In this chapter, we compare the results of the analyses given in chapter 5, which were obtained using the procedures given in chapter 4, with those obtained with Lawley's (1963) test for equal correlation and the likelihood ratio statistic.

In section 6.1 we discuss published approaches to testing of patterns in correlation matrices. In section 6.2, we compare the results of the analyses given in sections 5.1 and 5.2 which were obtained using the procedure of section 4.1, with the results obtained using Lawley's test and the likelihood ratio test. In section 6.3, we compare the results of the analyses given in sections 5.3, 5.4 and 5.5, with the results obtained using the likelihood ratio statistic. In section 6.4, a general discussion of three procedures compared in this chapter is given.

6.1 OTHER TESTS FOR PATTERNS IN CORRELATION MATRICES

The likelihood ratio statistic for testing patterns in variance-covariance matrices from a multivariate normal population has been utilised as a basis for deriving tests for patterns in correlation matrices (Bartlett (1950, 1951); Aitkin *et al.* (1968); McDonald (1975); Swain (1975, section 5.6) and Joreskog (1978)).

Tests for specific hypotheses have been derived, originally because of the difficulty in calculating the likelihood ratio statistic. Bartlett (1950, 1954) has given a test for independence between the variables and Bartlett (1950, 1951), Lawley (1963) and Aitkin *et al.* (1968) have proposed tests for the hypothesis that all correlations in a correlation matrix are equal. Of the tests proposed for equal correlation, it appears that Lawley's test has the best statistical properties. Aitkin *et al.* (1968) used empirical sampling studies to show that the asymptotic χ^2 distribution held, for Lawley's test, for $p \leq 6$, any ρ and $N \geq 25$. Gleser (1968) argues that satisfactory asymptotic rejection regions can be given for Lawley's test. Lawley (1963) claims the test is equivalent asymptotically to the likelihood ratio test. The test statistic for his test is given by

$$T_1 = (n'/\lambda^2) \{ \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (r_{ij} - \bar{\gamma})^2 - \mu \sum_{k=1}^{p} (\bar{r}_k - \bar{r})^2 \} \tag{6.1}$$

where

$$\bar{r}_k = \sum_{\substack{i=1 \\ i \neq k}}^{p} r_{ik}/(p-1),$$

$$\bar{r} = 2 \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} r_{ij}/\{p(p-1)\},$$

$$\mu = (p-1)^2(1-\lambda^2)/\{p-(p-2)\lambda^2\},$$

$$\lambda = 1 - \bar{r}.$$

More recently, an extremely general class of models for variance-covariance matrices has been described by Browne (1974), Swain (1975) and Joreskog (1978). The models are of the form

$$\Sigma = \Sigma(\underset{\sim}{\gamma}), \ \underset{\sim}{\gamma} \in \Gamma,$$

where Σ is a $p \times p$ variance-covariance matrix

and Γ is the parameter space which is contained in R^q, the space of all $q \times 1$ vectors.

In addition, certain regularity conditions on the model and $\underset{\sim}{\gamma}_0$, the population value of $\underset{\sim}{\gamma}$, are assumed.

Joreskog (1978) also points out that a general class of models for investigating correlation structure (which includes the patterns described in section 3.2) can be formulated as models for variance-covariance matrices as follows:

$$\Sigma = \Sigma(\underset{\sim}{\theta}),\ \theta \in \Phi$$
$$= DP(\underset{\sim}{\gamma})D$$

where $D = \text{diag}(\underset{\sim}{\sigma}) = \text{diag}(\sigma_1\sigma_2 \ldots \sigma_p)$,

$P(\underset{\sim}{\gamma})$ is a p x p correlation matrix, the form of which is a function of $\underset{\sim}{\gamma}$, and

$\underset{\sim}{\theta}' = (\underset{\sim}{\sigma}'\ \underset{\sim}{\gamma}')$.

That is, Σ is a function of the parameters $\underset{\sim}{\theta}$ which is comprised of both the population standard deviations, $\underset{\sim}{\sigma}$, and $\underset{\sim}{\gamma}$, all of which have to be estimated from an observed sample.

Several estimation procedures have been proposed for fitting the general class of models (Swain (1975)). However, we shall use only maximum likelihood procedures. The likelihood ratio test statistic, when applied to correlation structures, has been shown (Bock and Bergmann (1966)) to reduce to

$$T_2 = -N\log\{|S|/|\hat{\Sigma}|\} \qquad (6.2)$$

where $|\hat{\Sigma}| = \hat{\sigma}_1^2\hat{\sigma}_2^2 \ldots . \sigma_p^2|\hat{P}|$,

$|S| = s_1^2 s_2^2 \ldots . s_p^2 |R|$

$R = \{r_{ij}\}$ is the sample correlation matrix,

$\hat{P} = \{\hat{\rho}_{ij}\}$ is the maximum likelihood estimate of the population correlation matrix under the hypothesised pattern,

s_i^2 = sample variance of the i^{th} variable,

$\hat{\sigma}_i^2$ = maximum likelihood estimate of the population variance of the i^{th} variable under the hypothesised pattern.

Clearly,

$$T_2 = N\log\{|\hat{P}|/|R|\} + N \sum_{i=1}^{p} \log \frac{\hat{\sigma}_i^2}{s_i^2} \quad \text{(c.f. Aitkin *et al.* (1968))} \qquad (6.3)$$

McDonald (1975) has shown that $\hat{P}$ is invariant under changes of scale involving positive scale factors for each variable but that $\hat{\sigma}_i$ are appropriately rescaled. T_2 is in general invariant under such changes of scale (Joreskog (1978)). Thus, for testing patterns in correlation matrices, it is immaterial whether the variance-covariance or correlation matrix is subjected to the fitting procedure. If the fitting procedure is applied to the correlation matrix, then the diagonal elements of the estimated matrix will be the ratios $\hat{\sigma}_i^2/s_i^2$.

The program for fitting the general model of variance-covariance structure given by Swain (1975, appendix B) was used to fit the hypothesised patterns for correlation matrices. This program utilises the iterative Fletcher-Powell method (Fletcher and Powell (1963)) which minimizes the likelihood function using information about the first-order derivatives of the likelihood.

6.2 EQUALLY CORRELATED VARIABLES

In this section, we compare three test statistics, Lawley's statistic (T_1 given in equation (6.1)), the likelihood ratio statistic (T_2 given in equation (6.2)), and the least-squares-type statistic (T_3). This last statistic is equal to the corrected total χ^2 (the total χ^2 excluding the χ^2 for the Mean line) from the analysis procedure given in section 4.1.

The values of these three statistics for the situations analysed in section 5.1 and 5.2 are shown in Table 6.1. They are all asymptotically distributed as χ^2's with 9 degrees of freedom.

The values of the three test statistics are in all cases, except for Lawley's statistic for the Cabernet wines, in close agreement. I have no specific explanation for the difference between the value of Lawley's statistic and the other statistics for the Cabernet wines.

Table 6.1: Comparison of three tests for equal correlations

Experiment	Test Statistics Lawley's (T_1)	Likelihood (T_2)	Least squares (T_3)
Viticultural Field Experiment			
Yields	36.90	33.95	35.48
Residuals	9.07	9.10	9.15
Wine Experiment			
Cabernet	11.30	25.34	24.59
Riesling	4.69	5.10	5.55

6.3 MORE GENERAL EQUAL CORRELATION PATTERNS

In this section, the values obtained for the likelihood ratio statistic (T_2 given in equation (6.2)), under the hypothesis that the underlying form of the correlation matrix is the generalised pattern of equal correlation given in equation (3.5), are compared with those obtained for the least-squares-type statistic (T_4). This latter statistic is the sum of the three χ^2's for the Corrected Total lines of the analysis of variance table given in section 4.2.5.

The values of these two statistics for the situations analysed in section 5.3.1, 5.4 and 5.5 are given in Table 6.2. They are all asymptotically distributed as χ^2's with the degrees of freedom shown.

Table 6.2: Comparison of two tests for the more general equal correlation pattern

Experiment	Degrees of freedom	Test Statistics Likelihood (T_2)	Least squares (T_4)
Wine Experiments			
Experiment 1			
Original	63	n.a.	282.64
Residual	63	238.54	232.21
Experiment 2			
Original	63	101.32	110.28
Residual	63	85.06	81.42
Experiment 3			
Original	63	95.25	103.33
Residual	63	78.76	80.27
Experiment 4†			
Original	117	228.62	272.20
Residual	117	n.a.	130.73
Body Composition	42	95.89	155.03
Primary Ability Tests	42	n.a.	143.03

n.a. indicates that the value of the test statistic is not available in these instances due to the failure of the iterative procedure to converge or because the variance-covariance matrix is not positive definite.

† the model was fitted to the variance-covariance matrix formed from the scores to which uniform random variation had been added.

The values for the two statistics are in most cases in good agreement.

It is interesting to note that for the body composition data, the ratios $\hat{\sigma}_i^2/s_i^2$ (introduced in equation (6.3)) are .92, .93, .83, .96, 2.71, .81, .86, .76, .86 and 2.81; the values 2.71 and 2.81 are the values for muscle E from the left and right sides respectively. In section 5.4, we concluded that muscle E was poorly correlated with the other muscles, but that muscles A-D were equally correlated. It would appear that, in fitting the model for the variance-covariance matrix, the maximum likelihood procedure has given estimates for the variance of the measurement of muscle E that are inflated to compensate for the low covariances, and hence correlations, for that muscle. That is, the deviation of the estimated variance-covariance matrix from the observed variance-covariance matrix is less than it would be if it were impossible to adjust the variances to accommodate the low covariances. In the case of the generalised least squares statistic, the underlying model applies directly to the correlation matrix and does not involve the extra variance parameters. Clearly, an accommodation such as was mentioned above is not possible. This leads one to conjecture that the discrepancy between the two statistics for the body composition data (see Table 6.2) has largely resulted from the difference between the two sets of parameters involved.

6.4 DISCUSSION

The least squares procedures developed in chapter 4 involve the calculation of test statistics from the z-transforms of the elements of a correlation matrix. They are non-iterative procedures, there being explicit expressions for the associated statistics. The computations involved are not so difficult as to preclude one doing them with the aid of only a small calculator. The techniques are investigative in that, in the event that the observed matrix is judged not to be consistent with the hypothesised pattern, alternative models that should be further pursued are suggested by the results. The procedures are based on the assumption that the variables underlying the correlation matrix are multinormally distributed.

Lawley's (1963) test statistic for testing the hypothesis of equal correlation is calculated directly from the correlations. It also is a non-iterative procedure that can be performed with the aid of only a small calculator. However, it is specifically a test for equal correlation and is non-investigative. The assumption on which the test is based is that the variables underlying the correlations are multi-normally distributed.

The likelihood ratio test statistic is calculated from the variance-covariance matrix. The class of models on which it is based is quite general and not restricted to the two patterns investigated here; however, the technique is not investigative. The distribution of the test statistic is based on the assumption that the observed variance-covariance matrix is an observation from a Wishart distribution. The computations, which involve an iterative process, are daunting without the aid of a computer. Difficulty was experienced in getting the process to converge and, as can be seen from the results presented in section 6.3, in some instances a minimum was not found. Further, as is always the case with an iterative procedure, the value of the test statistic obtained in a particular case may correspond to a local, rather than a global, minimum. These computational problems cannot be encountered with the other two test statistics.

From the results presented in sections 6.2 and 6.3, it would seem that the results obtained with the least squares statistic are often the same as those obtained with the likelihood ratio statistic. However, it would also appear, from the results obtained for the body composition data (Table 6.2), that one can, in some instances, expect discrepancies between the results of the two procedures arising from differences in the quantities from which the statistics are computed.

7. DISCUSSION

The two major thrusts of the work in this thesis are the development of a least squares procedure for analysing the patterns in correlation matrices and methods for analysing the data from wine-tasting experiments. In section 7.1, the utility and the generalisation of the method are considered. In section 7.2, I discuss the implications of the results obtained using the new procedure for the design and analysis of wine-tasting experiments.

7.1 ANALYSIS OF PATTERNS IN CORRELATION MATRICES

In chapter 4, least squares procedures, applied to the z-transforms of an observed correlation matrix, were developed for testing the hypotheses that the observed matrix conforms to (i) equal correlation pattern and (ii) more general equal correlation pattern given in section 3.2.3. These procedures are analyses of variance and the associated models are similar to those for the diallel-cross analyses (Yates (1947)). They are relatively simple, non-iterative, investigative procedures.

The technique was used to analyse a range of examples and the results of these analyses were presented in chapter 5. It can be seen from them that the technique is useful in

(a) investigating the patterns in multitaster-multisession correlation matrices with a view to selecting a reliable subset of tasters who agree,

(b) determining the stability of the pattern in certain submatrices of correlation matrices calculated from the data of experiments that involve repetitions of a number of measurements, and

(c) investigating some of the necessary assumptions for valid application of an analysis of variance for mean differences.

The new method was compared, in chapter 6, with Lawley's test for equal correlation and the likelihood ratio test for the appropriate models for the variance-covariance matrix.

However, the procedure as developed here is restricted to the case where m = 2, i.e. where one of the factors cross-indexing the variables has only two levels. As was pointed out previously, this procedure could be utilised in situations where m > 2 by using it to do an analysis for each of the m(m-1)/2 pairs of levels of m. However, a more satisfactory approach would be to extend the procedure so that a combined analysis when m > 2 is possible. To do this requires only the extension of the third segment of the analysis. The initial step in the development of the analysis, via the analysis of variance technique, is to conjecture a set of linear models that generate the irreducible subspaces of V_3. As V_3 contains 14 distinct elements when m > 2 and p > 2, there will be 14 idempotents of the commuting algebra of V_3 and so we require 14 models. Following along the same lines as in previous cases (see chapter 4), I conjecture that the linear models that will generate the irreducible subspaces of V_3 are as follows:

M_1 : $E[Z_{(ik)(j\ell)}] = \mu$

M_2 : $E[Z_{(ik)(j\ell)}] = \mu + \beta_i + \beta_j = \mu_2$

M_3 : $E[Z_{(ik)(j\ell)}] = \mu_2 + \delta_{ij} = \mu_3$ $\quad(\delta_{ij} = \delta_{ji})$

M_4 : $E[Z_{(ik)(j\ell)}] = \mu + \alpha_k + \alpha_\ell = \mu_4$

M_5 : $E[Z_{(ik)(j\ell)}] = \mu_4 + \gamma_{kl} = \mu_5$ $\quad(\gamma_{k\ell} = \gamma_{\ell k})$

M_6 : $E[Z_{(ik)(j\ell)}] = \mu_2 + \mu_4 + \tau_{ik} + \tau_{i\ell} + \tau_{jk} + \tau_{j\ell} = \mu_6$

M_7 : $E[Z_{(ik)(j\ell)}] = \mu_5 + \mu_6 + \varepsilon_{ik\ell} + \varepsilon_{jk\ell} = \mu_7$ $\quad(\varepsilon_{ik\ell} = \varepsilon_{i\ell k})$

M_8 : $E[Z_{(ik)(j\ell)}] = \mu_3 + \mu_6 + \omega_{ijk} + \omega_{ij} = \mu_8$ $\quad(\omega_{ijk} = \omega_{jik})$

M_9 : $E[Z_{(ik)(j\ell)}] = \mu_7 + \mu_8 + \eta_{ijk\ell} = \mu_9$ $\quad(\eta_{ijk\ell} = \eta_{jik\ell} = \eta_{ij\ell k} = \eta_{ji\ell k})$

M_{10} : $E[Z_{(ik)(j\ell)}] = \mu_6 + \theta_{(ik)} + \theta_{(j\ell)} = \mu_{10}$

M_{11} : $E[Z_{(ik)(j\ell)}] = \mu_6 + \phi_{(i\ell)} + \phi_{(jk)} = \mu_{11}$

$$M_{12} : E[Z_{(ik)(j\ell)}] = \mu_5 + \mu_{10} + \lambda_{(ik)\ell} + \lambda_{(j\ell)k} = \mu_{12}$$

$$M_{13} : E[Z_{(ik)(j\ell)}] = \mu_3 + \mu_{11} + \psi_{(ik)j} + \psi_{(j\ell)i} = \mu_{13}$$

$$M_{14} : E[Z_{(ik)(j\ell)}] = \mu_9 + \mu_{12} + \mu_{13} + \zeta_{(ik)(j\ell)} = \mu_{14} \qquad (7.1)$$

The first 9 of these models have the property that $E[Z_{(ij)(k\ell)}] = E[Z_{(ik)(j\ell)}]$ and so are symmetric models. The remaining 5 models are asymmetric models.

If M_i denotes the subspace corresponding to the i^{th} model, then we require the orthogonal projection operators on the subspaces,

M_1,

$M_2 \ominus M_1$,

$M_3 \ominus M_2$,

$M_4 \ominus M_1$,

$M_5 \ominus M_4$,

$M_6 \ominus (M_2 \oplus M_4)$,

$M_7 \ominus (M_5 \oplus M_6)$,

$M_8 \ominus (M_3 \oplus M_6)$,

$M_9 \ominus (M_7 \oplus M_8)$,

$M_{10} \ominus M_6$,

$M_{11} \ominus M_6$,

$M_{12} \ominus (M_5 \oplus M_{10})$,

$M_{13} \ominus (M_3 \oplus M_{11})$,

$M_{14} \ominus (M_9 \oplus M_{12} \oplus M_{13})$.

Another development which might be considered is a combined analysis of the z-transforms of all the elements of a correlation matrix cross-indexed by two factors. This requires the analysis of the complete commuting algebra of the matrices of the form $V(\underset{\sim}{\rho})$ given in equation (3.7). It is not difficult to show that the matrix $V(\underset{\sim}{\rho})$, in general, contains 44 distinct elements (distributed amongst the submatrices of equation (3.7) as shown in Table 7.1) and so the algebra is of dimension 44.

Table 7.1: The number of distinct elements in each of the submatrices of $V(\underset{\sim}{\rho})$

V_1	V_2	V_3	V_{12}	V_{13}	V_{23}	$V(\underset{\sim}{\rho})$
6	6	14	4	7	7	44

To analyse this algebra, would be extremely difficult, irrespective of which of the algebraic or analysis of variance techniques of chapter 4 were employed. Further, it is most unlikely that an analysis involving 44 terms will be useful in interpreting the pattern in the correlation matrix.

7.2 WINE-TASTING EXPERIMENTS

The results of the analyses of the wine-tasting experiments presented in sections 5.1 and 5.3 provide further substantive evidence of the widely recognised fact that tasters can disagree in their evaluations of wines and differ in their reliability. It is also seen that a taster's reliability can vary from experiment to experiment.

The differences in reliability, in particular, have led experimenters to suggest that judges should be screened before they are used in taste panels (see Amerine *et al.* (1965, chapter 6 section IIA-C) and Amerine and Roessler (1976, pp.59-62)). However, these authors point out that the proposed screening methods are not completely satisfactory since (i) a judge's performance varies in time, with the quality of the product, and with different wine types, and (ii) tests of taste acuity and palatability (which are often used to screen judges) do not necessarily correlate with judges' performances as judgements involve many perceptual skills. Further, as Amerine and Roessler (*loc cit.*) comment, "We admit that qualification tests for selection of judges are usually impracticable and impolitic." CLearly, even if the membership of panels is restricted to "experts", it is highly likely that the panel for a particular tasting will include at least one member whose performance in that instance is poor. In a combined analysis for wine differences, the results will at best be less precise and may lead to erroneous conclusions, if the assessments of such tasters are not excluded. Thus, it is desirable that tasters' performances in a particular wine-tasting be checked and those giving poor performances in that tasting excluded. The method proposed here allows us to do this.

Because it would appear almost certain that the tasters will be heterogeneous in their performance, wine-tasting experiments should be designed so that the scores of each taster can be analysed for mean differences separately. This can be achieved by adequate replication of one or more of the following classes:

(i) field replication - batches of fruit from different localities are kept separate during the wine-making evaluation processes;

(ii) wine-making replication - a batch of fruit is subdivided into smaller lots each of which is made into a wine to be evaluated subsequently;

(iii) bottle replication - samples of a particular wine; and

(iv) session replication - bottles are repeatedly evaluated by a taster.

Otherwise, it may be impossible to perform a valid statistical analysis for mean differences. In addition, as reliability is clearly more important than agreement in assessing a taster's performance, it is desirable that session replicates be included. (There is no question that the results of an unreliable taster should be ignored; but ignoring the results of a reliable taster who tends not to agree with other tasters is not always desirable.) If session replicates are not included, it will be impossible to assess a judge's reliability independently of the other sources of variation.

Two session replicates were included for all wines in the first three CSIRO wine-tasting experiments. In the fourth CSIRO experiment, the tasters evaluated each wine twice but different glasses were used on each occasion; however, the wines in the clear glasses came from the same bottles as those in the black glasses.

A number of statistics which can be calculated from scores have been utilised in the selection of taste panel members from amongst the tasters who have participated in previous wine-tasting experiments. These include:

(i) Means and variances (for use of variances, Overman and Li (1948), and Bennett *et al.* (1956));

(ii) F-statistic from an analysis of variance (Overman and Li (1948), Girardot *et al.* (1952), Krumm (1955) and Wiley *et al.* (1957));

(iii) Intra-class correlations (Sawyer *et al.* (1962));

(iv) Correlation of each taster's scores with means over tasters (Hopkins (1946) and Moser *et al.*(1950));

(v) Effective range (Ough and Baker (1961), Ough (1964), Ough and Winton (1976) and Tromp and Conradie (1979));

(vi) Frequency of specified, or smaller, differences between duplicate samples (Liming (1966) and Tromp and Conradie (1979));

(vii) Distribution of scores (Ough and Baker (1961), Ough and Winton (1976) and Tromp and Conradie (1979)).

Statistics (i) and (vi) measure the reliability or consistency of the tasters, while statistics (ii), (v) and (vii) measure the discrimination of the tasters (i.e., the magnitude of the differences between the wines relative to the random variability). The intra-class correlation coefficient (statistic (iii)) is a monotonic function of the F-statistic from the analysis from which it is calculated and so is equivalent to the second statistic. The fourth statistic is a measure of conformity with the overall assessment of the wine whose major use would appear to be in determining a group of tasters who agree.

In comparison with the method of analysing multitaster and multitaster-multisession correlation matrices (developed herein):

(a) Means reflect the origin and variances the reliability and scale of the taster's scores, whereas the correlation coefficient reflects only the reliability. Thus, means and variances are only important when absolute, rather than relative, measures of wine quality are required as differences in origin and scale are unimportant in the assessment of a taster when relative measures are sufficient.

(b) The F-statistic appears to be complementary to the information provided by the analysis of correlation matrices. However, the two methods are to some extent related in that it is clear that an unreliable taster will have difficulty discriminating and that a taster who is unable to discriminate is likely to be unreliable.

(c) It would appear preferable to examine the correlations measuring agreement in the multitaster and multitaster-multisession matrices rather than the correlations of a taster's scores with the overall means for the wines. The latter statistic, to be effective, requires that the effect of individual tasters on the consensus opinion of the wine is minimal and this is unlikely when a panel of around six non-homogeneous tasters is used.

(d) The last three statistics are of more use when several experiments are involved as they generally require a larger number of scores for each taster. They are clearly of limited interest in the present context.

In the original report of the four CSIRO wine-tasting experiments (May *et al.* (1976), May (1977) and Hale and Brien (1978)), the results of the separate analyses, for treatment differences, of each of the taster's scores were presented as it was evidently unlikely that an overall analysis would be valid. As noted previously, the condition that the form of the population correlation matrix conforms to the more general equal correlation pattern given in section 3.2.3, together with the assumption of homoscedastic variances and equality of the population correlation matrix for all observations of the variables, is a sufficient, but not necessary, condition for a combined analysis of a tasting to be valid. However, it is desirable, if the investigation of individual effects is not to be complicated by differing variances of linear contrasts, that the population variance-covariance matrix be of this more restricted form.

To examine the degree to which the residual variance-covariance matrices for the four CSIRO wine-tasting experiments conform to this pattern, the χ^2's from the fitting, by maximum likelihood estimation, of the more general equal correlation pattern with either unequal or equal variances to the matrices for both the full sets and subsets of tasters are presented in Table 7.2. Although the more general equal correlation pattern is tenable for the third experiment, this pattern with equal variances is not tenable in any experiment, either for the full sets or the subsets of tasters. Thus, it would appear that the most appropriate analysis for treatment differences is a separate analysis of variance for each taster in the experiment. In drawing conclusions from these analyses, one should have a greater regard for the results of the selected subsets, these being the more reliable tasters who agree to a greater extent than the remaining tasters.

Table 7.2: χ^2's, and their degrees of freedom, for the deviation of the residual variance-covariance matrices, of both the full sets and the subsets of tasters from the four CSIRO wine-tasting experiments, from the more general equal correlation pattern with either unequal or equal variances.

Experiment	Full Sets of Tasters					Subsets of Tasters				
	Equal Correlation		Equal Correlation & Variance		Difference	Equal Correlation		Equal Correlation & Variance		Difference
	df	χ^2	df	χ^2	χ^2	df	χ^2	df	χ^2	χ^2
1	63	238.54***	74	280.20	41.66***	25	61.05***	32	87.43***	26.38***
2	63	85.06*	74	175.25***	90.19***	-	-	-	-	-
3	63	78.76n.s.	74	99.10*	20.34*	25	30.23n.s.	32	46.54*	16.31*
4	117	n.a.	132	n.a.	n.a.	25	44.67**	32	63.21***	18.54**

n.a. not available as variance-covariance matrix is not positive definite.

n.s. not significant

* significant at $p = 0.05$

** significant at $p = 0.01$

*** significant at $p = 0.001$

APPENDIX A

ORIGINAL SCORES FOR FOUR DUPLICATE-EVALUATION WINE EXPERIMENTS

TABLE A.1 ORIGINAL SCORES FOR EXPERIMENT 1

TREAT-MENTS	FERMEN-TATION REPLICATES	TASTERS / SESSIONS / BOTTLES	A 1	A 2	B 1	B 2	C 1	C 2	F 1	F 2	G 1	G 2	I 1	I 2
1	1	1	13.0	12.5	13.0	13.0	10.5	11.0	12.5	12.5	15.5	14.5	14.5	16.5
		2	12.0	12.0	13.0	14.0	11.0	11.0	13.0	13.5	15.5	13.5	15.5	17.0
	2	1	13.5	13.0	15.5	13.5	14.5	14.5	14.0	14.0	15.5	15.0	17.0	15.5
		2	12.0	12.0	12.5	14.0	12.0	12.0	12.0	13.5	15.0	13.0	16.0	16.5
	3	1	13.0	13.0	14.0	14.5	12.0	12.0	14.0	14.0	15.5	14.5	15.0	17.0
		2	13.0	12.0	14.0	16.0	13.5	13.5	14.5	13.5	15.5	14.5	17.5	17.5
2	1	1	14.5	14.5	17.0	15.0	15.5	15.5	16.0	16.0	16.5	16.0	14.0	14.5
		2	14.5	13.0	16.5	17.5	16.0	16.5	16.5	17.0	15.0	16.0	14.0	15.0
	2	1	14.0	13.5	18.0	18.0	15.5	16.0	15.0	15.0	16.0	15.5	14.5	14.5
		2	14.5	13.5	16.5	17.0	16.0	16.5	16.0	15.5	16.5	16.5	14.5	14.0
	3	1	13.5	14.0	16.5	15.5	14.5	14.5	15.0	14.5	15.5	16.5	14.0	14.5
		2	14.0	13.0	17.5	17.0	16.5	16.0	16.0	15.5	15.0	15.5	14.5	15.0
3	1	1	14.0	13.5	14.0	14.0	12.5	11.5	12.5	11.5	14.5	15.0	12.0	14.0
		2	14.0	11.5	15.0	14.5	13.5	13.0	11.5	12.0	15.5	15.0	15.0	15.5
	2	1	12.5	11.5	15.5	13.0	11.0	11.5	11.5	11.5	14.0	14.0	12.0	13.0
		2	11.5	11.0	13.5	16.5	12.5	12.5	11.5	12.0	14.0	14.0	12.5	13.5
	3	1	13.5	13.5	16.0	17.5	14.0	15.5	15.0	15.0	15.5	16.0	14.5	14.5
		2	14.0	13.5	16.5	15.0	14.5	15.5	14.5	14.5	15.0	15.5	13.5	13.5
4	1	1	14.5	14.5	17.0	17.0	13.0	13.0	14.0	14.0	17.0	14.5	14.5	15.5
		2	13.5	14.5	15.5	14.5	12.0	14.0	15.0	16.0	15.0	14.0	15.5	16.5
	2	1	13.0	13.0	14.5	13.0	10.0	10.5	12.0	12.0	14.0	14.0	11.5	14.0
		2	13.0	12.0	13.0	14.0	14.0	13.0	15.5	13.0	14.0	15.0	14.5	14.0
	3	1	15.0	14.5	18.0	18.0	15.0	15.0	15.0	15.0	16.5	16.5	14.5	14.5
		2	14.0	13.5	16.0	17.5	17.0	15.0	15.0	15.5	14.5	16.0	14.5	15.0

TABLE A.2 ORIGINAL SCORES FOR EXPERIMENT 2

TREATMENTS	FIELD-REPLICATES	FERMENTATION REPLICATES	A 1	A 2	B 1	B 2	C 1	C 2	D 1	D 2	E 1	E 2	H 1	H 2
1	1	1	18.0	18.5	15.0	15.0	16.0	15.0	18.5	16.5	13.0	16.5	16.0	16.0
		2	18.5	18.5	15.5	13.5	16.5	13.0	15.5	18.0	16.0	15.0	14.5	13.5
		3	18.5	18.5	16.0	16.5	16.5	15.0	20.0	18.0	17.0	17.0	14.0	14.5
		4	16.5	17.5	16.5	14.0	15.5	15.5	16.0	19.0	17.0	16.0	14.5	15.0
		5	18.0	17.5	16.0	16.0	14.5	14.5	19.0	18.0	16.5	15.0	13.0	14.0
		6	18.5	14.5	15.5	16.0	14.0	14.5	18.0	17.0	16.0	16.5	16.5	13.0
	2	1	13.5	13.5	15.0	15.0	13.0	13.5	16.0	15.0	15.0	13.5	11.0	14.0
		2	13.0	13.0	13.5	12.0	13.0	13.0	15.5	14.0	10.5	12.5	9.0	10.5
		3	13.0	13.5	14.5	15.0	12.0	13.5	15.0	14.0	11.5	13.5	11.0	13.5
		4	17.5	18.5	15.0	14.0	16.5	15.0	18.5	14.5	15.5	15.0	15.0	14.5
		5	13.0	18.5	14.5	14.5	17.0	17.0	15.0	19.0	15.5	14.5	12.0	15.0
		6	13.0	15.0	14.0	14.0	14.5	14.5	14.0	14.0	13.0	12.5	12.0	12.5
2	1	1	18.5	15.5	15.0	15.0	14.0	16.0	17.0	18.0	15.0	17.5	13.0	13.5
		2	14.0	16.5	16.5	16.0	15.0	14.5	18.0	16.0	13.5	16.0	10.5	13.5
		3	17.0	17.0	17.0	15.5	15.0	15.0	18.0	16.5	15.0	16.5	16.0	16.5
		4	15.0	17.0	16.0	16.0	15.0	16.0	17.5	16.0	16.5	15.0	11.0	13.5
		5	16.5	15.5	14.0	18.0	13.5	13.5	18.0	15.5	17.0	16.5	12.5	13.5
		6	16.0	17.5	16.5	16.0	15.5	17.5	17.5	16.5	16.5	17.0	16.0	15.0
	2	1	17.0	17.5	16.0	17.0	15.5	15.5	18.5	18.5	18.0	17.0	17.5	14.5
		2	16.0	17.5	17.0	15.0	15.0	16.0	15.5	16.0	15.5	16.5	17.0	13.0
		3	15.5	16.5	15.5	14.5	17.0	16.0	18.5	16.0	17.0	17.0	13.5	13.0
		4	14.0	13.5	13.0	13.0	15.0	15.0	16.5	14.5	18.0	16.0	13.0	13.5
		5	14.0	14.5	14.5	14.5	14.5	15.0	17.5	15.0	18.0	16.5	14.5	13.0
		6	17.0	18.5	15.0	16.0	15.0	14.0	19.0	16.0	17.5	17.5	14.5	14.5

/CONT'D...

TABLE A.2 CONT'D

TREATMENTS	FIELD-REPLICATES	FERMENTATION REPLICATES	A 1	A 2	B 1	B 2	C 1	C 2	D 1	D 2	E 1	E 2	H 1	H 2
3	1	1	17.0	16.0	16.0	18.0	16.0	14.5	20.0	17.0	18.0	16.0	16.5	15.5
		2	17.5	15.0	17.0	14.5	15.5	17.0	20.0	18.5	17.0	17.5	15.0	14.0
		3	16.0	17.5	18.5	18.0	16.5	16.5	17.5	18.5	17.0	17.5	14.0	15.0
		4	15.5	16.5	17.5	17.5	17.0	15.5	19.0	16.5	16.5	18.0	15.5	14.5
		5	17.0	17.0	17.5	17.5	16.5	14.0	16.5	17.0	17.5	15.5	13.5	14.0
		6	17.5	18.5	17.5	18.5	16.5	14.5	18.0	19.0	17.5	17.0	15.0	14.5
	2	1	15.0	14.5	15.0	16.0	17.0	16.5	16.5	15.0	16.0	17.0	18.5	14.0
		2	14.5	15.0	17.5	15.0	17.0	15.0	17.0	14.0	15.0	16.0	15.0	15.5
		3	15.0	16.0	16.0	15.5	17.0	16.0	16.0	14.5	15.5	15.0	12.0	13.5
		4	14.5	13.5	17.5	17.5	15.5	16.5	15.5	15.5	16.5	17.5	16.0	16.0
		5	15.5	16.0	17.5	16.5	16.5	17.0	15.0	14.5	16.5	17.5	12.0	17.0
		6	16.5	15.0	16.5	17.5	14.5	16.5	16.0	15.5	15.0	15.5	15.5	16.5
4	1	1	14.5	14.0	11.0	18.0	13.5	13.5	18.0	15.0	14.0	13.5	12.5	14.0
		2	15.0	14.5	17.0	16.0	16.5	16.0	18.0	17.0	14.5	14.0	14.0	17.0
		3	14.5	15.5	18.0	17.5	14.0	14.5	18.5	14.5	14.0	14.0	15.5	14.0
		4	13.0	15.5	16.5	16.0	15.0	14.0	17.0	18.0	15.5	12.0	11.0	13.0
		5	15.0	13.5	16.5	16.0	15.5	15.0	14.5	15.5	15.5	13.0	15.5	11.5
		6	14.5	13.0	16.5	14.0	17.0	15.0	16.0	14.5	15.5	14.0	13.0	16.5
	2	1	14.5	15.0	16.0	16.0	14.0	16.0	18.5	16.0	17.0	14.0	13.5	15.5
		2	17.0	16.0	17.0	18.0	14.5	15.5	17.0	15.5	13.5	16.0	13.5	14.0
		3	14.0	13.5	17.0	15.5	14.0	14.0	16.5	14.5	13.5	14.0	14.0	16.0
		4	14.0	13.0	16.5	17.5	15.5	14.0	16.5	14.5	14.5	15.0	17.5	17.5
		5	14.0	14.5	15.0	15.0	14.0	16.5	14.5	16.5	15.5	14.5	13.0	16.5
		6	16.0	14.0	18.0	17.5	15.5	15.0	15.0	15.0	15.5	14.0	18.0	17.5

TABLE A.3 ORIGINAL SCORES FOR EXPERIMENT 3

TREAT-MENTS	FIELD-REPLICATES	FERMEN-TATION REPLICATES	A 1	A 2	B 1	B 2	C 1	C 2	D 1	D 2	E 1	E 2	F 1	F 2
1	1	1	11	12	11	12	19	18	12	13	10	14	11	13
		2	11	12	9	13	12	10	14	13	10	7	8	10
		3	20	20	15	17	12	16	18	14	18	17	17	13
	2	1	16	17	13	16	15	15	15	17	16	15	15	17
		2	16	18	14	16	18	16	18	18	17	18	12	11
		3	16	14	15	13	14	15	16	17	15	15	12	12
	3	1	18	18	15	17	18	17	19	14	17	12	17	16
		2	13	14	15	11	13	11	12	17	12	13	13	12
		3	16	16	13	15	14	13	14	17	11	11	12	15
	4	1	14	12	13	12	15	14	17	14	16	18	17	17
		2	16	15	13	12	17	14	17	16	14	12	14	13
		3	16	18	14	11	14	17	15	15	15	17	15	14
2	1	1	18	18	14	14	18	17	14	15	16	14	14	12
		2	18	18	14	15	19	16	19	19	17	17	17	17
		3	15	18	14	15	13	15	15	15	15	14	13	15
	2	1	15	16	15	16	13	14	17	17	18	15	18	16
		2	12	14	13	15	10	12	19	12	11	9	12	13
		3	13	15	13	15	16	15	17	14	15	14	12	14
	3	1	13	15	15	15	13	16	19	15	17	12	13	16
		2	13	15	13	15	16	16	17	18	11	12	15	12
		3	18	18	15	13	15	12	16	16	15	9	15	15
	4	1	16	17	14	17	17	17	17	17	13	15	10	15
		2	15	14	17	14	19	14	18	13	11	12	15	13
		3	15	18	14	12	9	15	14	18	13	14	15	12
3	1	1	16	14	16	16	12	17	15	18	12	14	13	15
		2	18	15	16	16	14	12	15	17	14	16	13	13
		3	14	15	17	17	17	16	17	16	14	14	15	17
	2	1	11	13	18	13	9	11	13	12	8	8	10	11
		2	15	17	17	12	15	14	16	17	14	15	11	15
		3	11	14	17	11	14	13	17	13	13	9	11	10
	3	1	13	13	15	16	12	15	13	12	11	8	13	13
		2	15	13	13	13	14	14	16	16	17	14	13	15
		3	12	15	15	13	14	17	17	18	14	14	12	13
	4	1	13	14	4	17	11	15	12	17	6	14	13	13
		2	14	12	12	14	17	12	12	12	7	11	13	11
		3	18	20	17	16	14	17	17	16	18	13	13	15
4	1	1	14	15	15	13	14	14	16	16	11	12	14	15
		2	11	13	9	10	9	8	12	11	4	6	10	9
		3	13	13	15	15	15	13	15	14	12	8	12	15
	2	1	10	10	10	10	7	9	11	11	6	5	9	10
		2	18	16	15	16	15	17	18	18	16	15	17	17
		3	14	16	17	16	16	15	17	14	11	8	17	17
	3	1	18	17	17	14	17	16	16	19	16	14	17	15
		2	14	17	11	15	12	12	12	16	13	13	13	12
		3	18	17	15	17	16	15	17	17	16	14	14	15
	4	1	12	12	13	11	9	11	11	11	6	7	13	12
		2	13	12	13	12	13	11	15	13	11	9	13	11
		3	11	15	12	13	10	13	12	10	10	10	12	12

TABLE A.4 ORIGINAL SCORES (BOUQUET + PALATE) FOR EXPERIMENT 4

ROOTSTOCK	TRELLIS	FIELD-SAMPLES	FERMEN-TATION REPLICATES	A C	A B	D C	D B	F C	F B	G C	G B	J C	J B	K C	K B	L C	L B	M C	M B
			TASTERS / GLASS-TYPE*	C	B	C	B	C	B	C	B	C	B	C	B	C	B	C	B
1	1	1	1	8	8	7	8	9	9	6	10	7	7	9	9	6	7	6	8
			2	11	9	9	8	10	10	8	7	9	7	10	10	8	8	8	8
		2	1	11	11	9	10	10	10	6	7	10	9	10	11	8	8	8	8
			2	9	10	9	10	10	10	8	7	9	9	10	11	10	8	8	7
	2	1	1	10	10	11	15	10	10	10	13	8	7	12	11	12	10	8	8
			2	8	8	14	14	9	9	8	10	9	9	10	14	11	12	8	7
		2	1	12	12	14	15	11	11	12	14	10	11	10	9	8	10	10	11
			2	12	12	11	13	10	10	10	10	10	9	14	13	10	13	12	11
	3	1	1	10	10	10	11	10	10	9	9	8	9	10	12	11	11	7	7
			2	12	12	11	11	10	10	10	10	10	8	9	10	12	12	8	7
		2	1	12	13	10	11	10	10	10	10	10	10	11	10	10	10	9	8
			2	8	7	11	9	9	9	11	9	8	9	12	11	10	11	6	6
2	1	1	1	15	15	14	14	13	13	15	14	13	12	16	15	10	11	13	11
			2	15	15	14	14	13	13	11	10	11	12	14	12	12	11	11	12
		2	1	13	13	15	15	13	13	15	10	9	11	14	13	11	11	12	11
			2	13	13	15	15	13	13	12	11	11	10	15	13	11	13	12	10
	2	1	1	15	15	15	14	13	13	10	16	12	11	10	14	14	9	12	9
			2	15	14	15	11	13	13	9	15	11	10	12	11	10	10	11	10
		2	1	14	15	13	11	13	13	16	16	11	12	11	9	9	8	9	9
			2	14	14	12	13	13	13	14	13	9	12	11	13	9	10	10	11
	3	1	1	14	13	14	15	13	13	14	13	10	12	15	15	8	8	12	12
			2	13	15	14	14	13	13	12	14	12	11	14	14	9	9	11	10
		2	1	15	14	15	15	13	13	16	13	11	12	13	11	7	9	8	8
			2	15	15	15	14	13	13	13	12	13	9	11	13	8	7	11	10

*C = CLEAR GLASSES; B = BLACK GLASSES

APPENDIX B

LISTINGS OF GENSTAT INSTRUCTIONS FOR THE ANALYSIS OF THE z-TRANSFORMS OF THE ELEMENTS OF CORRELATION MATRICES

```
'REFE/LINE=6000,NID=425,NUNN=175' CABERNET
''
  AN EXAMPLE MAIN PROGRAM THAT SETS THE STRUCTURES NEEDED FOR THE MACRO
  ZEQUAL. THE INSTRUCTIONS THAT COMPRISE THE MACRO ARE PLACED IMMEDIATEL`
  BEFORE THE LAST 'RUN' STATEMENT. P, MDF AND THE VECTOR OF Z-TRANSFORMS
  ARE READ FROM A SEPARATE FILE.
''
'SCALAR' P,N,MDF
'INPUT' 2 'READ/PRIN=DEM' P,MDF 'INPUT' 1
'CALC' N=P*(P-1)/2
'RUN'
'START'
'UNITS' U1=1...N
'FACTOR' ROWS$P=2,2(3),3(4),4(5),5(6),6(7),7(8),8(9),9(10),10(11),
                11(12),12(13),13(14)
: COLS$P=1,1,2,1,2,3,(1...4),(1...5),(1...6),(1...7),(1...8),(1...9),
         (1...10),(1...11),(1...12),(1...13)
'HEADING' LAB=
'' ANALYSIS OF Z-TRANSFORMS OF CORRELATIONS FROM CABERNET TASTING''
'INPUT' 2
'READ/P' Z$ F,8(10),/
'CALC' Z=0.5*LOG((1+Z)/(1-Z))
'PRINT/P' U1,ROWS,COLS,Z$ 3(6.1),6.4
'INPUT' 1
'PAGE' 'PRINT' LAB 'LINE' 2
'USE/R' ZEQUAL$
'RUN'
'CLOSE'
'STOP'
```

```
'REFE/LINE=6000,NID=425,NUNN=175' MUSCLE
''
  AN EXAMPLE MAIN PROGRAM THAT SETS THE STRUCTURES NEEDED FOR THE MACRO
  ZCROSS. THE INSTRUCTIONS THAT COMPRISE THE MACRO ARE PLACED IMMEDIATEL
  BEFORE THE LAST 'RUN' STATEMENT. M, P, MDF, THE VECTOR OF Z-TRANSFORMS
   AND THE FACTORS INDEXING THE VECTOR ARE READ FROM A SEPARATE FILE.
''
'SCALAR' M,P,N,MDF
'INPUT' 2 'READ/PRIN=DEM' M,P,MDF 'INPUT' 1
'CALC' N=M*P*(M*P-1)/2
'RUN'
'START'
'UNITS' U1=1...N
'FACTOR' AROWS,ACOLS$M : BROWS,BCOLS$P : SETS$3
'HEADING' LAB=
'' ANALYSIS OF Z-TRANSFORMS OF CORRELATIONS FROM MUSCLE MEASUREMENTS''
'INPUT' 2
'READ/P' SETS,AROWS,ACOLS,BROWS,BCOLS,Z$ F,5(2),14,/
'PRINT/P' U1,SETS,AROWS,ACOLS,BROWS,BCOLS,Z$ 6(6.1),6.4
'INPUT' 1
'PAGE' 'PRINT' LAB 'LINE' 2
'USE/R' ZCROSS$
'RUN'
'CLOSE'
'STOP'
```

```
'MACRO' ZEQUAL$
''
  ZEQUAL - MACRO TO PERFORM A DIALLEL-TYPE ANALYSIS ON A SET OF
           Z-TRANSFORMS OF THE ELEMENTS OF A CORRELATION MATRIX, THE
           UNDERLYING VARIABLES OF WHICH ARE ASSUMED TO BE EQUALLY
           CORRELATED.

  INPUT:-  Z       A VECTOR CONTAINING THE P*(P-1)/2 Z-TRANSFORMS
           ROWS    A FACTOR INDEXING Z AND SPECIFYING, FOR EACH ELEMENT
                   OF Z, THE ROW OF THE OBSERVED CORRELATION MATRIX
                   (ORDER P) FROM WHICH IT CAME. ROWS HAS P LEVELS.
           COLS    A FACTOR SPECIFYING THE COLUMNS HAVING P LEVELS.
           P       THE NO. LEVELS OF THE FACTORS AND THE ORDER OF THE
                   OBSERVED CORRELATION MATRIX.
           MDF     THE DEGREES OF FREEDOM OF THE OBSERVED CORRELATIONS.
''
'LOCAL' SSQ(1...4),DF(1...4),DIV(1...4),CHI(1...4),H(1...4),GH,GM,X(1,2),
        V(1,2,3),ROW,COL,VS,RHO,T,P2
'SCALAR' GM,SSQ(1...4),DF(1...4),DIV(1...4),CHI(1...4),V(1,2,3),RHO,P2,T
'HEADING' VS=''ZDIALLEL - VERSION 2.0''
'PRINT' VS
''
  CALCULATE ESTIMATE, UNDER HYPOTHESIS OF EQUAL CORRELATION, OF
  CORRELATION BETWEEN VARIABLES
''
'CALC' RHO=MEAN(Z) : RHO=EXP(2*RHO) : RHO=(RHO-1)/(RHO+1)
'PRINT/P' RHO$ 10.4
'CALC' T=P*(P-1)/2 : P2=P-2
''
  CALCULATE TOTALS CORRESPONDING TO CALCULATION MATRICES
''
'TABLES' ROW$ROWS : COL$ COLS
'TABU' Z;ROW : Z;COL
'VARI' X(1,2)$P
'EQUATE' X(1,2)=ROW,COL
'CALC' X(1)=X(1)+X(2) : GM=SUM(Z)
'EQUATE' ROW=X(1) 'DEVA' X(1,2),COL
''
   OBTAIN SSQ CORRESPONDING TO QUADRATIC FORMS IN CALCULATION MATRICES
''
'CALC' SSQ(1)=GM*GM : SSQ(2)=SUM(ROW*ROW) : SSQ(4)=SUM(Z*Z)
''
   OBTAIN PARTITION OF VARIABILITY
''
'CALC' SSQ(2)=(P.EQ.2)*P2/P2+(P.NE.2)*(SSQ(2)-SSQ(1)*4/P)/P2
:       SSQ(1)=SSQ(1)/T
:       SSQ(3)=SSQ(4)-SSQ(1)-SSQ(2)
''
   CALCULATE DF
''
'CALC' DF(1)=1 : DF(2)=P-1 : DF(3)=T-DF(1)-DF(2)
:       DF(4)=T
```

```
""
   CALCULATE DIVISORS AND CHI-SQUARE
""
"CALC" V(1)=1/MDF
:       V(2)=V(1)*RHO*(3*RHO+2)/(1+RHO)/(1+RHO)/2
:       V(3)=V(1)*2*RHO*RHO/(1+RHO)/(1+RHO)
:       DIV(1)=V(1)+2*P2*V(2)+(P-1)*P2/2*V(3)
:       DIV(2)=V(1)+(P-4)*V(2)-(P-3)*V(3)
:       DIV(3)=V(1)-2*V(2)+V(3)
:       V(1...3)=DIV(1...3)*DF(1...3) : DIV(4)=VSUM(V(1...3))
:       DIV(1)=1/MDF
:       CHI(1...3)=SSQ(1...3)/DIV(1...3)
:       CHI(4)=CHI(2)+CHI(3)
""
   OUTPUT SSQ, DF, DIVISORS AND CHI-SQUARES
""
"HEADING" H(1)=""MEAN           ""          : H(2)=""BETW. ENTITIES""
:          H(3)=""ENTITY INTERACTION"" : H(4)=""CORR. TOTAL""
:          GH=""GRAND MEAN""
"LINE" 5
"CAPTION" ""  ** ANALYSIS OF VARIANCE TABLE **"" "LINE" 3
"LINE" 1
"CAPTION"
    ""SOURCE                    DF          SSQ          DIV          CHI""
"FOR" HEAD=H(1...4);SS=SSQ(1...4);DF=DF(1...4);DIV=DIV(1...4);
      CHI=CHI(1...4)
"LINE" 2
"PRINT/C,VAR=1,LABR=1,LABC=1" HEAD,DF,SS,DIV,CHI$
    20.0,2X,4.0,(2X,10.4)2,2X,10.2
"REPE"
"DEVA" H(1...4),SSQ(1...4),DF(1...4),DIV(1...4),CHI(1...4)
""
   CALCULATE AND OUTPUT SMOOTHED MEANS
""
"CALC" GM=MEAN(Z) : ROW=ROW/(P-1) : ROW=ROW-GM
:       ROW=(P.EQ.2)*P2/P2+(P.NE.2)*ROW*(P-1)/P2
:       ROW=GM+ROW
"LINE" 5 "CAPTION" ""
  ** TABLES OF SMOOTHED MEANS **""
"LINE" 3 "PRINT/C,LABR=1,LABC=1,VAR=1" GH,GM$ 5X,10,5X,10.4
"LINE" 2 "CAPTION" ""  ** ADDITIVE ENTITY MEANS **""
"PRINT/LABR=1,LABC=1" ROW$ 8.4
"ENDM"
```

```
"MACRO" ZCROSS$
""
  ZCROSS - MACRO TO PERFORM A DIALLEL-TYPE ANALYSIS ON A SET OF
           Z-TRANSFORMS OF THE ELEMENTS OF A CORRELATION MATRIX, THE
           VARIABLES OF THE CORRELATION MATRIX BEING CROSS-INDEXED BY TWO
           FACTORS.

  INPUT:-  Z       A VECTOR CONTAINING THE MP*(M-1)*(P-1)/2 Z-TRANSFORMS
           SETS    A FACTOR HAVING THE SAME LEVEL FOR Z-TRANSFORMS OF
                   CORRELATIONS HAVING THE SAME EXPECTATION.
           AROWS,  TWO FACTORS INDEXING Z AND SPECIFYING, FOR EACH
            BROWS  ELEMENT OF Z, THE VALUES OF THE TWO FACTORS THAT
                   CROSS-INDEX THE OBSERVED CORRELATION MATRIX (ORDER MP).
                   AROWS HAS M LEVELS AND BROWS HAS P LEVELS.
           ACOLS,  TWO FACTORS INDEXING Z AND SPECIFYING THE COLUMNS
            BCOLS  RATHER THAN THE ROWS OF THE OBSERVED CORRELATION
                   MATRIX. ACOLS HAS M LEVELS AND BCOLS HAS P LEVELS.
           M       THE NO. LEVELS OF THE A FACTOR (=2) CROSS-INDEXING THE
                   OBSERVED CORRELATION MATRIX.
           P       THE NO. LEVELS OF THE B FACTOR CROSS-INDEXING THE
                   OBSERVED CORRELATION MATRIX.
           MDF     THE DEGREES OF FREEDOM OF THE OBSERVED CORRELATIONS.

  THIS MACRO CALLS SETDIAL AND INTDIAL WHICH, IN TURN, CALL SETD1,
  SETD2, SETD3 AND INTD1, INTD2, INTD3, REPECTIVELY.

""
"LOCAL" VS,I,AN,AN(1,2,3),AR,BR,BC,S,G,RHO(1,2,3),RHOS(1,2,3),GZ
"TABLE" GZ$ SETS "SCALAR" RHO(1,2,3) "TABU" Z;;;GZ
"CALC" GZ=EXP(2*GZ) : GZ=(GZ-1)/(GZ+1) "EQUATE" RHO(1,3,2)=GZ
"PRINT/P" RHO(1,2,3)$ 10.4
"HEADING" VS=""ZDIALLEL - VERSION 2.0"" : AN(1)=""BETWEEN B WITHIN A
------------------"" : AN(2)=""BETWEEN A WITHIN B
------------------"" : AN(3)=""BETWEEN A AND B
-----------------""
"PRINT" VS
"FOR" I=1,2;AN=AN(1,2);AR=AROWS,BROWS;BR=BROWS,AROWS;BC=BCOLS,ACOLS;
      S=M,P;G=P,M;RHOS(1)=RHO(1,3);RHOS(2)=RHO(2,2);RHOS(3)=RHO(3,1)
"REST" Z$ SETS=I
"USE/R" SETDIAL$
"REPE"
"REST" Z$ SETS=3
"USE/R" INTDIAL$
"REST" Z
"ENDMACRO"
```

```
"MACRO" SETDIAL$
""
   SETDIAL - MACRO TO PERFORM DIALLEL ANALYSIS ON S SETS OF T (=G(G-1)/2)
             OBSERVATIONS.
""
"LOCAL" SSQ(1...7),DF(1...7),DIV(1...7),CHI(1...7),H(1...7),GH,GM,GS,
        X(1...4),SET,ROW,COL,AROW,ACOL,RC,T,G2,DM,DCOM
"SCALAR" GM,SSQ(1...7),DF(1...7),DIV(1...7),CHI(1...7),GS,G2,T
"MATRIX" DM$ 1,6 : DCOM$6,6
"CALC" GS=G*S : T=G*(G-1)/2 : G2=G-2
"USE/R" SETD1$
"USE/R" SETD2$
"USE/R" SETD3$
"ENDMACRO"
```

```
"MACRO" SETD1$
""
  SETD1 - MACRO TO OBTAIN TOTALS CORRESPONDING TO CALCULATION MATRICES
          FOR SETDIAL
""
"TABLES" ROW$BR : COL$ BC : AROW$ AR,BR : ACOL$ AR,BC : RC$ BR,BC
         : SET$ AR
"TABU" Z;ROW : Z;COL : Z;AROW : Z;ACOL : Z;RC : Z;SET
"VARI" X(1,2)$G : X(3,4)$GS
"EQUATE" X(1...4)=ROW,COL,AROW,ACOL
"CALC" X(1,3)=X(1,3)+X(2,4) : GM=SUM(Z)
"EQUATE" ROW,AROW=X(1,3) "DEVA" X(1...4),COL,ACOL
"ENDMACRO"
```

```
"MACRO" SETD2$
""
  SETD2 - MACRO TO PRODUCE AN AOV FOR SETDIAL
""
""
   OBTAIN SSQ CORRESPONDING TO QUADRATIC FORMS IN CALCULATION MATRICES
""
"CALC" SSQ(1)=GM*GM          : SSQ(2)=SUM(ROW*ROW)    : SSQ(3)=SUM(RC*RC)
:      SSQ(4)=SUM(SET*SET)  : SSQ(5)=SUM(AROW*AROW)  : SSQ(7)=SUM(Z*Z)
""
   OBTAIN PARTITION OF VARIABILITY
""
"CALC" SSQ(5)=(SSQ(5)-SSQ(2)/S-SSQ(4)*4/G+SSQ(1)*4/GS)
:      SSQ(5)=(G.EQ.2)*G2/G2+(G.NE.2)*SSQ(5)/G2
:      SSQ(4)=(SSQ(4)-SSQ(1)/S)/T
:      SSQ(3)=(G.EQ.2)*G2/G2+(G.NE.2)*
              (SSQ(3)-(SSQ(2)-SSQ(1)*2/(G-1))/G2)/S
:      SSQ(2)=(G.EQ.2)*G2/G2+(G.NE.2)*
              (SSQ(2)-SSQ(1)*4/G)/S/G2
:      SSQ(1)=SSQ(1)/T/S
:      SSQ(6)=SSQ(7)-SSQ(1)-SSQ(2)-SSQ(3)-SSQ(4)-SSQ(5)
```

```
""
   CALCULATE DF
""
"CALC" DF(1)=1 : DF(2)=G-1 : DF(3)=T-DF(1)-DF(2)
:       DF(4)=S-1 : DF(5)=DF(2)*DF(4) : DF(7)=T*S
:       DF(6)=DF(7)-T-DF(4)-DF(5)
""
   CALCULATE DIVISORS AND CHI-SQUARE
""
"CALC" DIV(1)=1 : DIV(2)=2*G2 : DIV(3)=G-4 : DIV(4)=-2
:       DIV(5)=G2*(G2-1)/2 : DIV(6)=-(G2-1)
"EQUATE" DCOM$ 18=DIV(6(1),(2,3,4)2,(5,6,1)2)
"CALC" DIV(1...6)=-DIV(1...6)
"EQUATE" DCOM$ 18X,(3X,3)3=DIV(1,1,1,2...6,1)
"CALC" DIV(1...6)=-(S-1)*DIV(1...6)
"EQUATE" DCOM$ 18X,(3,3X)3=DIV(1,1,1,2...6,1)
"CALC" DIV(1)=1/MDF : DIV(2)=DIV(1)*RHOS(1)*(3*RHOS(1)+2)/2/(RHOS(1)+1)
   (RHOS(1)+1) : DIV(3)=DIV(1)*2*RHOS(1)*RHOS(1)/(RHOS(1)+1)/(RHOS(1)+1
:       DIV(4)=(1+RHOS(1)*RHOS(1))*(RHO(2)*RHOS(2)+RHOS(3)*RHOS(3))
:       DIV(4)=DIV(1)*(DIV(4)-4*RHOS(1)*RHOS(2)*RHOS(3))
:       DIV(4)=DIV(4)/(1-RHOS(1)*RHOS(1))/(1-RHOS(1)*RHOS(1))
:       DIV(5)=RHOS(2)*(1-2*RHOS(1))*(RHOS(2)+RHOS(3))
:       DIV(5)=DIV(5)
              +0.5*RHOS(1)*RHOS(1)*(RHOS(3)*RHOS(3)+3*RHOS(2)*RHOS(2))
:       DIV(5)=DIV(5)*DIV(1)/(1-RHOS(1)*RHOS(1))/(1-RHOS(1)*RHOS(1))
:       DIV(6)=DIV(1)*2*RHOS(2)*RHOS(2)/(1+RHOS(1))/(1+RHOS(1))
"EQUATE" DM=DIV(1...6) "CALC" DM=PDT(DM;DCOM) "EQUATE" DIV(1...6)=DM
"CALC" DIV(1...6)=DIV(1...6)*DF(1...6) : DIV(7)=VSUM(DIV(1...6))
"EQUATE" DIV(1...6)=DM "CALC" DIV(1)=1/MDF
:       CHI(1...6)=SSQ(1...6)/DIV(1...6)
:       CHI(7)=VSUM(CHI(2...6))
""
   OUTPUT SSQ, DF, DIVISORS AND CHI-SQUARES
""
"HEADING" H(1)=""MEAN          ""          : H(2)=""BETW. ENTITIES""
:          H(3)=""ENTITY INTERACTION"" : H(4)=""BETW. SETS ""
:          H(5)=""ENTITIES.SETS""         : H(6)=""ENTITY INTER.SETS""
:          H(7)=""CORR. TOTAL""
:          GH=""GRAND MEAN""
"LINE" 5 "PRINT" AN
"CAPTION "" ** ANALYSIS OF VARIANCE TABLE **"" "LINE" 3
"LINE" 1
"CAPTION"
    ""SOURCE                    DF           SSQ           DIV           CHI""
"FOR" HEAD=H(1...7);SS=SSQ(1...7);DF=DF(1...7);DIV=DIV(1...7);
      CHI=CHI(1...7)
"LINE" 2
"PRINT/C,VAR=1,LABR=1,LABC=1" HEAD,DF,SS,DIV,CHI$
    20.0,2X,4.0,(2X,10.4)2,2X,10.2
"REPE"
"DEVA" H(1...7),SSQ(1...7),DF(1...7),DIV(1...7),CHI(1...7)
"ENDMACRO"
```

```
'MACRO' SETD3$
''
  SETD3 - MACRO TO CALCULATE AND OUTPUT SMOOTHED MEANS FOR SETDIAL
''
'CALC' GM=MEAN(Z) : ROW=ROW/S/(G-1) : SET=SET/T : AROW=AROW/(G-1)
:       RC/S : AROW=AROW-ROW-SET+GM
:       AROW=(G.EQ.2)*G2/G2+(G.NE.2)*AROW*(G-1)/G2
:       ROW=ROW-GM : ROW=(G.EQ.2)*G2/G2+(G.NE.2)*ROW*(G-1)/G2
:       AROW=ROW+SET+AROW : ROW=GM+ROW
'LINE' 5
'CAPTION' ''  ** TABLES OF SMOOTHED MEANS **''
'LINE' 3 'PRINT/C,LABR=1,LABC=1,VAR=1' GH,GM$ 5X,10,5X,10.4
'LINE' 2 'CAPTION' ''  ** ADDITIVE ENTITY MEANS **''
'PRINT/LABR=1,LABC=1' ROW$ 8.4
'LINE' 2 'CAPTION' ''  ** ENTITY INTERACTION MEANS **''
'PRINT/LABR=1,LABC=1' RC$ 8.4
'LINE' 2 'CAPTION' ''  ** SET MEANS **''
'PRINT/LABR=1,LABC=1' SET$ 8.4
'LINE' 2 'CAPTION' ''  ** SET X ADDITIVE ENTITY MEANS **''
'PRINT/LABR=1,LABC=1' AROW$ 8.4
'ENDM'
```

```
"MACRO" INTDIAL$
""
   INTDIAL - MACRO TO PERFORM DOUBLE DIALLEL ANALYSIS ON DATA INDEXED BY
             TWO FACTORS A (K,L) AND B (I,J). FACTOR A HAS 2 LEVELS AND
             FACTOR B HAS P LEVELS.

                                     T=P(P-1)/2
""
"LOCAL" SSQ(1...6),DF(1...6),DIV(1...6),CHI(1...6),H(1...6),GH,GM,
        X(1...4),BR,BC,BRC,ABR,ABC,T,MP,P2,M2,AR1,AC1,BR1,BC1,W,L2,
        V(1...5),O,V,R1,R2,R3
"SCALAR" GM,MP,T,P2,M2,R,V(1,2,3...5) : O=1
"CALC" MP=M*P : T=P*(P-1)/2 : P2=P-2
"USE/R" INTD1$
"USE/R" INTD2$
"USE/R" INTD3$
"ENDM"
```

```
"MACRO" INTD1$
""
  INTD1 - MACRO TO OBTAIN TOTALS CORRESPONDING TO CALCULATION MATRICES
          FOR INTDIAL
""
"FACTORS" AR1,AC1$M : BR1,BC1$P
"CALC" W=(FLOAT(AROWS).GT.FLOAT(ACOLS))*FLOAT(ACOLS)
:       W=W+(FLOAT(AROWS).LE.FLOAT(ACOLS))*FLOAT(AROWS)
"GROUPS" AR1=INTPT(W)
"CALC" W=(FLOAT(AROWS).GT.FLOAT(ACOLS))*FLOAT(AROWS)
:       W=W+(FLOAT(AROWS).LE.FLOAT(ACOLS))*FLOAT(ACOLS)
"GROUPS" AC1=INTPT(W)
"CALC" W=(FLOAT(BROWS).GT.FLOAT(BCOLS))*FLOAT(BCOLS)
:       W=W+(FLOAT(BROWS).LE.FLOAT(BCOLS))*FLOAT(BROWS)
"GROUPS" BR1=INTPT(W)
"CALC" W=(FLOAT(BROWS).GT.FLOAT(BCOLS))*FLOAT(BROWS)
:       W=W+(FLOAT(BROWS).LE.FLOAT(BCOLS))*FLOAT(BCOLS)
"GROUPS" BC1=INTPT(W)
"DEVA" W
"TABLES" BR$ BROWS : BC$BCOLS : BRC$ BR1,BC1 : ABR$ AROWS,BROWS
:         ABC$ ACOLS,BCOLS
"FOR" TAB=BR,BC,BRC,ABR,ABC "TABU" Z;TAB "REPE"
"VARI" X(1,2)$P : X(3,4)$MP
"EQUATE" X(1...4)=BR,BC,ABR,ABC
"CALC" X(1,3)=X(1,3)+X(2,4) : GM=SUM(Z)
"EQUATE" BR,ABR=X(1,3) "DEVA" X(1...4),AC1,BC1,BC,ABC
"ENDM"
```

```
'MACRO' INTD2$
''
  INTD2 - MACRO TO PRODUCE AOV FOR INTDIAL
''
''
   OBTAIN SSQ CORRESPONDING TO QUADRATIC FORMS IN CALCULATION MATRICES
''
'SCALAR' SSQ(1...6),DF(1...6),DIV(1...6),CHI(1...6)
'CALC' SSQ(1)=GM*GM : SSQ(2)=SUM(BR*BR) : SSQ(3)=SUM(BRC*BRC)
:       SSQ(4)=SUM(ABR*ABR) : SSQ(6)=SUM(Z*Z)
''
   OBTAIN PARTITION OF VARIABILITY
''
'CALC' SSQ(5)=SSQ(6)-SSQ(4)/P-SSQ(3)/2+SSQ(2)/2/P
:       SSQ(4)=(SSQ(4)-SSQ(2)/2)/P
:       SSQ(3)=SSQ(3)/2-(SSQ(2)/2-SSQ(1)/(P-1))/P2
:       SSQ(2)=(SSQ(2)/2-SSQ(1)*2/P)/P2
:       SSQ(1)=SSQ(1)/T/2
''
   CALCULATE DF
''
'CALC' DF(1)=1 : DF(2)=P-1 : DF(3)=T-DF(1)-DF(2)
:       DF(4)=P-1 : DF(6)=P*(P-1) : DF(5)=DF(6)-T-DF(4)
''
   CALCULATE DIVISORS AND CHI-SQUARE
''
'FOR' V=V(1...5);R1=0,RHO(1),0,RHO(2),RHO(1);R2=0,RHO(1,1,3,1);
      R3=RHO(2,3,2,1,2)
'CALC' V=(R1*R2+R3*R3)-RHO(2)*2*(R1*R3+R2*R3)
:       V=V+0.5*RHO(2)*RHO(2)*(R1*R1+R2*R2+2*R3*R3)
:       V=V/(1-RHO(2)*RHO(2))/(1-RHO(2)*RHO(2))/MDF
'REPE'
'CALC' DIV(1)=V(1)+V(2)+P2*(2*(V(3)+V(4))+(P2-1)*V(5))
:       DIV(2)=V(1)+V(2)+(P2-2)*(V(3)+V(4))-2*V(5)*(P2-1)
:       DIV(3)=V(1)+V(2)-2*(V(3)+V(4)-V(5))
:       DIV(4)=V(1)-V(2)+P2*(V(3)-V(4))
:       DIV(5)=V(1)-V(2)-2*(V(3)-V(4))
:       V(1...5)=DF(1...5)*DIV(1...5) : DIV(6)=VSUM(V(1...5))
:       DIV(1)=1/MDF
:       CHI(1...5)=SSQ(1...5)/DIV(1...5)
:       CHI(6)=VSUM(CHI(2...5))
''
   OUTPUT SSQ, DF, DIVISORS AND CHI-SQUARES
''
'HEADING' H(1)=''MEAN          ''          : H(2)=''B              ''
:          H(3)=''B INTERACTION''          : H(4)=''A.B            ''
:          H(5)=''A INTER.B INTER''        : H(6)=''CORR. TOTAL''
:          GH=''GRAND MEAN''
'PRINT' AN(3) 'LINE' 3
'CAPTION' ''  ** ANALYSIS OF VARIANCE TABLE **'' 'LINE' 3
'LINE' 1
'CAPTION'
    ''SOURCE                    DF          SSQ          DIV          CHI''
'FOR' HEAD=H(1...6);SS=SSQ(1...6);DF=DF(1...6);DIV=DIV(1...6);
      CHI=CHI(1...6)
'LINE' 2
'PRINT/C,VAR=1,LABR=1,LABC=1' HEAD,DF,SS,DIV,CHI$
     20.0,2X,4.0,(2X,10.4)2,2X,10.2
'REPE'
'DEVA' H(1...6),SSQ(1...6),DF(1...6),DIV(1...6),CHI(1...6),V(1...5)
'ENDMACRO'
```

```
'MACRO' INTD3$
''
  INTD3 - MACRO TO CALCULATE AND OUTPUT SMOOTHED MEANS FOR INTDIAL
''
'CALC' GM=MEAN(Z) : BR=BR/2/(P-1) : ABR=ABR/(P-1)
:       BRC=BRC/2 : ABR=(ABR-BR)*(P-1)/P
:       BR=(P.EQ.2)*P2/P2+(P.NE.2)*(BR-GM)*(P-1)/P2
:       ABR=GM+ABR
'LABEL' L1 'JUMP' (P.EQ.2)*L2 'CALC' ABR=ABR+BR
'LABEL' L2 'CALC' BR=GM+BR
'LINE' 3
'CAPTION' ''  ** TABLES OF SMOOTHED MEANS **''
'LINE' 3 'PRINT/C,LABR=1,LABC=1,VAR=1' GH,GM$ 5X,10,5X,10.4
'LINE' 2 'CAPTION' ''  ** ADDITIVE B MEANS **''
'PRINT/LABR=1,LABC=1' BR$ 8.4
'LINE' 2 'CAPTION' ''  ** B INTERACTION MEANS **''
'PRINT/LABR=1,LABC=1' BRC$ 8.4
'LINE' 2 'CAPTION' ''  ** ADDITIVE A.B MEANS **''
'PRINT/LABR=1,LABC=1' ABR$ 8.4
'ENDMACRO'
```

APPENDIX C

CORRELATION AND z-TRANSFORM MATRICES FOR THE FIVE-YEAR VITICULTURAL FIELD EXPERIMENT

Table C.1 Correlation Matrix

		(i) for Yields ($\nu^{\dagger}$= 161)					(ii) for Residuals ($\nu^{\dagger}$= 90)				
		Year					Year				
		1	2	3	4	5	1	2	3	4	5
	1	1.00	.33	.25	.40	.40	1.00	-.01	-.01	.10	-.06
	2		1.00	.41	.56	.49		1.00	.05	.26	.13
Year	3			1.00	.39	.45			1.00	.19	.19
	4				1.00	.65				1.00	.19
	5					1.00					1.00

† ν = degrees of freedom of the observed variances and covariances

Table C.2 Matrix of z-Transforms

		(i) for Yields					(ii) for Residuals				
		Year					Year				
		1	2	3	4	5	1	2	3	4	5
	1		.34	.26	.42	.43		-.01	-.01	.10	-.06
	2			.43	.64	.54			.05	.26	.13
Year	3				.41	.48				.19	.19
	4					.77					.19

APPENDIX D

CORRELATION AND z-TRANSFORM MATRICES FOR SINGLE EVALUATION WINE EXPERIMENTS INVOLVING CABERNET AND RIESLING WINES.

Table D.1 Correlation Matrix

		(i) Cabernet (ν† = 12)					(ii) Riesling (ν† = 16)				
		Taster					Taster				
		1	2	3	4	5	1	2	3	4	5
	1	1.00	.07	.55	.90	.34	1.00	.54	.75	.58	.70
	2		1.00	.37	.34	.24		1.00	.52	.46	.51
Taster	3			1.00	.50	.12			1.00	.44	.70
	4				1.00	.43				1.00	.54
	5					1.00					1.00

†ν is the degrees of freedom of the observed variances and covariances

Table D.2 z-Transform Matrix

		(i) Cabernet					(ii) Riesling				
		Taster					Taster				
		1	2	3	4	5	1	2	3	4	5
	1		.07	.62	1.48	.35		.60	.97	.66	.86
	2			.39	.35	.25			.58	.50	.56
Taster	3				.55	.12				.48	.86
	4					.45					.61
	5										

APPENDIX E1

CORRELATION AND z-TRANSFORM MATRICES, CALCULATED FROM THE ORIGINAL SCORES, FOR THE FOUR DUPLICATE EVALUATION WINE EXPERIMENTS

Table E1.1

Matrices for experiment 1 calculated from the original scores

(ν= 23= degrees of freedom of the observed variances and covariances)

i) Correlation matrix

Session	1						2							
Taster	A	B	C	F	G	I	A	B	C	F	G	I		
	1.00	0.79	0.67	0.63	0.57	0.00	0.74	0.52	0.67	0.58	0.78	-0.20	A	1
		1.00	0.69	0.61	0.50	-0.09	0.68	0.66	0.78	0.64	0.73	-0.37	B	
			1.00	0.78	0.32	0.20	0.39	0.73	0.91	0.72	0.81	-0.25	C	
				1.00	0.40	0.29	0.61	0.60	0.83	0.88	0.71	-0.02	F	
					1.00	0.44	0.57	0.42	0.41	0.47	0.39	0.30	G	
						1.00	0.00	0.10	0.22	0.34	-0.06	0.77	I	
							1.00	0.38	0.49	0.64	0.55	-0.06	A	2
								1.00	0.74	0.63	0.62	-0.14	B	
									1.00	0.84	0.79	-0.24	C	
										1.00	0.58	0.12	F	
											1.00	-0.40	G	
												1.00		
Variances	0.80	2.81	3.91	2.57	0.72	2.04	1.05	2.93	3.65	2.42	0.99	1.54		

Table E1.1 (cont'd)

ii) <u>z-transform matrix</u>

Sessions	1						2							
Tasters	A	B	C	F	G	I	A	B	C	F	G	I		
		1.08	0.81	0.75	0.65	0.00	0.96	0.57	0.82	0.66	1.05	-0.21	A	1
			0.85	0.70	0.55	-0.09	0.82	0.80	1.04	0.75	0.92	-0.39	B	
				1.03	0.33	0.21	0.41	0.94	1.55	0.90	1.13	-0.25	C	
					0.43	0.30	0.71	0.69	1.19	1.39	0.90	-0.02	F	
						0.47	0.65	0.44	0.43	0.50	0.42	0.31	G	
							0.00	0.10	0.22	0.36	-0.06	1.02	I	
								0.40	0.54	0.76	0.62	-0.06	A	2
									0.96	0.75	0.73	-0.14	B	
										1.21	1.08	-0.24	C	
											0.67	0.12	F	
												-0.42	G	
													I	

Table E1.2 Matrices for experiment 2 calculated from the original scores

(ν=47= degrees of freedom of the observed variances and covariances)

i) Correlation matrix

Sessions	1						2							
Tasters	A	B	C	D	E	H	A	B	C	D	E	H		
	1.00	0.21	0.25	0.50	0.40	0.44	0.44	0.23	0.07	0.53	0.58	0.09	A	
		1.00	0.42	0.07	0.20	0.38	-0.05	0.40	0.34	0.23	0.28	0.42	B	
			1.00	0.14	0.44	0.35	0.38	0.06	0.42	0.27	0.39	0.29	C	1
				1.00	0.37	0.21	0.14	0.30	-0.02	0.35	0.45	0.02	D	
					1.00	0.35	0.33	0.25	0.33	0.47	0.56	0.12	E	
						1.00	0.00	0.35	0.26	0.13	0.44	0.40	H	
							1.00	-0.07	0.17	0.50	0.39	0.03	A	
								1.00	0.06	0.15	0.26	0.34	B	
									1.00	0.24	0.45	0.33	C	2
										1.00	0.35	0.02	D	
											1.00	0.18	E	
												1.00	H	
Variances	2.84	2.09	1.59	2.40	2.81	4.50	8.33	2.27	1.28	2.35	2.48	2.29		

Table E1.2 (cont'd)

ii) <u>z-transform matrix</u>

Sessions		1						2						
Tasters		A	B	C	D	E	H	A	B	C	D	E	H	
1	A		0.22	0.26	0.55	0.42	0.47	0.47	0.23	0.07	0.58	0.67	0.09	A
	B			0.45	0.07	0.20	0.40	-0.05	0.43	0.36	0.23	0.28	0.44	B
	C				0.14	0.47	0.36	0.40	0.06	0.45	0.28	0.41	0.30	C
	D					0.39	0.21	0.14	0.31	-0.02	0.37	0.48	0.02	D
	E						0.36	0.34	0.25	0.34	0.51	0.64	0.12	E
	H							0.00	0.36	0.26	0.13	0.47	0.42	H
2	A								-0.07	0.17	0.54	0.41	0.03	A
	B									0.06	0.16	0.27	0.35	B
	C										0.24	0.49	0.34	C
	D											0.37	0.02	D
	E												0.18	E
	H													H

Table E1.3 Matrices for experiment 3 calculated from the original scores

(ν=47 = degrees of freedom of the observed variance and covariances)

i) Correlation matrix

Sessions	1						2							
Tasters	A	B	C	D	E	F	A	B	C	D	E	F		
	1.00	0.38	0.48	0.49	0.71	0.62	0.76	0.51	0.51	0.60	0.62	0.51	A	1
		1.00	0.31	0.50	0.49	0.32	0.35	0.16	0.31	0.23	0.17	0.37	B	
			1.00	0.51	0.49	0.38	0.32	0.34	0.58	0.40	0.49	0.43	C	
				1.00	0.71	0.47	0.47	0.45	0.51	0.40	0.43	0.54	D	
					1.00	0.54	0.68	0.41	0.62	0.57	0.70	0.59	E	
						1.00	0.47	0.37	0.46	0.39	0.46	0.63	F	
							1.00	0.47	0.56	0.54	0.55	0.41	A	2
								1.00	0.52	0.38	0.36	0.53	B	
									1.00	0.53	0.68	0.56	C	
										1.00	0.68	0.47	D	
											1.00	0.46	E	
												1.00	F	
Variances	6.21	6.42	8.61	5.36	12.45	5.31	5.40	4.22	5.53	5.67	10.63	4.71		

Table E1.3 (cont'd)

ii) <u>z-transform matrix</u>

Sessions	1						2							
Tasters	A	B	C	D	E	F	A	B	C	D	E	F		
		0.40	0.52	0.53	0.88	0.72	0.99	0.56	0.57	0.69	0.72	0.56	A	1
			0.32	0.55	0.54	0.34	0.36	0.17	0.32	0.23	0.17	0.39	B	
				0.57	0.54	0.40	0.33	0.36	0.67	0.42	0.53	0.47	C	
					0.89	0.51	0.50	0.49	0.57	0.43	0.45	0.61	D	
						0.60	0.83	0.44	0.72	0.65	0.86	0.67	E	
							0.51	0.39	0.49	0.41	0.50	0.74	F	
								0.51	0.63	0.60	0.61	0.43	A	2
									0.58	0.40	0.38	0.59	B	
										0.59	0.82	0.64	C	
											0.84	0.50	D	
												0.59	E	
													F	

Table E1.4 <u>Matrices for experiment 4 calculated from the original scores</u>

(ν=23= degrees of freedom of the observed variances and covariances)

i) <u>Correlation matrix</u>

Glass Type	Clear								Black									
Tasters	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M		
	1.00	0.71	0.91	0.66	0.81	0.49	0.07	0.76	0.95	0.53	0.91	0.63	0.72	0.34	-0.05	0.70	A	Clear
		1.00	0.79	0.67	0.68	0.54	0.25	0.70	0.67	0.80	0.79	0.66	0.68	0.53	0.24	0.55	D	
			1.00	0.74	0.72	0.58	0.05	0.75	0.89	0.60	1.00	0.67	0.78	0.44	-0.09	0.72	F	
				1.00	0.47	0.59	-0.06	0.50	0.64	0.60	0.74	0.58	0.76	0.30	0.07	0.51	G	
					1.00	0.43	0.09	0.71	0.84	0.46	0.72	0.54	0.56	0.42	-0.04	0.48	J	
						1.00	0.08	0.69	0.46	0.58	0.58	0.27	0.54	0.61	0.35	0.64	K	
							1.00	0.25	0.12	0.27	0.05	0.09	0.02	0.30	0.59	-0.06	L	
								1.00	0.76	0.66	0.75	0.49	0.59	0.66	0.23	0.85	M	
									1.00	0.55	0.89	0.66	0.74	0.34	-0.05	0.68	A	Black
										1.00	0.60	0.53	0.56	0.59	0.31	0.60	D	
											1.00	0.67	0.78	0.44	-0.09	0.72	F	
												1.00	0.57	0.21	-0.08	0.45	G	
													1.00	0.40	0.05	0.63	J	
														1.00	0.26	0.44	K	
															1.00	0.08	L	
																1.00	M	
Variances	5.85	6.07	2.78	8.56	2.48	4.35	3.41	4.34	6.43	5.57	2.78	7.37	2.86	3.45	3.10	3.16		

Table E1.4 (cont'd)

ii) <u>z-transforms matrix</u>

Glass Type	Clear								Black									
Tasters	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M		
		0.88	1.54	0.79	1.12	0.53	0.07	0.98	1.81	0.58	1.54	0.75	0.90	0.35	-0.05	0.87	A	Clear
			1.06	0.81	0.83	0.61	0.26	0.87	0.81	1.09	1.06	0.79	0.82	0.59	0.24	0.63	D	
				0.94	0.91	0.66	0.05	0.98	1.44	0.69	*	0.80	1.06	0.48	-0.09	0.90	F	
					0.51	0.67	-0.06	0.54	0.76	0.70	0.94	0.67	0.99	0.31	0.07	0.56	G	
						0.46	0.09	0.88	1.21	0.50	0.91	0.60	0.63	0.45	-0.04	0.52	J	
							0.08	0.85	0.50	0.66	0.66	0.27	0.60	0.71	0.36	0.76	K	
								0.26	0.12	0.28	0.05	0.09	0.02	0.31	0.67	-0.06	L	
									0.99	0.80	0.98	0.54	0.68	0.78	0.23	1.25	M	
										0.62	1.44	0.79	0.96	0.35	-0.05	0.83	A	Black
											0.69	0.58	0.63	0.67	0.32	0.69	D	
												0.80	1.06	0.48	-0.09	0.90	F	
													0.65	0.21	-0.08	0.48	G	
														0.42	0.05	0.73	J	
															0.26	0.47	K	
																0.08	L	
																	M	

* value undefined as observed correlation coefficient was exactly 1.

Table E1.5 Matrices for experiment 4 calculated from the original scores after uniform random variation has been added.

(ν= 23 = degrees of freedom of the observed variances and covariances)

i) Correlation matrix

Glass Type	Clear								Black									
Tasters	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M		
	1.00	0.68	0.85	0.69	0.77	0.39	0.06	0.67	0.90	0.54	0.90	0.56	0.69	0.26	-0.02	0.61	A	Clear
		1.00	0.78	0.61	0.61	0.46	0.15	0.67	0.58	0.68	0.76	0.66	0.66	0.44	0.30	0.59	D	
			1.00	0.68	0.67	0.58	0.00	0.76	0.75	0.69	0.89	0.60	0.75	0.56	0.03	0.63	F	
				1.00	0.41	0.59	-0.01	0.53	0.65	0.64	0.70	0.52	0.71	0.25	0.17	0.50	G	
					1.00	0.39	0.13	0.62	0.78	0.40	0.73	0.47	0.47	0.35	0.07	0.32	J	
						1.00	0.04	0.66	0.39	0.49	0.47	0.26	0.42	0.59	0.39	0.57	K	
							1.00	0.27	0.20	0.14	0.09	0.07	0.02	0.31	0.51	0.04	L	
								1.00	0.71	0.62	0.64	0.51	0.58	0.64	0.23	0.74	M	
									1.00	0.58	0.85	0.57	0.68	0.33	0.11	0.53	A	Black
										1.00	0.57	0.54	0.64	0.60	0.33	0.54	D	
											1.00	0.63	0.78	0.41	0.10	0.49	F	
												1.00	0.57	0.22	0.07	0.40	G	
													1.00	0.39	0.04	0.53	J	
														1.00	0.38	0.33	K	
															1.00	0.20	L	
																1.00	M	
Variances	6.00	6.14	2.93	9.17	3.12	4.89	4.31	4.60	5.42	4.77	2.81	7.18	2.81	3.61	3.39	3.46		

Table E1.5 (cont'd)

ii) <u>z-transform matrix</u>

Glass Type	Clear								Black									
Tasters	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M		
		0.84	1.26	0.84	1.02	0.41	0.06	0.81	1.48	0.60	1.47	0.63	0.85	0.27	-0.02	0.71	A	Clear
			1.04	0.71	0.71	0.50	0.15	0.81	0.66	0.83	1.00	0.80	0.78	0.48	0.31	0.68	D	
				0.82	0.80	0.67	0.00	1.00	0.98	0.85	1.43	0.69	0.96	0.63	0.03	0.74	F	
					0.44	0.68	-0.01	0.59	0.77	0.75	0.88	0.57	0.89	0.26	0.17	0.55	G	
						0.41	0.13	0.72	1.04	0.43	0.94	0.52	0.51	0.36	0.07	0.33	J	
							0.04	0.79	0.41	0.54	0.51	0.26	0.45	0.68	0.42	0.65	K	
								0.28	0.21	0.14	0.09	0.07	0.02	0.32	0.56	0.04	L	
									0.89	0.73	0.76	0.56	0.66	0.77	0.23	0.94	M	
										0.66	1.26	0.65	0.83	0.34	0.11	0.59	A	Black
											0.64	0.60	0.76	0.70	0.34	0.60	D	
												0.74	1.04	0.43	0.10	0.54	F	
													0.64	0.22	0.07	0.42	G	
														0.41	0.04	0.59	J	
															0.41	0.34	K	
																0.20	L	
																	M	

APPENDIX E2

CORRELATION AND z-TRANSFORM MATRICES, CALCULATED FROM RESIDUALS, FOR THE FOUR DUPLICATE-EVALUATION WINE EXPERIMENTS.

Table E2.1 Matrices for experiment 1 calculated from residuals

(ν=16 = degrees of freedom of the observed variances and covariances)

i) Correlation matrix

Sessions	1						2							
Tasters	A	B	C	F	G	I	A	B	C	F	G	I		
	1.00	0.74	0.14	0.18	0.49	-0.09	0.33	-0.20	0.22	-0.33	0.61	-0.64	A	1
		1.00	-0.06	-0.12	0.49	-0.04	0.30	-0.13	0.08	-0.31	0.23	-0.33	B	
			1.00	0.72	-0.07	0.67	-0.51	0.40	0.83	0.40	0.34	0.08	C	
				1.00	-0.15	0.54	-0.13	0.13	0.75	0.59	0.50	-0.10	F	
					1.00	0.09	0.27	0.11	-0.08	-0.53	0.41	-0.30	G	
						1.00	-0.64	0.33	0.81	0.48	0.22	0.55	I	
							1.00	-0.51	-0.49	-0.44	0.30	-0.74	A	2
								1.00	0.32	0.13	-0.04	0.39	B	
									1.00	0.61	0.43	0.24	C	
										1.00	-0.05	0.51	F	
											1.00	-0.48	G	
												1.00	I	
Variances	0.22	0.82	1.18	0.66	0.38	0.90	0.37	1.23	0.70	0.30	0.27	0.22		

Table E2.1 (cont'd)

ii) <u>z-transform matrix</u>

Sessions	1						2							
Tasters	A	B	C	F	G	I	A	B	C	F	G	I		
		0.95	0.14	0.19	0.53	-0.09	0.34	-0.20	0.22	-0.35	0.70	-0.76	A	
			-0.06	-0.12	0.53	-0.04	0.31	-0.13	0.08	-0.32	0.24	-0.35	B	
				0.92	-0.07	0.81	-0.56	0.42	1.19	0.43	0.36	0.08	C	1
					-0.16	0.60	-0.13	0.13	0.98	0.68	0.54	-0.10	F	
						0.09	0.28	0.11	-0.08	-0.58	0.44	-0.31	G	
							-0.76	0.35	1.12	0.52	0.22	0.61	I	
								-0.57	-0.54	-0.48	0.30	-0.96	A	
									0.33	0.13	-0.04	0.41	B	
										0.71	0.46	0.24	C	2
											-0.05	0.57	F	
												-0.52	G	
													I	

Table E2.2 Matrices for experiment 2 calculated from residuals

(ν=40= degrees of freedom of the observed variances and covariances)

i) Correlation matrix

Sessions	1						2							
Tasters	A	B	C	D	E	H	A	B	C	D	E	H		
	1.00	0.10	0.11	0.28	0.11	0.41	0.20	0.24	0.08	0.14	0.39	0.13	A	1
		1.00	0.30	-0.09	0.00	0.24	-0.07	0.06	0.24	0.17	0.14	0.24	B	
			1.00	-0.01	0.28	0.22	0.48	-0.20	0.37	0.22	0.24	0.33	C	
				1.00	0.07	0.10	-0.06	0.25	-0.02	-0.05	0.31	0.14	D	
					1.00	0.09	0.21	0.11	0.28	0.36	0.08	0.21	E	
						1.00	-0.10	0.24	0.17	0.01	0.36	0.32	H	
							1.00	0.00	0.20	0.43	0.02	0.17	A	2
								1.00	-0.17	0.08	0.22	0.13	B	
									1.00	0.50	0.42	0.26	C	
										1.00	0.16	0.16	D	
											1.00	0.36	E	
												1.00	H	
Variances	1.45	1.58	1.29	1.79	1.60	3.76	6.34	1.54	1.15	1.43	0.77	1.80		

Table E2.2 (cont'd)

ii) <u>z-transform matrix</u>

Sessions	1						2							
Tasters	A	B	C	D	E	H	A	B	C	D	E	H		
		0.10	0.11	0.28	0.11	0.43	0.21	0.24	0.08	0.14	0.41	0.13	A	
			0.31	-0.09	0.00	0.24	-0.07	0.06	0.25	0.18	0.15	0.25	B	
				-0.01	0.29	0.22	0.52	-0.20	0.39	0.22	0.24	0.34	C	1
					0.07	0.10	-0.06	0.25	-0.02	-0.05	0.32	0.14	D	
						0.09	0.21	0.11	0.28	0.38	0.08	0.21	E	
							-0.10	0.25	0.17	0.01	0.37	0.34	H	
								0.00	0.20	0.46	0.02	0.17	A	
									-0.17	0.09	0.23	0.13	B	
										0.55	0.45	0.26	C	2
											0.16	0.16	D	
												0.37	E	
													H	

Table E2.3 Matrices for experiment 3 calculated from residuals

(ν=32= degrees of freedom of the observed variances and covariances)

i) Correlation matrix

Sessions	1						2							
Tasters	A	B	C	D	E	F	A	B	C	D	E	F		
	1.00	0.55	0.30	0.57	0.73	0.60	0.72	0.45	0.40	0.43	0.50	0.47	A	1
		1.00	0.41	0.55	0.64	0.51	0.46	0.27	0.43	0.12	0.23	0.43	B	
			1.00	0.52	0.37	0.33	0.11	0.31	0.50	0.20	0.32	0.41	C	
				1.00	0.71	0.49	0.47	0.56	0.52	0.33	0.41	0.52	D	
					1.00	0.54	0.68	0.44	0.55	0.46	0.62	0.61	E	
						1.00	0.47	0.41	0.45	0.35	0.50	0.57	F	
							1.00	0.47	0.53	0.42	0.49	0.41	A	2
								1.00	0.52	0.16	0.30	0.58	B	
									1.00	0.44	0.61	0.59	C	
										1.00	0.67	0.39	D	
											1.00	0.50	E	
												1.00	F	
Variances	5.75	6.08	9.54	5.58	11.04	5.71	5.42	3.52	5.77	4.58	7.98	5.33		

Table E2.3 (cont'd)

ii) z-transform matrix

Sessions	1						2							
Tasters	A	B	C	D	E	F	A	B	C	D	E	F		
		0.62	0.31	0.64	0.93	0.69	0.90	0.49	0.42	0.46	0.54	0.51	A	1
			0.43	0.61	0.76	0.56	0.49	0.28	0.47	0.12	0.23	0.46	B	
				0.58	0.39	0.35	0.11	0.32	0.55	0.20	0.33	0.43	C	
					0.89	0.53	0.51	0.63	0.57	0.35	0.43	0.58	D	
						0.61	0.83	0.47	0.63	0.50	0.73	0.71	E	
							0.51	0.44	0.49	0.36	0.55	0.64	F	
								0.51	0.58	0.45	0.53	0.44	A	2
									0.58	0.16	0.31	0.67	B	
										0.47	0.71	0.67	C	
											0.81	0.42	D	
												0.55	E	
													F	

Table E2.4 <u>Matrices for experiment 4 calculated from Residuals</u>

(ν=12=degrees of freedom of the observed variances and covariances)

i) <u>Correlation matrix</u>

Glass Type	Clear								Black									
Taster	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M		
	1.00	-0.06	0.73	0.12	0.44	0.05	0.14	0.58	0.78	0.36	0.73	0.04	-0.03	-0.19	-0.12	0.50	A	Clear
		1.00	0.10	0.20	0.30	-0.53	-0.13	-0.17	-0.19	-0.04	0.10	0.00	0.43	-0.14	-0.04	-0.31	D	
			1.00	0.32	0.25	-0.16	0.08	0.26	0.62	0.51	1.00	0.31	0.11	-0.42	-0.46	0.33	F	
				1.00	-0.06	0.19	-0.04	-0.04	-0.07	0.21	0.32	0.46	0.35	-0.15	-0.30	-0.28	G	
					1.00	-0.17	0.20	0.55	0.60	-0.07	0.25	0.25	-0.25	0.04	0.03	-0.18	J	
						1.00	-0.11	0.08	-0.20	-0.37	-0.16	-0.14	-0.28	0.22	0.48	-0.11	K	
							1.00	0.31	0.25	0.24	0.08	-0.29	-0.25	0.24	0.06	-0.11	L	
								1.00	0.54	0.18	0.26	-0.17	-0.33	0.51	0.00	0.49	M	
									1.00	0.34	0.62	0.27	-0.12	-0.26	-0.25	0.30	A	Black
										1.00	0.51	0.25	0.48	0.07	-0.30	0.27	D	
											1.00	0.31	0.11	-0.42	-0.46	0.33	F	
												1.00	0.11	-0.46	-0.52	-0.30	G	
													1.00	-0.08	-0.08	-0.19	J	
														1.00	0.39	0.08	K	
															1.00	-0.28	L	
																1.00	M	
Variances	1.58	1.04	0.17	2.54	1.46	1.54	1.50	1.42	2.21	1.00	0.17	2.71	0.92	2.92	1.25	0.88		

Table E2.4 (cont'd)

ii) <u>z-transform matrix</u>

Glass Types	Tasters	Clear A	Clear D	Clear F	Clear G	Clear J	Clear K	Clear L	Clear M	Black A	Black D	Black F	Black G	Black J	Black K	Black L	Black M
Clear	A		-0.06	0.93	0.13	0.47	0.05	0.14	0.67	1.05	0.38	0.93	0.04	-0.03	-0.20	-0.12	0.54
Clear	D			0.10	0.21	0.31	-0.58	-0.13	-0.17	-0.19	-0.04	0.10	0.00	0.46	-0.14	-0.04	-0.32
Clear	F				0.33	0.26	-0.17	0.08	0.26	0.72	0.56	*	0.32	0.11	-0.45	-0.49	0.34
Clear	G					-0.07	0.19	-0.04	-0.04	-0.07	0.21	0.33	0.50	0.37	-0.15	-0.31	-0.29
Clear	J						-0.17	0.20	0.62	0.70	-0.07	0.26	0.26	-0.26	0.04	0.03	-0.19
Clear	K							-0.11	0.08	-0.21	-0.39	-0.17	-0.14	-0.29	0.22	0.52	-0.11
Clear	L								0.33	0.26	0.24	0.08	-0.30	-0.25	0.24	0.06	-0.11
Clear	M									0.61	0.18	0.26	-0.17	-0.34	0.57	0.00	0.53
Black	A										0.35	0.72	0.28	-0.12	-0.27	-0.26	0.31
Black	D											0.56	0.26	0.52	0.07	-0.31	0.27
Black	F												0.32	0.11	-0.45	-0.49	0.34
Black	G													0.11	-0.50	-0.58	-0.31
Black	J														-0.08	-0.08	-0.19
Black	K															0.42	0.08
Black	L																-0.29
Black	M																

* value undefined as observed correlation coefficient was exactly 1.

Table E2.5 Matrices for experiment calculated from residuals from the original scores after uniform random variation had been added.

(ν=12 = degrees of the observed variances and covariances)

i) Correlation matrix

Glass Types	Clear								Black									
Tasters	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M		
	1.00	0.05	0.41	0.31	0.43	-0.14	0.25	0.34	0.72	0.32	0.56	-0.24	-0.06	-0.25	-0.09	0.41	A	Clear
		1.00	0.09	0.05	-0.03	-0.58	-0.25	-0.27	-0.21	-0.19	0.08	0.04	0.14	-0.31	0.01	0.13	D	
			1.00	0.41	-0.10	0.20	-0.31	0.34	-0.07	0.22	0.03	-0.10	-0.05	0.05	-0.23	0.50	F	
				1.00	-0.11	0.20	0.07	0.12	0.07	0.25	0.16	0.42	0.30	-0.32	-0.36	-0.11	G	
					1.00	0.03	0.34	0.41	0.74	-0.10	0.35	0.03	-0.36	0.04	0.34	-0.25	J	
						1.00	-0.16	0.20	-0.14	-0.44	-0.01	-0.03	-0.44	0.29	0.31	-0.13	K	
							1.00	0.36	0.42	0.29	0.01	-0.33	0.03	0.27	-0.13	-0.18	L	
								1.00	0.49	0.35	-0.13	-0.12	-0.10	0.61	-0.01	0.11	M	
									1.00	0.30	0.48	0.05	-0.05	-0.13	-0.03	-0.08	A	Black
										1.00	-0.20	0.17	0.43	0.12	-0.47	0.17	D	
											1.00	-0.18	0.14	-0.44	0.40	-0.14	F	
												1.00	0.16	-0.43	-0.30	-0.43	G	
													1.00	-0.14	-0.22	-0.07	J	
														1.00	0.34	-0.03	K	
															1.00	-0.28	L	
																1.00	M	
Variances	1.96	1.69	0.38	3.37	1.73	2.04	2.44	1.71	2.52	1.09	0.34	3.25	1.62	2.74	1.51	1.32		

Table E2.5 (cont'd)

ii) <u>z-transform matrix</u>

Glass Type	Clear								Black									
Tasters	A	D	F	G	J	K	L	M	A	D	F	G	J	K	L	M		
		0.05	0.44	0.32	0.46	-0.14	0.25	0.36	0.90	0.34	0.63	-0.25	-0.06	-0.25	-0.09	0.44	A	Clear
			0.09	0.05	-0.03	-0.66	-0.25	-0.27	-0.22	-0.19	0.08	0.04	0.14	-0.32	0.01	0.13	D	
				0.44	-0.10	0.20	-0.32	0.35	-0.07	0.22	0.03	-0.10	-0.05	0.05	-0.24	0.55	F	
					-0.11	0.20	0.07	0.12	0.07	0.25	0.16	0.45	0.31	-0.33	-0.38	-0.11	G	
						0.03	0.36	0.44	0.95	-0.10	0.37	0.03	-0.37	0.04	0.36	-0.25	J	
							-0.16	0.20	-0.14	-0.47	-0.01	-0.03	-0.47	0.30	0.32	-0.13	K	
								0.38	0.44	0.30	0.01	-0.34	0.03	0.28	-0.13	-0.18	L	
									0.53	0.37	-0.13	-0.13	-0.10	0.70	-0.01	0.11	M	
										0.31	0.52	0.05	-0.05	-0.13	-0.03	-0.08	A.	Black
											-0.20	0.17	0.46	0.12	-0.51	0.17	D	
												-0.18	0.14	-0.47	0.42	-0.15	F	
													0.16	-0.46	-0.31	-0.46	G	
														-0.15	-0.22	-0.07	J	
															0.35	-0.03	K	
																-0.28	L	
																	M	

APPENDIX F

ANALYSIS OF VARIANCE TABLES FROM THE ANALYSIS OF THE z-TRANSFORMS FROM THE FOUR DUPLICATE-EVALUATION WINE EXPERIMENTS.

Table F.1 Analysis of variance of the z-transforms of the elements of the taster-session correlation matrices calculated from the original scores and residuals from experiment 1.

	ORIGINAL				RESIDUAL			
	DF	SSQ	DIV	χ^2	DF	SSQ	DIV	χ^2
BETWEEN TASTERS WITHIN SESSIONS								
Mean	1	7.52	.04	172.99	1	0.59	.06	9.51
Tasters	5	3.82	.07	55.19	5	0.88	.09	9.80
Tasters Interaction	9	.32	.03	12.06	9	2.58	.06	44.41
Sessions	1	.04	.04	.97	1	.60	.10	5.92
Sessions. Tasters	5	.41	.03	14.14	5	1.07	.06	19.02
Sessions. Tasters Interaction	9	.68	.01	47.51	9	1.22	.04	31.99
Corrected Total	29	5.27	-	129.87	29	6.35	-	111.14
BETWEEN SESSIONS WITHIN TASTERS								
Mean	1	6.27	.04	144.23	1	1.64	.06	26.18
Tasters	5	.84	.04	22.53	5	.95	.06	15.53
Corrected Total	5	.84	-	22.53	5	.95	-	15.53
BETWEEN TASTERS AND SESSIONS								
Mean	1	8.10	.04	186.23	1	.23	.06	3.61
Tasters	5	4.10	.07	58.74	5	1.30	.09	14.24
Tasters Interaction	9	.36	.03	13.26	9	2.33	.06	41.10
Sessions. Tasters	5	.38	.03	11.73	5	1.82	.07	26.12
Sessions. Tasters Interaction	10	.69	.01	46.52	10	.89	.04	24.07
Corrected Total	29	5.53	-	130.24	29	6.35	-	105.54

Table F.2 Analysis of variance of the z-transforms of the elements of the taster-session correlation matrices calculated from the original scores and residuals from experiment 2.

	ORIGINAL				RESIDUAL			
	DF	SSQ	DIV	χ^2	DF	SSQ	DIV	χ^2
BETWEEN TASTERS WITHIN SESSIONS								
Mean	1	2.42	.02	113.97	1	.95	.02	37.93
Tasters	5	.14	.03	4.97	1	.16	.03	5.34
Tasters Interaction	9	.39	.01	28.45	9	.22	.02	12.11
Sessions	1	.07	.04	1.66	1	.02	.05	.44
Sessions.Tasters	5	.08	.02	3.50	5	.17	.03	5.80
Sessions. Tasters	9	.12	.01	9.78	9	.24	.02	13.40
Corrected Total	29	.80	-	48.35	29	.81	-	37.10
BETWEEN SESSIONS WITHIN TASTERS								
Mean	1	1.28	.02	60.05	1	.17	.02	6.91
Tasters	5	.04	.02	2.10	5	.15	.02	6.18
Corrected Total	5	.04	-	2.10	5	.15	-	6.18
BETWEEN TASTERS AND SESSIONS								
Mean	1	2.17	.02	102.20	1	.97	.02	38.87
Tasters	5	.23	.03	7.83	5	.14	.03	4.76
Tasters Interaction	9	.35	.01	26.13	9	.25	.02	13.59
Sessions. Tasters	5	.18	.03	6.35	5	.06	.04	1.72
Sessions. Tasters Interaction	10	.24	.01	19.52	10	.33	.02	18.07
Corrected Total	29	1.00	-	59.83	29	.77	-	38.14

Table F.3 Analysis of variance of the z-transforms of the elements of the taster-session correlation matrices calculated from the original scores and residuals from experiment 3.

	ORIGINAL				RESIDUAL			
	DF	SSQ	DIV	χ^2	DF	SSQ	DIV	χ^2
BETWEEN TASTERS WITHIN SESSIONS								
Mean	1	9.55	.02	448.78	1	9.38	.03	300.13
Tasters	5	.29	.03	11.27	5	.15	.04	4.06
Tasters Interaction	9	.10	.01	10.48	9	.19	.01	13.69
Sessions	1	.00	.05	.06	1	.04	.09	.40
Sessions. Tasters	5	.13	.02	6.36	5	.32	.03	9.99
Sessions. Tasters Interaction	9	.12	.01	13.77	9	.15	.01	11.60
Corrected Total	29	.64	-	41.95	29	.85	-	39.75
BETWEEN SESSIONS WITHIN TASTERS								
Mean	1	2.48	.02	116.36	1	1.98	.03	63.39
Tasters	5	.45	.02	26.55	5	.27	.02	11.03
Corrected Total	5	.45	-	26.55	5	.27	-	11.03
BETWEEN TASTERS AND SESSIONS								
Mean	1	7.43	.02	349.10	1	6.35	.03	203.22
Tasters	5	.33	.03	12.35	5	.17	.04	4.29
Tasters Interaction	9	.10	.01	11.80	9	.14	.01	11.75
Sessions. Tasters	5	.19	.03	6.23	5	.34	.05	7.17
Sessions. Tasters Interaction	10	.03	.01	4.45	10	.07	.01	6.29
Corrected Total	29	.65	-	34.83	29	.72	-	29.49

Table F.4 Analysis of variance of the z-transforms of the elements of the taster-session correlation matrices calculated from the original scores and residuals from experiment 4. Uniform random variation was added to the original scores as an empirical correction for continuity.

	ORIGINAL				RESIDUAL			
	DF	SSQ	DIV	χ^2	DF	SSQ	DIV	χ^2
BETWEEN TASTERS WITHIN GLASS TYPES								
Mean	1	16.93	.04	389.34	1	.06	.08	.74
Tasters	7	4.09	.08	51.05	7	.74	.10	7.67
Tasters Interaction	20	.82	.02	36.61	20	.89	.08	11.30
Glass Types	1	.09	.08	1.09	1	.24	.10	2.43
Glass Types.Tasters	7	.27	.05	6.02	7	.86	.09	9.42
Glass Types.Tasters Interaction	20	.36	.02	22.42	20	1.88	.08	24.25
Corrected Total	55	5.64	-	117.20	55	4.60	-	55.07
BETWEEN GLASS TYPES WITHIN TASTERS								
Mean	1	6.11	.04	140.51	1	.14	.08	1.73
Tasters	7	1.04	.04	28.81	7	1.17	.08	14.03
Corrected Total	7	1.04	-	28.81	7	1.17	-	14.03
BETWEEN TASTERS AND GLASS TYPES								
Mean	1	16.94	.04	389.54	1	.18	.08	2.17
Tasters	7	3.86	.08	48.22	7	.73	.09	7.70
Tasters Interaction	20	1.30	.02	57.78	20	2.01	.08	25.52
Glass Types.Tasters	7	.22	.06	4.06	7	.47	.09	5.08
Glass Types.Tasters Interaction	21	.26	.02	16.13	21	1.82	.08	23.33
Corrected Total	55	5.65	-	126.19	55	5.04	-	61.63

APPENDIX G

CORRELATION AND z-TRANSFORM MATRICES FOR BODY COMPOSITION MEASUREMENTS ON CALVES

Table G.1 Matrices from body composition measurements on left and right hind legs of calves

(ν=38 = degrees of freedom of the observed variances and covariances)

i) Correlation matrices

Side	Left					Right						
Muscle	A	B	C	D	E	A	B	C	D	E		
	1.00	.59	.61	.58	-.05	.93	.65	.62	.64	.35	A	Left
		1.00	.48	.51	.27	.56	.92	.43	.60	.43	B	
			1.00	.60	.02	.66	.51	.95	.61	.21	C	
				1.00	.04	.60	.58	.62	.92	.33	D	
					1.00	-.13	.21	-.05	-.01	.60	E	
						1.00	.67	.72	.68	.26	A	Right
							1.00	.53	.70	.45	B	
								1.00	.67	.15	C	
									1.00	.29	D	
										1.00	E	

Table G.1 (cont'd)

ii) <u>z-Transform matrix</u>

Side	Left					Right						
Muscle	A	B	C	D	E	A	B	C	D	E		
		.68	.71	.66	-.05	1.65	.77	.73	.76	.37	A	Left
			.52	.57	.27	.63	1.58	.47	.70	.46	B	
				.69	.02	.80	.56	1.83	.71	.22	C	
					.04	.70	.67	.72	1.60	.35	D	
						-.13	.21	-.05	-.01	.69	E	
							.81	.90	.82	.26	A	Right
								.58	.87	.49	B	
									.81	.15	C	
										.29	D	
											E	

APPENDIX H

CORRELATION AND z-TRANSFORM MATRICES FOR THURSTON

PRIMARY ABILITY TESTS ADMINISTERED ON TWO OCCASIONS

Table H.1 Matrices for Primary Ability Tests Administered on Two Occasions

(ν=109 = degrees of feedom of the observed variance and covarances)

i) Correlation matrix

Occasion	1					2						
Test	1	2	3	4	5	1	2	3	4	5		
	1.00	.37	.42	.53	.38	.81	.35	.42	.41	.24	1	1
		1.00	.33	.14	.10	.35	.65	.32	.14	.15	2	
			1.00	.38	.20	.49	.20	.75	.40	.17	3	
				1.00	.24	.58	-.04	.46	.73	.15	4	
					1.00	.32	.11	.26	.19	.43	5	
						1.00	.34	.46	.56	.24	1	2
							1.00	.18	.06	.15	2	
								1.00	.54	.20	3	
									1.00	.16	4	
										1.00	5	

SOURCE: R.P. McDonald (1974).

Table H.1 (cont'd)

ii) <u>z-Transform matrix</u>

Occasion	1					2					
Test	1	2	3	4	5	1	2	3	4	5	
		.39	.45	.59	.41	1.14	.37	.45	.43	.25	1
			.34	.14	.10	.36	.78	.33	.14	.15	2
				.40	.21	.53	.21	.97	.38	.17	3
					.24	.67	-.04	.49	.93	.15	4
						.33	.12	.27	.19	.46	5
							.36	.49	.63	.24	1
								.18	.06	.15	2
									.61	.20	3
										.16	4
											5

BIBLIOGRAPHY

Aitkin, M.A., W.C. Nelson and K.H. Reinfurt (1968). Tests for correlation matrices. *Biometrika* 55, 327-333.

Alvey, N.G., C.F. Banfield, R.I. Baxter, J.C. Gower, W.J. Krzanowski, P.W. Lane, P.K. Leech, J.A. Nelder, R.W. Payne, K.M. Phelps, C.E. Rogers, G.V.S. Ross, H.R. Simpson, A.D. Todd, R.W.M. Wedderburn and G.N. Wilkinson (1977). *GENSTAT: A General Statistical Program.* Rothamsted Experimental Station, Harpenden, Herts.

Amerine, M.A. and E.B. Roessler (1976). *Wines: Their Sensory Evaluation.* Freeman, San Francisco.

Amerine, M.A., R.M. Pangborn and E.B. Roessler (1965). *Principles of Sensory Evaluation of Food.* Academic Press, New York.

Anderson, T.W. (1958). *An Introduction to Multivariate Statistical Analysis.* Wiley, New York.

Antcliff, A.J. and G.H. Kerridge (1976). Developments in small scale winemaking. *Food Technology in Australia* 27, 523-525, 536.

Baker, G.A. and M.A. Amerine (1953). Organoleptic ratings of wines estimated from analytical data. *Food Research* 18, 381-389.

Bartlett, M.S. (1950). Tests of significance in factor analysis. *British Journal of Psychology, Statistics Section* 3, 77-85.

Bartlett, M.S. (1951). A further note on tests of significance in factor analysis. *British Journal of Psychology, Statistics Section* 4, 1-2.

Bartlett, M.S. (1954). A note on the multiplying factors for various χ^2 approximations. *Journal of the Royal Statistical Society, Series B* 16, 296-298.

Bennett, G., B.M. Spahr and M.L. Dodds (1956). The value of training a sensory test panel. *Food Technology* 10, 205-208.

Bliss, C.I. (1967). *Statistics in Biology* Vol. I. McGraw-Hill, New York.

Bock, R.D. and R.E. Bargmann (1966). Analysis of covariance structures. *Psychometrika* 31, 507-534.

Browne, M.W. (1974). Generalised least squares estimators in the analysis of covariance structures. *South African Statistical Journal* 8, 1-24. (Reprinted in D.J. Aigner and A.S. Goldberger (1977) *Latent Variables in Socio-Economic Models.* North Holland Publishing Co., New York.)

Campbell, D.T. and D.W. Fiske (1959). Convergent and discriminant validation by the multitrait-multimethod matrix. *Psychological Bulletin* 56, 81-105.

Cochran, W.G. and G.M. Cox (1957). *Experimental Designs.* Wiley, New York (2nd edition).

Corballis, M.C. and R.E. Traub (1970). Longitudinal factor analysis. *Psychometrika* 35, 79-98.

Fisher, R.A. (1921). On the "probable error" of a coefficient of correlation deduced from a small sample. *Metron* 1, 1-32.

Fletcher, R. and M.J.D. Powell (1963). A rapidly convergent descent method for minimisation. *Computer Journal* 2, 163-168.

Giffing, B. (1956a). A generalised treatment of the use of diallel crosses in quantitative inheritance. *Heredity* 10, 31-50.

Giffing, B. (1956b). Concept of general and specific combining ability in relation to diallel crossing systems. *Australian Journal of Biological Sciences* 9, 463-493.

Girardot, N.F., D.R. Peryam and R. Shapiro (1952). Selection of sensory testing panels. *Food Technology* 6, 140-143.

Gleeson, A.C. and C.A. McGilchrist (1978). An introduction to relationship algebras. *Communications in Statistics, Theory and Methodology* A7, 1053-1078.

Gleser, L.J. (1968). On testing a set of correlation coefficients for equality: some asymptotic results. *Biometrika* 55, 513-517.

Hale, C.R. and C.J. Brien (1978). Influence of Salt Creek rootstock on composition and quality of Shiraz grapes and wine. *Vitis* 17, 139-146.

Hopkins, J.W. (1946). Precision of assessment of palatability of foodstuffs by laboratory panels. *Canadian Journal of Research Section F* 24, 203-214.

Hunyh, H. and L.S. Feldt (1970). Conditions under which mean square ratios in repeated measurements designs have exact F-distributions. *Journal of the American Statistical Association* 65, 1582-1589.

James, A.T. (1957). The relationship algebra of an experimental design. *Annals of Mathematical Statistics* 28, 993-1002.

Joreskog, K.G. (1978). Structural analysis of covariance and correlation matrices. *Psychometrika* 43, 443-477.

Kendall, M.G. (1952). *The Advanced Theory of Statistics. Vol. I.* Griffin, London. 5th edition.

Krumm, J.K. (1955). Sensory panel testing. *Food Engineering* 27 (July), 74-83.

Lawley, D.N. (1963). On testing a set of correlation coefficients for equality. *Annals of Mathematical Statistics* 34, 149-151.

Liming, N.E. (1966). Consistency of a trained taste panel. *Journal of Dairy Science* 49, 628-630.

McDonald, R.P. (1974). Testing pattern hypotheses for covariance matrices. *Psychometrika* 39, 189-201.

McDonald, R.P. (1975). Testing pattern hypotheses for correlation matrices. *Psychometrika* 40, 253-255.

Marie, R., D. Boubals and P. Galzy (1962). Sur la dégustation rationnelle des boissons. *Bulletin de l'Office International de la Vigne at du Vin* 35, 756-786.

May, P. (1977). Mechanical harvesting of grapes in Australia, with special reference to its effect on grape quality. Presented at *International Symposium on the Quality of Vintage*, Cape Town, South Africa.

May, P., P.R. Clingeleffer, P.B. Scholefield and C.J. Brien (1976). The response of the grape cultivar Crouchen (Australian syn. Clare Riesling) to various trellis and pruning treatments. *Australian Journal of Agricultural Research* 27, 845-856.

May, P., M.R. Sauer and P.B. Scholefield (1973). Effect of various combinations of trellis, pruning, and rootstock on vigorous Sultana vines. *Vitis* 12, 192-206.

Moser, H.A., H.J. Dutton, C.D. Evans and J.C. Cowan (1950). Conducting a taste panel for the evaluation of edible oils. *Food Technology* 4, 105-109.

Nair, K.R. (1951). Rectangular lattices and partially balanced incomplete block designs. *Biometrics* 7, 145-154.

Ogawa, J. and G. Ishii (1965). The relationship algebra and the analysis of variance of a partially balanced incomplete block design. *Annals of Mathematical Statistics* 36, 1815-1828.

Ough, C.S. (1964). *Selection of Judges and Sensory Evaluation of Wines*. Israel Wine Institute.

Ough, C.S. and G.A. Baker (1961). Small panel sensory evaluations of wines by scoring. *Hilgardia* 30, 587-619.

Ough, C.S. and H.W. Berg (1971). Simulated mechanical harvest and gondola transport. II. Effect of temperature, atmosphere, and skin contact on chemical and sensory qualities of white wines. *American Journal of Enology and Viticulture* 22, 194-198.

Ough, C.S., H.W. Berg and M.A. Amerine (1969). Substances extracted during skin contact with white musts. II. Effect of bentonite additions during and after fermentation on wine composition and sensory quality. *American Journal of Enology and Viticulture* 20, 101-107.

Ough, C.S. and W.A. Winton (1976). An evaluation of the Davis wine-score card and individual expert panel members. *American Journal of Enology and Viticulture* 27, 136-144.

Overman, A. and J.C.R. Li (1948). Dependability of food judges as indicated by an analysis of scores of a food-tasting panel. *Food Research* 13, 441-449.

Rankine, B.C. (1974). Wine tasting and judging. *Food Technology in Australia* 26, 443-453.

Rao, C.R. (1965). *Linear Statistical Inference and Its Applications.* Wiley, New York.

Rouanet, H. and D. Lépine (1970). Comparison between treatments in a repeated-measurements design: ANOVA and multivariate methods. *The British Journal of Mathematical and Statistical Psychology* 23, 147-163.

Sawyer, F.M., H. Stone, H. Abplanalp and G.F. Stewart (1962). Repeatability estimates in sensory-panel selection. *Journal of Food Science* 27, 386-393.

Swain, A.J. (1975). Analysis of parametric structures for variance matrices. Unpublished Ph.D. thesis, University of Adelaide.

Tromp, A. and W.J. Conradie (1979). An effective scoring system for sensory evaluation of experimental wines. *American Journal of Enology and Viticulture* 30, 278-283.

Tudor, G.D. (1971). The effect of pre-natal and early post-natal nutrition on the subsequent growth and body composition of beef cattle. Unpublished M. Ag. Sc. thesis, University of Queensland.

Venables, W.N. (1980) *personal communication.*

Wiley, R.C., A.M. Briant, I.S. Fagerson, J.H. Sabry and E.F. Murphy (1957). Evaluation of flavour changes due to pesticides - a regional approach. *Food Research* 22, 192-205.

Wilkinson, G.N. (1970). A general recursive procedure for analysis of variance. *Biometrika* 57, 19-46.

Yates, F. (1947). The analysis of data from all possible reciprocal crosses between a set of parental lines. *Heredity* 1, 287-30. (Reprinted in Yates, F. (1970). *Experimental Design.* Griffin, London.)