Option pricing using path integrals

by

Dr. Frédéric D.R. Bonnet

B.Sc. (Mathematical and Computer Science with Honours), 1998.
Ph.D. in Science (Theoretical and Astrophysics), 2002.
The University of Adelaide.

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Abstract

It is well established that stock market volatility has a memory of the past, moreover it is found that volatility correlations are long ranged. As a consequence, volatility cannot be characterized by a single correlation time in general. Recent empirical work suggests that the volatility correlation functions of various assets actually decay as a power law. Moreover it is well established that the distribution functions for the returns do not obey a Gaussian distribution, but follow more the type of distributions that incorporate what are commonly known as *fat–tailed* distributions. As a result, if one is to model the evolution of the stock price, stock market or any financial derivative, then standard Brownian motion models are inaccurate. One must take into account the results obtained from empirical studies and work with models that include realistic features observed on the market.

In this thesis we show that it is possible to derive the path integral for a non-Gaussian option pricing model that can capture fat–tails. However we find that the path integral technique can only be used on a very small set of problems, as a number of situations of interest are shown to be intractable.

Statement of Originality

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Signed

Date

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Conventions and formsetting

0.1 Typesetting

This thesis it typeset using the LaTeX2e software. WinEdt build 5.4 was used as an effective interface to LaTeX (Oetiker *et al.* 2000). Harvard style is used for referencing and citation in this thesis. Australian English spelling is adopted, as defined by the Macquarie English Dictionary (Delbridge 2001).

0.2 Software

The results were obtained using mathematical packages (Mathematica, Matlab), statistical packages (R).

The programming language used was Fortran 90 with parallel commands embedded in it. Also used was C++.

The reason being that it is more realistic in our days to have a wide range of computer skills instead of just a highly specialized one which may not be compatible elsewhere.

0.3 Data

The data was obtained from two different sources

- The yahoo finance website (*Yahoo Finance* 2008).
- The Tick Data website (*Tick Data Global Historical Data Solutions* 2008).

0.4 Mathematical Symbols

$\aleph$	The Kurtosis.
$\mathrm{E}[f(x)]$	The expected value of $f(x)$.
$\mathrm{Var}[f(x)]$	The variance of $f(x)$.
t	Time.
$S(t)$	The asset price at time t.
$\mathcal{O}(S(t),t))$	The option price of a given asset $S(t)$ at a time t.
$X(t), Y(t)$	The random process $X(t)$ and $Y(t)$.
$\mathcal{F}_t$	The filtration of t.
$\mathrm{E}[X(t)]$	The expected value of the random process $X(t)$.
$\mathrm{Var}[X(t)]$	The variance of the random process $X(t)$.
$L(\theta)$	The loglikelihood function a distribution with parameter set θ.
$\mathcal{L}(\dot{x}(t), x(t), t)$	The Lagrangian functional for the path integral.
$\mathcal{A}[x(t)]$	The action functional for the path integral.
$\mathcal{D}x(t)$	Integral measure for the path integral.

Publications

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