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Near threshold enhancement of the $p\bar{p}$ mass spectrum in J/Ψ decay

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We investigate the nature of the near-threshold enhancement in the $p\bar{p}$ invariant-mass spectrum of the reaction $J/\Psi \to \gamma p\bar{p}$ reported recently by the BES Collaboration. Using the Jülich $N\bar{N}$ model we show that the mass dependence of the $p\bar{p}$ spectrum close to the threshold can be reproduced by the S-wave $p\bar{p}$ final state interaction in the isospin $I = 1$ state within the Watson-Migdal approach. However, because of our poor knowledge of the $N\bar{N}$ interaction near-threshold and of the $J/\Psi \to \gamma p\bar{p}$ reaction mechanism and in view of the controversal situation in the decay $J/\Psi \to \pi^0 p\bar{p}$, where no obvious signs of a $p\bar{p}$ final state interaction are seen, explanations other than final state interactions cannot be ruled out at the present stage.

I. INTRODUCTION

Recently the BES Collaboration [1] reported a near-threshold enhancement in the proton-antiproton ($p\bar{p}$) invariant-mass spectrum, observed in the $J/\Psi \to \gamma p\bar{p}$ decay. Signs for a low mass $p\bar{p}$ enhancement had been already seen earlier by the Belle Collaboration in their study of the $B^+ \to K^+ p\bar{p}$ decay [2] as well as in the reaction $\bar{B}^0 \to D^0 p\bar{p}$ [3]. But because of the large statistical uncertainties of the Belle data it was difficult to draw concrete, quantitative conclusions about the extent of the near-threshold $p\bar{p}$ enhancement. The new data by Bai et al. [1], however, are of rather high statistical accuracy and therefore provide very precise information about the magnitude and the energy dependence of the $p\bar{p}$ mass spectrum very close to threshold.

The BES Collaboration [1] fitted their $p\bar{p}$ invariant-mass spectrum below 1.95 GeV by a Breit-Wigner resonance function. Assuming that the $p\bar{p}$ system is in an S-wave resulted in a resonance mass of $M = 1859^{+3+5}_{-10-25}$ MeV and a total width of $\Gamma < 30$ MeV. A comparable fit to the data could be achieved with a P-wave Breit-Wigner function with $M = 1876 \pm 0.9$ MeV and $\Gamma = 4.6 \pm 1.8$ MeV.

The proximity of these resonance masses to the $p\bar{p}$ reaction threshold (which is at 1876.54 MeV) nourished speculations that the observed strong enhancement could be a signal of an $N\bar{N}$ bound-state. While theoretical considerations of such $N\bar{N}$ bound states (or of baryonia, in general) abound in the literature [4–9] there is so far hardly any undisputed experimental information on the existence of such states [10–12]. Thus, the supposition that one has found here independent and possibly even more convincing evidence in support of $N\bar{N}$ bound states is certainly appealing.

An alternative explanation put forward by the BES collaboration invokes similarities of the observed enhancement in the $p\bar{p}$ mass spectrum near-threshold with the strong energy dependence of the electromagnetic form factor of the proton around $\sqrt{s} \approx 2m_p$, in the timelike region, as determined in the reaction $p\bar{p} \to e^+e^-$ [13]. In the latter case it was argued that the sharp structure seen in the experiment could be caused by a narrow, near-threshold vector-meson ($J^{PC}=1^{--}$) resonance [14] with $M = 1870 \pm 10$ MeV and $\Gamma = 10 \pm 5$ MeV. (See also Refs. [15,16] for a pertinent discussion.) One should keep in mind, however, that the quantum numbers of the J/Ψ particle ($J^{PC}=1^{--}$) would restrict such a resonance to occur in a pseudoscalar (0^{-+}) or scalar (0^{++}) state—should it be indeed responsible for the enhancement of the near-threshold $p\bar{p}$ mass spectrum in the decay $J/\Psi \to \gamma p\bar{p}$.

An entirely different and much more conventional interpretation of the observed enhancement was suggested in several recent works [17–19]. These authors argue that the enhancement is primarily due to the final state interaction (FSI) between the produced proton and antiproton. Specifically, it was shown within the scattering length approximation [17] that a calculation with a complex S-wave scattering length extracted from an effective-range analysis of $p\bar{p}$ scattering data can reproduce the shape of the $p\bar{p}$ mass distribution close to the threshold.

In the present paper we analyze the near-threshold enhancement in the $p\bar{p}$ invariant-mass spectrum reported by the BES Collaboration utilizing a realistic model of the $N\bar{N}$ interaction [20]. The elastic part of this model is the G-parity transform of the Bonn meson-exchange NN potential, supplemented by a phenomenological complex potential to account for $N\bar{N}$ annihilation.

As just mentioned, the investigations of Kerbikov et al. [17], as well as those of Bugg [18], rely on the scattering length approximation. But it remains unclear over which energy range this approximation can provide a reliable representation of the energy dependence of the $p\bar{p}$ amplitude. The enhancement seen in the BES data extends up to invariant masses of $M(p\bar{p}) \approx 2$ GeV, which corresponds to center-of-mass energies of around 120 MeV in the $p\bar{p}$ system. It is obvious that the simple scattering length

approximation cannot be valid over such a large energy region. Using the $p\bar{p}$ amplitude of our interaction model (which describes the available $N\bar{N}$ scattering data up to center-of-mass energies of 150 MeV [20]) we can examine to what extent the scattering length approximation can indeed reproduce the energy dependence of the $p\bar{p}$ scattering amplitude. Moreover, we can compare the energy dependence induced by the full $p\bar{p}$ amplitude with the BES data over the whole energy range where the enhancement was observed.

A microscopic $N\bar{N}$ model has a further advantage. It yields predictions for all possible spin- and isospin channels. Thus, we can investigate whether the energy dependence of the $^1S_0$ and $^3P_0$ $N\bar{N}$ scattering amplitudes in the $I = 0$ and $I = 1$ isospin channels is compatible with the BES data. (We use the nomenclature $^{(2I+1)(2S+1)}L_J$ but omit the isospin index when referring to both isospin channels or to isospin averaged results.) The analysis of Kerbikov et al. utilizes scattering lengths obtained from a spin-averaged effective-range fit to the low-energy $N\bar{N}$ data. Those values might be strongly influenced or even dominated by the $^3S_1 N\bar{N}$ partial wave, a channel which cannot contribute to the FSI in the $J/\Psi \to \gamma p\bar{p}$ decay because of charge-conjugation invariance.

Let us finally mention that the work of Zou and Chiang [19] does not employ the scattering length approximation but uses the K-matrix formalism. However, their interaction consists only of one-pion exchange and is therefore not realistic because it does not take into account the most striking feature of low-energy $N\bar{N}$ scattering, namely, annihilation. Recall that near the $p\bar{p}$ threshold the annihilation cross section is twice as large as the elastic $p\bar{p}$ cross section [20].

The paper is structured in the following way: In Sec. II we briefly review the $N\bar{N}$ potential model that is used in the present analysis and examine the reliability of the scattering length approximation to the $N\bar{N}$ amplitude for the partial waves relevant to the analysis of the BES data. The possible influence of the Coulomb interaction is discussed as well. In Sec. III we provide details of our calculation of the near-threshold $p\bar{p}$ mass spectrum for the reaction and we compare our results with the measurement of the BES collaboration. Sec. IV is devoted to a discussion of possible signals of $N\bar{N}$ bound states or (sub $p\bar{p}$ threshold) meson resonances in the $p\bar{p}$ mass spectrum. The paper ends with concluding remarks.

II. PROTON-ANTIPROTON SCATTERING AT LOW ENERGIES

In the present investigation we use one of the $N\bar{N}$ models developed by the Jülich group [20–22]. Specifically, we use the Bonn OBE (one-boson-exchange) model introduced in Ref. [21] whose results are also discussed in Ref. [20]. In the latter work the model is called A(OBE) and we also adopt this name in the present paper.

The model A(OBE) is based on the OBE version of the full Bonn potential derived in time-ordered perturbation theory, i.e., OBEPT in Ref. [23]. The G-parity transform of this interaction model constitutes the elastic part of the $N\bar{N}$ potential. A phenomenological spin-, isospin- and energy-independent complex potential of Gaussian form is added to account for the $N\bar{N}$ annihilation. With only three free parameters (the range and the strength of the real and imaginary parts of the annihilation potential) a good overall description of the low- and intermediate energy $N\bar{N}$ data was achieved. Results for the total and the integrated elastic and charge-exchange cross sections as well as angular dependent observables can be found in Refs. [20,21]. Here in Fig. 1 we show only the elastic $p\bar{p}$ cross section.

We also utilized the most complete $N\bar{N}$ model of the Jülich Group, model D published in Ref. [22]. The elastic part of this interaction model is derived from the G-parity transform of the full Bonn NN potential and the annihilation is described in part microscopically by $N\bar{N} \to 2$ meson decay channels—see Ref. [22] for details. But since the results turned out to be qualitatively rather similar to the ones obtained with A(OBE) we refrain from showing them here.

The differential cross sections for $p\bar{p}$ scattering [24–26] already indicate a substantial contribution from higher
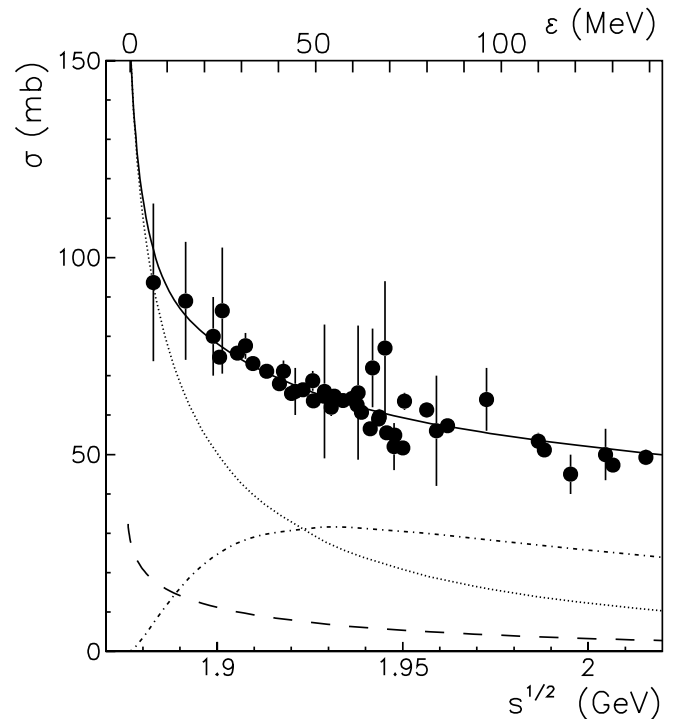

FIG. 1. The $p\bar{p}$ elastic cross section as a function of the invariant collision energy. The lines show the result of the Jülich model A(OBE) [20]. The dashed line is the contribution from the $^1S_0$ partial wave alone, dotted from all s waves, dashed-dotted from p waves and the solid line is the full calculation. The circles are experimental data taken from Ref. [48].

partial waves at around 10 MeV above the $p\bar{p}$ threshold. This feature is also reflected in the predictions of the $N\bar{N}$ model.

Since the scattering length approximation to the $N\bar{N}$ amplitude was used in two of the analyses of the BES data [17,18] we want to investigate the validity of this approximation. In the following we ignore the proton-neutron mass difference and also the Coulomb interaction in the $p\bar{p}$ system, in order to simplify the discussion. But we will come back to these issues later.

In the scattering length approximation the S-wave $p\bar{p}$ scattering amplitude T is given by

$$T = \frac{a}{1 + iaq_p}, \tag{1}$$

where the scattering length a is a complex number because of inelastic channels (annihilation into multimeson states) that are open already at the $p\bar{p}$ threshold. The proton momentum in the c.m. system is $q_p = \sqrt{s - 4m_p^2}/2$, where m_p is the proton mass.

Results for $|T|^2$ for the $^1S_0$ partial wave are presented in the upper part of Fig. 2. The solid lines are the result for the full amplitude while the dashed lines are based on the scattering length approximation given by Eq. (1). Note that the scattering lengths predicted by the $N\bar{N}$ model, which we use for the $^1S_0$ partial wave are $a_0 = (-0.18 - i1.18)$ fm and $a_1 = (1.13 - i0.61)$ fm for the isospin $I = 0$ and $I = 1$ channels, respectively. It is evident that the scattering length approximation does not reproduce the energy dependence of the scattering amplitude that well. For the $I = 1$ channel the difference at an excess energy of 50 MeV amounts to as much as 50%. The difference is even more pronounced for the $I = 0$ channel, where we already observe large deviations from the full result at rather low energies. This strong failure of the scattering length approximation is due to the much smaller scattering length predicted by our model for the $I = 0$ partial wave.

Results for the $^3S_1$ partial wave are shown in the lower part of Fig. 2. Here the scattering lengths predicted by the $N\bar{N}$ model are $a_0 = (1.16 - i0.82)$ fm and $a_1 = (0.75 - i0.84)$ fm for the $I = 0$ and $I = 1$ channels, respectively. As pointed out above, this partial wave cannot contribute to the reaction $J/\Psi \rightarrow \gamma p\bar{p}$. But it is still interesting to see how well the scattering length approximation works in this case. Obviously we also see similar shortcomings here. Thus, it is clear from Fig. 2 that the use of the scattering length approximation allows only a very rough qualitative estimate of the effects of FSI but it is definitely not reliable for a more quantitative analysis of the BES data.

Nevertheless, one has to realize that the main uncertainty in estimating $p\bar{p}$ FSI effects does not come from the scattering length approximation but from our poor knowledge of the $p\bar{p}$ $^1S_0$ amplitudes near-threshold and of the
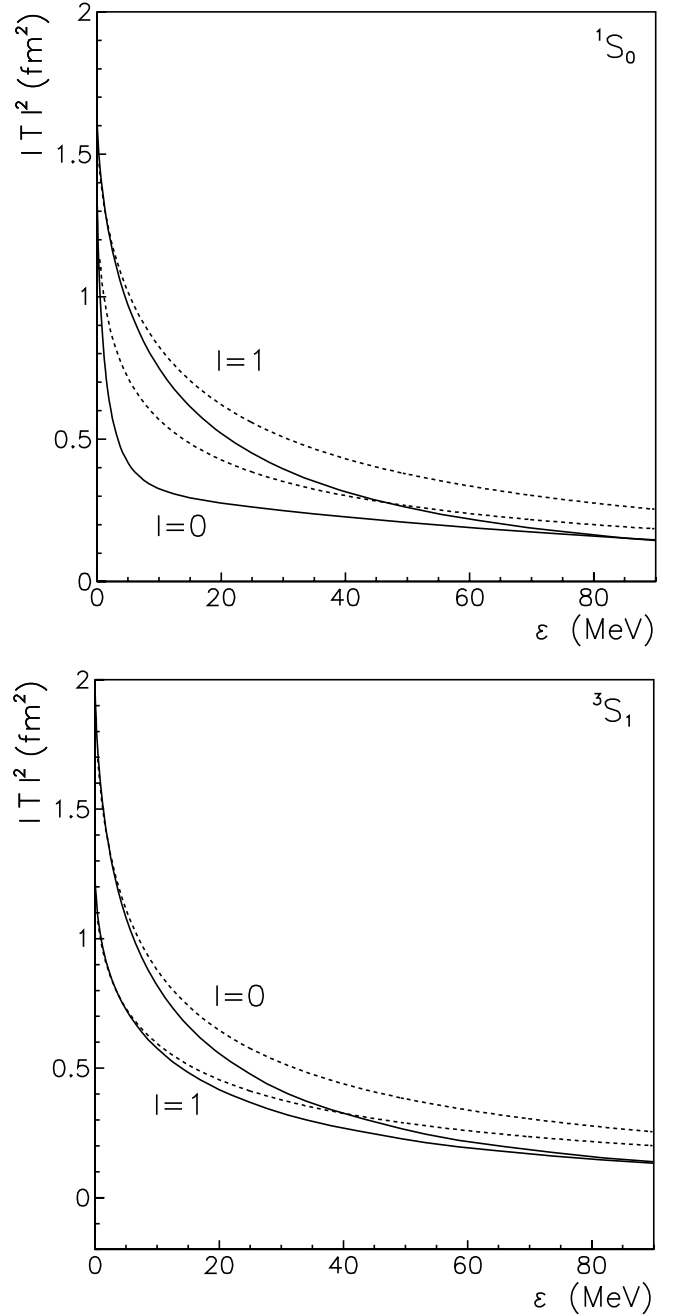

FIG. 2. The $p\bar{p}$ scattering amplitudes for the $^1S_0$ and $^3S_1$ partial waves as a function of the center-of-mass energy. The solid lines show the results of the Jülich model while the dashed lines indicate the scattering length approximation given by Eq. (1).

$J/\Psi \rightarrow \gamma p\bar{p}$ reaction mechanism. For example, the scattering lengths employed by Kerbikov et al. are spin-averaged values. Since the $^3S_1$ partial wave contributes with a weighting factor 3 to the $p\bar{p}$ cross sections (and there are no spin-dependent observables at low energies that would allow one to disentangle the spin-dependence) it is obvious that their value should correspond predominantly to the $^3S_1$ amplitude. Thus, it is questionable

whether it should be used for analyzing the BES data at all because the contribution of the $^3S_1$ partial wave to the decay $J/\Psi \rightarrow \gamma p\bar{p}$ is forbidden by charge-conjugation invariance. But even the availability of genuine $^1S_0$ amplitudes (which are necessarily model dependent) does not solve the problem. The reaction $J/\Psi \rightarrow \gamma p\bar{p}$ can have any isospin combination in the final $p\bar{p}$ state. A dominant reaction mechanism involving an intermediate isoscalar meson resonance (η, η_c, ...) cf. Figure 3(c), would yield a pure $I = 0$ final $p\bar{p}$ state, whereas an intermediate isovector meson resonance (π, $\pi(1800)$, ...) leads to pure $I = 1$. On the other hand, mechanisms like the ones depicted in Fig. 3(a) and 3(b) generate the usual equal weighting of the two isospin amplitudes.

In this context let us mention that the spin-averaged $p\bar{p}$ scattering length predicted by the model A(OBE), $a_S = 0.84 - i0.85$ fm, is in rough agreement with the value extracted from the level shifts of antiproton-proton atoms cf. Table 8 in Ref. [27]. On the other hand, this averaged value differs significantly from the scattering length in the $^{11}S_0$ partial wave—which is one of the possible final states in the decay $J/\Psi \rightarrow \gamma p\bar{p}$. At first glance one would tend to believe that the unusually small real part of a_0 is basically accidental and reflects the fact that the $^1S_0$ state enters with smaller statistical weight into

FIG. 3. Some possible reaction mechanisms for the decays $J/\Psi \rightarrow \gamma p\bar{p}$ and $J/\Psi \rightarrow \pi^0 p\bar{p}$. M indicates intermediate mesonic states and N ($\bar{N}$) intermediate nucleon (antinucleon) resonances.

the $p\bar{p}$ cross section and, therefore, is much less constrained by the $p\bar{p}$ data. However, exploratory calculations with the other $N\bar{N}$ models of the Jülich group revealed that the real part of the $^{11}S_0$ scattering length is always small (and often of a different sign from the other S waves). For example, the more involved $N\bar{N}$ model D published in Ref. [22], yields $a_0 = -0.25 - i1.01$ fm. Moreover, the $N\bar{N}$ models of Dover-Richard also and Kohno-Weise predict rather small values for the real part of the $^{11}S_0$ scattering length, as is evident from the results presented in Ref. [28]. The Paris $N\bar{N}$ potential, on the other hand, predicts the real parts of the $^1S_0$ scattering lengths to be of similar magnitude—cf. Table 4 in Ref. [29]. But Fig. 11 of that paper suggests that the central potential in the $I = 0$, $S = 0$ channel has been drastically modified inside 1 fm as compared to what follows from the G-parity transformation of the Paris NN potential. In the Jülich $N\bar{N}$ models no such changes are introduced when deriving the elastic part of the $N\bar{N}$ interaction via G-parity!

A closer examination of our $N\bar{N}$ models disclosed that here the smallness of $Re\, a_0$ is definitely caused by the one-pion exchange contribution, i.e., by long-range physics. Switching off this contribution always led to a large scattering length similar to the one in the $I = 1$ channel, whereas modifications of the short-range contributions to the elastic part of the $N\bar{N}$ potential had hardly any qualitative influence on the scattering lengths. Thus, it would be interesting to obtain experimental constraints on the $^1S_0$ scattering lengths. Corresponding measurements could be performed at the future facility FLAIR [30] at the FAIR project (GSI), where it is possible to have a polarized antiproton beam [31], as required for inferring the spin-triplet and spin-singlet amplitudes from the data.

III. J/Ψ DECAY RATE AND FSI EFFECTS

The $J/\Psi \rightarrow \gamma p\bar{p}$ decay rate is given as [32]

$$d\Gamma = \frac{|A|^2}{2^9 \pi^5 m_{J/\Psi}^2} \lambda^{1/2}(m_{J/\Psi}^2, M^2, m_\gamma^2)$$
$$\times \lambda^{1/2}(M^2, m_p^2, m_p^2) dM d\Omega_p d\Omega_\gamma, \quad (2)$$

where the function λ is defined by

$$\lambda(x, y, z) = \frac{(x - y - z)^2 - 4yz}{4x}, \quad (3)$$

M is the invariant-mass of the $p\bar{p}$ system, Ω_p is the proton angle in that system, while Ω_γ is the photon angle in the J/Ψ rest frame. After averaging over the spin states and integrating over the angles, the differential decay rate is

$$\frac{d\Gamma}{dM} = \frac{(m_{J/\Psi}^2 - M^2)\sqrt{M^2 - 4m_p^2}}{2^7 \pi^3 m_{J/\Psi}^3} |A|^2, \quad (4)$$

where A is the total $J/\Psi \rightarrow \gamma p\bar{p}$ reaction amplitude. Note that A is dimensionless. The differential rate $d\Gamma/dM$,
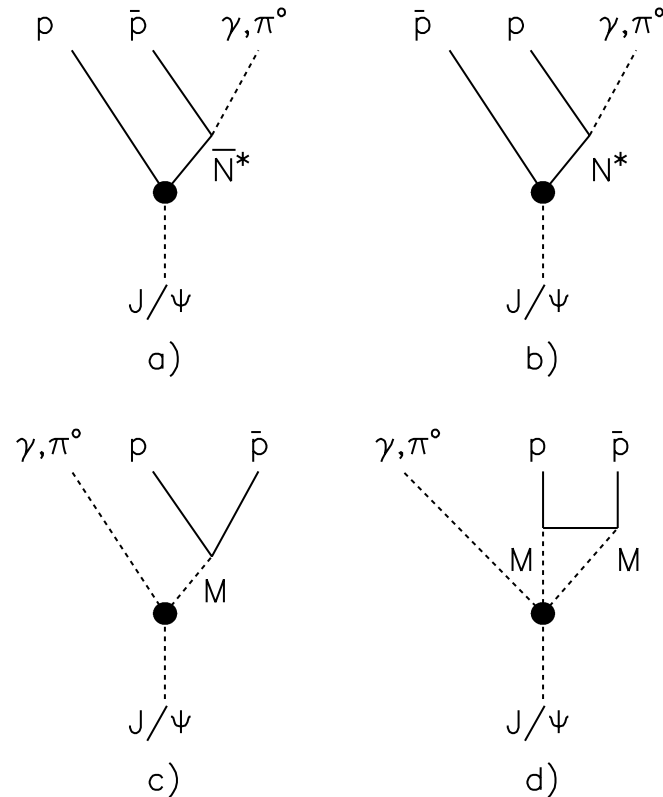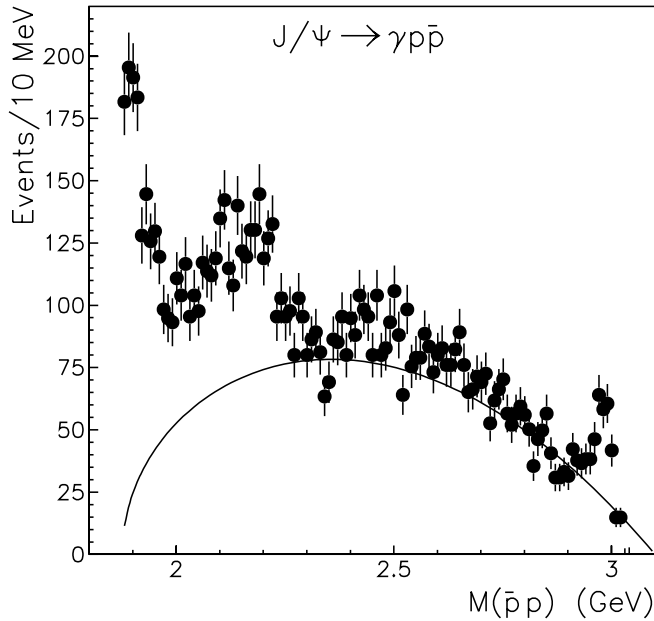
FIG. 4.   The $p\bar{p}$ mass spectrum from the decay $J/\Psi \to \gamma p\bar{p}$. The circles show experimental results of the BES Collaboration [1], while the solid line is the spectrum obtained from Eq. (4) by assuming a constant reaction amplitude A.

integrated over the range from $2m_p$ to $m_{J/\Psi}$, yields the partial $J/\Psi \to \gamma p\bar{p}$ decay width, $\Gamma = (3.5 \pm 0.9)10^{-2}$ keV [33].

The solid line in Fig. 4 is obtained with Eq. (4) by using a constant $|A|^2$. This result corresponds to the so-called phase-space distribution. The actual value of $|A|^2$ was adjusted to the data at $M \geq 2.8$ GeV, i.e., around the η_c resonance region. The circles in Fig. 4 show data for the $J/\Psi \to \gamma p\bar{p}$ decay published by the BES Collaboration [1]. Evidently, close to the $p\bar{p}$ threshold the data deviate substantially from the phase-space distribution.

Since close to the threshold the phase-space factor introduces a strong but trivial energy dependence of the mass distribution it is convenient to divide the experimental $p\bar{p}$ spectrum by the kinematical factors that appear on the right side of Eq. (4) and extract the invariant amplitude $|A|^2$ from the data. Corresponding results for the BES data [1] are shown by the squares in Fig. 5. It is obvious that the experimental reaction amplitude exhibits a very strong invariant-mass dependence near the $p\bar{p}$ threshold. In recent investigations [17–19] this feature has been attributed to a strong final state interaction (FSI) between the outgoing protons and antiprotons. Indeed, an enhancement and/or a strong energy dependence of near-threshold cross sections has been seen in many other reactions involving hadrons and is commonly seen as to be caused by FSI effects cf. [34–40]. Thus, it is plausible and it even has to be expected that FSI effects play a role in the decay $J/\Psi \to \gamma p\bar{p}$ as well. The interesting issue is primarily whether the proton-antiproton interaction is really strong enough to induce

such a sizeable enhancement in the invariant-mass spectrum.

A very simple and therefore also very popular treatment of FSI effects is due to Watson [41] and Migdal [42]. These authors suggested that the reaction amplitude for a production and/or decay reaction which is of short-ranged nature
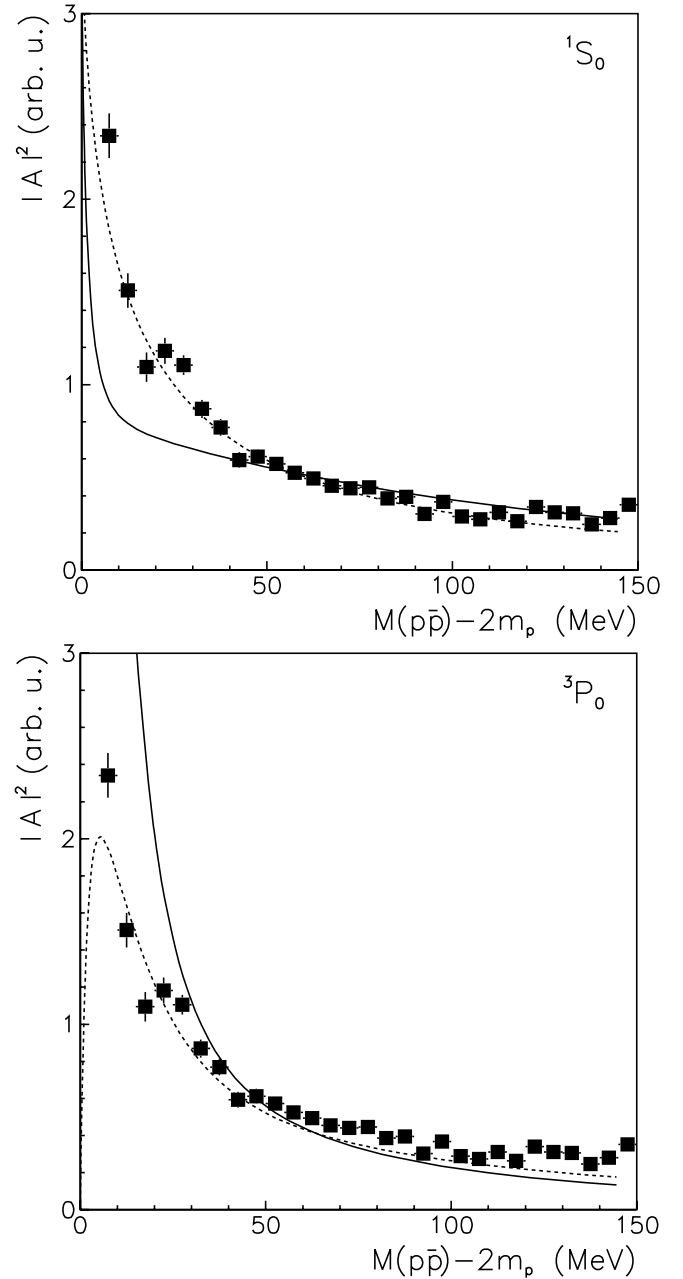

FIG. 5.   Invariant $J/\Psi \to \gamma p\bar{p}$ amplitude $|A|^2$ as a function of the $p\bar{p}$ mass. The squares represent the experimental values of $|A|^2$ extracted from the BES data [1] via Eq. (4). The curves are the scattering amplitude squared ($|T|^2$) predicted by the $N\bar{N}$ model A(OBE) for the $^1S_0$ and $^3P_0$ partial waves and the $I = 0$ (solid) and $I = 1$ (dashed) channels, respectively. Note that the latter results have been normalized to $|A|^2$ at $M(p\bar{p}) - 2mp = 50$ MeV.

can be factorized in terms of an elementary production amplitude A_0 and the $p\bar{p}$ scattering amplitude T of the particles in the final state,

$$A_{\text{prod}} \approx N A_0 \cdot T, \qquad (5)$$

where A_0 is given by diagrams like those shown in Fig. 3 and N is a normalization factor.

If the production mechanism is of short-range then the amplitude A_0 depends only very weakly on the energy and the near-threshold energy dependence of the reaction amplitude is driven primarily by the scattering amplitude, T, of the outgoing particles. This means that in the case of the reaction $J/\Psi \rightarrow \gamma p\bar{p}$ the near-threshold mass dependence of the $p\bar{p}$ spectrum should be dominated by the energy dependence of the $p\bar{p}$ scattering amplitude. In the analyses of the BES data by Kerbikov et al. [17] and by Bugg [18] the above treatment of FSI effects was adopted.

However, the prescription of Watson–Migdal is only valid for interactions that yield a rather large scattering length, like the $^1S_0$ NN partial wave where the scattering length a is in the order of 20 fm, and even then only for a relatively small energy range, as was pointed out in several recent papers [43–45]. Therefore, in the present case, where the scattering lengths are in the order of 1 fm one should be cautious with the interpretation of results obtained from applying Eq. (5). Rather one should start from the more general expression for the reaction amplitude,

$$A_{\text{prod}} = A_0 + A_0 G^{p\bar{p}} T, \qquad (6)$$

which corresponds to a distorted wave Born approximation. If one assumes that the production amplitude, A_0, has only a very weak energy and momentum dependence it can be factorized and one obtains

$$A_{\text{prod}} \approx A_0 [1 + G^{p\bar{p}} T] = A_0 \Psi_{q_p}^{(-)*}(0), \qquad (7)$$

where $\Psi_{q_p}^{(-)*}(\mathbf{r})$ is a suitably normalized $N\bar{N}$ continuum wave function and $\Psi_{q_p}^{(-)*}(0)$ is nothing else but the inverse of the Jost function J, i.e., $\Psi_{q_p}^{(-)*}(0) = J^{-1}(-q_p)$ [46]. Equation (6) itself can be cast into the form [44]

$$A_{\text{prod}} = A_0 [1 + (c - iq_p) T], \qquad (8)$$

where c is, in general, a complex number that represents the principal-value integral over the half-off-shell extension of the production amplitude A_0 and the $p\bar{p}$ scattering amplitude that appears on the very right hand side of Eq. (6). Obviously, Eq. (8) is formally equivalent to the Watson-Migdal prescription of Eq. (5) if $|c|$ is large. In case of the NN $^1S_0$ partial wave $|c|$ is indeed large, as has been demonstrated in Ref. [45].

In considering the S-wave interaction in the final $p\bar{p}$ system one should account for the centrifugal barrier between the photon and the $p\bar{p}$ state given by the factor

$$C_l = \left[\frac{m_{J/\Psi}^2 - M^2}{2m_{J/\Psi}^2} \right]^{l_\gamma}, \qquad (9)$$

with $l_\gamma = 1$ being the orbital momentum between the photon and the $p\bar{p}$ system. Around the threshold, $M = 2m_p$, the centrifugal correction depends only weakly on the invariant-mass of the $p\bar{p}$ system and basically does not modify the mass dependence of the FSI. Within the range $2m_p \leq M \leq 2.1$ GeV the factor C_1^2 varies between 0.23 and 0.27, which is not large enough to counterbalance the contribution from the strong S-wave FSI over the same invariant-mass range.

Note also that the Watson-Migdal formula (5) has to be modified when used for $p\bar{p}$ FSI effects in a P-wave. Since in the production reaction only the final momentum is on-shell one has to divide the on-shell T matrix by the factor q_p in order to impose the correct threshold behavior of the production amplitude.

IV. RESULTS

Let us first discuss calculations based on the Watson-Migdal approach given by Eq. (5). The solid lines in Fig. 5 show the $p\bar{p}$ invariant scattering amplitudes squared for the $^1S_0$ and $^3P_0$ partial waves and the $I = 0$ channel, the dashed lines correspond to those for the $I = 1$ channel. We consider the isospin channels separately because, as mentioned above, the actual isospin mixture in the final $p\bar{p}$ system depends on the reaction mechanism and is not known. Note that all squared $p\bar{p}$ scattering amplitudes $|T|^2$ were normalized to the BES data at the invariant-mass $M(p\bar{p}) - 2m_p = 50$ MeV by multiplying them with a suitable constant. The results indicate that the mass dependence of the BES data can indeed be described with FSI effects induced by the $^1S_0$ scattering amplitude in the $I = 1$ isospin channel. The $I = 0$ channel leads to a stronger energy dependence which is not in agreement with the BES data. We can also exclude dominant FSI effects from the $^3P_0$ partial waves. Here the different threshold behavior due to the P-wave nature cannot be brought in line with the data points very close to threshold. It should be clear, of course, that a suitable combination of several partial waves might as well reproduce the experimental results on the $p\bar{p}$ invariant-mass spectrum.

Note that we do not include the Coulomb interaction in our model calculation and we also ignore the difference in the $p\bar{p}$ and $n\bar{n}$ thresholds. Judging from the results shown by Kerbikov et al. [17] their influence is noticeable only for excess energies below say 5 MeV. Accordingly we do not consider the lowest data point of the BES experiment, which lies in the first (5 MeV) energy bin, in our discussion.

Our results support the conjecture of Kerbikov [17] and Bugg [18] that the enhancement seen in the near-threshold $p\bar{p}$ invariant-mass spectrum of the decay $J/\Psi \rightarrow \gamma p\bar{p}$

could be primarily due to FSI effects in the $p\bar{p}$ channel. But we should also consider the $p\bar{p}$ invariant-mass spectrum of the decay $J/\Psi \rightarrow \pi^0 p\bar{p}$ which was presented by the BES collaboration in the same paper [1]. Here the near-threshold mass distribution does not show any enhancement as compared to the phase-space distribution—though one would likewise expect strong FSI effects in the $p\bar{p}$ channel. In this reaction isospin is preserved and the possible partial waves in the final $p\bar{p}$ state, $^{33}S_1$ and $^{31}P_1$, differ from those available in the decay $J/\Psi \rightarrow \gamma p\bar{p}$. But the $^{33}S_1$ partial wave amplitude of the Jülich $N\bar{N}$ model, shown in the lower part of Fig. 2, leads to FSI effects that are comparable to those of the $^{31}S_0$ state—and in contradiction with the experimental $\pi^0 p\bar{p}$ mass distribution - as illustrated in Fig. 6. We should mention in this context that the issue of the $\pi^0 p\bar{p}$ mass distribution is not discussed in the work of Bugg, while Kerbikov argues that in the effective-range parameterization that he employs $|Re\, a_1|$ is much smaller than $|Re\, a_0|$ and therefore there should be much smaller FSI effects in the $I = 1$ channel. However, our experience is that $p\bar{p}$ amplitudes with a small $|Re\, a|$ often yield an even stronger energy dependence and therefore larger FSI effects—see the case of the $^{11}S_0$ partial wave in Fig. 2.

In our opinion these seemingly contradictory results can only be reconciled if we assume that the treatment of FSI
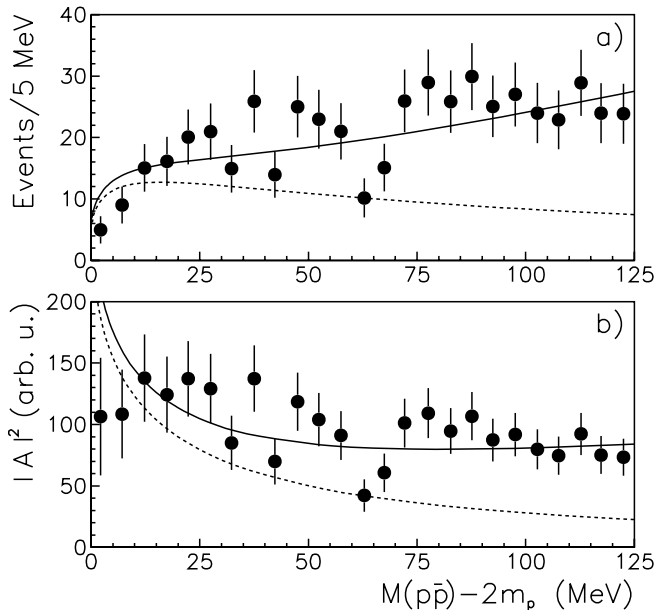

FIG. 6. (a) The $p\bar{p}$ mass spectrum from the decay $J/\Psi \rightarrow \pi^0 p\bar{p}$. The circles show experimental results of the BES Collaboration [1]. (b) Invariant $J/\Psi \rightarrow \pi^0 p\bar{p}$ amplitude $|A|^2$ as a function of the $p\bar{p}$ mass. The circles represent the experimental values of $|A|^2$ extracted from the BES data via Eq. (4). The curves are corresponding calculations using the T matrix predicted by the $N\bar{N}$ model A(OBE) for the $^{33}S_1$ partial wave. The dashed line is the results from the Watson-Migdal prescription Eq. (5), while the solid line is based on Eq. (8) with $c = -0.1$. Both curves are normalized arbitrarily.

effects by means of Eq. (5) is oversimplified and one should use Eq. (8) instead. Results based on the latter equation are shown by the solid lines in Fig. 6, where we set $c = -0.1$. This value was chosen after a short exploratory fit. It could be possible that a fine tuning of c could allow one to better reproduce the BES data for the $\pi^0 p\bar{p}$ channels, though in view of the rather large statistical variations in the data we refrain from doing so. In any case it is obvious that the more refined treatment of the FSI effects based on Eq. (8) leads, in general, to a weaker energy dependence of the $p\bar{p}$ mass spectrum.

V. DISCUSSION

It is important to note that the near-threshold contribution to the $J/\Psi \rightarrow \gamma p\bar{p}$ decay rate is relatively large. The contribution to $J/\Psi \rightarrow \gamma p\bar{p}$ up to invariant $p\bar{p}$ masses of $M(p\bar{p}) \approx 2$ GeV is roughly five times larger than the rate for the reaction $J/\Psi \rightarrow \gamma \eta_c$ followed by the $\eta_c \rightarrow p\bar{p}$ decay. Taking into account that the latter rate was recently published [47] as BR $= 1.9 \cdot 10^{-5}$ we can estimate that

$$BR(J/\Psi \rightarrow \gamma\,^1\tilde{S}_0) \times BR(^1\tilde{S}_0 \rightarrow p\bar{p}) \simeq 9.5 \cdot 10^{-5}, \quad (10)$$

where $^1\tilde{S}_0$ indicates the near-threshold contribution of this partial wave—which is indeed a large fraction of the total $J/\Psi \rightarrow \gamma p\bar{p}$ branching rate that amounts to $(3.8 \pm 1.0) \cdot 10^{-4}$.

This suggests that in this energy region the reaction $J/\Psi \rightarrow \gamma p\bar{p}$ could be indeed dominated by a resonance. Such a resonance should lie below the $p\bar{p}$ threshold but, unlike the resonance state which emerged from the Breit-Wigner fit of the BES collaboration [1], its mass could be significantly below the $p\bar{p}$ threshold and it could have a large width, like the $\pi(1800)$ or $\eta(1760)$ resonances that are listed in the Review of Particle Physics Physics [48]. We demonstrate the scenario in Fig. 7. For illustration we consider the $\pi(1800)$ meson with mass and width 1801 and 210 MeV, respectively. The corresponding resonance amplitude squared is shown by the dashed line in Fig. 7. Its energy dependence is, of course, too weak and does not agree with the near-threshold $p\bar{p}$ mass spectrum found by the BES collaboration. However, when the $\pi(1800)$ resonance decays into a proton and antiproton, a final state interaction should occur. The solid line in Fig. 7 shows the total reaction amplitude squared, taken now as the product of the resonance amplitude and the $p\bar{p}$ scattering amplitude in the $^{31}S_0$ partial wave as is given by Eq. (5). The corresponding result is roughly in line with the BES data.

Though this line of reasoning is certainly more a plausibility argument, rather than a solid calculation, we believe that it makes clear that one has to be rather cautious when trying to extract resonance properties by fitting the near-threshold $p\bar{p}$ mass spectrum with a resonance amplitude, because any final state interaction will necessarily and substantially distort the production amplitude.
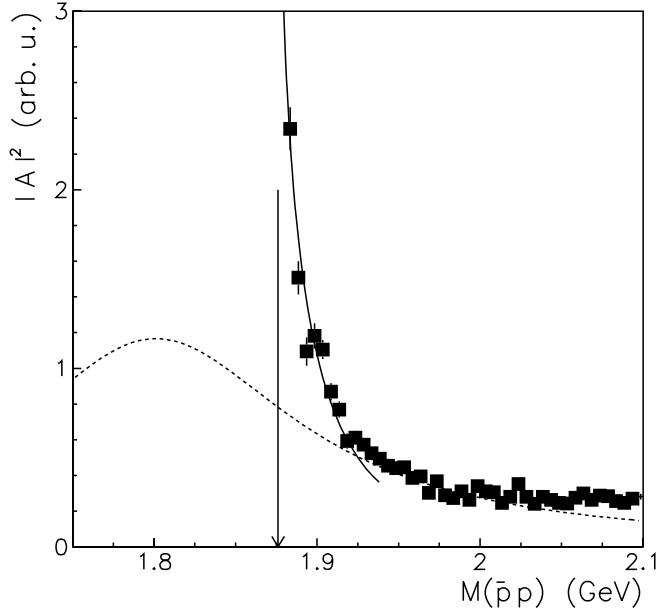
FIG. 7.   Invariant $J/\Psi \to \gamma p\bar{p}$ amplitude $|A|^2$ as a function of the $p\bar{p}$ mass. The squares represent the experimental values of $|A|^2$ extracted from the BES data[1] via Eq. (4). The dashed line shows the $\pi(1800)$ resonance amplitude squared, while the solid line indicates the square of the product of resonance and $p\bar{p}$ $^1S_0$ scattering amplitude given by Eq. (5). The arrow shows the $p\bar{p}$ threshold.

A different and alternative scenario consists in assuming the formation of many multimesonic intermediate states in the J/Ψ decay that couple strongly to the $p\bar{p}$ system where again a strong FSI occurs. The branching rates of the J/Ψ radiative decays into the $\pi^+\pi^-2\pi^0$, $\rho\rho$, $2\pi^+2\pi^-$ and $\omega\omega$ channels are larger than 10^{-3} and the transition of those mesonic states into the $p\bar{p}$ final state could be sufficiently strong [20,22]. Thus, such a scenario cannot be excluded by the available data.

Finally, let us come back to the other proposed explanation for the enhancement in the $p\bar{p}$ mass spectrum, namely, near-threshold $N\bar{N}$ bound states. Clearly a description of the experimental mass spectrum in terms of $p\bar{p}$ FSI effects does not contradict the existence of such states. Indeed, in case of the NN interaction the strong FSI effects are interconnected with the existence of a near-threshold bound-state (deuteron) or anti bound-state in the corresponding $^3S_1$ and $^1S_0$ partial waves. However, the $N\bar{N}$ model that we used in the present study does not lead to any near-threshold bound states. In fact, we found only two bound states for the model A(OBE) within 100 MeV from the threshold, namely, at $E = -04 - i413$ MeV in the $^{11}S_0$ partial wave and at $E = -24.2 - i107$ MeV in the $^{13}P_0$ partial wave. Obviously both bound states lie rather far away from the real axis and therefore should have practically no influence on the $p\bar{p}$ scattering amplitude near-threshold.

VI. CONCLUSION

We have investigated suggested explanations for the near-threshold enhancement in the $p\bar{p}$ invariant-mass spectrum of the $J/\Psi \to \gamma p\bar{p}$ decay, reported recently by the BES Collaboration. In particular, we showed that the near-threshold enhancement in the $p\bar{p}$ mass spectrum can, in principle, be understood in terms of a final state interaction in the outgoing proton-antiproton system. Within our model calculation it can be described with the scattering amplitude in the $^{31}S_0$ $p\bar{p}$ partial wave but disagrees with the energy dependence that follows from the $^{11}S_0$ and both $(I = 0, 1)$ $^3P_0$ amplitudes.

We showed that the scattering length approximation cannot be used for a more quantitative evaluation of the energy dependence of BES data. In general, it does not reproduce the energy dependence of the scattering amplitude reliably enough within the $p\bar{p}$ mass range required. At the same time, one has to concede that the lack of knowledge of the reaction mechanism for $J/\Psi \to \gamma p\bar{p}$ and the insufficient information on the $p\bar{p}$ interaction near-threshold precludes any more quantitative conclusions, even when microscopic models are used—as in the case of the present study.

We argued also that the use of the simple Watson-Migdal prescription for treating FSI effects has to be considered with caution for the present case in view of the small $p\bar{p}$ scattering lengths, whose real parts are typically in the order of only 1 fm. Our suspicion is nourished, in particular, by the observation that the experimental $p\bar{p}$ mass spectrum of the comparable decay $J/\Psi \to \pi^0 p\bar{p}$ does not show any obvious sign of a $p\bar{p}$ FSI, while applicaton of the Watson-Migdal prescription would yield a similar, strong, near-threshold enhancement as for the $\gamma p\bar{p}$ channel for any of the $N\bar{N}$ models we utilized. We demonstrated that a consistent qualitative description of the $p\bar{p}$ invariant-mass spectra from the decay reactions $J/\Psi \to \gamma p\bar{p}$ and $J/\Psi \to \pi^0 p\bar{p}$ can be achieved, however, within a more refined treatment of FSI effects as it follows from a DWBA approach.

Though our study shows that the enhancement seen in the decay $J/\Psi \to \gamma p\bar{p}$ could indeed be a result of the FSI in the $p\bar{p}$ system one has to admit that due to the uncertainties mentioned above and the controversial situation in the $\pi^0 p\bar{p}$ channel explanations other than final state effects cannot be ruled out at the present stage. Since the available data on the reaction $J/\Psi \to \pi^0 p\bar{p}$ are afflicted by large error bars it would be desirable to obtain improved experimental information here that allows one to quantify the extent of $p\bar{p}$ FSI effects in this reaction. As already mentioned in the introduction, other reactions involving the $p\bar{p}$ system in the final state, such as $B^+ \to K^+ p\bar{p}$ decay [2] or $\bar{B}^0 \to D^0 p\bar{p}$ [3] do show some indications for $p\bar{p}$ FSI effects. But here too the quality of the available data is too poor to permit any reliable conclusions.

In this context let us emphasize that $p\bar{p}$ mass spectra for the reactions $J/\Psi \to \omega p\bar{p}$ or $J/\Psi \to \eta p\bar{p}$ would be rather interesting for clarifying the role of $p\bar{p}$ FSI effects in the decay of the J/Ψ meson. Both reactions restrict the isospin in the $p\bar{p}$ system to be zero so that one could explore the FSI for specific $p\bar{p}$ partial waves, namely $^{11}S_0$ and $^{13}S_1$.

During the preparation of this paper two eprints [49] appeared dealing with the same topic, having somewhat different conclusions.

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