Aspects of stochastic control and switching: from Parrondo's games to electrical circuits

by

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Contents

Abstract

The first half of this thesis deals with the line of thought that leads to the development of discrete games of chance as models in statistical physics, with an emphasis on analysis of Parrondo's games.

The second half of the thesis is concerned with applying discrete games of chance to the modelling of other phenomena in the discipline of electrical engineering. The important features being the element of *switching* that is implicit in discrete games of chance and the element of *uncertainty*, introduced by the random aspect of discrete games of chance.

Statement of Originality

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published written by another person, except where due reference has been made in the text.

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Whenever one writes about a scientific subject, one stands on the shoulders of giants. Some of these people are famous, and appear in the literature and are cited. Some are less famous, for their contributions to the work, and deserve special recognition here.

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"Writing a book is a horrible, exhausting struggle, like a long bout of some painful illness. One would never undertake such a thing if one were not driven on by some demon whom one can neither resist nor understand." George Orwell

Conventions

This thesis is typeset using LaTeX2e software, including the core packages tetex-base, tetex-bin and tetex-extra. All LaTeXsoftware was obtained from the Debian Archive at: `http://www.debian.org`.

Numerical calculations were carried out in Matlab, and in the equivalent open-source packages GNU Octave and Gnuplot, which were also obtained from the Debian archive.

Many of the Figures were generated in Matlab and in GNU Octave. The other figures were drawn, or post-processed, in a number of other drawing packages, including Corel-Draw 9 under Windows 2000, Adobe Creative Suite 3 under Mac OS-X, or in xfig and Inkscape, under LINUX. The last two packages were downloaded from the Debian archive. All drawings have been converted or exported to encapsulated post-script (eps) format.

The complete editing environment, Emacs21 (Editing with MACroS, version 21.4.1) was used as an effective interface to LaTeX. The idiomatic conventions, for LaTeX, conform to standard described in (Lamport 1994).

Harvard style is used for referencing and citation in this thesis. British spelling is adopted, consistent with the Ispell package, using the British dictionary, in Emacs21. Additional words have been traced back to their original sources. Where we have needed to quote works in other languages, including works in US English, we have used the original spelling.

Publications

Book Chapters

ALLISON-A., ABBOTT-D. & AND PEARCE-C. E. M. (2005) State-space visualisation and fractal properties of Parrondo's games, *in* A. S. Nowak., and K. Szajowski. (eds.), *Proceedings of the Ninth International Symposium on Dynamic Games and Applications 2000, Advances in Dynamic Games: Applications to Economics, Finance, Optimization, and Stochastic Control.*, Vol. 7, The International Society of Dynamic Games (ISDG), Birkhauser, pp. 613–633.

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