

# **A Stochastic Space-Time Rainfall Model for Engineering Risk Assessment**

by Michael Leonard

Submitted in fulfilment of the requirements for the degree of  
**DOCTOR OF PHILOSOPHY**

April 14, 2010

FACULTY OF ENGINEERING, COMPUTER AND MATHEMATICAL SCIENCES

---

School of Civil Environmental and Mining Engineering



# A Stochastic Space-Time Rainfall Model for Engineering Risk Assessment

By: Michael Leonard, *B.E. Civil (Hons)*

April 14, 2010

Thesis submitted in fulfilment of the requirements for the degree of  
**Doctor of Philosophy**

School of Civil Environmental and Mining Engineering  
Faculty of Engineering, Computer and Mathematical Sciences  
The University of Adelaide SA 5005 Australia

Telephone: +61 8 8303 5451

Facsimile: +61 8 8303 4359

Web: [www.ecms.adelaide.edu.au/civeng](http://www.ecms.adelaide.edu.au/civeng)

Email: [mleonard@civeng.adelaide.edu.au](mailto:mleonard@civeng.adelaide.edu.au)

# Table of Contents

---

<b>Table of Contents</b>	<b>iii</b>
<b>List of Figures</b>	<b>vii</b>
<b>List of Tables</b>	<b>xi</b>
<b>Abstract</b>	<b>xiii</b>
<b>Statement of Originality</b>	<b>xv</b>
<b>Acknowledgements</b>	<b>xvii</b>
<b>List of Symbols</b>	<b>xix</b>
<b>1 Introduction</b>	<b>1</b>
1.1 The Rational Method . . . . .	2
1.2 Design Events . . . . .	2
1.2.1 AEP Neutrality . . . . .	3
1.2.2 Intensity Frequency Duration curves . . . . .	4
1.2.3 Temporal patterns . . . . .	4
1.3 Monte Carlo Simulation . . . . .	6
1.4 Continuous Simulation . . . . .	7
1.5 The Australian Climate . . . . .	8
1.6 Rainfall Measurement . . . . .	9
1.6.1 Daily rainfall gauges . . . . .	11
1.6.2 Pluviograph rainfall gauges . . . . .	13
1.6.3 Radar rainfall reflectivity . . . . .	14
1.6.4 Summary of rainfall data . . . . .	15
1.7 Thesis Objectives . . . . .	16
1.8 Thesis Structure . . . . .	18

1.9	Overview of Australian Case studies . . . . .	19
<b>2</b>	<b>Literature Review</b>	<b>23</b>
2.1	Rainfall Homogeneity . . . . .	24
2.1.1	Spatial homogeneity . . . . .	24
2.1.2	Temporal homogeneity . . . . .	25
2.2	Single-site Rainfall Models . . . . .	26
2.2.1	Event-based models . . . . .	27
2.2.2	Poisson-cluster models . . . . .	28
2.2.3	Disaggregation models . . . . .	32
2.2.4	Regionalised single-site models . . . . .	33
2.3	Multi-site Models . . . . .	34
2.4	Space-time Models . . . . .	35
2.4.1	Radar-based models . . . . .	35
2.4.2	Gauge-based models . . . . .	38
2.5	Summary of Literature . . . . .	41
<b>3</b>	<b>Model Formulation and Calibration</b>	<b>43</b>
3.1	Model Formulation . . . . .	43
3.2	Calibration Statistics . . . . .	48
3.3	Calibration Summary . . . . .	54
<b>4</b>	<b>Simulation Involving a Regional Boundary</b>	<b>57</b>
4.1	Introduction . . . . .	57
4.2	Algorithm for Simulation of Cells Using a Buffer Region . . . . .	58
4.3	Algorithm for Direct Simulation of Cells . . . . .	61
4.3.1	Number of Outer Cells Intersecting Target . . . . .	61
4.3.2	Cell Centre Conditioned on Intersecting Target . . . . .	63
4.3.3	Cell Radius Conditioned on Location . . . . .	63
4.4	Results and Discussion . . . . .	64
4.5	Conclusion . . . . .	66
<b>5</b>	<b>Calibration Involving a Monthly Boundary</b>	<b>67</b>
5.1	Introduction . . . . .	67
5.2	Derivation of Monthly Bias . . . . .	68
5.3	Case-study . . . . .	71
5.4	Conclusion . . . . .	73

---

<b>6</b>	<b>Calibration Using Simulated Moments</b>	<b>75</b>
6.1	Introduction . . . . .	75
6.2	Calibration methodology . . . . .	76
6.3	Case-study . . . . .	78
6.4	Conclusion . . . . .	84
<b>7</b>	<b>Spatial Storm Extent</b>	<b>87</b>
7.1	Introduction . . . . .	87
7.2	Probability of storm occurrence at a point . . . . .	87
7.3	Model having storm extent . . . . .	89
7.4	Model calibration . . . . .	92
7.5	Model simulation . . . . .	93
7.6	Case-study application . . . . .	94
7.6.1	Modelled region and calibrated parameters . . . . .	94
7.6.2	Interpolation of Scale Parameters . . . . .	96
7.6.3	Model Comparison to Calibration Statistics . . . . .	100
7.6.4	Model Comparison to Other Statistics . . . . .	101
7.7	Visualisation of Spatial Storm Process . . . . .	103
7.8	Conclusion . . . . .	107
<b>8</b>	<b>Climatic and Seasonally Partitioned Extreme Rainfall</b>	<b>109</b>
8.1	Introduction . . . . .	110
8.2	Methodology . . . . .	114
8.3	Urban Design Application - Scott Creek . . . . .	117
8.4	Climate Conditioned SNSRP Methodology . . . . .	123
8.5	Bourke Case Study . . . . .	126
8.6	Observed Partitioned Extremes . . . . .	127
8.7	Comparison of Observed and Simulated Extremes . . . . .	134
8.8	Conclusions . . . . .	135
<b>9</b>	<b>Spatially Inhomogeneous Neyman-Scott Model</b>	<b>137</b>
9.1	Introduction . . . . .	137
9.2	Model Development . . . . .	139
9.2.1	Spatial process . . . . .	139
9.3	Integral Approximation . . . . .	139
9.3.1	Temporal process . . . . .	146

9.3.2	Matrix Inversion to Find Spatial Rate Surface . . . . .	149
9.4	Simulation Technique . . . . .	154
9.5	Case Study . . . . .	157
9.5.1	Observed Data . . . . .	157
9.5.2	Calibration to Regional Statistics . . . . .	163
9.5.3	Estimation of number of cells . . . . .	164
9.6	Results . . . . .	167
9.7	Conclusions . . . . .	171
<b>10</b>	<b>Summary and Recommendations</b>	<b>177</b>
10.1	Model Review . . . . .	177
10.2	Recommendations . . . . .	181
	<b>References</b>	<b>185</b>
	<b>Appendices</b>	<b>197</b>
	<b>Appendix A Spatial Storm Extent</b>	<b>199</b>
A.1	List of rainfall gauges . . . . .	199
A.2	Simulated Annual Totals . . . . .	202
A.3	Simulated Extreme Values . . . . .	212
	<b>Appendix B Bourke Case Study</b>	<b>227</b>
B.1	Observed SOI Partitioned Annual Extremes . . . . .	227
B.2	Observed SOI Partitioned Summer Extremes . . . . .	231
B.3	Observed SOI Partitioned Winter Extremes . . . . .	235
B.4	Observed Seasonally Partitioned SOI+ Extremes . . . . .	239
B.5	Observed Seasonally Partitioned SOI- Extremes . . . . .	243
B.6	Comparison of Observed and Simulated Extremes . . . . .	247

# List of Figures

---

1.1	24 hour design storm temporal patterns for 8 different zones of Australia.	5
1.2	Mean rainfall in millimetres depth across Australia. . . . .	10
1.3	Climatic Zones of Australia. . . . .	11
1.4	Distribution of rainfall measuring devices about Australia. . . . .	12
1.5	Number of commissioned gauges/radars in time. . . . .	16
2.1	Schematic diagram of an alternating renewal model of storm events. . . .	27
2.2	Schematic diagrams for various point-process formulations. . . . .	30
3.1	Schematic diagram of the Neyman-Scott Rectangular Pulse model. . . . .	44
3.2	Schematic diagram of spatial cell process. . . . .	46
4.1	Schematic diagram of cells generated inside and outside the target region.	58
4.2	Proportion reduction in simulated rainfall depth. . . . .	60
4.3	Efficiency of direct-algorithm, proportion of memory usage and run-time.	65
5.1	Schematic of rainfall landing outside the month boundary. . . . .	69
5.2	Bias in mean due to monthly boundary. . . . .	72
5.3	Change in parameters accounting for monthly bias. . . . .	74
6.1	Spatial coordinates of observation gauges about Launceston, Tasmania. .	78
6.2	Observed and fitted temporal statistics, Launceston case-study. . . . .	80
6.3	Observed and fitted spatial statistics, Launceston case-study. . . . .	81
6.4	Observed and simulated annual maxima, Launceston. . . . .	83
6.5	Observed and simulated annual totals, Launceston. . . . .	84
7.1	Schematic diagram showing the cause of spurious cross-correlations. . . .	88
7.2	Example of variation in cross correlation at the 24 hour aggregate. . . . .	91
7.3	Sydney 24 hr spurious cross-correlations. . . . .	93
7.4	Sydney case study, gauge locations. . . . .	95
7.5	Variation in mean rainfall across Sydney. . . . .	98

7.6	Simulated values for statistics used in calibration. . . . .	99
7.7	Comparison of model to observed statistics, Sydney. . . . .	102
7.8	Model visualisation without storm-envelope. . . . .	104
7.9	Model visualisation including storm-envelope. . . . .	105
8.1	Comparison of distributions of summer and winter seasonal maxima about Australia. . . . .	113
8.2	Average monthly rainfall, Adelaide, South Australia. . . . .	118
8.3	Maximum water depths in Scott Creek Basin. . . . .	119
8.4	Maximum rainfall intensities in neighbouring catchment to Scott Creek. .	120
8.5	Annual maximum water depths in Scott Creek Basin, composite distribu- tion from seasonal maxima. . . . .	121
8.6	Maximum water depths in Scott Creek Basin, adjusted composite distri- bution. . . . .	122
8.7	Southern Oscillation Index. . . . .	124
8.8	ACF and PACF of the SOI . . . . .	124
8.9	Pattern of two indicator states based on partition of SOI. . . . .	125
8.10	Locations of gauges used for Bourke case-study. . . . .	127
8.11	Fitted correlation statistics for Bourke. . . . .	129
8.12	Fitted coefficient of variation for Bourke. . . . .	130
8.13	Fitted skewness for Bourke. . . . .	131
8.14	SOI+/SOI- partitioned, observed annual extremes, site 4, Bourke. . . . .	133
8.15	SOI+/SOI- partitioned, observed extremes, site 4, Bourke. . . . .	133
8.16	Seasonally partitioned, observed SOI+ extremes, site 4, Bourke. . . . .	134
8.17	Comparison of observed and simulated extremes, site 4, Bourke. . . . .	135
9.1	Avon Basin from <i>Jothityangkoon et al.</i> [2000]. . . . .	138
9.2	Schematic of rainfall process. . . . .	140
9.3	Schematic of two pixels from gridded rainfall data. . . . .	140
9.4	Schematic of distance between two points $(u,v)$ and $(x,y)$ . . . . .	141
9.5	Integration regions used for $h^-$ case with $i = k; j = l$ . . . . .	142
9.6	Integration regions used for $h^-$ case with $ i - k  =  j - l  \geq 0$ . . . . .	143
9.7	Effectiveness of hypotenuse approximations for various distances. . . . .	146
9.8	1-D line example of spatial cell process. . . . .	150
9.9	1-D circular line example of spatial cell process. . . . .	151
9.10	Inhomogeneous model simulation schematic . . . . .	155



9.11	Inhomogeneous model accept-reject schematic . . . . .	156
9.12	Location of gauges for the Avon basin case-study. . . . .	157
9.13	Variation in mean rainfall over the Avon Basin. . . . .	159
9.14	Variation in the coefficient of variability over the Avon Basin. . . . .	160
9.15	Variation in the probability of a dry day over the Avon Basin. . . . .	161
9.16	Variation in the daily auto-correlation over the Avon Basin. . . . .	162
9.17	Fitted regional coefficient of variation, Avon Basin. . . . .	163
9.18	Fitted regional autocorrelation, Avon Basin. . . . .	163
9.19	Fitted regional skewness, Avon Basin. . . . .	164
9.20	Fitted variation in the number of cells over the Avon Basin. . . . .	166
9.21	Spline fitted number of cells over the Avon Basin. . . . .	168
9.22	Residuals of the number of cells from spline fitting. . . . .	169
9.23	Comparison of simulated and observed daily dry probability. . . . .	170
9.24	Comparison of simulated and observed daily coefficient of variation. . . .	172
9.25	Simulated spatial rate of cells over the Avon Basin. . . . .	173
9.26	Uncertainty in cross-correlation estimates. . . . .	174
9.27	Simulated and observed cross-correl. comparison, after spline interpolation.	175
B.1	SOI+/SOI- partitioned, observed annual extremes, sites 1 - 6, Bourke. . .	228
B.2	SOI+/SOI- partitioned, observed annual extremes, sites 7 - 12, Bourke. . .	229
B.3	SOI+/SOI- partitioned, observed annual extremes, sites 13 - 16, Bourke. .	230
B.4	SOI+/SOI- partitioned, observed summer extremes, sites 1 - 6, Bourke. . .	232
B.5	SOI+/SOI- partitioned, observed summer extremes, sites 7 - 12, Bourke. .	233
B.6	SOI+/SOI- partitioned, observed summer extremes, sites 13 - 16, Bourke.	234
B.7	SOI+/SOI- partitioned, observed winter extremes, sites 1 - 6, Bourke. . .	236
B.8	SOI+/SOI- partitioned, observed winter extremes, sites 7 - 12, Bourke. . .	237
B.9	SOI+/SOI- partitioned, observed winter extremes, sites 13 - 16, Bourke. .	238
B.10	Seasonally partitioned, observed SOI+ extremes, sites 1 - 6, Bourke. . . .	240
B.11	Seasonally partitioned, observed SOI+ extremes, sites 7 - 12, Bourke. . .	241
B.12	Seasonally partitioned, observed SOI+ extremes, sites 13 - 16, Bourke. . .	242
B.13	Seasonally partitioned, observed SOI- extremes, sites 1 - 6, Bourke. . . .	244
B.14	Seasonally partitioned, observed SOI- extremes, sites 7 - 12, Bourke. . . .	245
B.15	Seasonally partitioned, observed SOI- extremes, sites 13 - 16, Bourke. . .	246
B.16	Comparison of observed and simulated extremes, sites 1 - 3, Bourke. . . .	248
B.17	Comparison of observed and simulated extremes, sites 4 - 6, Bourke. . . .	249
B.18	Comparison of observed and simulated extremes, sites 7 - 9, Bourke. . . .	250

B.19 Comparison of observed and simulated extremes, sites 10 - 12, Bourke. . . . .	251
B.20 Comparison of observed and simulated extremes, sites 13 - 15, Bourke. . . . .	252
B.21 Comparison of observed and simulated extremes, site 16, Bourke. . . . .	253

# List of Tables

---

1.1	Summary statistics for varied case studies. . . . .	20
5.1	Bias in mean due to monthly boundary. . . . .	71
5.2	Bias in mean due to monthly boundary. . . . .	73
6.1	Launceston parameter estimates for each month, via analytic expressions. . . . .	82
6.2	Launceston parameter estimates for each month, via MCS. . . . .	82
7.1	Algorithm for efficiently simulating storm and cell properties. . . . .	94
7.2	Parameter estimates for each month . . . . .	97
8.1	List of gauges used for Bourke case-study. . . . .	128
8.2	Bourke regional parameter estimates, entire record. . . . .	128
8.3	Bourke regional parameter estimates, SOI-. . . . .	132
8.4	Bourke regional parameter estimates, SOI+. . . . .	132
9.1	Computational requirements of matrix inversion. . . . .	149
9.2	Conjugate Gradient Algorithm. . . . .	153
9.3	Avon Basin regional parameter estimates for each month. . . . .	165
A.1	List of pluviograph gauges from Sydney Water Observation Network . . . . .	199
A.2	List of Bureau of Meteorology daily rainfall gauges . . . . .	200



# Abstract

---

*The temporal and spatial variability of Australia's climate affects the quantity and quality of its water resources, the productivity of its agricultural systems, and the health of its ecosystems. This variability should be taken into account when assessing the risks associated with flooding. Continuous simulation rainfall models are one means for doing this, whereby sequences of storms are generated for an arbitrarily long time period and over some region of interest. The simulated rainfall should reproduce observed statistics in time and space so that it can be used as a suitable input for hydrologic models at the catchment scale, with particular emphasis on extreme events.*

*There are a variety of approaches to modelling rainfall, including a broad range of single-site and multi-site rainfall models. By way of contrast there are few models that aim to simulate rainfall across all points within a region at daily or sub-daily increments. This thesis focuses on models calibrated solely to rain gauges, and a specific type known as Neyman-Scott Rectangular Pulse (NSRP) models. Existing NSRP models have a mature history of modelling developments including calibration methodology and an ability to reproduce key statistics across a range of timescales. Nonetheless, these models also have several limitations (and other space-time models not withstanding) that are addressed in this thesis. These developments include improvements to the conceptual representation of rainfall and improvements to calibration and simulation techniques. Specifically these improvements include (i) the development of an efficient simulation technique, (ii) assessing the impact of monthly parameter changes on rainfall statistics, (iii) the use of simulated statistics within calibration to overcome reliance on derived model properties (iv) incorporating a storm extent parameter to better match spatial correlations, (v) incorporating long term climatic variability and developing a methodology to assess climatic and seasonal variability in simulated extremes (vi) incorporating inhomogeneity of rainfall occurrence across a region. Numerous case studies are used at various locations about Australia to illustrate these improvements and highlight the applicability of the model under varied climatic conditions.*



# Statement of Originality

---

*This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Michael Leonard and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. I also give permission for the digital version of my thesis to be made available on the web, via the Universitys digital research repository, the Library catalogue, the Australasian Digital Theses Program (ADTP) and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.*

*SIGNED: ..... DATE: .....*





# Acknowledgements

---

*Undertaking a thesis is a privileged opportunity. Amongst the many privileges, I consider the community of support I have received and the resulting friendships to be among the greatest.*

*In this spirit, I would like to thank my supervisors Prof. Martin Lambert and Assoc. Prof. Andrew Metcalfe and collaborators Prof. George Kuczera, Dr. Mark Thyer and Dr. Paul Cowpertwait for their friendship, time, advice and enthusiasm. For their numerous conversations, assistance and general interest I also thank Prof. Angus Simpson, Dr. Alan Seed, Dr. Theresa Heneker, Dr. Julian Whiting, Dr. Andrew Frost, Ms. Juan Qin, Mr. Alex Osti, Mr. Steven Need, Ms. Geraldine Wong. There are many other friends I would like to mention here who encouraged me but, for the sake of avoiding long lists, to whom I hope a simple ‘thank you’ suffices. I will single out only Mr. Rob May as he has been especially generous in his friendship.*

*I am thankful to all of the staff in the School of Civil, Environmental and Mining Engineering at the University of Adelaide for creating a genuinely positive and constructive research environment. I am also indebted to fellow postgraduate students for their positive and constructive research distractions.*

*I am grateful to my family for their ongoing support and to J.B. for her love and patient understanding. Lastly, I am thankful to a merciful God who ‘sends rain on the righteous and the unrighteous’ alike (Sermon on the Mount, Matthew 5:45).*



# List of Symbols

Symbol	Quantity	Units
<b>Functions</b>		
$A(h, l), B(h, l)$	Used to calculate SNSRP covariance	
$E[]$	Expected value	
$E_1()$	First-order, exponential integral function	
$f_{R_c}$	Probability density for random variable $R_c$	
$f_{R_{xy}}$	Probability density for random variable $R_{xy}$	
$f(), g()$	Used to calculate SNSRP skewness	
$\Gamma()$	Gamma function	
$p_h(t)$	Probability of no-rain in interval (t,t+h)	
$P(N_o = n_o)$	Probability of obtaining $n_o$ cells	
$\psi()$	Prob. of a cell overlapping a point, given it overlaps a region	
$P(\phi, d)$	Prob. of a cell overlapping a point at a distance $d$ away	
$Pr()$	Probability that ...	
<b>English</b>		
$A$	Rational method, catchment area	$km^2$
$A_K$	Area of target region	$km^2$
$b_r$	$r^{th}$ order, probability weighted moment	
$C$	Random variable, number of cells	
$C_Y$	Rational method, dimensionless runoff coefficient	
$d$	Distance between two generic points	$km$
$dN$	Number of cells 'alive' at time $t$	
$D_{i,j}$	Domain of points defining a pixel	
$D_{k,l}$	Domain of points defining a pixel	
$D \equiv I_{h,i,j,k,l}$	Indicator function for data element $x_{h,i,j,k,l}$ , (1=dry,0=wet)	
$F_1, F_2$	Least squares objective functions used in calibration	
$h$	Level of aggregation	$hr$

continued on next page

Symbol	Quantity	Units
$h, h^-, h^+$	Approximations for length of a hypotenuse	
$I \equiv I_{h,i,j,k,l}$	Indicator function for data element $x_{h,i,j,k,l}$ , (1=valid,0=corrupt)	
$I_{t_c,Y}$	Rational method, intensity at critical period	$mm/hr$
$k$	Month index, $k = 1 \dots 12$	
$K$	Set of points defining a region	
$l$	Index of individual data interval, $l = 1 \dots 31(24/h)$	
$L, L_{i,j}$	Random variable, cell lifetime, $i^{th}$ storm, $j^{th}$ cell,	$hr$
$L$	Length scale of a pixel	
$M$	Number of observed rainfall sites in region, $i = 1 \dots M$	
$n$	Number of storm types in mixture of SNSRP processes	
$n_{h,k}$	Number of data elements all sites, aggregate $h$ , month $k$	
$n_{h,i,k}$	Number of data elements, aggregate $h$ , site $i$ , month $k$	
$N_i$	Number of years in observed record at site $i$ , $j = 1 \dots N_i$	
$N_o$	Number of successful Bernoulli trials	
$N_{tot}$	Total number of Bernoulli trials	
$N_{xy}$	Number of concurrent years in records at sites $x$ , and $y$	
$p_o$	Arbitrary constant probability for Bernoulli trials	
$Q_Y$	Rational method, peak flow for ARI Y	$m^3 s^{-1}$
$r_K$	Radius of target region	$km$
$R_C$	Random Variable, radius of cell centre	$km$
$R_{xy}$	Random Variable, dist. from region edge to cell centre	$km$
$S, S_{i,j}$	Random variable, cell start-time, $i^{th}$ storm, $j^{th}$ cell,	$hr$
$t_3$	The L-skewness, calculated from L-moments	
$t_c$	Rational method, critical period	$s$
$T, T_i$	Random variable, arrival time of $i^{th}$ storm	$hr$
$u, v$	Dummy integration variables for a pixel	
$U$	Random variable, uniform distribution, $U \sim [0, 1]$	
$w$	Mixture ratio	
$w_\psi, \dots$	Relative weight values for the method of moments	
$x, y$	Dummy integration variables for a pixel	
$x_{h,i,j,k,l}$	Individual element of data, agg. $h$ , site $i$ , year $j$ , month $k$ , interval $l$	
$X, X_{i,j}$	Random variable, cell intensity, $i^{th}$ storm, $j^{th}$ cell,	$mm$
$X_j$	$j^{th}$ order statistic, arbitrary data $X$	
$Y(t)$	Instantaneous rainfall process at time $t$	$mm$
$Y_l^{(h)}$	Aggregate rainfall process, $l^{th}$ interval, agg. level $h$	$mm$
$Y_Z(t)$	Instantaneous rainfall process at time $t$ , spatial location $Z$	$mm$
$Z$	Generic location in $\mathbb{R}^2$ domain	

*continued on next page*

Symbol	Quantity	Units
$Z_C$	Random Variable, location of cell centre in $\mathbb{R}^2$ domain	
<b>Greek</b>		
$\alpha$	Storm intensity parameter	
$\alpha^{(p)}$	Storm intensity parameter, $SNSRP^{(p)}$ mixture	
$\beta$	Cell dispersion parameter	$hr^{-1}$
$\beta^{(p)}$	Cell dispersion parameter, $SNSRP^{(p)}$ mixture	$hr^{-1}$
$\gamma_{d,h,\tau}$	Covariance, distance $d$ ( $km$ ), aggregation $h$ , time-lag $\tau$	$mm^2$
$\gamma_{d,h,\tau}^{(p)}$	Covariance of $SNSRP^{(p)}$ mixture	$mm^2$
$\gamma_{0,h,\tau}$	Auto-covariance, from $\gamma_{d,h,\tau}$	$mm^2$
$\gamma_{0,h,0}$	Variance, from $\gamma_{d,h,\tau}$	$mm^2$
$\delta r$	Elemental radius length	$km$
$\delta \xi$	Elemental angle	$rad.$
$\epsilon_{i,\epsilon}$	Rainfall falling beyond month $i$ boundary	$mm$
$\zeta_h$	Unstandardised skewness, aggregation $h$	$mm^3$
$\zeta_h^{(p)}$	Unstandardised skew of $SNSRP^{(p)}$ mixture, agg. $h$	$mm^3$
$\eta$	Cell lifetime parameter	$hr^{-1}$
$\eta^{(p)}$	Cell lifetime parameter, $SNSRP^{(p)}$ mixture	$hr^{-1}$
$\theta$	Storm intensity parameter / scale parameter	$mm$
$\hat{\theta}_{h,i,k}$	Estimate of scale parameter, agg. $h$ , site $i$ , month $k$	$mm$
$\kappa_h$	Standardised skewness	
$\hat{\kappa}_{h,k}$	Non-dim., pooled skew estimate, agg. $h$ , month $k$	
$\lambda$	Storm rate parameter	$hr^{-1}$
$\lambda^{(p)}$	Storm rate parameter, $SNSRP^{(p)}$ mixture	$hr^{-1}$
$\mu_C$	Mean number of cells covering a point in space	
$\mu_C$	Other interpretation: Mean number of cells per storm	
$\mu_C^{(p)}$	Mean number of cells per storm, $SNSRP^{(p)}$ mixture	
$\mu_h$	Mean rainfall intensity at agg. level $h$	$mm$
$\mu_h^{(p)}$	Mean rainfall intensity of $SNSRP^{(p)}$ mixture, agg. level $h$	$mm$
$\hat{\mu}_{h,i,k}$	Estimate of mean, agg. $h$ , site $i$ , month $k$	$mm$
$\hat{\mu}_{h,k}$	Pooled estimate of mean, agg. $h$ , month $k$	$mm$
$\mu_y$	Mean rainfall of instantaneous rainfall intensity	$mm$
$\nu_h$	Coefficient of variation	

continued on next page

Symbol	Quantity	Units
$\hat{\nu}_{h,k}$	Pooled estimate of coefficient of variation, agg. $h$ , month $k$	
$\xi$	Arbitrary angle at which a cell is located	
$\Xi$	Random variable, uniform angle, $X_i \sim [0, 2\pi]$	
$\rho_h$	Lag-1 auto-correlation	
$\rho_{x,y,h}$	Lag-0 cross-correlation between site $x$ and site $y$ , agg. $h$	
$\hat{\rho}_{x,y,h,k}$	Estimate of lag-0 cross-correlation, sites $x$ & $y$ , agg. $h$ , month $k$	
$\sigma_h^2$	Variance, aggregation $h$	$mm^2$
$\hat{\sigma}_{h,k}^2$	Non-dim., pooled variance estimate, agg. $h$ , month $k$	
$\tau$	Correlation lag in time	
$\varphi_C$	Spatial rate parameter	$km^{-2}$
$\phi$	Generic radius parameter, either $\phi_c$ or $\phi_s$	$km^{-1}$
$\phi_c$	Cell radius parameter	$km^{-1}$
$\phi_c^{(p)}$	Cell radius parameter, $SNSRP^{(p)}$ mixture	$km^{-1}$
$\phi_s$	Storm radius parameter	$km^{-1}$
$\phi_s^{(p)}$	Storm radius parameter, $SNSRP^{(p)}$ mixture	$km^{-1}$
$\psi_h$	Dry portion / probability of a dry interval	
$\psi_h^{(p)}$	Dry portion of $SNSRP^{(p)}$ mixture	
$\Psi_i, \Psi$	Rainfall bias in mean / portion of month $i$ rainfall	
$\hat{\psi}_{h,k}$	Pooled estimate of dry portion, agg. $h$ , month $k$	
$\omega$	ratio of radius parameter $\phi$ to radius of target region $r_t$	
$\Omega$	Generic parameter of Poisson distribution	

### Acronyms

---

<i>ABARE</i>	Australian Bureau of Agriculture and Resource Economics
<i>ABS</i>	Australian Bureau of Statistics
<i>AMF</i>	Annual Maximum Frequency
<i>ARI</i>	Annual Recurrence Interval
<i>ARMA</i>	Auto-Regressive Moving-Average
<i>BLRP</i>	Bartlett-Lewis Rectangular Pulse
<i>BOM</i>	Bureau of Meteorology
<i>BTE</i>	Bureau of Transport Economics
<i>CV</i>	Coefficient of Variation
<i>ENSO</i>	El Niño Southern Oscillation
<i>FFT</i>	Fast Fourier Transform
<i>GDP</i>	Gross Domestic Product
<i>GRF</i>	Gaussian Random Field

*continued on next page*

<b>Symbol</b>	<b>Quantity</b>	<b>Units</b>
<i>IPO</i>	Interdecadal Pacific Oscillation	
<i>MTB</i>	Modified Turning Bands	
<i>NSRP</i>	Neyman-Scott Rectangular Pulse	
<i>MCS</i>	Monte Carlo Simulation	
<i>SNSRP</i>	Spatial Neyman-Scott Rectangular Pulse	
<i>SNSRP<sup>(p)</sup></i>	The $p^{th}$ superimposed SNSRP process	
<i>SOI</i>	Southern Oscillation Index	

