A Stochastic Space-Time Rainfall Model for Engineering Risk Assessment

by Michael Leonard

Submitted in fulfilment of the requirements for the degree of
DOCTOR OF PHILOSOPHY

April 14, 2010

FACULTY OF ENGINEERING, COMPUTER AND MATHEMATICAL SCIENCES

School of Civil Environmental and Mining Engineering

THE UNIVERSITY
OF ADELAIDE
AUSTRALIA

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Abstract

The temporal and spatial variability of Australia's climate affects the quantity and quality of its water resources, the productivity of its agricultural systems, and the health of its ecosystems. This variability should be taken into account when assessing the risks associated with flooding. Continuous simulation rainfall models are one means for doing this, whereby sequences of storms are generated for an arbitrarily long time period and over some region of interest. The simulated rainfall should reproduce observed statistics in time and space so that it can be used as a suitable input for hydrologic models at the catchment scale, with particular emphasis on extreme events.

There are a variety of approaches to modelling rainfall, including a broad range of single-site and multi-site rainfall models. By way of contrast there are few models that aim to simulate rainfall across all points within a region at daily or sub-daily increments. This thesis focuses on models calibrated solely to rain gauges, and a specific type known as Neyman-Scott Rectangular Pulse (NSRP) models. Existing NSRP models have a mature history of modelling developments including calibration methodology and an ability to reproduce key statistics across a range of timescales. Nonetheless, these models also have several limitations (and other space-time models not withstanding) that are addressed in this thesis. These developments include improvements to the conceptual representation of rainfall and improvements to calibration and simulation techniques. Specifically these improvements include (i) the development of an efficient simulation technique, (ii) assessing the impact of monthly parameter changes on on rainfall statistics, (iii) the use of simulated statistics within calibration to overcome reliance on derived model properties (iv) incorporating a storm extent parameter to better match spatial correlations, (v) incorporating long term climatic variability and developing a methodology to assess climatic and seasonal variability in simulated extremes (vi) incorporating inhomogeneity of rainfall occurrence across a region. Numerous case studies are used at various locations about Australia to illustrate these improvements and highlight the applicability of the model under varied climatic conditions.

Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Michael Leonard and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. I also give permission for the digital version of my thesis to be made available on the web, via the Universitys digital research repository, the Library catalogue, the Australasian Digital Theses Program (ADTP) and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

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List of Symbols

Symbol	Quantity	Units
Functions		
$A(h,l)$,B(h,l)	Used to calculate SNSRP covariance	
$E[\,]$	Expected value	
$E_1()$	First-order, exponential integral function	
f_{R_c}	Probability density for random variable R_c	
$f_{R_{xy}}$	Probability density for random variable R_{xy}	
$f()$,$g()$	Used to calculate SNSRP skewness	
$\Gamma()$	Gamma function	
$p_h(t)$	Probability of no-rain in interval (t,t+h)	
$P(N_o = n_o)$	Probability of obtaining n_o cells	
$\psi()$	Prob. of a cell overlapping a point, given it overlaps a region	
$P(\phi, d)$	Prob. of a cell overlapping a point at a distance d away	
$Pr()$	Probability that ...	
English		
A	Rational method, catchment area	km^2
A_K	Area of target region	km^2
b_r	r^{th} order, probability weighted moment	
C	Random variable, number of cells	
C_Y	Rational method, dimensionless runoff coefficient	
d	Distance between two generic points	km
dN	Number of cells ‘alive’ at time t	
$D_{i,j}$	Domain of points defining a pixel	
$D_{k,l}$	Domain of points defining a pixel	
$D \equiv I_{h,i,j,k,l}$	Indicator function for data element $x_{h,i,j,k,l}$, (1=dry,0=wet)	
F_1,F_2	Least squares objective functions used in calibration	
h	Level of aggregation	hr

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Symbol	**Quantity**	**Units**
h, h^-, h^+	Approximations for length of a hypotenuse	
$I \equiv I_{h,i,j,k,l}$	Indicator function for data element $x_{h,i,j,k,l}$, (1=valid,0=corrupt)	
$I_{t_c,Y}$	Rational method, intensity at critical period	mm/hr
k	Month index, $k = 1 \ldots 12$	
K	Set of points defining a region	
l	Index of individual data interval, $l = 1 \ldots 31(24/h)$	
$L, L_{i,j}$	Random variable, cell lifetime, i^{th} storm, j^{th} cell,	hr
L	Length scale of a pixel	
M	Number of observed rainfall sites in region, $i = 1 \ldots M$	
n	Number of storm types in mixture of SNSRP processes	
$n_{h,k}$	Number of data elements all sites, aggregate h, month k	
$n_{h,i,k}$	Number of data elements, aggregate h, site i, month k	
N_i	Number of years in observed record at site i, $j = 1 \ldots N_i$	
N_o	Number of successful Bernoulli trials	
N_{tot}	Total number of Bernoulli trials	
N_{xy}	Number of concurrent years in records at sites x, and y	
p_o	Arbitrary constant probability for Bernoulli trials	
Q_Y	Rational method, peak flow for ARI Y	m^3s^{-1}
r_K	Radius of target region	km
R_C	Random Variable, radius of cell centre	km
R_{xy}	Random Variable, dist. from region edge to cell centre	km
$S, S_{i,j}$	Random variable, cell start-time, i^{th} storm, j^{th} cell,	hr
t_3	The L-skewness, calculated from L-moments	
t_c	Rational method, critical period	s
T, T_i	Random variable, arrival time of i^{th} storm	hr
u, v	Dummy integration variables for a pixel	
U	Random variable, uniform distribution, $U \sim\ [0,1]$	
w	Mixture ratio	
$w_\psi, \ldots$	Relative weight values for the method of moments	
x, y	Dummy integration variables for a pixel	
$x_{h,i,j,k,l}$	Individual element of data, agg. h, site i, year j, month k, interval l	
$X, X_{i,j}$	Random variable, cell intensity, i^{th} storm, j^{th} cell,	mm
X_j	j^{th} order statistic, arbitrary data X	
$Y(t)$	Instantaneous rainfall process at time t	mm
$Y_l^(h)$	Aggregate rainfall process, l^{th} interval, agg. level h	mm
$Y_Z(t)$	Instantaneous rainfall process at time t, spatial location Z	mm
Z	Generic location in $\mathbb{R}^2$ domain	

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Symbol	Quantity	Units
Z_C	Random Variable, location of cell centre in $\mathbb{R}^2$ domain	

Greek

α	Storm intensity parameter	
$\alpha^{(p)}$	Storm intensity parameter, $SNSRP^{(p)}$ mixture	
β	Cell dispersion parameter	hr^{-1}
$\beta^{(p)}$	Cell dispersion parameter, $SNSRP^{(p)}$ mixture	hr^{-1}
$\gamma_{d,h,\tau}$	Covariance, distance d (km), aggregation h, time-lag τ	mm^2
$\gamma_{d,h,\tau}^{(p)}$	Covariance of $SNSRP^{(p)}$ mixture	mm^2
$\gamma_{0,h,\tau}$	Auto-covariance, from $\gamma_{d,h,\tau}$	mm^2
$\gamma_{0,h,0}$	Variance, from $\gamma_{d,h,\tau}$	mm^2
δr	Elemental radius length	km
$\delta\xi$	Elemental angle	$rad.$
ϵ_i,ϵ	Rainfall falling beyond month i boundary	mm
ζ_h	Unstandardised skewness, aggregation h	mm^3
$\zeta_h^{(p)}$	Unstandardised skew of $SNSRP^{(p)}$ mixture, agg. h	mm^3
η	Cell lifetime parameter	hr^{-1}
$\eta^{(p)}$	Cell lifetime parameter, $SNSRP^{(p)}$ mixture	hr^{-1}
θ	Storm intensity parameter / scale parameter	mm
$\hat{\theta}_{h,i,k}$	Estimate of scale parameter, agg. h, site i, month k	mm
κ_h	Standardised skewness	
$\hat{\kappa}_{h,k}$	Non-dim., pooled skew estimate, agg. h, month k	
λ	Storm rate parameter	hr^{-1}
$\lambda^{(p)}$	Storm rate parameter, $SNSRP^{(p)}$ mixture	hr^{-1}
μ_C	Mean number of cells covering a point in space	
μ_C	Other interpretation: Mean number of cells per storm	
$\mu_C^{(p)}$	Mean number of cells per storm, $SNSRP^{(p)}$ mixture	
μ_h	Mean rainfall intensity at agg. level h	mm
$\mu_h^{(p)}$	Mean rainfall intensity of $SNSRP^{(p)}$ mixture, agg. level h	mm
$\hat{\mu}_{h,i,k}$	Estimate of mean, agg. h, site i, month k	mm
$\hat{\mu}_{h,k}$	Pooled estimate of mean, agg. h, month k	mm
μ_y	Mean rainfall of instantaneous rainfall intensity	mm
ν_h	Coefficient of variation	

continued on next page

Symbol	**Quantity**	**Units**
$\hat{\nu}_{h,k}$	Pooled estimate of coefficient of variation, agg. h, month k	
ξ	Arbitrary angle at which a cell is located	
Ξ	Random variable, uniform angle, $Xi \sim\ [0, 2\pi]$	
ρ_h	Lag-1 auto-correlation	
$\rho_{x,y,h}$	Lag-0 cross-correlation between site x and site y, agg. h	
$\hat{\rho}_{x,y,h,k}$	Estimate of lag-0 cross-correlation, sites x & y, agg. h,month k	
σ_h^2	Variance, aggregation h	mm^2
$\hat{\sigma}_{h,k}^2$	Non-dim., pooled variance estimate, agg. h, month k	
τ	Correlation lag in time	
φ_C	Spatial rate parameter	km^{-2}
ϕ	Generic radius parameter, either ϕ_c or ϕ_s	km^{-1}
ϕ_c	Cell radius parameter	km^{-1}
$\phi_c^{(p)}$	Cell radius parameter, $SNSRP^{(p)}$ mixture	km^{-1}
ϕ_s	Storm radius parameter	km^{-1}
$\phi_s^{(p)}$	Storm radius parameter, $SNSRP^{(p)}$ mixture	km^{-1}
ψ_h	Dry portion / probability of a dry interval	
$\psi_h^{(p)}$	Dry portion of $SNSRP^{(p)}$ mixture	
Ψ_i,Ψ	Rainfall bias in mean / portion of month i rainfall	
$\hat{\psi}_{h,k}$	Pooled estimate of dry portion, agg. h, month k	
ω	ratio of radius parameter ϕ to radius of target region r_t	
Ω	Generic parameter of Poisson distribution	

Acronyms	
ABARE	Australian Bureau of Agriculture and Resource Economics
ABS	Australian Bureau of Statistics
AMF	Annual Maximum Frequency
ARI	Annual Recurrence Interval
ARMA	Auto-Regressive Moving-Average
BLRP	Bartlett-Lewis Rectangular Pulse
BOM	Bureau of Meteorology
BTE	Bureau of Transport Economics
CV	Coefficient of Variation
ENSO	El Niño Southern Oscillation
FFT	Fast Fourier Transform
GDP	Gross Domestic Product
GRF	Gaussian Random Field

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Symbol	**Quantity**	**Units**
IPO	Interdecadal Pacific Oscillation	
MTB	Modified Turning Bands	
NSRP	Neyman-Scott Rectangular Pulse	
MCS	Monte Carlo Simulation	
SNSRP	Spatial Neyman-Scott Rectangular Pulse	
SNSRP$^{(p)}$	The p^{th} superimposed SNSRP process	
SOI	Southern Oscillation Index	