November 3, 1937

Dear Andrade,

Stevens has carried out and checked the computation on your oscillation measurements, with results which I think you will find remarkably interesting. You will remember my mentioning to you that I made a preliminary estimate of the accuracy, based on the sums instead of the differences of the alternate measurements, and that this indicated that the first group of 11 tested showed discrepancies much greater than any which followed. We therefore carried out the longer computations for the succeeding 55 differences, and, on considering the matter, I decided finally not to include the first group, even with reduced weight. The first page I shall enclose, therefore, refers to this preliminary test. The second page is set out as a model for routine computation, and in this I should only like to stress the fact that in view of the actual accuracy of the readings I have not retained any unnecessary figures, but that the heavy column for wy is needed to the full accuracy shown; the logarithms (y) are needed to 5 figures, and the means of x and y both to 10 significant

figures. The reason is that the exponential curve as fitted accounts for very nearly the whole of the variation among the readings, so that a good six digits disappear on subtraction, as shown in the analysis of variance, which is simply a subdivision of the total weighted variability into the two portions (a) represented by an exact exponential curve, and (b) differences from this curve. The standard error of the regression calculated from the data is extraordinarily low, being little more than 1/10,000 of the value estimated. This, I imagine, is so low as to throw all the weight of further enquiry into precision on to factors other than those causing these deviations.

We did, however, take up two further points in respect to these: first, it appears that the deviations from the curve, slight as they are, are certainly larger than those obtained when examining the sums of the successive readings. Actually the mean square is about double what we obtained from t these; consequently there must be causes of disturbance present somewhat greater than the random errors of the readings. Secondly, it occurred to me that if the system had not entirely reached a steady state at the time the reading was begun, e.g., if the proportion of the energy of the system due to kinetic energy of the fluid within the sphere were increasing toward a steady value, this

might show itself in a slight persistent curvature of the line through the logarithms. Taking the sums of the residuals by groups of 11 there does appear to be such a curvature, which is significant, in that ~~this value~~ it exceeds its standard error rather more than three-fold. In view of these facts there may be a small systematic error in the viscosity inferred, owing to a portion of the damping observed among the earlier observations, to which greatest weight is attached, being partly due to a re-distribution of energy between one part of the system and another. Hence the amazingly high precision which the data show cannot be taken absolutely at its face value, and indeed I suppose when it comes to one part in ten thousand there may be some other determinants of time constants of the system which you would like to tune up to a level worthy of the actual working of the apparatus.

Yours sincerely,

3 Tables