October 25, 1937

Dear Professor Andrade,

I find the theory of your problem quite manageable, as the enclosed sheets show. I should, however, like to set out an example of the actual computations required, as these may prove troublesome without such guidance.

Yours sincerely,

P.S. Continued

1. The range of amplitudes to be measured to yield the highest precision for a given number of measurements.

If r is the square of the ratio of each measured amplitude to the one before, the amount of information respecting viscosity obtainable from n successive measurements of the same precision is found to be proportional to

$$I = (\log r)^2 \left\{ \frac{r(1-r^n)}{(1-r)^2} - \frac{n^2 r^n}{(1-r)(1-r^n)} \right\}$$

Choosing $n = 31$, and using common logarithms, I find

$-\log r$	$\log I$
·056	·0262246
·057	·0267051
·058	·0269766
·059	·0270429
·060	·0269213
·061	·0266198
·062	·0261515

The values are not correct to the last figure, but are sufficient to locate the maximum value nearly at $-\log r = .058852$, for which the ratio, r^{15}, between the last measured amplitude and the first is 13·1 %.

2. The limiting ratio when the number of measurements is increased indefinitely

If n is increased indefinitely and we substitute

$$\alpha = -n \log_e r$$

the amount of information is given by

$$I = n \left\{ \frac{1-e^{-\alpha}}{\alpha} - \frac{\alpha e^{-\alpha}}{1-e^{-\alpha}} \right\}.$$

The ratio of the last to the first amplitude is then $e^{-\alpha/2}$. I find

α	I/n
4·03	·1708017
4·04	·1708017
4·05	·1708000
4·06	·1707967

The minimum is evidently near to x = 4·035, for which the ratio of amplitudes is 13·3%.

When a reasonable number of measurements are taken, the actual ratio for the last to the first measured is evidently nearly independent of the number of measurements. It would therefore seem to be good practice to measure all successive amplitudes down to ~~that~~ one which is about 1/8 of the first.

The precision of the experiment, so far as it is limited by accuracy of measurement, will then be nearly proportional to the number of measurements taken. E.g. if ~~the~~ each amplitude is only 2% less than the last, 100 would be needed before the last would be reached; if ~~they~~ each were 10% less than the last only about 20 would be needed, but only about 1/5 of the precision would be obtained by the smaller number of readings. Some estimate of the probable magnitude of other ~~errors~~ sources of error would be needed for determining how small it is worth while to make that further than the errors of measurement. It may be that these other sources may be eliminated more successfully in obtaining comparative than in obtaining absolute values, in which case very small damping and very numerous measurements might be worth while for comparative purposes.