1st. May 1947.

Dear Anscombe,

If you agree that the sampling process you have in view is the same as mine, in which *

$$\text{Variance of } N = E(N) = M$$

$$\text{Variance of } S = \alpha \log \frac{2M+\alpha}{M+\alpha}$$

$$\text{Covariance of } S \text{ and } N = \frac{\alpha M}{M+\alpha}$$

and if α is expressed implicitly in terms of N and S by the equation

$$S = \alpha \left\{ \log(N+\alpha) - \log\alpha \right\}$$

then putting M for N after differentiation

$$\log \frac{M+\alpha}{\alpha} - \frac{M}{M+\alpha} \, d \quad = dS - \frac{\alpha}{M+\alpha} dN$$

whence

$$\log \frac{M+\alpha}{\alpha} - \frac{M}{M+\alpha} \; V(\alpha)$$

$$= V(S) - \frac{2\alpha}{M+\alpha} CV(S,N) + \frac{\alpha^2}{(M+\alpha)^2} V(N)$$

$$= \alpha \log \frac{2M+\alpha}{M+\alpha} - \frac{\alpha^2 M}{(M+\alpha)^2} ,$$

[* cf CP 173. J.H.F.]

leading to my formula

$$V(\alpha) = \frac{\alpha^{3}\left\{(M+\alpha)^{2}\log\frac{2M+\alpha}{M+\alpha} - \alpha M\right\}}{(SM + S\alpha - M\alpha)^{2}}$$

where N observed is to be used for its expectation M, and , of course, the observed values of S and α are also to be inserted.

Yours sincerely,