6 July 1933.

W. Whately Carington, Esq.,
Calandstraat 64,
ROTTERDAM,
Holland.

Dear Whately Carington:

I have taken 24 imaginary readings of psycho-galvanic reaction to show the arithmetic. It is considerably shorter than the correlational procedure, and what is more important, it helps the logic more.

One repeatedly uses the fact that

$$S(x-\bar{x})^2 = S(x^2) - \bar{x}S(x)$$

where $S(x)$ is the sum and $\bar{x}$ the corresponding mean of the items dealt with. The sum of the 24 readings is 465, the mean therefore 19.125, and

$$\bar{x}\,S(x) \quad = \quad 8893.125.$$

The sum of the squares of all 24 readings ignoring occasion and personality is 11693, so that

$$S(x-\bar{x})^2 = 11693 - 8893.125 = 2799.875$$

and this corresponds to the 23 independent comparisons.

Take now the personality totals found by summing for all occasions. The sum of the products of totals by means is

$\frac{1}{6}(46^2 + 206^2 + \ldots + \ldots) = 11266.833$, so that subtracting 8893.125 again, we have 2373.7083 for differences between personalities. Notice that this is only an arithmetical short cut to calculating

$$6\left\{(7.\dot{6} - 19.125)^2 + (34.\dot{3} - 19.125)^2 + \ldots\ldots + \ldots\ldots\right\}$$

from the personality means. The point is, however, that out of a total of about 2800 in 23 degrees of freedom, the 3 degrees for differences between personality have secured 2374 leaving only 426 for the other 20. Of these the 5 for occasions get 225 or more than half the remainder, leaving only 201 for the 15 degrees of freedom of errors.

If on this one word you want to test significance, you calculate in successive columns the mean square, $\frac{1}{2}$ its natural log., and, by subtracting the error value, the test z, the distribution of which in random samples depends only on the degrees of freedom and is therefore known exactly. Thus for 3 degrees of freedom v. 15, the value z = .8448 is exceeded by chance only once in 100 trials, so the observed value 2.0392 is very markedly significant, and would give fine long odds if you calculated them, as indeed is obvious from the data. The effect of occasions is more doubtful, z = .6060 exceeds the 5 per cent. point, but is less than the 1 per cent. point. It would appear that the second occasion scored highest for personalities X, Y, Z.

That, at least as far as analysing the sum of squares is what

I should do for each single word, giving 25 tables, and one more for the 24 totals of all words. This last you analyse like the others dividing the sum of squares entries by 25, to give the three contrasts involving all words together. Subtracting these values from the totals of the 25 other tables gives the three contrasts involving differences between words, including the 360 for pure error.

Special queries:-

(2) I think you would be right to analyse readings already fully corrected, as far as this is possible. It is a useful resource though, if you are doubtful as to the value of any such correction to see if it really reduces the residual error, i.e. the sum of squares for the 360 degrees of freedom. If not it is, empirically speaking, not doing any real good.

(3) Gaps are really troublesome. I should analyse all words for which the record is perfect, and if the others seem to you too valuable to be ignored, I will go into the question of gap filling.

(4) I will do my best about exact high odds if later you think they will be useful. Significance is only consistency of experience, and many people get a clearer hold of an experimental fact through realising how consistently it is found.

(5) If you get all four personalities on each occasion, it would not be troublesome to work the whole thing each time. The

arithmetical procedure really does treat each occurrence as a separate experiment, i.e. a 4 x 25 table, for what it is worth, that is

Personalities	3
Words	24
Words x personalities	72
	99

a single occasion is useless because W x P is just what you are after, and one occasion gives you nothing to compare it with. After the second sitting, you have

	Sums of sittings		Difference between sittings
P	3	P x O	3
W	24	W x O	24
W x P	72	W x P x O	72
		O	1

at which point you can begin to recognise consistency of experience, if it is there, by W x P exceeding W x P x O.

After three sittings, of course

	Sum of sittings		Variance among sittings
P	3	P x O	6
W	24	W x O	48
W x P	72	W x P x O	144
		O	2

and so on.

Yours sincerely,

C_A_L_C_U_L_A_T_I_O_N_S.

Suppose "Wolf" means nothing to W, a lot to X and variable amounts to Y and Z one might get:-

Occasion	1	2	3	4	5	6	Total t_2	Mean
W	8	4	10	7	6	11	46	$7.\dot{6}$
X	35	40	31	38	32	30	206	$34.\dot{3}$
Y	15	32	20	18	23	12	120	20.0
Z	18	20	13	10	15	17	93	15.5
Total t_1	76	96	74	73	76	70	465	
	19.0	24.0	18.5	18.25	19.0	17.5		19.125

$S(x^2)$		$\frac{1}{4}S(t_1^2)$		$\frac{1}{6}S(t_2^2)$	
	11693		9118.25		11266.833
	8893.125		8893.125		8893.125
	2799.875		225.125		2373.7083

Variance due to	Degrees of freedom	Sum of Squares	Mean Square	$\frac{1}{2}\log_e$	z	
Personalities	3	2373.708	791.236	3.3368	2.0392	> .8448 (1%
Occasions	5	225.125	45.025	1.9036	0.6060	> .5326 (5% < .7382 (1%
Error	15	201.042	13.403	1.2976		
Total	23	2799.875				