22 June 1932.

Snr. A.O. Franco,
    Ministerio da Agricultura,
        Rio de Janeiro,
            Brazil.

My dear Franco:

I was much interested in your letter, and in the manurial experiment you included.

The analysis of the actual yields has been checked and is quite correct up to the test of significance; at this point there is an error in the decimal point so that .2574 has been written for $2.574 = \frac{1}{2} \log$ 172. z is therefore small .0392, but not so small as was thought.

With respect to the analysis of the yields per 85 plants an error of 5.09 has occurred in each entry of the Sums of Squares. In the result the Mean Square for error is reduced to 1.21 from 1.59, but the effects of both of blocks and of treatments are reduced still further. The point gained here is that the precision can be increased by taking account of plant number.

As an illustration of the procedure which is being found
rather widely valuable, I have had some further analysis
made on the same lines.   At the bottom, analysis (A)
is an analysis of plant number, exactly analogous to the
analysis of yield on your first page.   Next, analysis (B)
at the top of the page is an analysis of covariance in
which, instead of using square of yields, or of plant number,
we use at every stage products of the two.   It appears that
the products are positive in each line of the table.   The
best linear correction for plant number is now found by
dividing 173.19 from Table (B) by 2551.05 from Table (A)
giving an allowance of .0679 per plant.

Each individual yield may now be corrected to a fixed
plant number (e.g. 85, *or any other number*) and the corrected values may be
analysed.   We need not. however, go to all this labour
but can write down from the calculations already made the
analysis of yield corrected for plant number, merely using the
fact that

$$S(y - bx)^2 = S(y^2) - 2b\,S(xy) + b^2\,S(x^2).$$

This is done in Table (C) where you will notice that we
have 29 degrees of freedom instead of 30, though otherwise
it closely resembles your table for yield per 85 plants.

This analysis of covariance has been developed in the 4th edition (1932) of my book on Statistical Methods.

Personally, I should always use the analysis of covariance to allow for variation in plant number, for it will often give a materially better analysis than any other mode of correction, and never, I think, will give a worse analysis.

Remember that I am always happy to hear from you, and to hear of the progress you are making in organised experimentation.

Yours sincerely,

Tables and analysis sheet returned with this letter no copy left.